Microscale Transport Processes

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Module No. # 01

Lecture No. # 31

Immiscible Flow in Microchannel (Contd.)

Welcome to this lecture of microscale transport process, what we have been discussing is immiscible flow through a microchannel.We have introduced several dimensionless numbers that controls the flow of bubbles, flow of two phases, flow of bubbles or droplets through a microchannel.Now, there are certain issues concerning this flow and we have talked about what difficulties are, in introducing this meniscus or inserting the meniscus of a bubble or a droplet inside the microchannel. What kind of pressure drop one would encounter if they want to introduce this meniscus.

Now, these we have discussed. So, what we will do in today's class is we will go into something called a Bretherton analysis where we would like to find out, how the pressure drop would vary; what would be the expression for pressure drop how pressure drop will change with velocity or surface tension between the two phases. So, what would be the theory how would; how can we theorize this flow process and conclude what would be the pressure drop. So, let us get into this analysis now.

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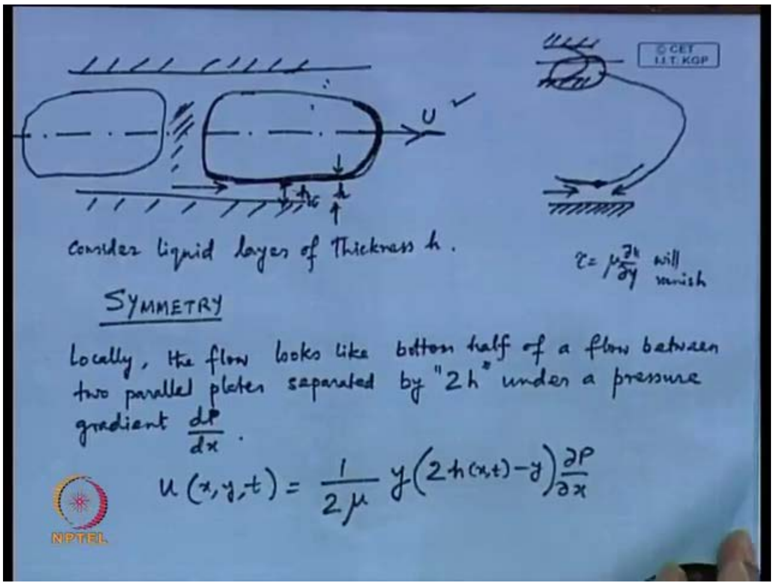

What we have been talking about is a microchannel and then, we have bubbles placed axisymmetrically that is the idea, what we mean by axisymmetricplaced,axisymmetrically means that you have this is the axis. So, bubble is symmetric around this axis that is what we mean by axisymmetric bubbles. So, bubbles are placed axisymmetrically and these bubbles are moving with a velocity u.Now, we would like to find out what would be the thickness of this now, what thickness, thickness of this liquid layer, thickness of this annular film.Now, this thickness would be varying in fact, if we look at how the thickness would be varying, thickness would be minimum here and we will be calling this the h infinity.

So, this is the minimum thickness that you have and then there would be a curvature and finally, here we are; this we are referring as front cap and similarly, on the backside we have a back cap. So, the thickness of this film it is h infinity at the place where at around the mid plane and then it changes this thickness changes and finally, this curvature merges with the curvature of the front cap. So, that is the idea.

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Now, here what you have is we have a flow going; flow of liquid going on through this through this annular film and then liquid is sandwiched between these two bubbles as well. So, you have; and the bubbles are moving at a velocity u and the net flow of liquid and liquid and gas is maintained.Now, here we; so, we are calling at any arbitrary location we are calling this thickness of this annular film as h.Now, what we are;so, the assumptions that we have here is the bubble is moving at a velocity u that I mentioned then we have the bubbles placed axisymmetrically within the channel that we have talked about and waiting of annular film.The annular; the liquid is completely wetting the wall that is the other thing that we are assuming here, the liquid that is there that is completely wetting the wall.

So, we do not any issue of contact angle here I mean it is just a interface between the gas and the; for a bubble it is gas and the liquid or between the two phases, that we are talking about here. So, what we have I mean let me write it down, here we are considering a liquid layer of thickness h now, we are talking about only the horizontal velocity here. So, the; what we will have is the shear stress on the free surface, what is the free surface here as far as this liquid film is concerned, if we isolate the liquid film on this side it has the wall and on this side it has the bubble. So, if we are talking about the liquid film on this there is this wall and this side there is the bubble.

So, you have the upper surface you can say that this is called a free surface, at the free surface the shear stress tau which is mu d u d y that will vanish, where it will vanish this will vanish here at the free surface. So, this is something which is which is happening here and we are talking about only the horizontal velocity, we do not have any reason to believe that there would be a vertical velocity. So, under this condition we can assume a symmetry I mean what is the implication of it, we can say that locally the flow whose flow we are talking about flow of this liquid in that annular film.Locally, the flow looks like bottom half of a flow between two parallel plates separated by 2 h distance under a pressure gradient d p d x.

What; that means is that; what would be the flow between two parallel plates separated by 2 h under a pressure(( )) what kind of flow we are talking about (( )).If there are two parallel plates fixed plates and then you have a flow, what kind of flow you get?You get a parabolic velocity profile right, what you are getting here actually is half of it. So, this is basically see this distance is h, we are talking about this distance.We are talking about this annular film as h, what they are saying is the flow would be locally utilizing this symmetry, you can see that the flow looks like bottom half of a flow between two parallel plates.That means, this flow this part of the flow simulates the flow here in that annular portion.

So, that is what you are; I mean that you can assume and in that case you can take straightaway the velocity profile for flow between two parallel plates and work with the; I mean flow between two parallel plates which has an aperture of 2 h instead of h. So, if we try to write that velocity; this would be something like this u.

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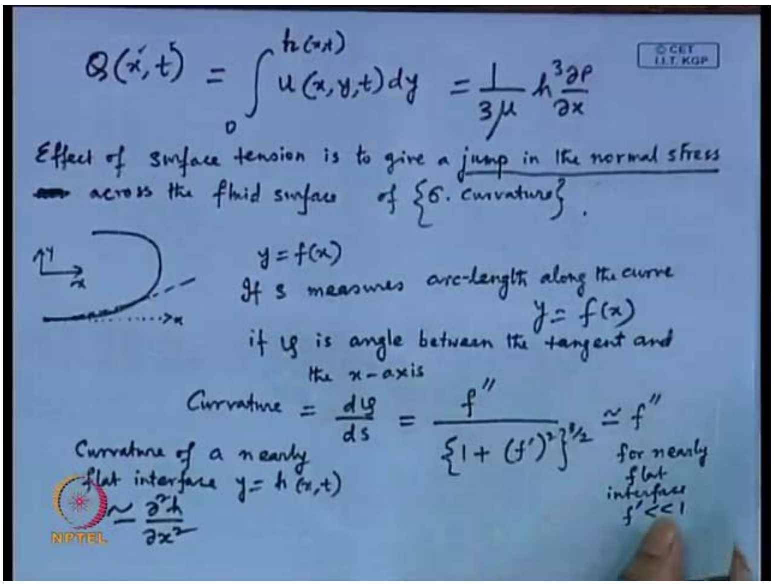

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1 by 2 mu y 2 h minus y del p del x all right. So, this is coming from a flow between two parallel plates they are separated by distance 2 h.Now, then in this case if somebody wants to find out what is the volumetric flow per unit depth of microchannel; depth in the sense per unit width I should say if you; if somebody wants to know what would be the flow per unit width of the microchannel. So, then in that case what he needs to do is he needs to integrate this further. So, what he would say in that case is, he will write a person who wants to do this, he will write q at any position x t, see y is gone we had u as a function of x y t. So, now you can write Q x t that is equal to integration 0 to h x t u x y t d y all right.

So, what are doing is; you are integrating from 0 to h u d y that means, the Q that you are getting it is basically a flux per unit width perpendicular to the paper so, that is something which you are referring to.Now, if you take this expression for u, as I said that you can simulate the flow in this annular portion,the annular portion around the bubble if you can simulate that flow by the flow between two parallel plates.You taking the bottom symmetric half of it so, if that can be done here.So, then you can write this as a velocity profile, because this is the velocity profile for flow between two parallel plates.

And then, you can take this velocity profile, take this expression for u and integrate it, you can integrate it and if one integrates this what he will get is something like this 1 by 3 mu h cube del p del x so, that is what he will get as cube. Now,so what you are doing here is basically you if somebody knows what is h at a particular x, see here q x t so, q is a function of x and t, what is x?X is the distance in this direction, x is this position x is here and y is here. So, you know the x position, if you know the x position if you know the time so, you can find out what would be the flux per unit depth per unit width and that is equal to 1 by 3 mu h cube del p del x and what is h?H is a function of x and t mind it I did not write it here.

So, h is a function of x and t so, h varies with position h varies with time and then on top of this you have del p del x. So, this is an expression for q what we are trying to do here, let me recapitulate once again.We are trying to find out if we introduce bubbles or two phases basically bubbles are droplets.Basically if we introduce two phases inside a microchannel then, we would encounter a pressure drop and it was found that inserting a meniscus into the microchannel is not an easy job. So, if you really insert that and if want to know what would be the pressure drop then how would you approach it. So, one thing is there that first of all, you would like to know what would be this thickness of this annular film.

And once you have that probably, you can get some feel what would be the pressure drop. So, that is what you are try that is; what you are approaching that is why we are getting into this analysis. So, for that what we have said is that if we have this; if we have this bubble, if we have this wall and if we have this as the annular film, the flow because liquid will be flowing through this annular portion.Bubble would be moving at a velocity u, but liquid is also be flowing, because you are introducing liquid and gas at the inlet. So, liquid would be flowing and it would be flowing in through this direction as well.

So, how would you find out the; what would be the velocity profile and one said that; this can be symmetric bottom half of flow between two parallel plates and that is how you have gotten u and that is how you are getting integrating it to obtain q so, that is what you have done. Now, the issue here is how you introduce the effect of surface tension in this process, this is all nice and you have the velocity profileas a function; or you have a flow rate as a function of the; and thickness annular thickness and the pressure gradient.Now, how; what sigma has to do with this, we write here that the effect of surface tension is to give a jump in the normal stress across the fluid surface of sigma into curvature.

What it says is effect of surface tension is to give a jump in the normal stress across the fluid surface this jump in normal stress the value is sigma into curvature, we have sigma divided by r by two and all kinds of things we have. So, sigma into curvature is that is the effect of surface tension. So, here we are talking about this; where is the curvature here we have a bubble or a droplet so, this is the x direction and this is the y direction. So, we can consider this arc and we can write this arc as y is equal to f x. So, if s measures arc length along the curve y is equal to f x.So, s measures the arc length along the curve and if psi is equal to; psi is angle between the tangent and the x axis, the x is this direction this is x and tangent is; this is the tangent.

So, the angle is psi angle between this two is psi so, in that case you can write the curvature,curvature is written as equal to d psi d s and this is written further as if y is equal to f x is the original function I mean we are just talking about the geometry here. So, we; this can be written as f double prime divided by 1 plus f prime square to the power 3 by 2, that is; that comes from geometry, you can checkthe geometry and this can be approximated as f double prime for nearly flat interface, where f prime is much less than 1. So, you can consider the curvature to be equal to f double prime if that is what the understanding is then; in your case the y is basically the h, you remember what is h in this whole analysis we are talking about this height as h.

So, h is changing with x that is exactly what we have done, see here we put h as the function of x and t, here we have; here also in integration limit we put h as a function of x and t. So, h is changing with x and in your case basically y is synonymous to h. So, in that case that curvature;so, you can write, the curvature of a nearly flat interface y is equal to instead of f you would write h x and t. So, then in that case you can write this curvature is equal to curvature is approximately equal to del square h del x square.F double prime and f is basically your saying f and; I mean this is synonymous so, you can write this as curvature as del square h del x square.Now, if that is what the curvature is; now you started with what you said that effect of surface tension is to give a jump in the normal stress across the fluid surface and that jump is; the magnitude of this jump is sigma into curvature all right.

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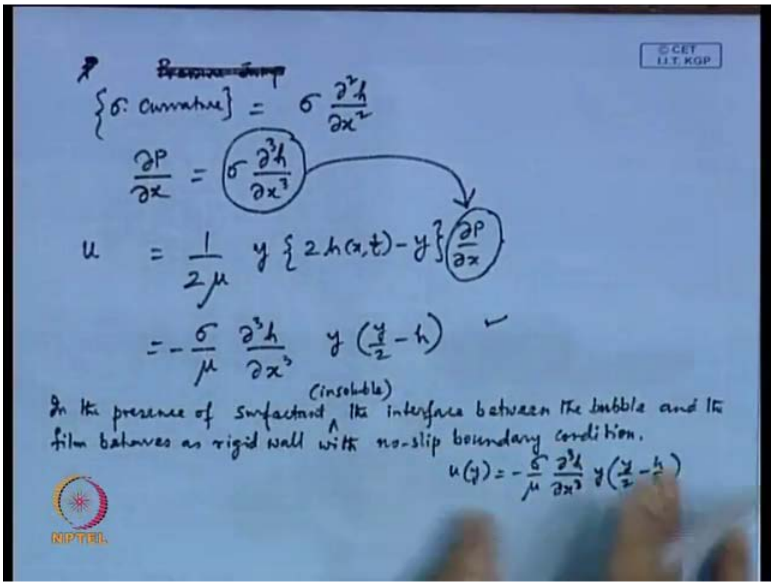

So, in that case what you can write is the pressure p that jump you can write that jump to be equal to sigma; that pressure jump is;so, this is basically; so, you are talking about sigma into curvature. The sigma into curvature would be equal to in this case sigma into curvature this would be; this case would be sigma into del square h del x square. So, if somebody wants to know, what is del p del x in this case? So, del p del x would be equal to sigma del cube h del x cube, because sigma is constant. So, now if somebody walks with these del p del x that means, the surface tension driven del p del x. So, what is the original expression for u we have been working with?We have been working with u, as u of the original expression for u was that this is equal to 1 by 2 mu y into 2 h x t minus y into del p del x.

So, this here you have instead of del p del x, you will bring in instead of this del p del x term you will be bringing in this quantity, this expression for u came from that flow between two parallel plates zero to; between with the thickness being zero to; the distance between two parallel plates is two h. So, based on that, you have a velocity profile now instead of;so, only thing what you are doing is now, you are changing this del p del x by this quantity. So, you get u is equal to this quantity and if you simplify this further what you should be getting is sigma minus sigma by mu del cube h del x cube into y into y by 2 minus h.If you simplify this further that means, this 2 is goes in there so, you will have h minus y by 2,if you write it instead of this, if you write y by 2 minus h so, you get a minus sign here and this is the expression.

And this expression people who are; the students or the researchers who are working on this lubrication they might have seen this kind of expression before this is a common expression that is; this expression is there in theory of lubrication you have this expression. But anyway we do not; let us not divert our attention to anything else so, we have a velocity profile. So, what is the basis of this velocity profile you are talking about that flow in that annular portion?That annular portion that is the liquid; annular liquid layer around the bubble,only thing what you are doing is the del p del x term you are obtaining using this sigma into curvature. So, using that expression you are getting the del p del x so, that is how you are getting the velocity profile.

I must point out couple of things here at this point is that; this is a velocity profile which is as per the theory of lubrication.However, when there is surfactant present I mean this is reported that when in the presence of surfactant.In the presence of surfactant which is not; in the presence of surfactant this; I mean typically surfactants are divided into two categories insoluble and soluble. So, surfactant which is insoluble I mean which is the common surfactant that we work with surfactant.In presence of surfactant the interface let us write it insoluble, but that is the common one we have the interface between the bubble and the film behaves as rigid wall with no slip boundary condition.

In the presence of surfactant, I mean when their surfactant molecules you know surfactant molecules they have their own alignments and all.And how they have head they have tail and they will align in certain manner so, in the presence of surfactant interface between the bubble and the film behaves as rigid wall with no slip boundary condition. So, in that case what you would do is instead of talking about the flow between two parallel plates two h distance apart probably you would be looking at h distance apart. So, in that case this expression here it remains very much same, the expression that we have here remains very much same only thing is this h would be replaced by h by 2.

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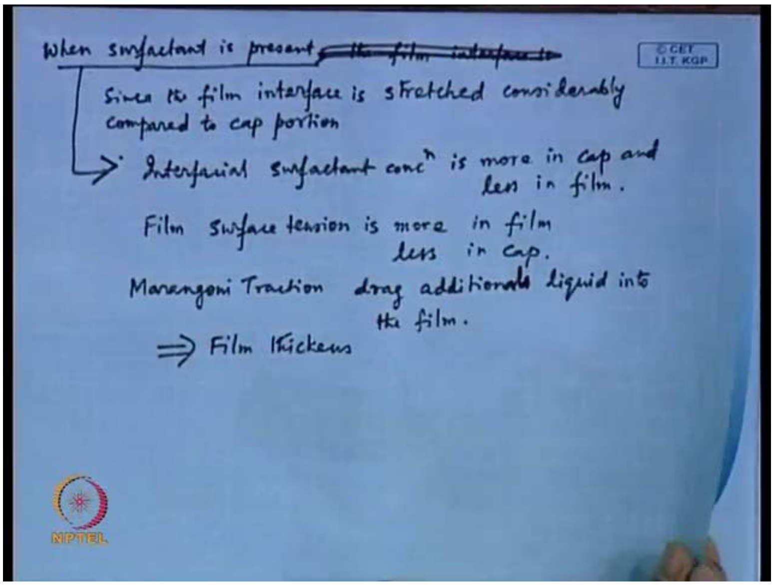

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Now, we are not going into that expression now, but let me let me write it down in a very small font that this is (No Audio From: 28:01 to 28:09) this would be the expression for u in that case.Now, we would be working with this expression for u and this is very common expression in; you will see in connection with lubrication.And now, that is one thing another thing is when surfactant is present that is there is another issue which we are not going to consider here. But this may matter if one has surfactant in the system is that when surfactant is present, the film interface when surfactant is present now, the; I mean probably; I mean when surfactant is present since, I should write the since, the film interface is stretched.

The film interface is stretched irrespective of whether the surfactant is present or not since, the film interface is stretched considerably compared to cap portion so, by now what you understand?What we mean by film and the cap.This is what the film part and this is what the cap part, what they are saying is that this part is more stretched compared to this part, I think we have we have discussed this in previous class that why this would be the case. So, this is stretched compared to the cap portion so, what you will see is that when surfactant is present, because of this effect the interfacial surfactant concentration is more in cap and less in film.The effect of this is that film surface tension; the effect of surfactant is to reduce the surface tension.

So, film surface tension is more or more in film less in cap so, if this happens then, you have a process by which additional liquid will be dragged into the film.So, this implies film thickness so, this is another effect of having surfactant in the system, which we are not going to consider here. So, this is these are couple of effects in this condition which you would be; which you may be bothered with when you have surfactant present in the system.

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Now, let us go back to our original expression I mean just we had deviated we just said that if one has this expression for u, u is what u is the velocity profile in that annular portion, the liquid portion annular portion around the bubble. So, in that case this is the case which we are not considering we are considering this case now, this is the velocity profile.Now, how to go about it, because our original aim is to find out what is that thickness h?And what is the pressure drop?Because that is what we would be concerned with if there is microchannel and if you want to design this microchannel for a flow of liquid and gas, if you have two immiscible phases flowing through that microchannel.We would like to know how these bubbles or droplets, how they distribute themselves inside the microchannel and what would be the pressure drop.

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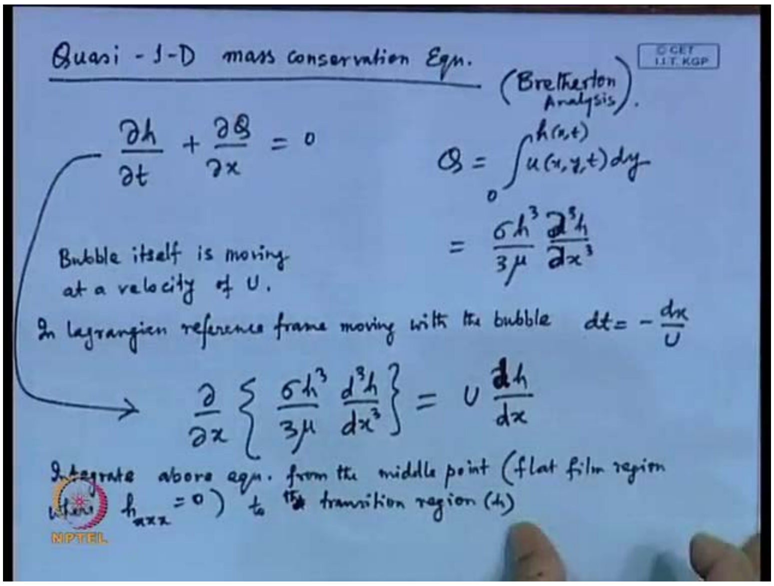

Now, if this is the velocity profile we have already talked about then, what we will do is we will do another aspect which I have pointed out earlier also that this curvature in the cap and the front cap and the back cap this curvatures are not same. In fact, they have to be different apparently that is how the bubbles movesso this is also another issue.Anyway now, what we will do to find that annular thickness and the pressure drop, what we would do is next we will solve something called a material balance.Which we call Quasi-1-D mass conservation equation, this is given by; this is basically this mass conservation equation, I have seen in lubrication literature.

But, this is for bubbles there is one researcher I mean these analysis is given; this analysis is recognizes one particular scientists who is recognized for this analysis, his name is Bretherton. So, this analysis is also referred as Bretherton analysis Quasi-1-D mass conservation equation and this mass conservation equation appears like this del h del t plus del q del x that is equal to 0.What is h? H you know; by now h is a function of x and t and all this we have seen, h is basically the; this annular thickness h, h is changing with x, h is changing with t. So, you have on one hand del h del t and the on the other hand you have something called q which is basically you have taken the velocity profile and then integrated it, but mind it is not exactly the flow rate, it is probably flow rate per unit depth per unit width of the microchannel perpendicular to the paper.

So, this is; what is Q?Q is also a function of x and t and since that is how it is done. So, Q will not have the unit of meter cube per second rather it will meter square per second.And then when you divide it by meter it would be meter per second and here also del h del t so, these are these are unit wise this is consistent. So, what you are assuming here is that the flow that over x so, you take a differential element d x and then you see how much of Q is going in, how much of Q is coming out. So, difference of this is calling; you are calling d Q divide by d x and that change in Q is reflected by the change in height something of that sort. So, you are writing this as quasi one d mass conservation equation, but the problem here is that the bubble equation is not fix bubble is also moving.

So, I mean we just; this is an approximate analysis, but try to understand how it is being; how it this analysis is handled.Now, let us what is Q here?Q is basically we said earlier that Q is equal to integration 0 to h x t u of x y t d y and that is equal to; we said sigma h cube divide by 3 mu d cube h d x cube.We should not be writing it this way let us write it del cube h del x cube, because still we have we kept h as a function x and t so, del cube h del x cube. Now, what you have is that the bubble itself is moving at a velocity u and if the bubble moves then that is defining what should be h, because if the bubble moves so, bubble has; I said the bubble has here it is h infinity and then this h is changing with position if the bubble is static fine we can do that analysis.

But, if the bubble is moving itself, is moving at velocity u how will you justify this analysis, what is done here is in Lagrangian reference frame moving with the bubble.You write d t is equal to minus d x by u and then in that case, you basically do not have t anymore in this entire analysis.You would be putting this here you will get del x of sigma h cube by 3 mu d cube h d x cube so, this is basically nothing but del q del x.And this you write as u del h del x or rather you should be writing since, we are writing everything t is gone now, you are writing u as d h d x u as d h d x. So, this is what you do in Lagrangian reference frame moving with the bubble, you would be writing it like this.

And this equation would be integrated to obtain a pressure drop so, this is known as in fact, this is not the final form what you would do is now, you make certain changes within this make some terms dimensionless.How you do it?Is first of all if you integrate the above equation you first; this equation is integrated, integrate above equation from the middle point which is middle point, is this point here, this is the middle point I am sorry, this is the middle point here. So, from the middle point that means, the flat film regionwhere h x xx that means, d cube h is equal to 0, that means d cube h d x cube is equal to 0.

And at that place you are calling h as h infinity to some other h to a transition region to a; is not the word to the transition region which is some h. So if you do that so that means, you are integrating from this point onwards and going up not all the way to the cap, but to the transition region where the film changes to the cap.

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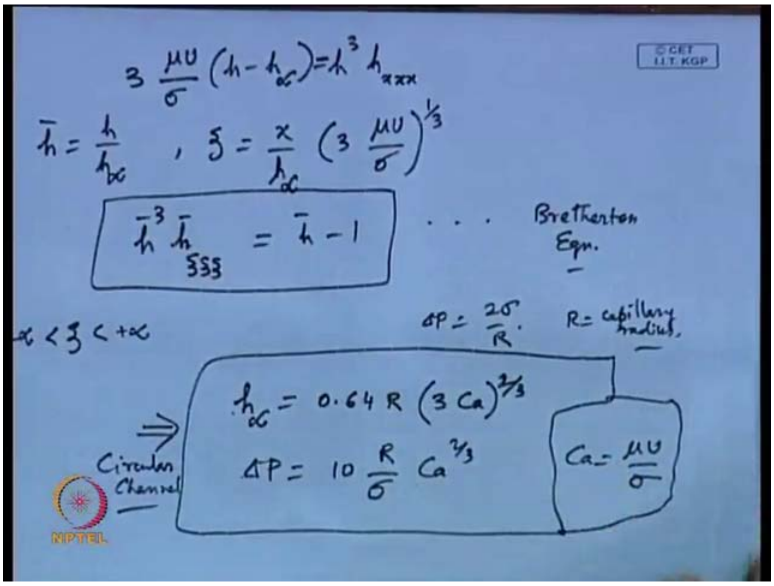

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So, this if you do in that case you will be; the solution is I mean what Bretherton has obtained is 3 mu u divided by sigma h minus h infinity into h cube h x x sorry, this is equal to h cube into h x xx. So, here that you it was dimensionless in a way that h bar is written as h by h infinity, zeta is written as x divided by h infinity x is the distance from that middle point, that flat film region,where h is infinity. So, that place x by h infinity 3 mu u divided by sigma to the power one-third. So, if this is the case then one can write h bar cube h zeta third derivative that is equal to this quantity, this is famously known as Bretherton equation.Ideally this equation should be solved numerically, but originally when this equation was proposed there are, some analytical solution available.

People have integrated this expression zeta changing from minus infinity to plus infinity that people have; that has been integrated and then on top of that.This equation was matched with what; I mean there was the other; at the other end you have this cap portion.And the cap portion has it is own Laplace young equation to satisfy; that means, delta p is equal to 2 sigma divided by R, where R is equal to capillary radius. So, that is equal to that part that is hold; that is held at the cap means in this case the integration that you are doing here at the front cap. So, this equation has been solved with this conditions I mean this typically, this is solved numerically however, originally it was done analytically. But the bottom line here is; one has the final result that Bretherton has given and which is; supposed to be a very powerful expression is something like this.

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This is the most; this is the outcome I mean of doing all this analysis, I mean what; this is the outcome this is what you have, this is what you are waiting for I mean this is the final expression that he had given that with the solution of this.Now, this is ideally this is to be solved numerically and then probably getting such kind of expression is difficult, but he had shown using some kind of asymptotic analysis and using some approximate analysis he showed that this differential equation can be solved, to obtain this final form.Now, what does this say h infinity which is nothing but this height here, h infinity is, this thickness in this mid plane that liquid film.

So, this is the h infinity so, h infinity is 0.64 R, R is the radius of the capillary for a circular capillary it is;R is the radius and then three capillary number which is mu u by sigma to the power two-third. So, that is how h infinity is; and then delta p the pressure drop would be 10R by sigma capillary number to the power tow-third. So, (No Audio From: 46:41 to 46:50) this is a very important formula which is considered kind of the apex formula for the flow of immiscible phases through a microchannel. Now, there are certain modifications further corrections that were done, this expression that I have given is for a circular channel.

This expression that one has obtained after solving similar the equation system that I have gone through, what was equation system we had come up with a velocity profile in that annular region.We obtained q and then we have used;that is what Bretherton has used Quasi-1-D mass conservation equation.And then he had obtained this expression and then he integrated this and finally, he obtained an equation system which is like this h infinity and delta p is given by this. So, this is for a circular channel that the results that I showed here, this results is meant for a circular channel now, there is immediately there was a correction made by another researcher.

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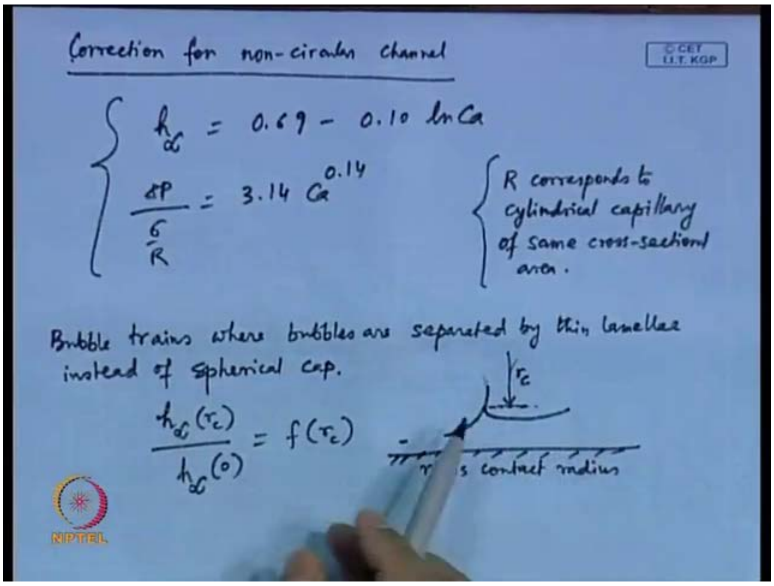

So, after Bretherton what he did is, he has done a correction, further correction for number one, the correction for noncircular channel. So, there what you have is, you have h infinity given by 0.69 minus 0.10 l n c a and delta p divided by sigma by R that is equal to three point one four capillary number to the power 0.14,this is the expression for h infinity and delta p.Now, here R corresponds to; because we are talking about noncircular channel R corresponds to cylindrical capillary of same cross-sectional area. (No Audio From: 49:34 to 49:39) So, R is not exactly they have not related it with hydraulic diameter this is how they have related R for a noncircular channel.

So, this is the expression that was given I do not remember the name of the researcher who has proposed this, but there was a paper where there they have came up with this extension of Bretherton analysis for noncircular channel. There was another issue here which is, if you have bubble train that means, one bubble coming after the other, there could; there was a possibility that the bubbles are separated.Basically bubbles are separated by another liquid film so that means, you have a front you have a cap and that is not affected by the next bubble that is what you are assuming here.

Now, if the gas fraction is much more, if the gas fraction is high then, in that case this interface and this interface may interfere that could be a possibility, in that case also people; the researchers have come up with some correction. So, what they have done is bubble trains; where bubbles are separated by thin lamella instead of spherical cap.Instead of a spherical cap if that is separated by thin lamella, in that case what you can do is one can come up with a radius say for example, this is one bubble and another bubble starts from here itself. So, here I mean this should have been a cap complete cap, but that is not happening, that just a thin lamella between the two bubbles.

So, in that case what they do is the researchers have done is this radius they call it r c.And then, they have done; they have proposed some correlation of h infinity for r c divided by h infinity for no such situation; that means, the original cap intact that as a function of r c, where r c is basically contact radius.You understood what we are talking this is the channel; this is one bubble that is the other bubble. But the bubbles are not separated properly; it is just a thin lamella which is separating the bubble in that case you call another radius.Which is called contact radius where basically this line breaks here you have the film, but beyond this point it supposed be spherical cap and this should be a spherical cap separated, but that has not happened.

So, in that case one can come up with some correlation in terms the; this is there in the literature researchers have proposed correlations. So, what we are heading to basically what we have done is the most important expression that you should take from here is the one that I proposed earlier for a circular channel.This is the expression which is important in many literature in research papers in book you will see this expression is coated.Finally, this is the expression where you have h infinity and delta p and this is the expression for this particular research paper, I have seen where this for noncircular channel such expression has been derived and obtained.

Now,this expressions are important to you as far as your work is concerned or this class is concerned.However, I thought I should give you a brief background of what is the origin of this Bretherton analysis, how what equation what is a governing equation that Bretherton is trying to solve and this is something which you have.Basically this is the final form I mean, I have seen some places this is the form and they said this is by solving this I got this, but this has a big history behind this expression. So, I try to give you a flavour of what kind of analysis Bretherton has gone through when he first solved this problem.

So, in a next class we will probably; there are some unique issues of; if somebody wants to electroosmotic flow, we have; this is nothing to do with this flow. But in case of an electroosmotic flow if you have a bubble introduced in a microchannel, you will be surprised to see that liquid will be drawn like this. But the bubble will stay there sitting there and that is the physics defines that problem, it is the physics that indicates that it should be that way.That the bubble will be sitting in the channel static and the liquid will be just flowing around the bubbles if one has two electrodes placed at the two end of the channel.

So, this is a very unique phenomenon I will try to show, I will go to the equation for electroosmotic flow and I will introduce a bubble inside the channel and I will show how this is happening. So, in the next class I will start with that and then I will probably go into other topics as time permits that is all I have for today’s class.Thank you very much.