Microscale Transport Processes

Prof. S. Ganguly

Department of Chemical Engineering

Indian Institute of Technology, Kharagpur

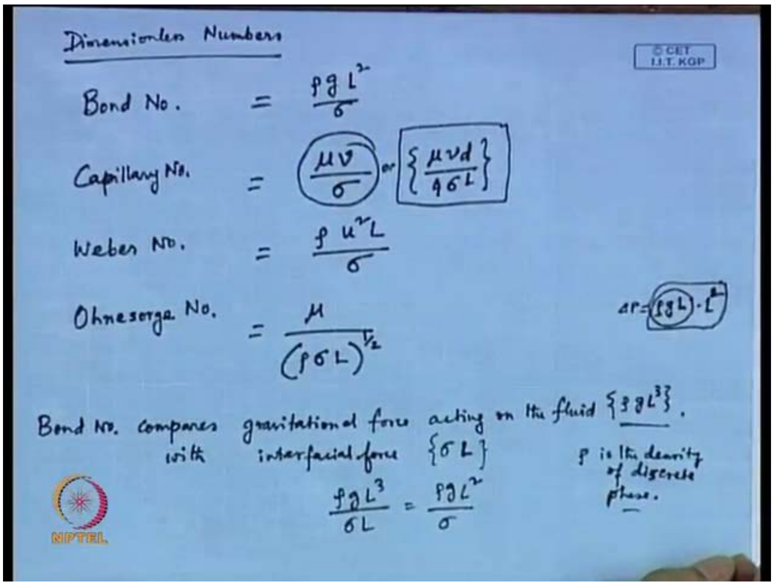
Module No. # 01

Lecture No. # 30

Immiscible Flow in Microchannel (Contd.)

Welcome to this lecture of microscale transport process. What we have been discussing is immiscible flow through microchannel. We have introduced Laplace pressure and we have introduced certain stability criteria when you have two phases present together. I mean when they split into, when they break into droplets or bubbles that we have briefly touched upon. What I will do in today's class first, first what I will do is I will introduce some dimensionless numbers. These dimensionless numbers are extremely important in this context. And then I will try to find out how we can calculate the pressure drop through a microchannel when we have this immiscible flow happening. What would be the pressure drop and what would be the size of the bubble or what would be the size of that annular outer liquid so, these issues we will address in today's class.

(Refer Slide Time: 01:19)



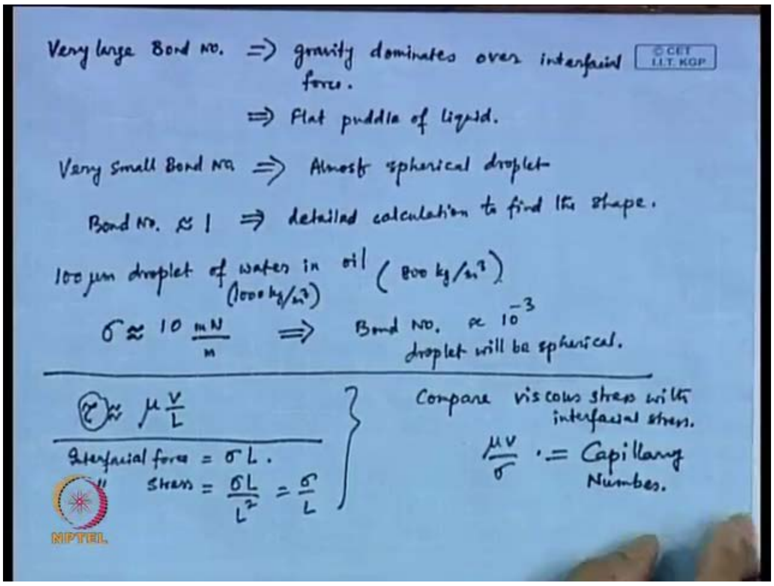
So, first let me introduce some dimensionless numbers which are very important in this context. (No audio from 01:18 to 01:33) What we will be talking about here is something called bond number, capillary number, Weber number and this is the fourth one. Bond number, let me let me just give the (()) first with symbols let me put it how bond number looks and then we will try to find out what is the significance of this. (No audio from 02:10 to 02:31) Rho is the density, g is the gravity of course, you can say density of which phase. So, this is this is probably density of the discrete phase. So, let us and let me point out there is another version of capillary number, this is not the end of this capillary number, there is another version of capillary number.

In fact, the alternative definition of capillary number would be mu v d divided by 4 sigma L, this is another definition, this is a different definition, this is not a product, this is one definition or this can be another definition. Here, you can this is these are dimensionless numbers, here also it is dimensionless you have multiplied it by d by 4 L d by 4 and L d by 4 divided by L. So, that is what you multiplied this way here d is the diameter of the droplet and L is the so, 1 is 1 is the characteristic dimension of the droplet and other is the characteristic dimension of the channel. So, this is an alternative definition which could be useful.

Now, let us focus one by one on what we have what do they mean. Bond number basically compares gravitational force acting on the fluid with compare something with something so, with interfacial force. The way this us conceptualized is that the gravitational force, this would be dimensionally it would be rho g L q, how is it? So, we know that you for gravity for a hydrostatic heat, the pressure, the delta p is written as rho g into L that is how delta p is written h rho g so, delta p is rho g L so, this is the so, rho g L signifies the pressure. So, now if you multiply this with L square; that means, the area so then only you get a force so, if you want to call it gravitational force, then it will be rho g L into L square that is what here we have written here.

Whereas, in the interfacial force, what do we have? Interfacial force is I mean how is interfacial force written it is written as sigma into L, you all know Newton per meter surface tension the unit of surface tension is Newton per meter and that has to be multiplied by the meter. So, you Newton so, you get it in Newton. So, you got to multiply by the L dimension L then only you get the force. And here you got to this represents a gravitational force. So, the ratio of these two; that means, rho g L cube divided by sigma L or in other words rho g L square divided by sigma. So, that is exactly what is bond number.

(Refer Slide Time: 07:13)



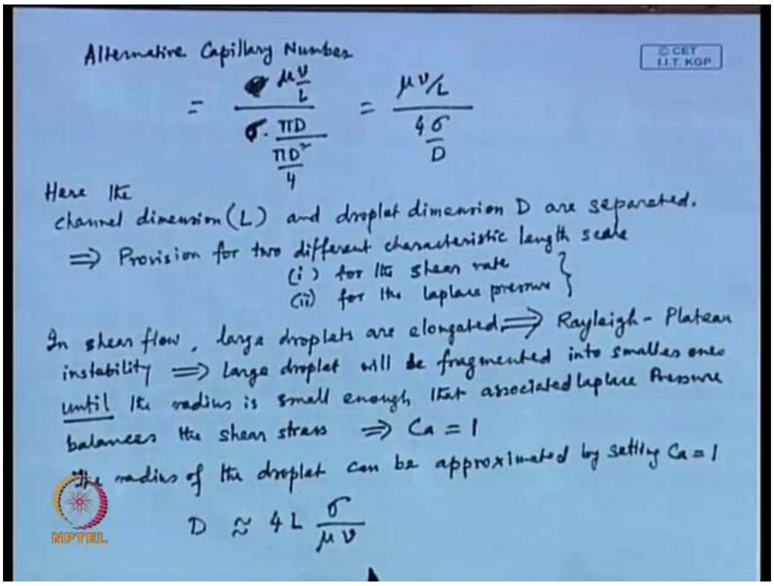
So, what does this mean? This means that when bond number is of course, I mean you got to understand here is that rho is the density of discrete phase, discrete phase means you are producing oil droplets in water or you are producing bubbles in water. Then the discrete phase here is air and the continuous phase is water. Discrete phase is the droplet or the bubble that phase is discrete phase so, rho represents that density. So, what this means is a very large bond number implies gravity dominates over interfacial force. (No audio from 07:34 to 07:41) So, very large bond number means a flat puddle of liquid you understand what that means a flat puddle of liquid.

On the other hand I mean let me point out what happens if one number is very small then you will understand what this puddle of liquid means very small bond number this implies almost droplet spherical, spherical droplet or bubble. So, by looking at the bond number if the bond number is large, you can expect that it will be a puddle of liquid it will not form a sphere, it will not form a droplet, it will not take a spherical shape. On the other hand if the bond number is very small, invariably you know that there would be this there would be the shape of this discrete phase would be spherical because interfacial phase will dominate over the gravity force.

Now, the problem would be if bond number is close to 1 then it needs detailed calculation to find the shape, you cannot predict the shape. Also in your calculation if you want to ignore gravity, then probably one justification of doing that would be that you are working with a small bond number. So, these are these are the few thing you can have. Now, let us get a feel for what could be a bond number in case of a microchannel, if we look at say a 100 micrometer droplet of water in oil, water density you assumed is 1000 k g per meter cube and oil density you assume as 800 k g per meter cube. This and if you assume the sigma to be equal to sigma is of the order of say what is the sigma we mentioned in the last class or water in oil we have not measured mentioned anything.

If we assume close to say 10 milli Newton per meter, I mean it if it of the order of 10 milli Newton per meter, what would be the bond number, in that case? I think bond number would be of the order of 10 to the power minus 3. So, you can you can conclude that the droplet will be spherical. So, this is these are the some of the numbers that you can work with.

(Refer Slide Time: 13:52)



The next dimensionless number we had defined here is capillary number which we said is mu v by sigma and alternative definition would be mu v d by 4 sigma L. Now, here we have we do not have rho, here we had rho and we said rho is the density of the discrete phase, here we have mu. Let us first point out what capillary means because we have to you have to mention whenever such thing happens we have to mention that mu of which phase because we are working with two phases. So, here think of it this way that tau dimensionally tau is what? Tau is mu d u d y you call it. Now, d u d y if at the wall it is 0 and if at distance L away from the wall it is v so, you can call this mu v by L. So that is the shear stress and the interfacial stress I mean if we look at the interfacial force, interfacial force is equal to sigma into L.

You understand Newton per meter into meter, meter meter cancels, if it is Newton interfacial force. So, if somebody wants to find out what is interfacial stress? You will write this sigma L divided by L square force per unit area. So, this becomes equal to sigma by L. So, if somebody wants to find out the ratio of viscous stress, if somebody wants to compare say ratio compare viscous stress with interfacial stress, then this would be equal to mu v by L divided by sigma by L, L cancels out and so, invariably you end up with mu v by sigma. So that is the that is basically the definition of capillary number. (No audio from 13:34 to 13:41) That is basically the definition of capillary number.

Now, here there the we have already mentioned that there is an alternative definition, the meaning of alternative definition would be that you are working with the alternative. Let us write this as alternative capillary number that is equal to (No audio from 14:11 to 14:17) you continue with the shear stress term, the tau and you (No audio from 14:23 to 14:32) here you would write this as sigma into pi D divided by pi D square by 4, what do we do here? On the top we instead of tau we can write this as mu instead of tau, you can write this as mu v by L that is that is what we have done earlier here, that tau is equal to mu v by L.

However, here the interfacial stress is written as sigma by L square. So, what did we do here, which we are not doing here, you understand that. Here, we are keeping the dimension L that is used for viscous stress and the dimension L that is used to calculate interfacial stress as same. But for a droplet or a bubble flowing from microchannel the dimension of the droplet and dimension of the channel this is the length scales the characteristic length scales would be different. So, what you are doing here is, you are continuing here with mu v by L, but here instead of sigma by L what you are writing is sigma into pi D divided by pi D square by 4. That means, if so, this is how we are we are defining the interfacial stress.

So, what you would get in this case is mu v by L divided by 4 sigma by D.

(No audio from 16:19 to 16:45)

Is there any other way you can get this 4 sigma by D? I think this is, what is the interpretation of 4 sigma by D, think about it. Now, the channel so, here what you are doing is here the channel dimension L and droplet dimension (No audio from 17:19 to 17:26) D are separated. So, this implies provision for two different characteristic length scale; one for the shear rate and other for the Laplace the pressure so, this is this is an alternative definition. Now, there are couple of things you need to appreciate here regard to capillary number, before we proceed to the next dimensionless number we must point this out is that in shear flow large droplets are elongated. I mean it is understandable I mean shear we have already talked about this elongational, what we call it linear stretching and all kinds of things.

So, basically you are stretching the fluid so, if instead of the fluid there is a droplet so, droplet will also be stretched. So, large droplets are elongated and so, when large droplets are elongated they; that means, that they will undergo Rayleigh-Plateau instability that we have talked about in the last class. That this will the moment you stretch this moment you have a droplet, moment you have an immiscible cylinder, when moment you have a cylinder that cylinder will break into smaller droplets because of certain instability. Now, so that instability will come into play there.

Now, if you have this happening so; that means, I can say the large droplet will be fragmented into smaller ones till what time? You it will be fragmented, but fragmented to what length scale? So, it will be into smaller ones until this is important, until the radius is small, small enough. That the curvature with associated Laplace pressure balances the shear stress, what that means is essentially the capillary number is equal to 1. So, until the radius is small enough that associated Laplace pressure (No audio from 21:07 to 21:14) balances the shear stress. And this happens when capillary number is equal to 1.

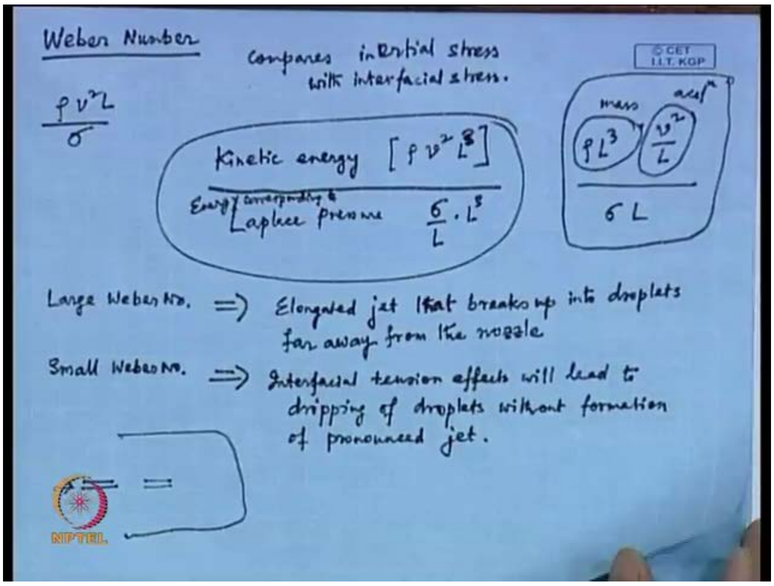
So, if somebody wants to find out what would be that limiting radius that this droplet will achieve I mean I have put two phases together and I am having that to having those two phases flowing through a tube, flowing through a small tube. I would like to know what would be that limiting radius to which this droplets would be fragmented that radius can be obtained by equating capillary number is equal to 1.

So that means, the radius of the droplet can be so, what we mean here is that the radius of the droplet can be approximated by setting capillary number is equal to 1. And capillary number is equal to 1, in if you look at this if this is the alternative capillary number definition of alternative capillary number. And if this is equal to 1; that means, you get some idea of what is D, what is D? D would be close to 4 L sigma divided by mu u not mu v. So that would be the diameter of the droplet. So, this would be the diameter of the droplet, you can expect this is the limiting diameter of the droplet.

Another issue is that you can check the capillary number and whether the capillary number is greater than 1. If you if you have a higher capillary so, in a particular flow when you have a flow through a microchannel or a flow through a channel let us put it and you calculate the capillary number. And if you see that the capillary number is greater than 1 or capillary number is high, in that case you can conclude that the shear can dominate in this fragmentation process. On the other hand if you have a flow through a channel where you see that the capillary number is low, capillary number is not significant, capillary number is less than 1. So, in that case you would be you can conclude that you cannot expect much if a breakup arising from the shear flow, by imposing shear you cannot break it into you cannot get into this fragmentation.

So, this capillary number is an indication how effective shear will be to break the droplets. So, if you get a good capillary, if you get a good microchannel, if you get if you see that your capillary number is pretty high there so, you know one thing for sure that if you introduce the if you or if you can if you know that you have some shear environment in which you can introduce these immiscible phases and you can elongate it. Then you know that you will be able to fragment it. But the capillary number would determine whether you will be able to fragment it or not, then and you can see what is the limiting value of diameter that you can expect.

(Refer Slide Time: 25:26)



Now, let us go to the next dimensionless number which we called Weber number. Weber number first of all this compares, the Weber number this compares inertial stress with interfacial stress. Inertial stress this may arise in case of a jet, a elongated jet that breaks down it is not a viscous stress, it is inertial stress so that is how it is different from capillary number. So, if somebody is observing a jet and the jet breaking down so that is probably the case where you have this Weber number. Let us see how we get into this Weber number is there are two, there can be two interpretations possible; first of all Weber number is rho we said that rho v square L divided by sigma that is the form.

There are two interpretations possible; one is that it is written as (No audio from 26:51 to 26:57) it is written as kinetic energy which is given by rho v square that is per unit. So, rho v square is typically per unit volume. So, this multiplied by L cube so that is the kinetic energy, this divided by Laplace pressure which is basically not exactly the Laplace pressure. The way we interpret this is it is sigma divided by L into L cube. So, how do we look at it? Sigma by L is basically the pressure this multiplied by L cube.

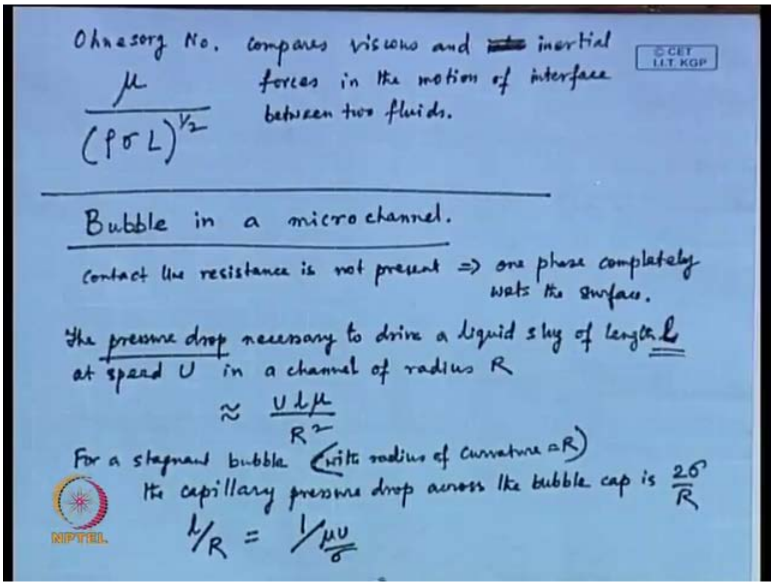
So, energy are we talking about the energy associated with Laplace pressure probably something that is what we are looking at. Or the other interpretation could be that you think of it as rho L cube, this gives you the mass and then you have v square divided by L this gives you the acceleration divided by sigma into L. So, this is an interpretation of inertial force divided by interfacial force. So, this is one interpretation you can think of, this also leads to rho v square by rho v square L by sigma. Or you can write this as energy corresponding to this quantity so, this is also another interpretation possible. Anyway the large Weber number implies elongated jet that breaks into breaks up into droplets far away from the nozzle. (No audio from 29:51 to 30:04)

Now, if it is a small Weber number, this implies interfacial tension effects will lead to dripping of droplets without formation of pronounced jet. So, large Weber number means elongated jet that breaks up into droplets far away from the nozzle. And small Weber number means interfacial tension effects will lead to dripping of droplets without formation of pronounced jets. You do you understand what we are talking about? Suppose, I mean it will think of this way that I have a pool of some phase and you are introducing say another phase it cause be that it you are introducing air into a pool of water, it could be that you are introducing water in a pool of oil. I mean it could be one phase you are introducing into the another phase.

So that would be in the form of some tube would be inserted into that reservoir into that pool. And then you are forming, you are calling a droplets bubble because the you know that moment you are introducing a cylindrical, you are introducing a cylinder that is if you have a flow, you are introducing a cylinder. If there would have been no other obstruction, the flow would have flow should have continued like a cylinder. But due to instability these cylinders will not it cannot continue that way, instead this cylinder breaks down. Now, where it breaks whether it breaks immediately next to the nozzle where it enters the where the nozzle enters into the pool or whether it will drip from the nozzle or it will go and break far away from the nozzle that effect can be found out from this from this dimensionless number.

Probably, I here I am just briefly touching these I am just mentioning or I am trying to give briefly the significance of these dimensionless numbers. When you are actually working with it, probably the people who are really working on this dimensionless numbers they probably have better interpretation of it, but for that you need to play with this number for long. I mean you have to do theoretical study with these numbers then probably you can have that kind of handle. Here, I just touch up on the significance of these numbers in nutshell.

(Refer Slide Time: 33:26)



The next number that I have here of that last one in this slot is, this number here we mentioned that this number the expression is mu divided by rho sigma L to the power half. And this number compares viscous and inertial forces (No audio from 34:00 to 34:11) in the motion of interface between two fluids. I have not I mean so far the work that I have done. I have not used this number if any of you have done that probably you can share your thoughts about this number. Basically, I just mentioned here that this number is there and there is a basically, these dimensionless numbers I mean if you come up with a system which were you think that there would be a dimensionless number would be appropriate. You can create your own I mean basically see dimensional you introducing dimensionless number helps you in expressing the results helps you in analyzing the results.

So, I mean it is it is very open forum I mean people the researchers have studied several systems and they have introduced those numbers. In fact, if you go to internet these days, you can see earlier it used to be in the backside of the book the list of entire list of dimensionless numbers that are already people have which are recognized dimensionless numbers. Now, the system that you are working with you can find some dimensionless number will help you analyzing the results and that you are free to use that number. The point is that if people find that the systems other people are doing, other people are researching and they are, they continue to I mean the research they are doing, they need that same dimensionless number in various places.

So, then probably this dimensionless number will be important and the person who had floated, who had put forward this number he would be recognized. So that is that is the whole idea. So, it is a very open forum and keep an open mind when you are studying a system you see what dimensionless number would should be appropriate. In fact, it would be a very it would be a puzzle, it would be a creative work if you can come up with the right definition of the system through these numbers. That is what we had as far as these dimensionless numbers are concerned. So, what I will do next is, I will get into the I mean first I before I get into the pressure drop calculation, let me point out couple of details a bubble in a let us say microchannel. (No audio from 37:58 to 37:05)

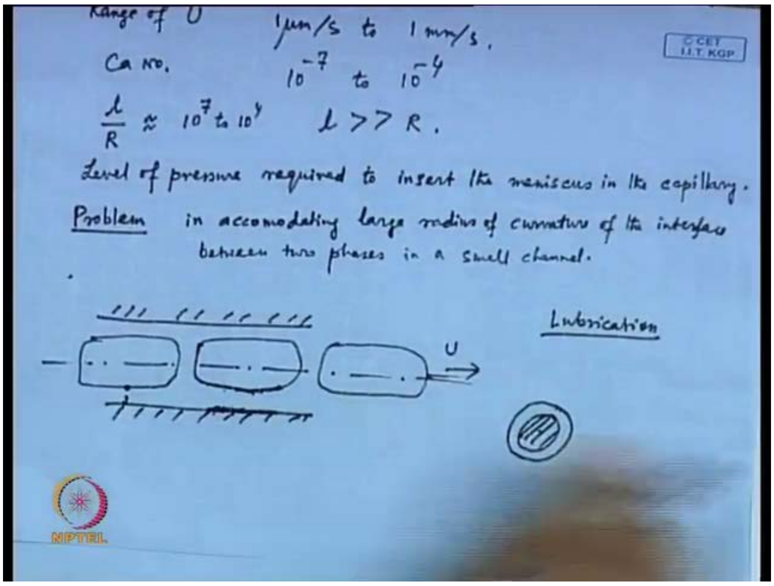
Now, one thing at the that the analysis that I am going to present here, one thing I would like to point at the very outset is that the contact line resistance is not present, here that means, one phase completely wets the surface (No audio from 37:34 to 37:47) That the other point I mean I would like to work with these some of the I would like to play with numbers to see, what would be the importance of Laplace pressure and the pressure drop for a flow through a pipe? Pressure drop for flow through a pipe what was that? Hagen-Poiseuille's equation or what you have if you take that pressure drop and if you compare it with a Laplace pressure for that same channel? I mean for the similar dimensions I mean how these numbers look that let me let me point out here.

If I write the pressure drop (No audio from 38:31 to 38:37) necessary to drive a liquid slug of length l, I am not talking about two phase I am talking about one phase, liquid slug of length l. Let us say small l at speed say capital U in a channel of radius. We are not a we are talking about a circular channel, circular cross section channel of radius R. You will find that this pressure drop is of the order of U l mu divided by R square. Now, if we consider the for a stagnant bubble that has that radius of curvature of R so with radius of curvature is equal to R. So, for a stagnant bubble, the capillary pressure drop (No audio from 40:23 to 40:32) across the bubble cap is 2 sigma by R that you already know.

Now, if we set these two pressure drops I mean if we try to find out how these, what should be this l for example? Or in other words what I am trying to do is, I am trying to find out if somebody if some length of slug provides a pressure drop which is equal to this pressure drop. If you try to find out what would be that length of the slug length of the liquid slug that can for driving that length of the slug, you need a pressure drop. And you are trying to equate that pressure drop.

So, you want to get a feel for what would be the length equivalent liquid lengths, liquid slug length equivalent of this Laplace pressure. If you want to do that you will find that l divided by R that is equal to 1 divided by mu U divided by sigma. In fact, you are now you are getting another interpretation of capillary number do you do not you, I mean l divided by R is equal to 1 by mu U by sigma. This you are talking this we said is the capillary number. So, this is another interpretation of capillary number that capillary number gives you comparison here. If you had what is the, what would be the length of the liquid slug that you can drive with the same pressure as the capillary pressure for that stagnant bubble. So, this is this is the value.

(Refer Slide Time: 42:35)



Now, if somebody finds out say for the velocity range let us talk about a range see U is say 1 micrometer per second to 1 millimeter per second. Let us put this as the range for U. You will find that if you take water and air and their corresponding interfacial tension and everything, you will find that the capillary number would be equal to 10 to the power minus 7 to 10 to the power minus 4 corresponding to this range of U, 10 to the power minus 7 to 10 to the power minus 4. So, can you imagine what would be the case in this case? Now, l divided by R is equal to what we are saying is 10 to the power 7 to 10 to the power 4 that is what we are saying. So, can you imagine what would be the length of the slug we are looking at? Length of that liquid slug which is equivalent to the Laplace pressure corresponding to the or the capillary pressure corresponding to that bubble that is of this order I mean what this means is l is much higher than R.

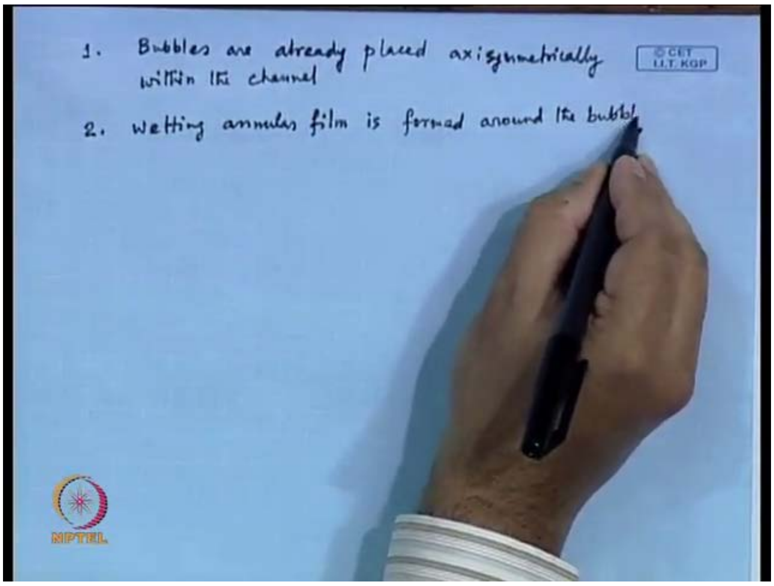
So, this basically shows the level of pressure required to insert the meniscus in the capillary. (No audio from 44:16 to 44:27) So, the problem here I mean when it comes to having this two phase flow, the problem here is to, the problem lies in accommodating large radius of curvature (No audio from 44:49 to 44:55) of the interface between two phases in a small channel. (No audio from 45:10 to 45:16) Now, the calculation that I showed is not I mean what we are interested in at this point is probably the for a flow of bubble. That means, what we are we are interested in here is that if you have a if this is the wall and if you have train of bubbles; this is one bubble, this is another bubble then there will be another one like this. So, you have a train of bubble flowing.

So, for the pressure drop for the flow of this train of bubbles that is again some more different the mechanism would be some more different because we need to understand how these annular region will behave that is also important. How this part will behave so that needs to be understood. So that and in fact, the pressure drop calculation for the flow over train of bubbles that should be that should come from that understanding of how these annular region behaves. So that is not been accounted so, what I mentioned here is probably the what you call back of the envelop calculations so, this not actually the rigorous one what we are interested in this.

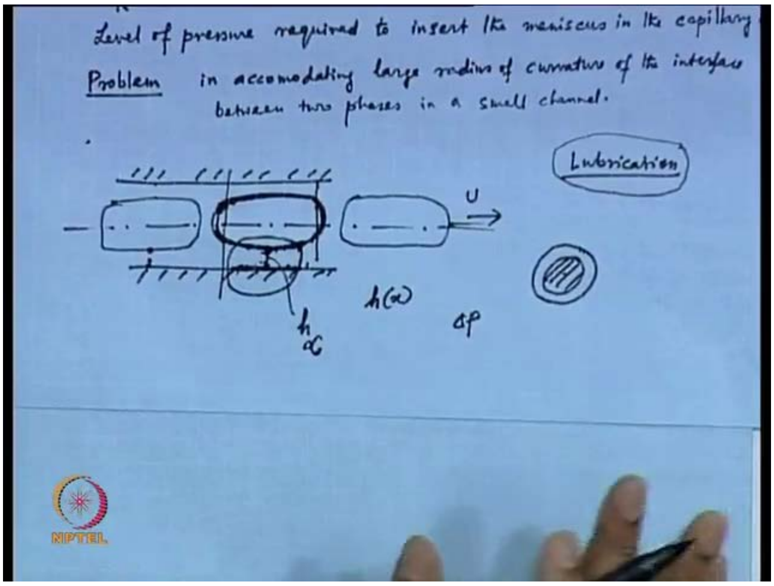
But I would like to point out I mean if somebody compares the Laplace pressure and the pressure drop, conventional pressure drop of a single phase and particularly in the context of flow in a microchannel, the what they will end up with these numbers that is what I would I am just trying to emphasize. Now, here we I mean what I will be taking up next is how to address this bubbles are moving at a velocity U and in this annular region here of course, at the wall the velocity has to be 0. And then there would be a velocity profile from here to here, mind it that these dimensions are small. So, there is there are certain existing theories I mean nobody is inventing new things here already there has been good amount of work done in the area of lubrication.

So that those some of those theories will be invoked to understand this annular part, this annular part how these theories on lubrication can be, we will look into that how that can be invoked here.

(Refer Slide Time: 48:18)



(Refer Slide Time: 51:26)



Now, there are certain considerations we need to have here; one is that bubbles are already placed. I mean when you get into this pressure drop calculation your frame of reference is that bubbles are already placed axisymmetrically within the channel. (No audio from 48:48 to 48:53) What does axisymmetrically means is that if this is the axis

(No audio from 49:00 to 49:06) if this is the axis then this bubble is axisymmetrically placed. That means, it is not that one bubble is sitting in one corner and then the annular region is it is perfectly, it is the cylindrical, it is perfectly I would say two circles they are forming the annular region and inside this is this is the bubble.

So, this is axisymmetrically this bowls are axisymmetrically placed. The other assumption that you have is (No audio from 49:42 to 49:48) wetting annular film is formed around the bubble (No audio from 50:05 to 50:11) a wetting annular film is formed around the bubble and this is that annular film we are talking about. And this is wetting the wall of the microchannel. So, this is what you are assuming and you are assuming that a bubble is moving at a velocity U.

Now, what we will do at this point I mean for probably in the next class what we will do is we will focus on this region. We will come up with a velocity profile for the liquid layer. Considering the this wall is moving so, you using this theory of lubrication we will be finding out the velocity profile within this layer. And that is going to be going to going to affect significantly the final derivation of pressure drop because what we are interested in here is the pressure drop to have these train of bubbles flowing through this.

You now, what you if you look at these bubbles what you have to find what do you see here is that there is a curvature, I mean this is uniform this is symmetric. Now here it looks like a cap. So, this we will call a front cap and this we will call the back cap because bubble is moving in this direction. And as a matter of act this front cap and back cap they cannot be symmetric because they have to be asymmetric otherwise the bubble will not flow that is that is one condition you have. And then you have this part, this thickness we will call this as h infinity and then we will be considering h at various points. So, we will be considering h as a function of x and finally, the profile that you have that has to match with the cap part.

Now, there are there are lot of other issues involved. In fact, the bubble here you will find that this the that is the it the force arising from surface tension that is not uniform everywhere, some places it is more and some places it is less. And that will be causing the surfactants to be loaded in one place and surfactant the absence of surfactant in other places. So, the there would be some imbalances coming up there.

So, it is a complex I mean if you get into the complexity of course, there will be complexity if somebody wants to treat this as a black box. I just find out delta p empirically with the (( )) and with the with the flow rate I can do that. And if you want to get into the complexity there are several such complexities possible. I will try to address them as far as possible and my final M is to come up with a delta p that is delta p in terms of probably capillary number. Probably, in terms of things which for example, if I say it is in terms of h then you will ask me how will I measure h in the microchannel I cannot made the dimension even. So, how do you expect me to measure that? So, I will not do that, but come up with an expression that is the job of the researchers is to simplify the complex volt. So, we will we will continue this lecture in the in the next class. That is all I have for today.