Microscale Transport Processes

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Module No. # 01

Lecture No. # 29

Immiscible Flow in Microchannel

Welcome to this lecture of MicroscaleTransport Process.In today's lecture, we are going to introduce a new topic, which is basically the immiscible flow in microfluidic channels, by immiscible flow I mean when there are two phases.And the two phases are flowing through the microchannel and these two phases are not miscible, like water or oil or water or gas.

So so there could be droplets or bubbles, so that that is how the two phases can coexist and if you want to have these two phases flowing through a microchannel, the physics would be somewhat different from what you have for a single phase the in this context couple of things needs to be defined first.

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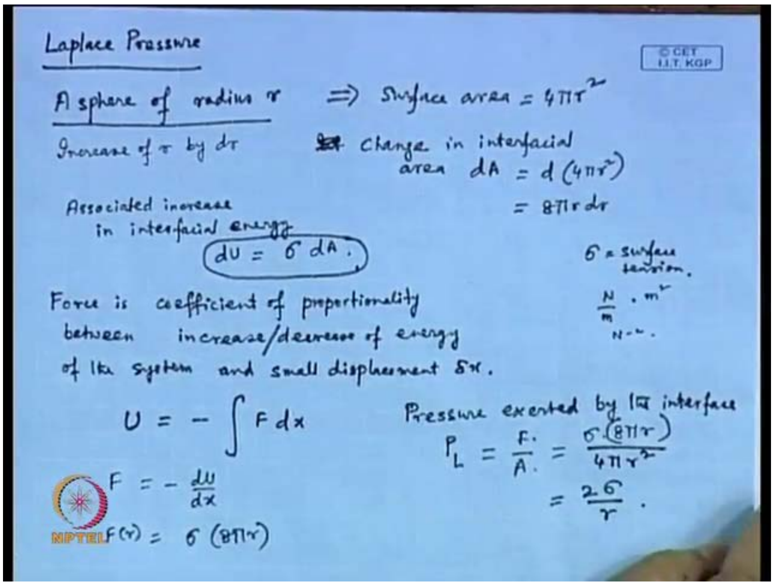

So, what we are we are working with is immiscible flow and first topic, that we briefly touch upon is something called Laplaceequation. This is, this I presume you are already familiar with in other context, I justbriefly touch upon this,let us think of a sphere let us think of a sphere of radius r, so the surface area is equal to 4 pi r square. So, if you have if this if this radius changes by d r, increaseof r by d r this changes the interfacial area increase of r by d r, this changes the interfacial area.So, if I if Iwrite change in interfacial area d A that is equal to d of 4 pi r square, that is equal to 8 pi r d r and associated increase associated increase in interfacial energyassociated increase in interfacial energy that is d U. Let us say and that d U would be equal to sigma into d A (No audio from 03:24 to 03:33) what is sigma?Sigma is the surface tension.

So, what does this star mean d U is equal to gamma d U equal to sigma d A, what doesthis mean what this means is that U what is basically the energy energy is equal to the work done that is equal to force into displacement.all rightForce into displacementand go to the unit of interfacial tension do you do you do you remember how the how the interfacial tension was derived at that time.What what is the unit of inter surface tension that is basically Newton per meter rightthat is the unit that is that is the that is the unit of surface tension Newton per meter.

So,Newton per meter you are multiplying it with d A, which is an area term I mean to say meter square, so essentially what you are getting is Newton meter,Newton meter is the force into displacement.So,that is that is the energy so associated energy, associated increase in interfacial energy can be given by d U which is equal to sigma d A.

Now, if you have if if Iif I say that the force force is coefficientof proportionalityproportionality between increaseor decrease of energy of the systemand small displacement delta x.So, force is considered it is the coefficient of proportionality between increase or decrease of energy of the system and the small displacement that means force is basically the energy divided byd x the displacement.If we if we look at it this way then, what we write here is U is equal to the energy minus integration F d x basically, the increase or decrease of energy that is defined by the sign.

So, U is equal to integration of F d x, so or in other words you can write F is equal to minus d U d x all right.So, in this case with this d U this information available to you d U is equal to sigma d A, what do you have for this sphere, we started with a sphere of radius r. So, we are we are talking about a bubble of radius r inside say water and we are trying to find out, what would be the force in this case; let me let me let me put it here if I write thispressure exertedpressure exerted by the interface, we call this a P L that is equal to force divided by area.And force in this case for the sphere it would be sigma into 8 pi r, d U d x; that means this is equal to sigma d A d x right and what or d A d r I mean if you if you want to write this in r coordinate.If you want to write this F r, what would be F r, F r would be equal to sigma into 8 pi r right from this expression.

So, what you have is sigma into 8 pi r and in that case P L would be equal to F by A that means is equal to sigma into 8 pi r divided by, what is the area? 4 pi r square that means equal to 2 sigma by r. So, what is I mean why why are we getting into this and what is the what is the I mean what are we trying to conclude.

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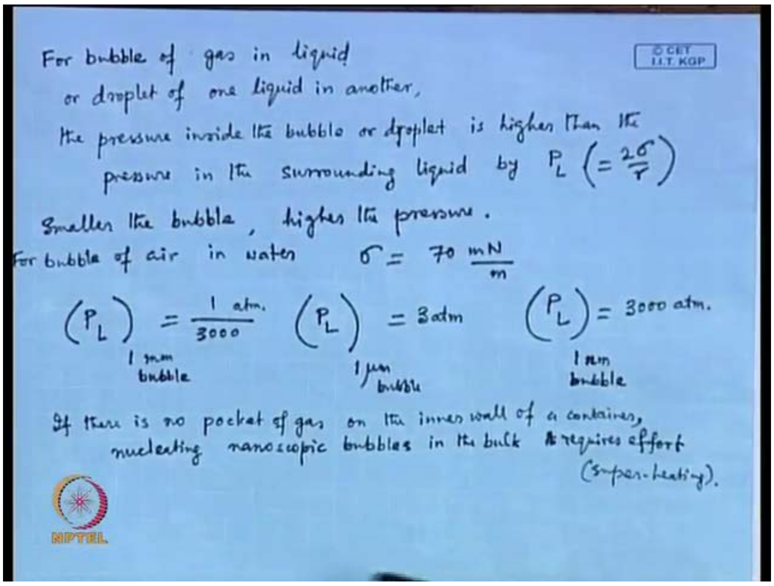

The conclusion that I draw here is for first I write it and then let us see,if we can if it make sense for bubble of gas in liquidor droplet of one liquid in another, the pressureinside the bubble or droplet is is higher than the pressure in the surrounding liquid by P L.

So,what is what are we what are we up to here (Refer Slide Time: 09:48) we said that we started with a sphere of radius r and I said that if somebody wants to change the interfacial area d A, change the interfacial area by d Athis sphere of radius r a bubble of radius r that had an area that area is 4 pi r square, where r is the radius of the bubble.So, if there is a change in interfacial area and if we call this d A, thenbecause of that there would be associated increase in interfacial energy,and because of that there would be a force existing.So, there will be some extra pressure pressure exerted by the interface would be this quantity.

So,what you what we what we write here is for bubble of bubble of gas in liquid or droplet of one liquid in another, the pressure inside the bubble or droplet is higher pressure inside the bubble or droplet is higher than the pressure in the surrounding liquid by this quantity which is P L. Nowthat means, nowP L is P L isequal to what P L is equal to 2 sigma by r, that meanssmaller the bubble smaller the bubble higher the pressure pressure inside the bubble.smaller the bubble higher the pressureSo, if somebody assumes bubble of air in water for for bubble of air in water you can assume sigma to be equal to 70 mille Newton per meter.

Now, if somebody tries to find outthis quantity P L for 1 millimeter bubble1 millimetersize is of the order of 1millimeter if somebody tries to find out P L for size of the order of 1 micrometer.And if somebody tries to find out this pressure, this extra pressure that that will be there inside the bubble for size of the order of one nanometer, one will see that this quantity for 1 millimeter bubble is 1 by 3000 atmosphere.This quantity is for 1 micrometer bubble it is 3 atmospheres and this quantity here is 3000 atmosphereall right.

So, this is the difference and what is what is what would be the repercussion, onerepercussion I mean one oneoneimportant significance of these data that I sighted just now is that.Ifthere isno pocket of gas on the inner wall of a container (No audio from 13:15 to 13:25) nucleating nanoscopic bubbles in the bulk is I meanrequires effort.Let us put it this way probably, I should write here is thatit requires superheatingeffort in a sense I iIwrite this as superheating what Iwhat I try to write here is if there is no pocket of gas on the inner wall of a containerif some.

Suppose you want to boil water in a pot and if there is no such support from no such artificial nucleation done, I mean no suchno such arrangements, no suchplatforms present for nucleation.Then if it has to nucleate on it is own inside the bulk, that meansyou have to the bubble has to form inside the bulk of the liquid and thatthat has to be a nanoscopic bubble.it requires so you have to you have to You have to break this barrier you have to you have to provide that kind of pressure then only the bubble would be generated, so that isthat is that is what I would like you toappreciate here is that this pressure can be quite significant, when the dimension of the bubble is small. Now, we have been talking about only one radius, we we said that this P L is equal to 2 sigma by r, I mean we are just working with the with the sphere, just a single sphere but, you may not have a sphere all the way.

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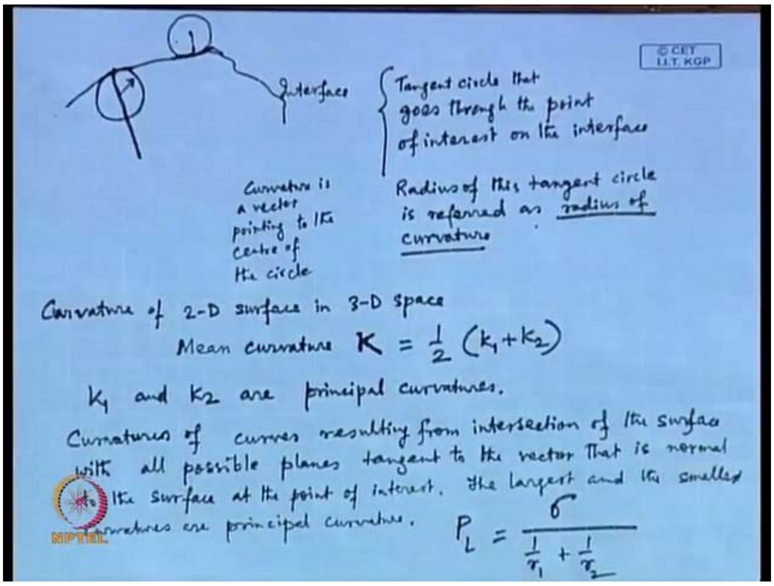

You can have a curvature, which will run like this or you you may have an interface,which will be something whichwhich will be something like this this may be the interface.

So,you have to you have to make note that you you will you can have something called a curvature and what would be that curvature, you will be you can form suppose this is the interface, then in that case you can draw something called a tangent circle tangentcircle that goes through that goes through the point of interest (No audio from 16:24 to 16:36) on the interface. Now, the radius of this tangent circle radius of this tangent circleis referred asradius of curvature radius of curvaturethis is a vectorfirst of all this is a vector.

So, the curvature as such the curvature is a vector and this is pointing to the centre of the circle, so curvature is a vector is a vector pointing to the centre of the circle what that means is if you want to draw another another say say these portion here, the curvature is differentI can I can see the curvature is is little turned turned like this; so it may be that the circle goes in the other direction, it may not come inside.And so this would be the this would be that tangent circleand then that would be the radius of curvature all right.

Now, there is another issue here is that, when it comes to 2 Dsurface in a 3 D space, when we are talking about curvature when we are talking about curvature of 2 D surface in 3 D space, this can be expressed in many different ways2 d. I mean curvature of 2 D surface in 3 D space, basically what you what you write here is something called a mean curvature mean curvature, which is given by this term let us call it this K, which is equal to half of k 1 plus k 2, where these two quantities k 1 and k 2 are principle curvatures.

Now, the question would be what is the principle curvature? Think of curvatures of curve curvatures of curves resulting from intersection of the intersection of the surfaceintersection of the surface of interest, with all possible planestangent to the vector, that is normal to the surfaceat the point of interest (No audio from 20:24 to 20:36). The largest and the smallestcurvatures are principle curvatures. So what what I mean here is that, when it comes toa 2 D surface in a 3 D plane, so in in a in a 3 D space you have a 2 D surface.

So, you can have curvatures of curves resulting from intersection of the surface with all possible planes tangent to the vector,so suppose this is the point of interest, say let us think of this to be the point of interest, so you have something perpendicular to this point of interest. Now when it comes to a 2 D surface this I have drawn here is 1 D just a single line when it comes to a 2 D surface, in that case there could be several such planes tangent to the vector that is normal to the surface at the point of interest, that means there could be there could be you have drawn a circle, you could have drawn a circle like this, you could have drawn a circle like this (Refer Slide Time: 21:57) there could be several such circles possible and you canhave several such curvatures also possible.

So, the largest and the smallest are referred as principle curvature and you take the mean of that and call is mean curvature, now you so you need to understand there are two two terms, we have already we are we are trying with, one is the curvature and other is the radius of curvature and one is the reciprocal of the other, that you need to understand.

So, what we do here is, if we want to write what is P L, in this case we would be writing P L as sigma divided by 1 by r 1 plus 1 by r 2, there was 2 sigma by r right, we had what was there for the sphere, for the sphere we had it as 2 sigma divided by r.Instead of r wehave 2 into instead of r we, we had we had that time as 2 sigma divided by r, instead of that, we have now that r is replaced by or rather r by 2 is replaced by 1 by r 1 plus 1 by r 2, so basically your k 1 here is 1 by r 1.

(( )) sigma into 1 by r 1 plus (( )) sigma into 1 by r 1 is that so, I am sorry yes yes.

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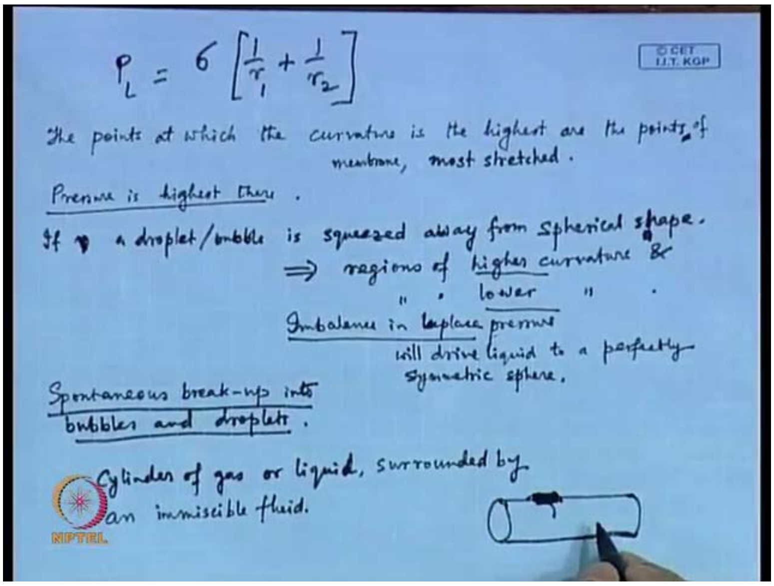

It is sigma by r exactly it is it is sigma by r, so it should be P L is equal to sigma into 1 by r 1 plus 1 by r 2, so that is it, so earlier you had 2 sigma divided by r, now it should be sigma into 1 by;so thesethis represents, what the so 1 by r 1 represents the curvature here all right.Now now (No audio from 24:11 to 24:28) the fact is that the points the points at which the curvature is the highestare the points most stretched points moststretched are the points of membrane most stretched.

So, the pressure would be highest there (No audio from 25:34 to 25:42), so ifso if you ifififif a droplet or bubble is squeezed away from spherical shape if it is squeezed away from spherical shape then then the result would be you will have higher regions of higher curvature and regions of lower curvature.That was not the case with that was that was not the case with the sphere,when when it was originally in it is in it is when the when itis originally in its spherical shape, when the droplet or the bubble is originally in it is spherical shape it is it is sphere and a curvature issame everywhere.Now, if somebody squeezes the droplet or bubble to a non-spherical shape, that means in some places there will be higher curvature and in some places there will be lower curvature.

So, automatically points at which the curvature is highest are the points of the membrane most stretched, so pressure is the highest there, so automatically regions of higher curvature and lower curvature,there you will have an imbalance there will you have an imbalance in Laplace pressure.There would be an imbalance means at higher higher curvature you have different pressure than the lower curvature,and this imbalance willdrive the liquid this imbalance in Laplace pressure, this imbalance will drive liquid to a perfectly symmetric sphere.

So, this is this is one point you need to remember that it is it is it is the spherical form itis the it is the perfectly symmetric spherical form that a droplet or a bubble will acquire, if you squeeze and then let it leave on it is own it will it will acquire that spherical shape.The other point here is what we call spontaneous break-up into bubbles and dropletsspontaneous break-up into bubbles and droplets think of a cylinder of gas or liquid think of a cylinder of gasor liquid surrounded by surrounded by an immiscible fluid.So, we are talking about a cylindera cylinder surrounded by another fluid, so it can be it can be a cylinder of air surrounded by water or it could be cylinderofoil surrounded by water.

Now, if somebody tries to find out, what would be the radius of curvature if he if he picks up some some element and if he picks up the radius of curvature of that of that element then, what he will find is there is a curvature in this direction I canI can understand.However there is no curvature in this direction, so you can you can consider that to be that that that radius to be infinity or curvature to be 0, that can be considered but, you haveoneone curvature is already there, where the radius is simply the radius of the cylinder.So, if you if you look at the curvature of the cylinder, that is how you would fit this.

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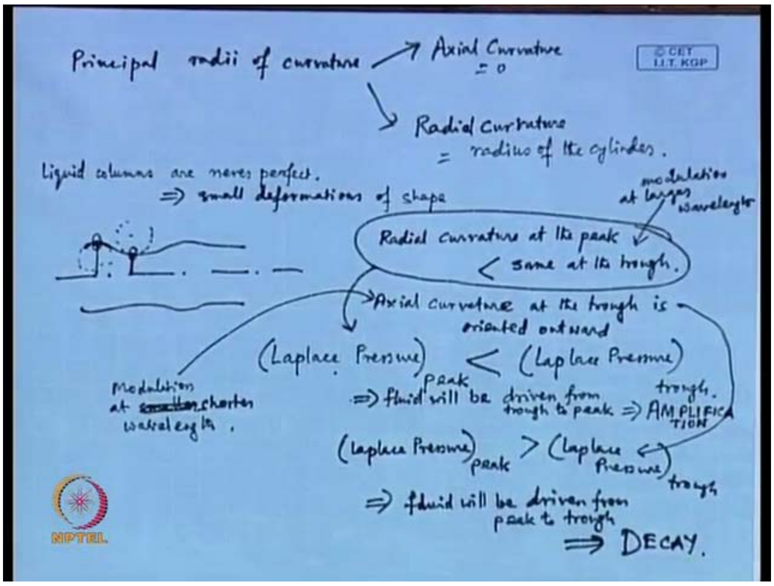

So, you have when we talk about the principle radii of curvaturesprinciple radii ofcurvature radii itself isplural, then you have one which is axial curvature and the other one is radial curvature.This axial curvature for the cylinder, this is equal to 0 and for radial curvature for radial curvature this is equal to the radius of the cylinder.Now there is one point here is that this liquid columns are never perfect and there will be small deformationsthere would be small deformations of shape, due to various reasons if not anything due to thermal fluctuations.

So,if there is if there is small deformations of shape, then it will take it will take it will it will appear something like this, instead of the cylinder that wall of the cylinder will appear something like this. So, in that case if somebody looks into the peak if if this is called a peak and if this is called a trough, so they will find that in this modulation, the radial curvature radial curvatureat the peak is smaller than the same at the trough.

Why is it possible, because here the radial curvature means, we are talking about this distance and here the radial curvature means we are talking about this distance I mean curvature I meanI mean the radius of curvature by radius of curvature at this point.We are talking about this distance and here radius of curvature, we are talking about this distance (Refer Slide Time: 33:41).So, when it comes to one by this quantity you will find that curvature is basically 1 divided by and so it it is just the reciprocal of it, so radial curvature at the peak would be less than the same at the trough. So (No audio from

33:55 to 34:07) and and if somebody tries to find outthere would be an axial curvature being generated, because of these modulation.

Earlier when you had a cylinder there was no axial curvature,it was it was all straight, so that is why you considered that axial curvature to be 0, the radius of actual curvature is infinity but, now since there is an undulation, so there would be an actual curvature present.So, if you try to find out, you will find that the axial curvature for that what you need to do is, you need to draw a circle and find the tangent here, you need to draw a circle, which is in the circle would be in other direction the center of the circle would be outside the cylinder.

So, what you will find is that the axial curvature (No audio from 34:56 to 35:06) axial curvature at the trough is oriented outward, so when it comes to the Laplace pressure ifyou if you if you compare Laplace pressureat the peak and if you compare Laplace pressure at the trough.You will find that the this particular aspect, this will cause Laplace pressure at the peak to be lessthan the Laplace pressure of trough, whereas this particular aspect we bring Laplace pressureat peak to be greater than pressure at at the trough.

So, the the influence of this particular aspect is like this and influence of the second aspect is like this, now this first one this will this will happen where for for a modulation or for forwe call it modulation modulation at larger wavelength,where as this particular aspect will happen formodulation at smaller wavelength a shorter or oror should I call it instead of smaller it should be shorter wavelength.In this case basically this they I mean why why I am writing this is probably this thisLaplace pressure will become negative, since the circle goes in the for the trough, there circle goes outside the cylinder, so probably this will become negative.

Now, if younow nownow the fact here is that when this happens Laplace pressure of peak is less than Laplace pressure of trough, what that means is fluid will be driven the this condition implies fluid will be driven fluid will be drivenfluid will bedriven from troughto peak, because trough is at higher pressure.So, fluid will be driven from trough to peak,so if the fluid is driven from trough to peak so that means you are encouraging the perturbation you are you are amplifying the perturbation, so this implies amplification of perturbation.

On the other hand the second case means that the fluid will be drivenfluid will be driven from peak to trough this implies Decay. So, depending on the wavelength of this modulation either the perturbation may amplify or the perturbation may decay (No audio from 39:16 to 39:43).

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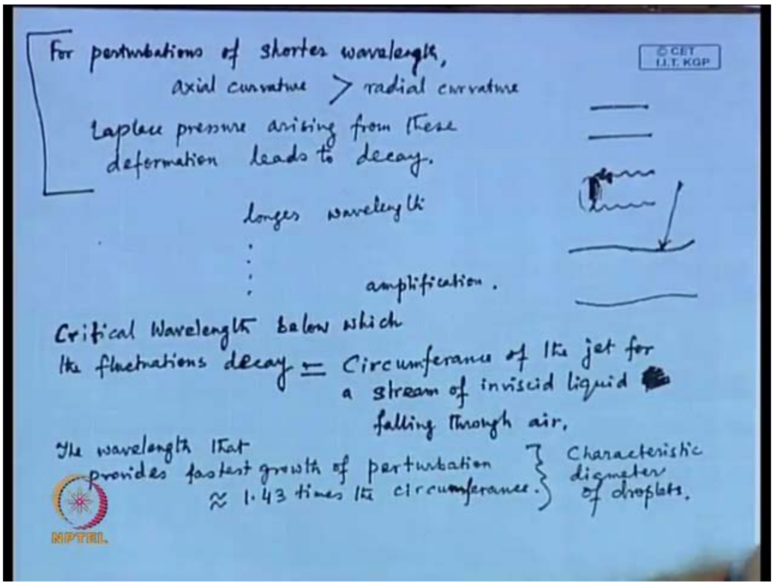

Now, for perturbations of shorter wavelength for perturbations of shorter wavelengthaxial curvature is greater than radial curvature, for perturbations of shorter wavelength; what does shorter wavelength means, if this was the cylinder for shorter wavelength meansand what is the larger wavelength meanslike this right.So, for perturbations of shorter wavelength the axial curvature is greater than the radial curvature, what does this mean radialaxial curvature is greater than the radial curvature.

You for for shorter wavelength, you havethe what is the axial curvature in this case,for example, this one the axial curvature is here but, for here this one the axial curvaturewould be the radius of curvature would be this this much here,the radius of curvature would be shorter right.So, this is this is what we are referring to the axial curvatures and the radial curvature is this quantity, this this this curvature we are referring to, so this is the radius of that cylinder at the at the peak and at the trough, whatever the variation is that we are calling radial curvature.

And this we are calling axial curvature, here this muc, here this much (Refer Slide Time: 42:01).So, so here in this case of this is the shorter wavelength and this is the this is the longer wavelength, so we are trying to conclude that for perturbation of shorter wavelength, the axial curvature curvature is basically the inverse or the reciprocal of the radius of curvature.

So, axial radius of curvature is basically less than the radial radialradial radius of curvature, in this case that means what I am saying is this is the case of a shorter wavelength.And in this case the axial radius of curvature, which is this quantity this much is much less than the radial radius of curvature, which is this quantity whereas in this case it is the other way.

So, for perturbations of shorter wavelength the axial curvature is greater than the radial curvature and so you are saying that the Laplace pressureLaplace pressurefinally, I mean this is the conclusion we are making, that the Laplace pressure arising from these deformations leads to Decay all right.And the the converses the converse would be for longer wavelength for longer wavelength we go through the same argument, I mean I i hope you understood, what I said for longer wavelength the deformation leads to amplification.

So, in this case there should be a critical wavelength, there should be an existence ofcritical wavelengthcritical wavelength below whichthe fluctuations Decay, soI meanyou are saying what you are trying to say here is that for shorter wavelength the that wouldif if there is perturbation that will that will Decay. So, that means it will restore it is cylindrical configuration and if the wavelength is large, how large, so there is there hasto be some critical there has to be some limit, if the wavelength is beyond that limit, then it will not Decay anymore rather the perturbation will amplify all right.

So, what is that critical wavelength below which the fluctuations Decay above which the fluctuations amplify here,this is this is I mean this this has been this has been shown this has been derivedby by something called Reyleighplateau, this is this is referred as Reyleigh-plateau instability and it it has been derived that, this critical wavelength below which the fluctuation Decay is equal to circumferenceof the jet fora stream of inviscid liquid falling throughfalling through air.

So,inviscid liquid if there is a stream of inviscid liquid and that is falling through air, then critical wavelength below which the fluctuations Decay is equal to the circumference of the jet for a stream of inviscid liquidequal to the circumference of the jet, what you can and an another anotheranother I mean this this has been derived this has been derived I mean somebody, I meanas I said in Reyleigh-plateau instability, there it has been derived using a sinusoidal modulation and thenthis has been shown that this is the critical wavelength.

Another another aspect here that I must mention is that the wavelengththis has been also derived the wavelength that provides fastest growth of perturbationisis almost equal to 1.43 times the circumference the circumference the wavelength that provides fastest growth of perturbation is close to 1.43 times the circumferenceof that of that cylinder.So,what is this these two more importantly the second criteria what this defines is characteristicdiameter of droplets, what this means is think of there there is a there is a problem of dripping faucet.

Which is which you are you are many of you are already aware of that when when a water is dripping from the tap if it is if there is a thin thread that comes out from the beyond a distance, it break downs and forms a spherical shape it it takes a spherical shape if the cylinder breaks. Beyond what distance it breaks and what wouldbe the characteristic diameter of that water droplet that forms, that can be I i mean the theory behind that instability, theory behind that breaking of that droplet that is covered by these Reyleigh-plateau instabilitycriteria, rather Reyleigh-plateau instability theory.

So these this has been these these are not just empirical facts these these are all derivedthis derivation is already available.

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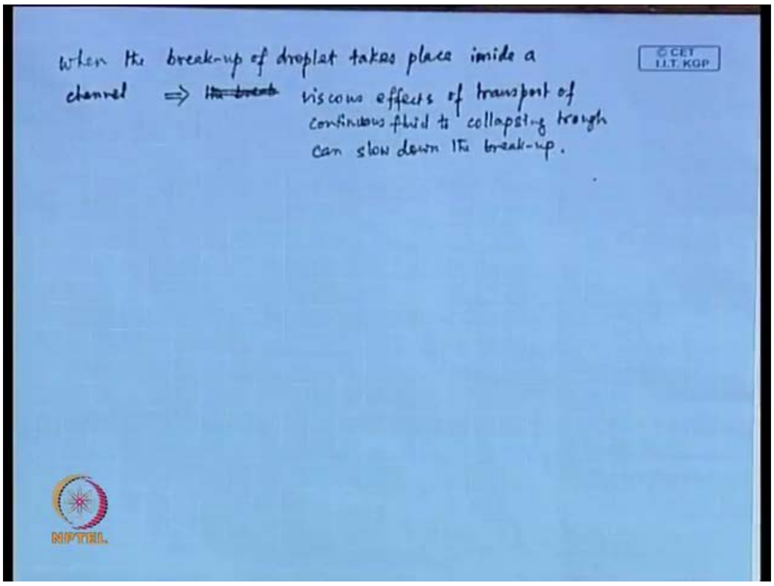

Now, when the breakup takes place inside a channel,when the breakup when the breakup of droplettakes placeinside a channel, the breakup process can be differentor or when it happens, what you can write is that, viscous effect of transport of continuous fluid to collapsingcollapsing troughcan slowdown the breakup.Do you do you understand, what I am trying to say here, that in a dripping faucet, you have an infinite medium and throughwhich the water is simply dripping.

Now, if it does not happen that way instead if you havechannel in which there is some continuous liquid is there and within that you are trying to create droplets.In that case if you are relying on thisthisReyleigh-plateau instability to guide formation of droplets, you must appreciate the fact that these these breakup of droplet is taking place, because there is fluid being transported from the from the peak to the trough or from the to stabilize or from trough to the peak to amplify.

So that, transport here you have considered and what what did we say that word, that we used here is inviscid liquid, we called it called that inviscid liquid, so it is it is it is a potential flow problem but, when it is inside the channel, there will be viscous effect for that movement of liquid from trough to the peak. And that is basically instrumental in in in breakup of droplets, so these also needs to be considered, so it would it it is somewhat different I mean the breakup process,it will be it will slowdown I mean you cannot you cannot make your decision based on that these criteria that I mentioned that wavelength should be 1.43 times circumference, this cannot be generally applied to other situationsthat is all I have to say as far as today'slecture is concerned.

In the in the next class, what I will do is first I will define few dimensionless numbers, which are important for in immiscible flow throughmicrochannel, those are for example, capillary number, bond number, waver number.So, I i will first briefly introduce those dimensionless numbers and then I will go straight away to the motion of a droplet or bubble througha microchannel, because that will there you will find that the pressure drop.

If if you had them flowing separately,if you had if you had been if you are flowing just simply the gas or simply the liquid and you calculate the pressure drop and if we assume that since I am introducing bubbles, which are basically the mixture of two; so the pressure drop would be the half way it will not be the case rather there are lot of complexdynamics playing they coming into play.And we we will get into the movement of the bubbles and how the pressure drop is, how you derive the pressure drop in such situation.

So, we will first introduce the dimensionless numbers and then we will get into theget into finding how a bubble will move, that mean you are expecting the bubble to be there and surrounded by an annular liquidor a droplet to be there surrounded by the other liquid, which is a continuous phase.And now you are havinga motion, so what would be what would be the thickness of the annular film, let us say orwhat would be the pressure drop under such situation, so that we will get into in the next class.

Thank youthat is all, I have for today.