Microscale Transport Processes

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Lecture No # 25

Dielectrophoresis (Contd.)

I welcome you to this lecture of Micro Scale Transport Process.

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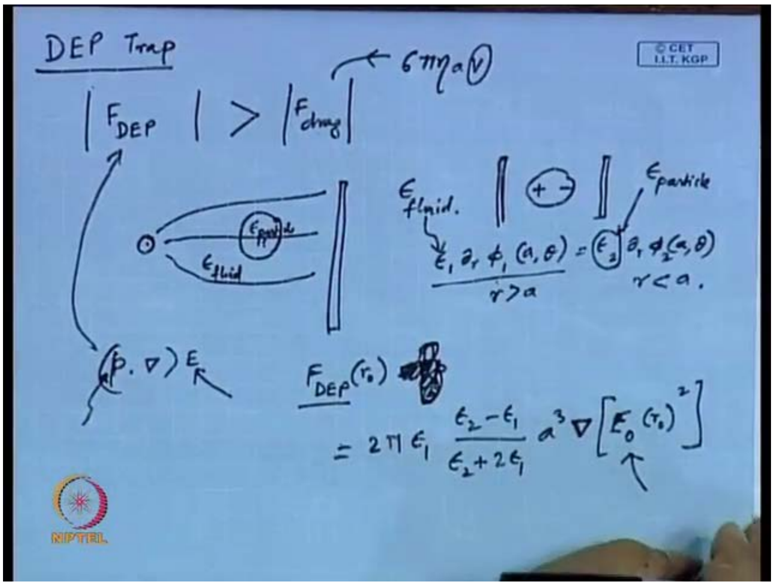

The topic that we have been covering in the last class was Dielectrophoresis, more precisely something called DEP trap where we were trying to trying to relate this DEP force, FDEP, we will we are we are trying to relate this FDEP and F drag. We we are we are trying to see how this FDEP this FDEP, the mode of FDEP and the mode of F drag, these two you are supposed to compare and we are supposed to find out, what should be the velocity of the fluid such that this particles can get trapped right.

And as as as a matter of fact, why are they why are they getting trapped, I mean let me let me repeat once again that you have a non-uniform electric field. That means, is a planner electrode and then electrode which is a point electrode, we are we are simulating it, by we are calling it is sphere and we are saying that this is this is the dielectric particle, it could be a biological cell and then we are saying that these electric field lines, the since this is a non-uniform electric field there are field lines traversing like this and there is an intensity, there is a there is a concentration of these field lines here.

So, there there there is a concentration of these electric field, this place here you have the highest density of electric field lines. So, you are expecting that here the electric, then here the if the electric field is stronger in this region, the electric field is stronger in this region, now then we said that these particle has an epsa, epsa particle, the dielectric constant and the fluid had its own epsa, epsa fluid.

Now, if epsa particle is more than epsa fluid, then the particle would be more polarized; that means, the charges on the surface, there would be more such more such charges on this particle on on the surface here and and I pointed out that, if you have a planner electrode I mean just the way of capacitor works; if you have planner electrode and if you had such cell there, what you expect is that, the charges will align itself such that or or the particle itself will align itself such that, they these these charges, they are consistent with the with the electric field.

So, these these alignment is always there, now to to achieve that kind of alignment, these particle has to move either I mean epsa particle is more than epsa fluid, then this particle has to move in these direction, because since field is here the field is uniform. So, this alignment is possible, but here it continues to moves towards this direction such that it satisfies that alignment criteria.

So, that is way these particle will continue to move towards left if epsa particle is more than epsa fluid and it is if it is other way, then the fluid would be moving towards this direction or in other wards of particle will be push towards the right side (Refer Slide Time: 3:42). So, this is this is something which we are trying to (()), this is something which we, which which is being utilized to trap a particle.

So, this is this is what we mentioned here as DEP trap and then we are trying to find out what is the FDEP and what is the F drag, what are the what are the, we are trying to find out expression, now F drag we have already pointed out, that F drag is very simple F drag follows this stokes law, and F drag is simply written as you you you have this 6, F drag this is written as 6 pi eta a into v and this v is given for a micro channel which is of rectangular channel, not not a circular capillary, we have already given an expression for v in the last class. So, F drag part we understand and we said that this FDEP, this FDEP has to be FDEP has to be greater than F drag, then only the then only this is, this DEP trap is going to trap the particle.

Now, for the FDEP part, now we have pointed out this FDEP as something called p dot delta of e, this this we already understood and in the last class, we are trying to find out some expression for p some expression for p and some expression for e right in the near the near the end of last class, we were trying to trying to come up with these these expression and what we ended up with, you remember in the last class if you go if you go to the notes, what we ended up with is that FDEP FDEP at r 0 that is equal to FDEP at r 0 that is equal to let me give the final expression, you had is 2 pi epsa 1 epsa 2 minus epsa 1 divided by epsa 2 plus 2 epsa 1 a cube delta of delta of e not r not whole square.

Now, you remember what are these epsa 2, epsa 1, basically these these are 1 epsa is for the particle, the other epsa is for the fluid right in the in the if you go back to notes of the last class, we had we had equated, I remember we had equated the continuity in epsa 1 del r phi 1 a theta, you remember with epsa 2 del r phi 2 a theta. That means, we are saying that at the at the surface of the particle, whatever is within the particle, whatever you are getting from the within the particle has to be equal to the equal to the equal to that outside the particle. So, that is there is continuity, continuity with the potential and continuity with the derivative of the potential.

Now, so, what we did in the last class, what was our definition for what is the one here, one is the case when r is greater than a, you remember r is this is for r is greater than a and this r less than a. So, that means, this epsa 2 is basically the epsa particle. So, this epsa 2 is epsa particle as per our old convention and this epsa 1 is epsa fluid as per our old convention right. So, this is this is what this is what we have we have done in the last class and this is something, here we had an identity to work with at near the end of the last class, we discussed about the identity and you go back and check this identity yourself. So, this is the final expression that you end up with.

Now, I said, so this is this is now FDEP now out of this here, what is left out here now is you have to give an expression for this and there is a, there is the sign. So, you have to take the derivative also, you have you have to come up with an expression for this and take the derivative of it and then you can put all of them together and come up with an expression for FDEP r 0 which can equate which you can equate with F drag. Now, I said, how you get this electric field, how will how will you find this out? Since, it is a non-uniform electric field, in the last class I mentioned that this non-uniform electric field can be simulated by considering a mirror electrode, you you remember let me see if I can get that yeah near the end of the last class.

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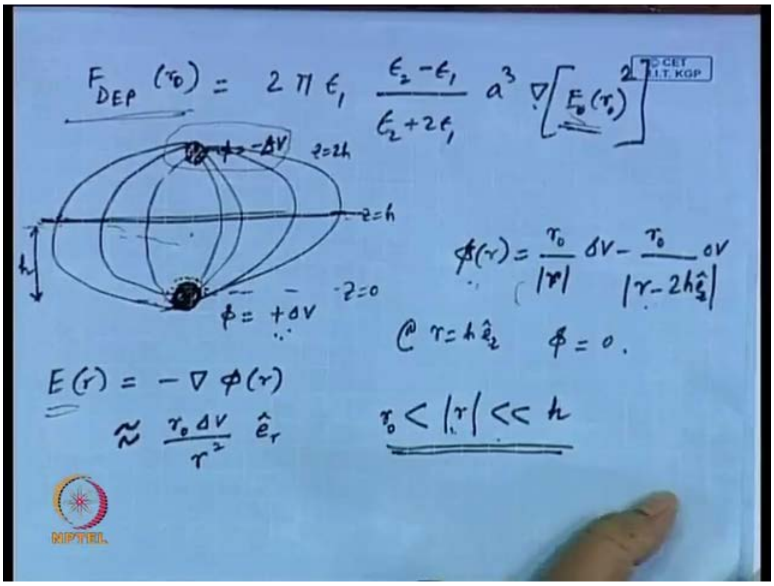

This is something which we have been talking about is that, here you have a point electrode and here you have a planner electrode, now instead of having point electrode and planner electrode, if you take away the planner electrode and instead think of a mirror electrode at minus delta v, here it was plus delta v and 0, but instead of that if you put plus delta v and here you have put minus delta v, as a mirror electrode and then if you look at the electric field lines, you will find they are very similar to the case where you have a planner electrode and a point electrode.

So, by this understanding, you can have the definition of phi is given as this quantity r 0 by r delta v minus r 0 divided by r minus 2 h e z delta v, you can check it yourself that at r equal to h e z; that means, at r is equal to; that means, at this position at z equal to h, at z equal to h you can have phi as 0, you can you can put it here and you can see that this is this is the case. So, this is an expression for the for non-uniform electric filed that we are talking about that is constituted by a planner electrode and a point electrode. So, with this phi, now what you need to do is you need to you need to find out what is e what is e because you are you are interested in what in the last last slide what we had FDEP r 0, we have e here and e you know is electric field is minus del phi. So, you if you have this as phi, if you have this as phi, now you take this and find out what are e, now so, e at r would be equal to minus of del phi of r this is this is by definition.

Now, here you make an approximation, I give you the final result and then you then you think about it how we get in get in there (No audio from 10:59 to 11:08) and the approximation here is something like this (No audio from 11:16 to 11:23), what we do here, here what we are doing here is we are saying that the location that we are interested in to find the electric the find the e, the location that we are interested in that r, r is at position where we want to find out the e, or r is the position where we want to find out the FDEP.

We are saying that this position is much less than h, what does that mean, this position is so; that means, this is much closer to the point at row what is r 0 here, r 0 is we have we have a sphere, this sphere has a radius r 0 that is what we are talking about, this sphere has a radius r 0 and these dimension is h (Refer Slide Time: 12:00). So, the position that we are interested in from the center of this point electrode, the position that we are interested in is here, because if the particle has to be trapped, this particle will be trapped here around this location. So, this position of course is greater than r 0; that means, it has to be outside the electrode that is what certain, but this is much less than h, I mean it is not here that you are trapping the particle, you are trapping the particle here all right.

So, this is an approximation you are making here and if you make this approximation look at this one, here if r is much less than h, then r would be much less than 2 h, then this term this term here whether you have r, whether you have do not have r does not make much of a sense (refer Slide Time: 12:40). So, it would be simply r 0 by 2 h, r 0 divided by minus 2 h or this minus will cancel out, it would be plus r 0 by 2 h, so this quantity becomes constant. So, when you take up derivative with respect to position, then this term will not contribute, this term will become 0.

So, these these assumption will help taking out this term in the derivative, this term remains in the phi, but when you take a derivative of it, when you take a del of this by this approximation, you are not considering this term because you are treating this as a constant (Refer Slide Time: 13:26). So, you are working with only the first term and we if we work with the first term, probably this is what you will end up with, minus sign cancels out alright. So, if that is so, so now, you would be working with this e, so this e has to go go back there and this e has to be put in there (Refer Slide Time: 13:52). So, what do you get in that case, what was the what was the original expression for FDEP, this is the expression for FDEP right and here this e not would be substituted.

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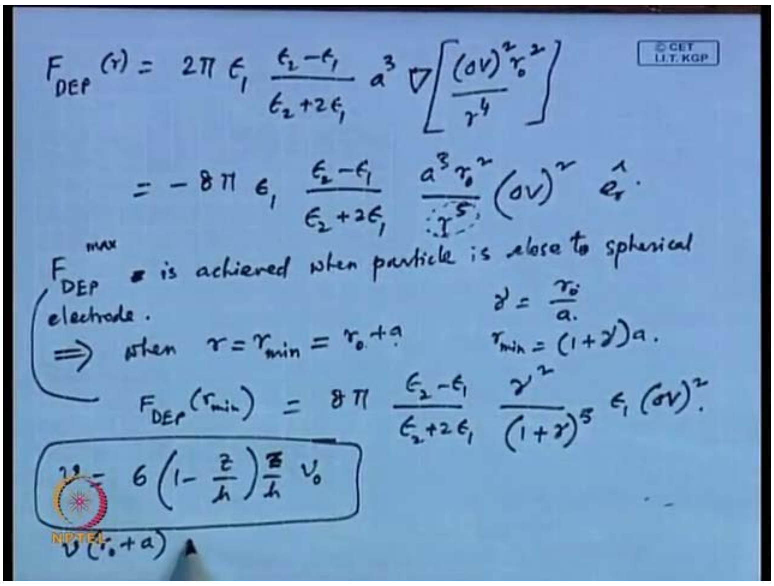

So, what you get finally is FDEP at r that is equal to 2 pi epsa 1 epsa 2 minus epsa 1 by epsa 2 plus 2 epsa 1 a cube del of this e not square that it was square right. So, this has to be delta v square r naught square divided by r to the power 4. So, it would be del of this quantity del of this quantity here, these are constant it will come out r to the power 4, r to the power 4 means r 2 the power minus 4. So, when you take the derivative with respect to r, what do you get, you get r to the power minus 4. So, it would be r to the power minus 4 minus 1 right and 4 will come out, outside minus 4 will come out. So, minus 4 divided by r to the power 5 that is what you will end up with so that minus 4 multiplied by 2, it would be minus 8.

So, I am writing it as 8 pi epsa 1 epsa 2 minus epsa 1 by epsa 2 plus 2 epsa 1 a cube r not square has come out, r to the power 5 and then delta v square and e r hat. So, this is this is this is the expression for FDEP this is the expression for FDEP, now you need to find out what is FDEP max, FDEP max this this is you you write FDEP max is achieved when particle is close to spherical electrode. That means, this means this implies r, when r is equal to r min and what is r min? r min would be equal to r 0 plus a you cannot reduce r below this, basically r 0 you have to satisfy, r 0 is the radius of the electrode and a is the radius of a particle.

So, you cannot go any foreclose, I mean you have to if the particle is attached to the electrode, then also you you have to satisfy this requirement. So, this is this is the case, so this is the r min and at r min, FDEP you can expect FDEP to be FDEP max. So, this FDEP max is equal to FDEP at r min and that r min, if you put, then it would be equal to, there is a there is a way to handle this further, you can you can take a ratio of this, you can take a ratio of this two, say let us let us call this say gamma is equal to r 0 divided by a.

So, this is basically the radius of the electrode divided by radius of a particle, let us let us consider this as a characteristic for the system, characteristic constant for this system. So, in that case r min would be equal to 1 plus gamma into a. So, now if you put this instead of this r, this r to the power 5; instead of r to the power 5, you write r min to the power five. So, if you want to put r min to power 5, what you get here is if you if you simplify this further it would be 8 pi epsa 2 minus epsa 1 by epsa 2 plus 2 epsa 1 gamma to the power square by 1 plus gamma to the power 5 epsa 1 into delta v square.

So, this is this is what is the final, this is the FDEP max possible, now if somebody wants to find out, what should be the value of the what should be the value of this FDEP drag at that point at that at that at that location, in that case what you would do is we consider this to be infinite parallel plate right, there we have considered this velocity to be equal to 6 into 1 minus z by h into z by h into v 0, this is the velocity this is the this is that this is that expression for the velocity, parabolic what you call it, parabolic velocity profile in a rectangular channel. If you would have been a circular channel you have that 2 v not 1 minus r by capital r whole square. So, here you have 6 into 1 minus z by h into z by h v naught, so this is this is this is the expression for velocity.

Now, if you want to find out the velocity at this location, I would say velocity at r 0 plus a, what would that be velocity at r 0 plus a, with this expression if you try to find out at r min what is the velocity, at r min the velocity would be then if you if you simplify this further, you should be getting this as 6 into 1 minus 1 plus gamma into a by h into 1 plus gamma, a by h into v not, this is the expression that you get at r 0 plus a you can you can you can (( )) with it and see how you get this expression. So, this is the expression you get for the velocity at that location, this is the velocity at that location, at that location means at r min and what is r min, r min is basically r 0 plus a r 0 is the radius of the electrode and a is the radius of particle. So, that is the minimum r you can get, I mean you cannot go any r below that that is not that is impossible and you expect that at r min the FDEP would be max maximum.

So, now, this expression this is this is this is approximated, this expression is approximated as 6 into. So, this this is this is this is approximated as 6 into 1 plus gamma a by h into v 0 this is approximated as 6 into 1 plus gamma a by h v 0, so this is this is the expression for v.

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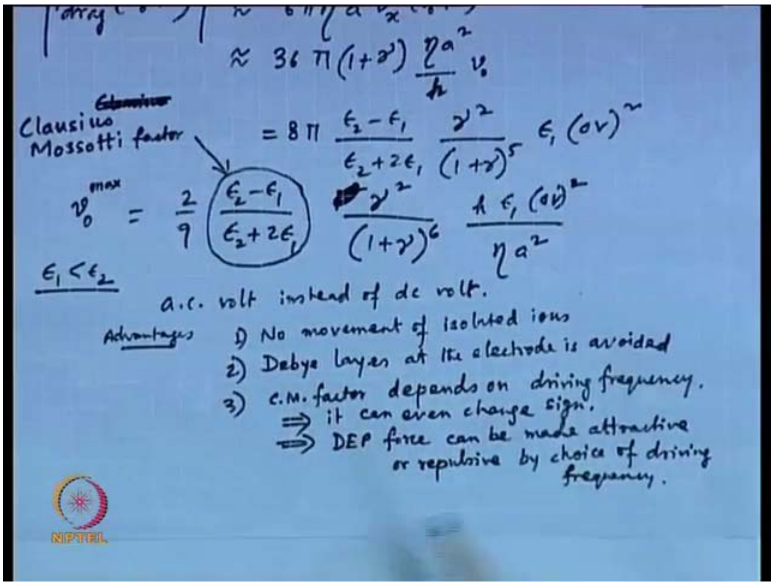

Now, if we go back to that, so what is what is now F drag with all these understanding what is F drag? F drag at r 0 plus a that is at r min location that if we if we look at the mode of it, we are interested in the magnitude, this would be close to 6 pi eta a v x at r 0 plus a. And if you if you if you bring in those expression for v that we discussed just now, you would be end up with 36 pi 1 plus gamma eta a square by h into v not.

So, this is the expression for F drag that you end up with, and this is what you are equating with FDEP max which is at r min and what is that r min? This is the expression FDEP r min right 8 pi epsa 2 minus epsa 1 (Refer Slide Time: 22:56). So, you are equating this with what is expression, this is equated with this is equated with 8 pi epsa 2 minus epsa 1 divided by epsa 2 plus 2 epsa 1 gamma square by 1 plus gamma to the power 5 epsa 1 delta v square. So, this is coming from this, this is coming from that FDEP max, this side and this is coming the upper one is coming from F drag. Now, you equate these two and you will get a fair idea of what should be v not, so this is v not, say the expression for v not would be something like this (No audio from 23:50 to 24:15).

This is the expression for v not (Refer Slide Time: 23:50), we can call this v not max, why v not max, because if v not exceeds these values, then you know for sure that this particle will not be trapped, it will go out. So, this is this is the maximum v not that you can you can permit for this system and to obtain trapping, of course epsa 1 has to be less than epsa 2, because you remember what what were the epsa 1 epsa 2 we define just now, we said that epsa 2 is epsa particle, epsa 1 is epsa fluid that is our convention and particle has to be greater than the fluid, particle epsa has to be greater than the particle, then only there could be trapping.

Now, so this is this is an expression you get, this this is the this is the velocity that is that is a maximum velocity that the fluid can have if somebody wants to trap a particle of of these these properties. That means, fluid has a property here, fluid has dielectric property given by epsa 1, particle has a dielectric property is given by epsa 2, gamma is the characteristic of system r 0 by, what was the definition of gamma we used? We we we have used gamma as r 0 by a; that means, radius of that spherical electrode divided by the radius of a particle.

So, that is what we have gamma, h is the micro channel that (()), the distance between the two electrodes epsa 1, you know epsa 1 is the dielectric property of the fluid, delta v square, delta v is the applied voltage and eta is the viscosity of the fluid, a is the radius of the particle. So, this is an expression you have, so the given these properties, you should have the v naught, this this is the maximum v naught you can have. So, if you expect that particle to be trapped by the, by this by this mechanism.

Now, I said at the very outset that this we this this entire analysis is based on DC field, now ideally it is it is used an AC as an AC field and in fact, if you if you look at some literature on this dielectrophoresis, this start with this AC field itself. Now, if you have an AC field, if you have an AC volt instead of DC volt (No audio from 26:53 to 26:59) DC volt, then the advantages are advantages are no movement of isolated ions (No audio from 27:10 to 27:10). Debye layer at the electrode is avoided, now this this this factor this factor is referred there is a name to it, this you you might have seen already, this factor we are carrying all the time I mean right from the beginning and this factor has a name (Refer Slide time: 27:58), the name is this Claudius c l a, let me right it clearly, Claudius M o s s o t t i factor, now this factor, this let me call it CM factor for for for for the brevity, I hope that is not that that that is that can be allowed here is this can depend on depends on in when when you have an AC volt, this depends on driving frequency.

Because if you have an AC AC voltage, if you have an alternating voltage, then it has a frequency attached to it and this CM factor then depends on the driving frequency. So, what what you will find is that, if you this entire analysis will be repeated with these various places, you will have this driving frequency inbuilt, I mean this entire analysis can be repeated for an AC electric field. So, you will see lot of parameters, the basic the framework remains same, but in many places you will find that that driving frequency goes in and most importantly the CM factor depends on driving frequency.

So, this offshoot is that this factor it can it can even change sign it can even change sign. So, depending on what driving frequency you have for the AC field, this CM factor can change sign and you can see the repercussion of this changing sign, if this sign gets changed; that means, a DEP force can be made attractive DEP force can be made attractive or repulsive by choice of driving frequency, see you you get this you get this flexibility.

So, for these reasons this this AC AC field would be more sort after than the DC field, but the analysis that we have done, the basic framework will remain same, only you will find that various constants we used, there this functionality with the driving frequency will go in there, you can you can check it out yourself, I am not covering this in the this class, because as such this derivation itself is quite complicated, but the framework remains same. So, that that is more important and I suggest if you are working in this field, you should looking to that aspect, because that is that is that is probably more important and that is in some literature I mean they start with that with an AC field, how this derivation would be, so that is how they have done.

So, I guess this is all I have as far as the dielectrophoresis is concerned. So, what we are doing basically is we are picking up one by one, various physicochemical processes that are that are that that can have tremendous importance in micro scale. So, what we have done is say, we picked up electric double layer, then we talked about electro osmosis, electro osmotic flow, electro osmotic pump, then we have a covered the electrophoresis this that is one way of differentiating mixture of particles. And then you have this Dielectrophoresis, this dielectrophoresis is well known for separating dead cells from live cells, probably these properties that we are talking about, this dielectric properties probably these dielectric properties they change in a major way if a cell is cell undergoes this kind of transformation.

So, these are, so dielectrophoresis is I mean if you if you look at popular literature, this dielectrophoresis initially was meant for separating the (( )). For example, you are you are trying to find out pathogen in a water sample, and this water sample may contain several live cells and dead cells, and live cells are going to contribute to the or not contribute, live cells would be the would be the problem area. So, you want to know how many live cells are there and not just how many cells are there. So, that is that is how in an in popular terms dielectrophoresis was projected that it is a remedy for separating those.

So, these are these are these are some of the mechanisms that are important and you these these can these can, I think you should know, one who is studying micro scale micro scale transport, these are the mechanisms which are unique and of course, these these Dielectrophoresis, electroosmotic flow, these have been studied, these these are these are in electrohydrodynamics in those areas, these are very established topics, but when it comes to your micro channel, then people found that these would be, this will have tremendous effect, this will have tremendous importance in micro scale transport, then these these topics are kind of given renewed thoughts and probably what we are doing is something similar.

With that, I mean I would be I would be done with this electro osmotic flow and electric double layer and this dielectrophoresis, the next topic that I pick up now is more important for flow of gas through a micro channel, not of not a liquid, what we would be discussing?

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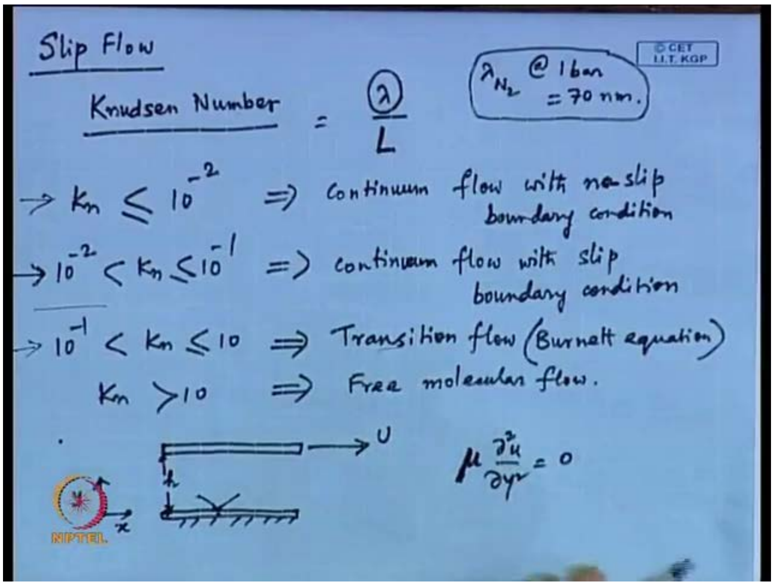


The next topic that I pick up is something called slip flow, that would be the next topic that I would be covering in micro scale transport. So, far we have been mostly focusing on the liquid and ions and ions getting pulled, but this is a this is a different this is this is something called the slip flow and this this this slip flow, the importance of slip flow will be, there is one factor which we I think we I briefly mentioned this to you that is that is given by this name Knudsen number right.

This Knudsen number and what is this Knudsen number? Knudsen number is basically lambda by L where lambda is the mean free path of the molecules and L is the characteristic length, may be the channel that the gas is, or the liquid or the gas is flowing that dimension of that the aperture of that channel that is that is what the characteristic length is, so this is the Knudsen number lambda by L.

Now, what is the lambda, what what is commonly what lambda you have for a liquid and what lambda you have for a gas? And we have we have done some scaling analysis to show that for a micro channel, when the L is of the order of 1 micrometer, then the lambda that you have for a liquid that is the, then this Knudsen number is not significant, whereas the lambda that you have for a gas, then this Knudsen number becomes important. And this Knudsen number defines, whether you have a whether whether you have a slip flow or a, or whether you have a slip flow or you you do not consider slip flow; that means, you do not consider slip flow means, you consider the no slip condition right.

So, in Navier-Stokes equation when you when you solve this, when boundary condition, we commonly employ is that at the wall there is no slip right. So, here you can consider this to be no slip depending on what is the value of Knudsen number, let me point out some numbers here, the numbers are like this that if Knudsen number is less or equal to 10 to the power of minus 2, this implies continuum flow with no slip with no slip boundary condition. If Knudsen number is between 10 to the power minus 2 and 10 to the power of minus 1, this implies continuum flow with slip boundary condition.

If the Knudsen number is between 10 to the power of minus 1 and 10, this implies transition flow and commonly, there is something called a Burnett equation that needs to studied, that needs to be that is that is the regime that you have. And if Knudsen number is greater than 10, then you have free molecular flow, it is not a continuum mechanics at all, it would be free molecular flow means what you have in molecular dynamics, collision of molecules and that is that is the regime you are in, so continuum flow Navier-stokes all these will be gone. So, if the Knudsen number is less or equal to 10 to the power of minus 2, then only you will have this continuum flow with no slip boundary condition.

Now, what we what we what we showed in the in the, what we showed in the last class is that, what we showed previously is that lambda of nitrogen at 1 bar pressure is about 70 nanometer lambda of nitrogen at one bar pressure is about 70 nanometer. So, you can calculate if have a 1 micrometer of the channel, then what would be the Knudsen number? And typically this 10 to the, I mean in this regime 10 to the power of minus 1 and above, this regime is I mean we, what we will be doing in this class is this part you understand, continuum flow with no slip boundary condition, you have studied time and again fluid mechanics transport phenomena everywhere.

So, this I am not going talk about, this part I am talking about continuum flow with slip boundary condition (refer Slide time: 39:20); that means, here what you are doing is this is this is the ultimate, free molecular flow, this is how you should be simulating at. But free molecular flow, I mean you understand, I mean how much computation will (( )) would be gone just to simulate event for so many seconds, for for just tracking the each and every molecule, and it requires huge amount of computational efforts, so you cannot afford that. So, here you are somewhere somewhere in between, you are not discarding continuum flow, but bringing in some aspects of free molecular flow so that you can make a makeshift arrangement.

So, this is then makeshift arrangement (Refer Slide time: 40:00), this is probably even more towards the free molecular flow, this transition flow equation, this we are not, these are these are very important important method and you you can you can you can go through this in your, I mean you can take this is an assignment, but what in this class we will cover is only this continuum flow with slip boundary condition and free molecular flow is something which is, which comes under the domain of molecular dynamics that that is also equally important, but I will not be taking it up, we are we are kind of we are we are restricting our self to this this particular this particular one all right.

Now, you you have to, you got to understand this I mean this this continuum description, continuum flow means you are using Navier-Stokes and heat and mass transfer equations now continuum field quantities that would be that would be considered in a continuum, continuum flow. So, so we would be we would be directly picking up this continuum flow with slip boundary condition and what we would be discussing probably we it will we will (( )) into the next, this will (( )) into the next class is something called a micro couette flow, micro couette flow means you this this you have already studied, when when we when we first time when we studied this viscosity, definition of viscosity.

The first thing what we studied is those two plates right (Refer Slide Time: 41:30), one plate is fixed and the other plate is moving at a constant velocity u, and this plate is fixed right and you have this separation between those these two plates is h, your system of coordinates is this is x, this is y and perpendicular to the perpendicular to the paper, it is z and this this is fixed this is fixed and this is moving, this this you have you have studied for the definition of viscosity right what we would do is, so what do you do generally?

I mean if you, if you let me let me tell you, because we we may not we may not be able to complete this analysis here, what we would do in this case is we will solve the Navier-Stokes equation which is which is very simple in this case, because you you take so many assumptions, you you take so many, when you when you define viscosity, you remember that basic definition of viscosity, there you end up with this the velocity would be linearly changing with distance. To come to that, you make certain assumption and start with the Navier-Stokes equation and all the components have gone and you you will you will only end up with one term and that is, so you may, you you get the velocity to be linearly changing with y and no variation in velocity in x or z direction.

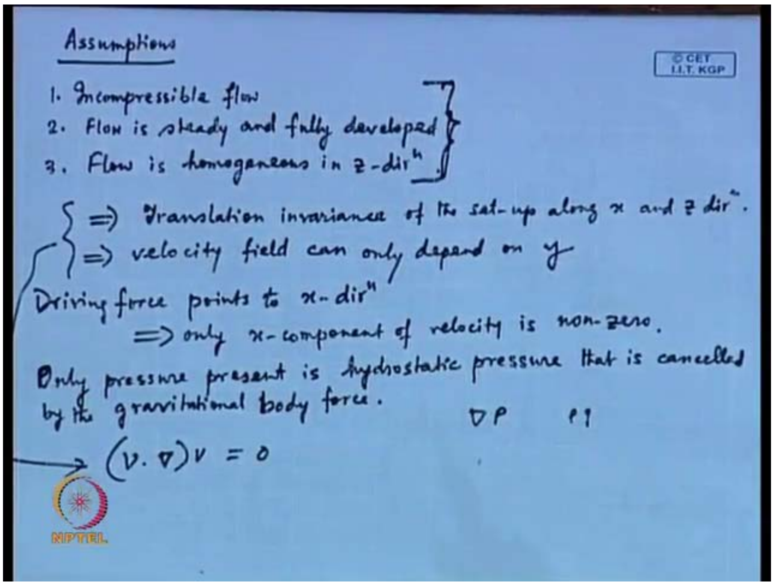
So, that is what you end up with, you you are familiar with that part, so that that we will follow the Navier-Stokes equation, only thing is we will not use the boundary condition, the boundary, what was the boundary condition there? Boundary condition was that u is equal to 0 at the wall, and u is the velocity is equal to 0 at the wall and velocity is equal to u at this wall that was the boundary condition. Instead of that, we will use a different boundary condition and that boundary condition will arise from the, that boundary condition will involve the mean free path that boundary condition, because because you will, you have to think about what happens at the boundary, you are assuming that these molecules they are flowing and the molecule is coming to a standstill at the wall.

But in case of a slip flow, what you will assume is that the molecule is coming, hitting the wall and bouncing off and whatever momentum it carries whatever momentum it carries it will it will this momentum there would be an interaction with the wall. If this wall is fixed, you have one situation; if this wall is moving, you have another situation, but this molecule will come and bounce back to back there and whatever happens due to this that would be taken into consideration.

So, let me let me first let me first write the Navier-Stokes equation I mean the basic equation that we that we use, but before that we need to we need to write those assumptions first, what all assumptions we make to get into that equation, may be may be the right one would be to give the final expression that would be that would be better. I write the final expression and then say what all assumptions we have there, we write typically as these mu into del square u del y square that is equal to 0 this is this is what you have, you have u in x direction. So, basically you have, I am coming back to this concept from this mu here, ideally we we have we have used eta in other in in electro hydrodynamics or electrodynamics, there we have used eta, because there we are using something called chemical potential mu was involved.

So, that is why for viscosity we are writing, eta let me go back to this old system. So, this is this is this is how it is mu into del square u del y square equal to 0, this is the equation that you basically solve if you if you go back, if you remember what we do. We start with the Navier-Stokes equation, but all the terms are gone all the terms are gone, because of certain assumptions and what are those assumptions? Let us list those assumptions now.

(Refer Slide Time: 46:09)



The assumptions here are incompressible flow (No audio from 46:19 to 46:30), flow is steady and fully developed (no audio from 46:36 to 46:50), number 3 is (No audio from 46:52 to 47:06) flow is homogeneous in z direction. So, the offshoot of these is, these or or I should say the entire the, if you use this assumption, this implies there is translation invariance, there is a very translation invariance of the setup along x and z direction, this implies velocity field can only depend on y, what is y? This is the y (refer Slide Time: 48:08).

So, velocity field can only depend on y, and in x and z there is a translation invariance, also the driving force the driving force points to x direction, this implies only x component of velocity is non zero that is exactly we have here, u is the x component of velocity x component, x is in this direction, x component of velocity. So, that is what you have, the other thing you are you are writing here is that the pressure, there is no presence of pressure, pressure gradient, only pressure present is hydrostatic pressure that is cancelled by the by the that is cancelled by the gravitational body force. So, in Navier-Stokes equation, you have that delta the del p term and you have the body force term right. So, they are gone, because where is as such I mean del p the only only pressure that is present is the hydrostatic pressure and so, this is this is what you have.

So, del del so you you remember in the Navier-Stokes equation, what all you have, Navier-Stokes equation you have, Navier-Stokes equation what all you have? You have one v dot delta v term that is equal to 0, because of this reason, this is only pressure present. So, delta p and that body force term rho g that is, they are cancelling out in Navier-Stokes equation. So, you are ending up with and the other terms are all the velocity field only depends on y and only x component of velocity is non zero. So, what what will you have, you will end up with this expression all righ, I mean I have this is clear, I mean I mean you can take it if you check this this has been done time and again, I mean I am just revising, so I think we should not be spending much time on this.

So, in that case if this is so, what would be the solution to this equation, u is equal to c 1 y plus c 2 right u is equal to c 1 y plus c 2 and what you do, when you when you when you have a no slip boundary condition, you say that at y is equal to 0, u is equal to 0, since it is fixed wall and at y is equal to h, u is equal to capital U, right that is how you do it. And so, what you get u at y is equal to 0 u is equal to 0, so automatically c 2 cancels out. So, c 2 becomes equal to 0 and at y is equal to h, u become U, so c 1 is basically nothing but, capital U divided by h. So, what you get is u is equal to capital U into y by h right and that is why we said the velocity profile is linear, velocity is changing linearly, u is equal to or you can write U by u is equal to y by h, dimensionless velocity is equal to dimensionless distance.

So, velocity profile is linear and that is what we have done for defining viscosity, remember when you if you go to any standard fluid mechanics book and look at the definition of viscosity, this is where they start with, we will continue this exercise, we will use this expression, but we cannot write c 2 is equal to 0 when you have a slip boundary condition, there we have to bring in that what we said the molecule is coming to the wall and getting reflected and this wall may have a velocity or may not have a velocity.

So, how that, so what happens? So, what we do is, we come up with an expression for u in that case and compare, we keep this as a standard (Refer Slide Time: 53:00), this is this is the no slip condition and where all we deviate in our understanding, where all we deviate from no slip, so that is what we would be doing in the next class. We will we will start with this u is equal to c 1 y plus c 2, but we will start imposing the different boundary condition. That is all I have for today’s class, thank you very much.