Microscale Transport Processes

Prof. S. Ganguly

Department of Chemical Engineering

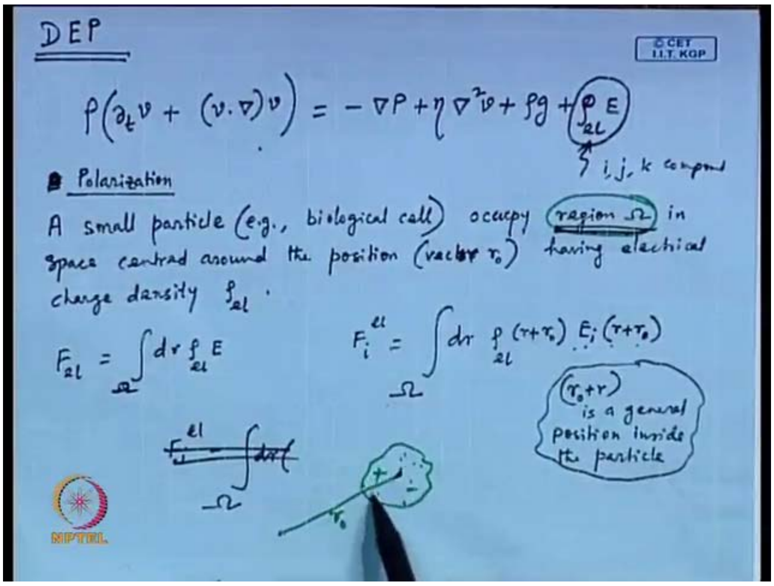
Indian Institute of Technology, Kharagpur

Lecture No. # 24

Dielectrophoresis (Contd.)

I welcome you to this lecture of Microscale Transport Process. What we have been discussing was Dielectrophoresis, we briefly introduced or we started the introduction of this dielectrophoresis in the last class.

(Refer Slide Time: 00:32)



What we mentioned in the last class, that we have this Navier-stokes equation here. Here, we have this Navier-stokes equation, where we have this body force term arising from this electric field, which has i j and k component. What we mentioned in the last class I mean we tried to, tried tried to describe something called a polarization, where we mentioned that, a small particle such as a biological cell occupy region omega in space centred around the position vector r 0 having electrical charge density rho e l.

So, it will have a force, integration d r rho basically this this body force term in the in the in the Navier-stokes equation, this will take the shape F e l is equal to integration for all.

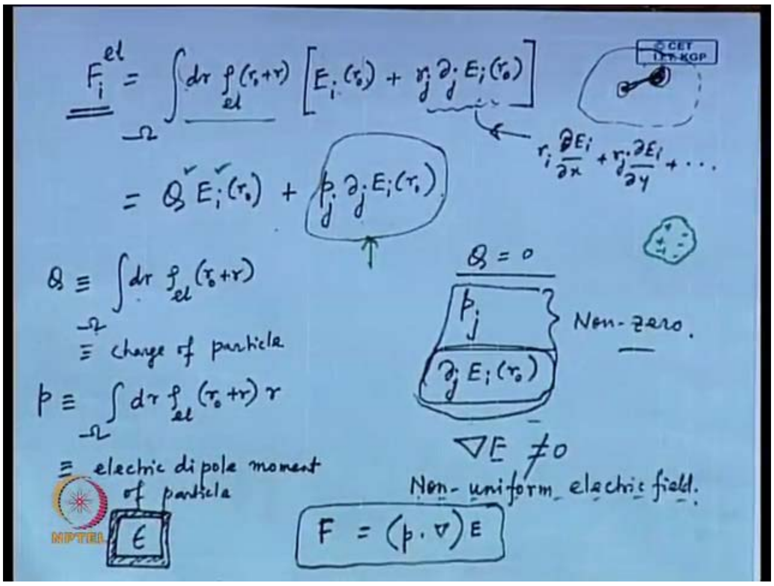
We are talking about a region omega, not just a point; and for this region omega, we had that integration d r rho e l E this quantity that is integrated over the entire region. By that, what we mean is, we have a particle we have a small particle and we in this particle we have charges present plus minus these charges are present.

And we we have an, if there is a net charge and this would be reflected there, now this what we are trying to find out here is, we are integrating rho e l E, this product for this entire region omega. So, for various positions here, here, here, here and we are integrating this product, that is what it means.

So, this is the electrical, this is the force component and the i-th component of this force was given last time as this. This this was given as the i-th component of the force for DEP, where r naught plus r is a general position inside the particle; that means, I have I have a reference point, from there I have located this particle at a distance r I mean, located this particle at a position vector r 0.

And then we have, then we have within around this particle we have constructed an volume or a or a region omega. So, that region is defined as this various point within this region is referred as r 0 plus r. So, r 0 plus r is a general position inside the particle, so this is the force term in that case.

(Refer Slide Time: 03:12)



Now, what we did in the last class was, we have we have expanded this using Tailor series, this E this E that E basically the electric field that term we have expanded using Tailor series. And only we have taken the first first order terms, I mean we we have not taken the terms beyond first order.

And since it is multivariable system, so we have I mean we have i j and k involved, so we have basically this is a Tailor series expansion in a multivariable sense. Then we have a we have written this product as Q into E i r 0 and this product, product of this and this that we have written as p j del j E i r 0. So, this this is this is one term and this is another term, this is how we have written this i-th component of the force arising from this electric field.

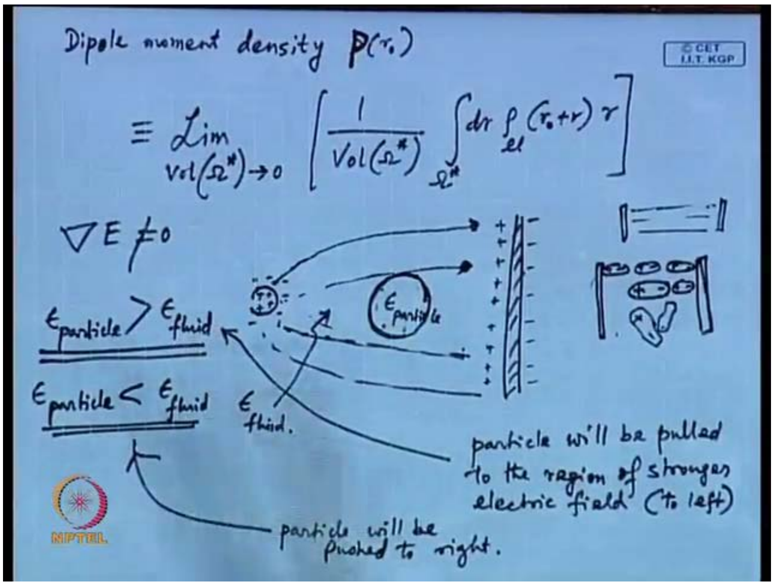
Now, we said what is Q? Q is basically integration of this d r rho e l r 0 plus r, and this is referred as charge of particle. On the other hand, this p which is basically referred as electric dipole moment of particle that is, this quantity d r rho e l r 0 plus r into r, so this is this is called electric dipole moment. And this Q is charge of particle, anyway these are these are immediate result of this expanding expansion of this expression.

Now, what we said is, in case of an (( )) in case of what we had done before say, electro osmosis and other places, there we have this Q present, there we have a Q there we have this Q present, there we have this E i present and they contributed to the force. But, in this case we at the very outset we said Q is equal to 0, so this term is 0. However, we said that, derivative of this E that means del of E that is non-zero that means, that means the electric field is non-uniform.

So, because of this non-uniform electric field and because of this non-zero p j, this term features, so we have a force. So, what this means is? There is a particle the net charge on this particle is 0, because the number of pluses that you have and number of minuses that you have, they summed up to be 0. So, the net charge on this particle is 0, but still some force is exerted on this particle by imposing a non-uniform electric field, that is what we are talking about.

And these force we are referring as dielectric force, and the definition of this force is p dot delta E, where p is this quantity, electric dipole moment of particle. So, this is what we have done, we have briefly introduced in the last class alright.

(Refer Slide Time: 06:32)



So now, if we if we if we try to if we try to write this, if we try to now before before we get into this further, I let me let me do one thing. Let me just define this dipole moment density, this is something which is there in the literature, dipole moment density P of r 0, let me let me put it, this is a capital P to differentiate this we had a small p here, which is electric dipole moment of particle. So, this is a small p to differentiate this, this is a small p.

So, we have to differentiate this we have capital P here, this is defined as limit volume of omega star tending to 0 1 divided by volume of omega star integration over omega star d r rho e l r 0 plus r into r, so this quantity is, so this is this is referred as dipole moment density. Now, I said that, we need to establish a non-uniform electric field non-uniform electric field such that, this del of E is non-zero, gradient of E is non-zero.

So, this type of field you can establish by having a planar electrode on one hand, this is a planar electrode (Refer Slide Time: 08:01), and a point electrode on the other side, this is a point electrode and this is a planar electrode, so you have this is the plus and this is the minus.

So, you will be pulling charges here and you will be having see, if somebody wants to draw the electric field it you can see the automatically it would be non-uniform in a sense that, these would be the, these would be the this is how the fields will be located or

I do not want to put the arrow here, but at least think of them. So, this is this is how the field, this is this how the field field is located alright.

So, what would have been in uniform field is that, you have one electrode here, another electrode here, which are both are of same size and you you have this electric field, this is known as this is uniform electric field. Here, you have a non-uniform electric field now, suppose you have a dielectric sphere present here.

In this non-uniform electric field, you have put a dielectric sphere or dielectric particle, which has certain epsa, so we call this epsa of particle. And on top of that, you have a medium in which these particles are suspended and let us call this as epsa fluid. And we note here that, because of this non-uniform nature of the electric field, the concentration of electric or the highest density of electric field lines, highest density of electric field lines is around this location. So, on the left of the particle, that is the place where the field, this electric field lines are most concentrated.

So, this is the region of strong E field strong electric field is in this region, you remember those stream lines getting converged and the place where the stream lines are coming close to each other, that is the place you have the highest velocity and things like that. So, this I mean you can you can think in that context as well.

So here, you have the electric field lines which are coming, electric field lines which are concentrated near this on the left hand side. So, the region of strong E field, strong electric field is on the left hand side. So, there is one issue here, either it could be that a epsa particle, either the epsa particle a is more than epsa fluid or the other other possibility could be that, epsa particle is less than epsa fluid. The dielectric constant, that for a particle and a fluid, they can be they can be they can be little, they they can be this different, one can be higher one can be larger than the other.

Now, if the epsa particle is more if epsa particle is more in that case, you can expect more polarization charges at the surface. If the under this condition, the epsa particle is greater than epsa fluid, there would be more polarization charges on the particle. What does that mean? There would be induced polarization that we have been talking about this electric dipole moment etcetera. So, there would be charges on the particle net charge is 0, but on the surface, there would be plus charges and minus charges.

Think of what what happens in a capacitor? I mean if you have if you have two electrodes and if you have dielectric say a dielectric particle present, what we what you will find is that, say plus and minus, this dielectric particle will align itself the charges will align itself such that, it remains this way; another dielectric particle will align again here plus and minus, another would align here.

So, originally when the when the when the voltage was not applied when the electric field was not applied, that time you have the, you you have this this this this oriented in a in an all in a in a random manner alright. But now, if you if you impose an electric field you will see that, the charges they get aligned in the direction of the electric field that, that is how it works alright.

Now, these charge these charges I am referring as, the surface the charges that is available on the particle, the net charge is 0. Now, if the epsa particle is more than epsa fluid in under this condition, you will find that more number of more charges is available on the surface of this particle; and in that case, this particle will be drawn towards the drawn towards the region of strong electric field.

Why, will it be drawn? I mean if the if the if this if this electric field would have been uniform if this electric field would have been uniform as is the case here, in this case just the particle will align itself and sit there. So, at the at the end of an end of the day, you will find all these particles they are aligned plus minus plus minus plus minus neatly, no moment of the particle nothing, that is that is what we understand.

Now here, you have a non-uniform electric field. So, if the particle tends to align in this way, it would be extremely difficult for the particle to maintain retain the equilibrium. So, the only way the particle can align itself is, by moving towards this. And since this electric field is non-uniform, so it will keep moving towards the left hand side such that, it can it can establish this kind of structure.

If this if this if the instead of this spherical electrode, if I would have had a planar electrode, particle would have aligned particle would have aligned this particle would have just simply aligned itself such that, the charges are corresponding all all the say, all the negative charges would have been on in this direction, and all the positive charges would have been or or away away in it it will be it will be it will be that way; but the particle will not move I mean depending on what what is the sign of this electrode, it will have all some particular charge will be on all on this that this side, some other charge would be on this side, but the particle will sit there, particle all the charges will be arranged, they will be aligned that way.

But here, in this case you have a non-uniform electric field not a planar electrode, but a point electrode. And because of this non-uniform electric field, this particle this person does not this particle does not have any anyway to go; only way it can, if it if it tends to satisfy this this sort of condition, only way it can satisfy itself is by, it will keep moving towards the left hand side and more it moves, again it is it is it is non-uniform that way.

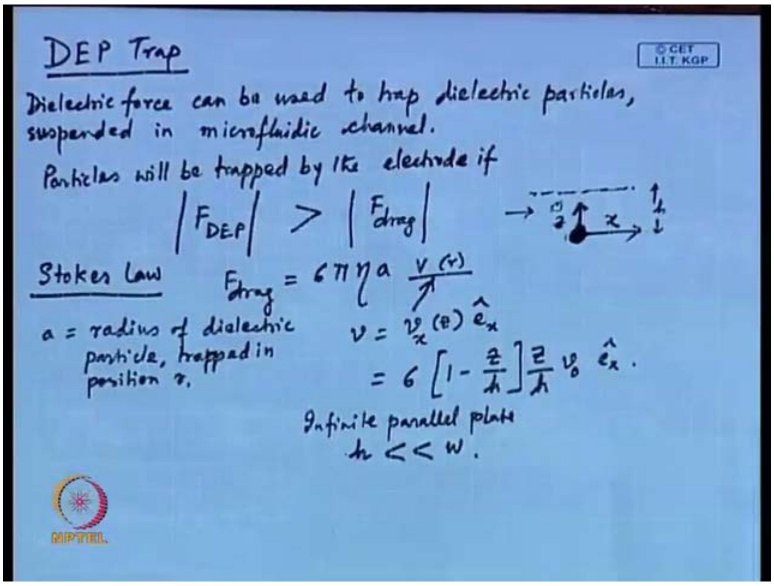
So, the only way it can restore the only way it can be, what it can do at this point is, it will keep moving on the left hand side. Now, think of the case when epsa particle is less than epsa fluid that means, this particle is less polarized and the fluid in which the particle is suspended, that is more polarized in that case, the surface charges on on the fluid that would be more than the particle.

This particle will have less charges, the fluid will have more charges. So, in that case, it would be easier for the fluid to move towards this direction than I mean with reference to the particle or in other words, we can say the particle will be pushed towards this direction.

So, when epsa particle is greater than epsa fluid the particle, so I should write this down clearly. When epsa particle is greater than epsa fluid, then the particle will be pulled to the region of stronger electric field that means, toward to left and when this is the case then particle will be pushed to right.

So, what you basically do here is, you are you you are having to, you you are creating a non-uniform electric field and by imposing a non-uniform electric field, you are pulling some particles and not pulling some other particles, depending on the dielectric constant of that particle. So that is that is what is the bottom line I mean what we discussed so far.

(Refer Slide Time: 17:50)



Now, the very common way of utilizing this process is, the there is there is something called a DEP trap. DEP trap means you have, basically you have a situation like this, where the fluid is flowing suppose you have a channel this is the, this is one wall of the channel and the opposite wall is the point there is this point electrode; one wall you have planar electrode and other wall is, point electrode. And you have particles different particles they are flowing through this flowing through flowing through this region.

So, the depending on the epsa of the particle, you expect some particle will be pulled towards the point electrode, and some particle will be pushed away from the point electrode. So, there is one thing you could be that, you can use this channel in a field flow fractionation mode. You are you are already familiar with what field flow fractionation means; that means you have a you are you are you are collecting the sample, but by the time it comes out, this would be; since it is since you have a parabolic velocity profile, so near the wall the velocity would be 0 and away from the wall, near the central part of the channel, the velocity would be highest.

So, the central part of the channel will come out first and layer that is next to the wall that is taking the longest time that is taking the most time to reach the outlet. So, if you if you if you create a fractogram; that means, if you have a detector connected at the outlet and if you collect, so it will it will continuously it will measure the average, average means average over this cross section, whatever it receives with time.

So in that case, you will see peaks one peak arising from the layer that is arriving first the central one, then the one that is coming later and the one that is placed next to the wall that will come that will come at the end, so you will see peaks. Now, if you can align particles depending on their dielectric property, if you can align the particles on within this channel at various locations; then you can you can at the at the at the outlet when you receive it, at the outlet you can you can detect them.

And against a pre-calibration, you can find out for an unknown sample, what all what all particles you have that is, that is one way of doing it using using field flow using this DEP in a field flow fractionation mode, which has been which which researchers have already studied.

The other possibility is that DEP trap that I said, that you trap the particle I mean suppose so many particles are flowing through this, but some particle satisfies this epsa, and those particles will be trapped next to the electrode. So, once you remove once the process is done, suppose you have a sample a slug and you flow the slug through this place, and within the slug whatever particle you have that satisfy this condition, you trap them next to the point electrode. So, this this could be another way of looking at it and this is referred as DEP trap.

Now to have this DEP trap working, so so what is this DEP trap? If I try to articulate it in in a sentence, it would be that the dielectric force dielectric force can be used to trap dielectric particles suspended in micro fluidic channel. Now you say, that this particle will be trapped when particles will be trapped by the electrode, if mod of F DEP, F DEP is that p dot delta E that we already described; this has to be greater than the drag force.

Your immediate response would be that, since we have a non-uniform field here or the way it operates here, I mean why we do not have an angle or anything I mean you can we just equate F DEP with F drag I mean that would be your immediate response. My belief is that, this is a limiting situation that they are talking about, the limiting situation next to the electrode.

So, this is this is this is this is how probably this can be reason I mean I would be I would probably like to have your views also on how it should be done. But, F DEP we have some idea, p dot delta E; and F drag will come from that Stokes law I mean that that that is that is perfectly applicable in this case. So, this is, so this inequality basically shows the limit of it I mean ideally there should be an angle involved.

Now, Stokes law says F drag is equal to 6 pi eta a v v at a position, v at r. What are these properties? 6 pi is 22 bar slash 7 (( )), eta is the viscosity of the fluid right, and a is the radius of the particle, and v is velocity of the particle. So, this is this is this is the Stokes law, this we already have talked about.

So, if you if you want to write I mean probably we need to note here, a is equal to radius of dielectric particle, trapped in position r. And what is v? v is, if this direction is x and if this is z we are talking about we are talking about a point a or a point electrode and a planar electrode.

So, this is the point electrode (Refer Slide Time: 25:02), this is the planar electrode and this direction is x, this direction is z; if that is so, that means the flow is taking place in this direction, and this is the point electrode and this is the planar electrode, and particles would be trapped in near the point electrode, that is the idea. Then how, what what would be v in that case? v would be given as, v x as a function of z e x hat I mean we are we are ignoring this. So so we have y is in perpendicular to the paper, and that part we are we are taking that it is it is indifferent.

So, v is equal to v x z e x hat and then these v x z, how will you write this v x z for a channel? Let me write this I mean what I got, this as an expression I think this should be you you can you can verify it yourself later; 6 into 1 minus z by h, h is the aperture of the channel right. So, 1 minus z by h into z by h v 0, v 0 is the average velocity e x hat. This is that same expression we had you remember, 2 v 0 into 1 minus r by r whole square for a laminar flow through a channel in circular channel.

So, I think if you go for a laminar flow through a rectangular channel, which is which happens to be micropedic channel, you you would get this expression probably, you you should you should look into this. Now here, one catch is there it, you are you are you are assuming this to be an infinite parallel plate; that means, h is much less than w. Infinite this this term itself is relative that means, if this w is 1 centimetre and if this h is 1 micrometre then, probably you can still consider that to be infinity I mean it is that way.

So, it is as long as this aperture is much smaller than the width of this perpendicular to the paper whatever we have, this as long as this h is much smaller than w you can assume that, this plates are infinite infinite parallel plate. And and of course, you you need to have the velocity field fully developed in this direction, that is that is there.

Now, what you are doing in this case here is that, we do not have any w in this expression for v. So, this is this is simply, this is the velocity field I mean we should be happy with it. There would be there, would be formation of Debye layer, next to the particle that is a possibility. You remember that, we when we talked about electrophoresis we said that, this Debye layer is basically instrumental in pulling, its still similar Debye layer around the particle could be, there is a possibility of formation of this Debye layer.

However, that that possibility would be gone, if somebody uses an AC electric field that means, alternating current here to be and that is that is very common. In fact, if you if you look into any advanced literature in DEP, you will find that, they will start with these AC electric field altogether. But I thought, it would be wise to first do it build it from scratch and then, probably point out which all parameters would be affected, if there is an AC electric field.

So, if somebody employs an AC electric field then, probably the formation of Debye layer around the particle that would be that would not be important. So, that is that is the that is the situation here. Now, how would you this is this is the F drag, so we have already figured out. How F drag could be 6 pi eta a into v, and that v would be, this v would be replaced by this quantity (Refer Slide Time: 29:48).

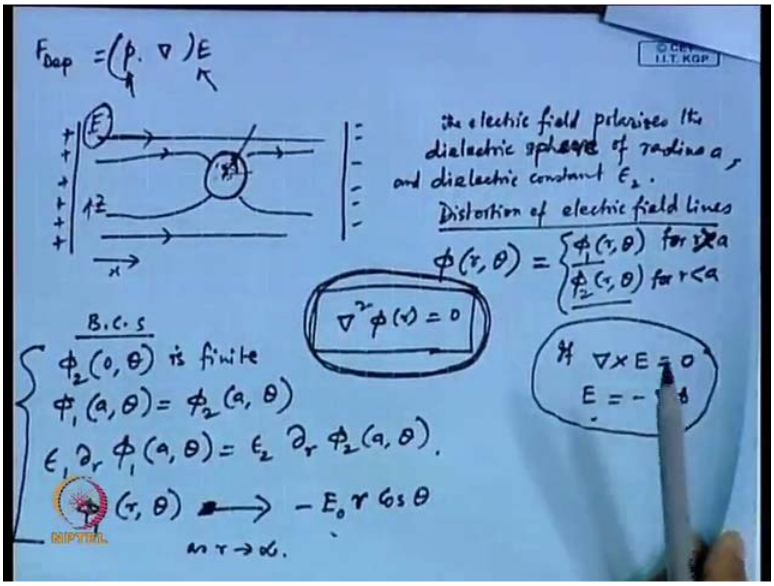
So, this drag part is simple the right hand side part of this inequality is simple, but for for left hand side you need to find out F DEP. Now you may say, I mean what what are we trying to, what are we trying to do I mean DEP trap I understand, I understand inequality, but what are we trying to solve by putting this writing this drag force.

Our objective is to find out, what should be the v naught, what should be the electric field such that, you can trap a particle of certain epsa. So so what should be that expression, how would you, so what should be? Suppose, somebody gives you the, I want to trap particle of this dielectric constant of this size; a is given, epsa is given and then now I have this much of electric field at my disposal, tell me what should be the v naught such that, I can trap the particle because, if you are beyond that v naught then, the you cannot trap the particle it will simply flow away. So, that is probably something which we are trying to establish, but by doing this what we would have is, we would have some clarification on how these DEP force etcetera are handled that is that is probably the objective.

Now, F drag part, the right hand side part of the inequality already we found out. Now, F DEP part which is nothing but, p dot delta E we have already we have already discussed this, right we have already established that this force is p dot delta E. So now, you have to focus on this part, how these relates to the F drag and how we can how we can find out, what should be the v naught.

Now we said that, it is an inhomogeneous electric field. Now, how will you create an inhomogeneous electric field? Before, before we get into this inhomogeneous electric field, what we do here is, first let us find out an expression for p; delta E is one thing we we will we will find out, how we how we handle this delta E, I mean basically look at look at this we have F DEP that is equal to p dot delta E, that is what we said.

(Refer Slide Time: 32:03)



Now, we have two issues in hand here, one is this p, another another is E. Now, how will you find E for this case point electrode and planar electrode, how will you find E that is one issue, the other issue is, how to find p. So, these are the two things you need to resolve, then only you can find out what is F DEP and then you can equate it with F drag to find out what should be v naught such that, you can trap the particle.

Now, before we get into this F before we get into finding what is p let me let me point out couple of things here. Suppose, we have an uniform electric field and we have a particle, how would we have done the polarization, the unperturbed electric field lines they would be something like this, right these are all plus, these are all minus. So, these would be the field lines and near the particle, there would be some twist like this, alright.

Now, how would you find p in this case, how do you first of all, let us let us find out how you can how you can solve this case of unperturbed electric field line. Now, unperturbed field lines would is this, there was no sphere it is all straight line, but because of this particle, there would be some twist here. How will you find out, what what governs this twist that is that is something which is important here.

Now here, we you can we can do one thing is that, you need to first of all I mentioned here that, the electric field polarizes the dielectric sphere the electric field polarizes the dielectric sphere sphe dielectric sphere, I write this as sphere of radius a, now radius a and dielectric constant epsa 2. Now, there would be distortion of electric field lines.

And, if somebody wants to theorise or or somebody wants to simulate in by theory, there were these these distortion of electric field lines. What he has to note here is that, v the potential in r theta coordinate, here we have one coordinate system, which was we we said that, this is x this direction is x, this direction is z, that is what we had originally right; instead of that, we have another one which is r theta coordinate, this angle I mean instead of instead of a Cartesian we can go to a go go to cylindrical or polar system.

So, if we if we go to this phi r theta this coordinate system, here we have to note that, this would be equal to phi 1 r theta for for r less than a or r sorry r greater than a we are calling it, and phi 2 r theta for r less than a. What is a? a is the radius of the particle. So, you have a potential you have a phi inside this particle, you have a phi outside the particle, so this this is this is the phi r theta. So, you have you have 1 phi one function, you have 1 phi 2.

Now, what you need to finally solve here to find this p, what you need to solve here is, this del square phi r that is equal to 0, this is the governing equation for the fluid and sphere, this is the governing equation that needs to be solved.

However, there are boundary conditions, the boundary conditions are phi 2 0 theta, phi 2 is inside the sphere inside the particle, phi 2 0 theta is finite that is one thing, other is continuity which is important; phi 1 a theta that is equal to phi 2 a theta that means where the at the end, the place where the sphere ends and the fluid begins, there has to be a continuity in phi, their their their derivative you need a continuity there as well you need a continuity there as well. And after all this phi has to be related to the electric field lines, because originally you have imposed the E there right, you have imposed some electric field E.

And how is this E and phi related, what is phi and what is E? You remember E what was what was phi is the potential, and E is the electric field. What we, how we define this? We said that, if del cross E is equal to 0, then E is equal to minus delta phi right. This is this is this is this is the definition at the very outset we said this, you you go through your earlier notes; you will see I mean in fact, you compare these with our velocity field and existence of potential function that exactly it is it is it is in a similar manner I mean we we discuss this before, how this how we obtained this in connection with electro electric double layer we discussed this in detail.

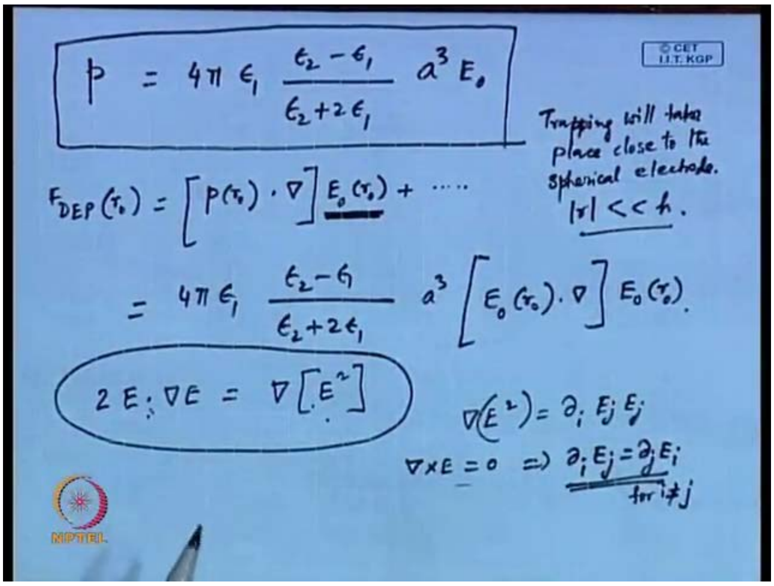
So, go back to that notes. So, this is how it is related and this E you have imposed. And E is equal to minus delta phi, so this has to be this has to be satisfied. Now, how will you how will you satisfy because you are you are talking about some r theta coordinate here and E is imposed in this direction. So, if you want to do that, you should be writing it as, so what you should be writing here is phi 1 r theta that is that tends to minus E naught r cos theta as r tends to infinity; that means you have, what is E? E is basically minus del phi del r all right.

So, you have to you had to write this at far away at far away from the origin, you will not have any of these twists felt. So, if you write it that way, so you you have to you have to you have to write it phi 1 r theta is equal to minus E naught r cos theta. So, this is this is that so far away from this particle, it would be simple unperturbed electric field lines, so that is what this boundary condition states.

So now, if you have these boundary conditions in place and if you solve these governing equation if you solve these governing equation and with this boundary conditions in place; and then, what was p originally I mean I am not going to I am not going to talk about these these how the solution is done. I will give the final form here, at least you want you to appreciate what all steps we have gone through, this is the electric dipole movement here, and this is the electric dipole movement here.

And you know how these rho e l term how these rho e l term is how this rho e l term is related to phi that that we have already discussed, in connection with electric double layer. So, if you if you solve this equation and finally, get to that p, what you would end up with is p.

(Refer Slide Time: 41:41)



p is equal to p p near p near r is 0 p p near r 0 I I will I will get back to that, 4 pi let me write the expression first, this is the expression that you get in that case. If you if you solve that equation that is that is the that is the idea, solving that governing equation with these boundary conditions, you end up with this expression. And what you write here is, F DEP mind it, the trapping will take place, the trapping trapping will takes trapping will take place.

Trapping will take place close to the close to the we are calling it spherical electrode, that point electrode we call it spherical electrode, close to the spherical electrode, spherical close to this spherical electrode; that means the r value that we work with is much less than h, r value that you work with is much less than h. So, what you write here, F DEP at r 0 that is equal to p of r 0 dot delta into E 0 r 0 plus, there are other higher order terms that you are not considering, what was what was your (No audio from 44:04 to 44:40).

Basically this this F DEP r 0 here, there should be there should be some other some other higher order terms, because what you are doing here is this, you are writing this as E 0 r 0, this this E 0 is with reference to unperturbed electric field. So, so basically this was this was your this was the E 0 term, you remember this was the unperturbed electric field line (Refer Slide Time: 45:14).

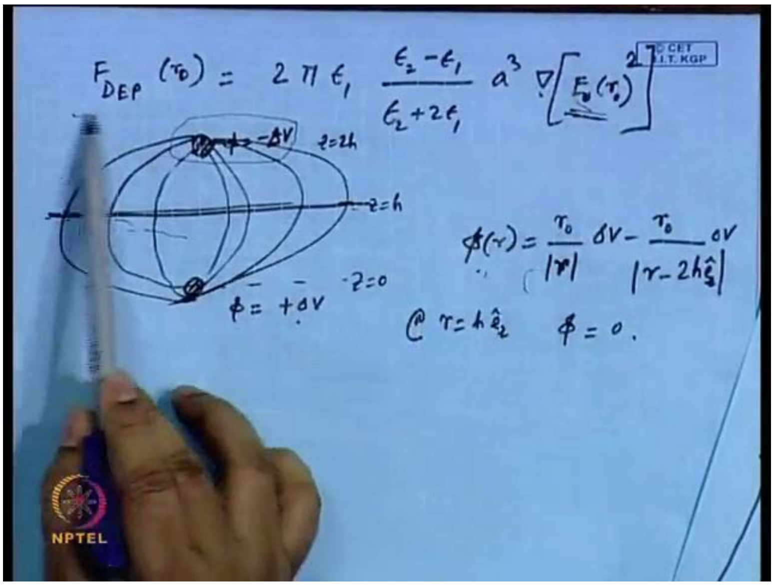
Now here, if you you cannot ideally I do not think, you can write this in terms of E 0. So, there should be other higher order terms which you are not considering in this process. So, you are only focusing on this p r 0 dot delta E 0 r 0, that is what you are saying here; and then, you are bringing in these expression for p inside. So, what you are getting in that case is, 4 pi epsa 1 epsa 2 minus epsa 1 divided by epsa 2 plus 2 epsa 1, there is a name given to this factor a cube and then, you have E 0 r 0 dot delta E 0 r 0.

Now, there is an identity here to to solve this equation further, there is an identity which needs to be utilized, that identity is that 2 E dot delta E is equal to delta of E square. Have you have you seen this identity, this identity is possible, because this is this is based on a I mean I mean let me let me point out that this; what is this delta of E square? delta of E square is equal to del of i E j E j that is what it mean.

And on the other hand, the left hand side you can equate this to the left hand side, only if you have del cross E is equal to 0, what this means is, del of i E j is equal to del of j E i. If del cross E is equal to 0, you know how a cross product of a vector is and that means that this has to be satisfied (Refer Slide Time: 47:44); and if this is satisfied, then only you can equate the left hand side with the right hand side, because right hand side is basically this quantity.

So, other cross terms can be taken of, I mean if you if you have, this this is for i is not equal to j, this this this thing is for I is not equal to j. So, this is an identity and that identity needs to be used here in this case.

(Refer Slide Time: 48:20)



To come up with this final expression for F DEP which appears like this, F DEP at r 0 that is equal to 2 pi epsa 1 epsa 2 minus epsa 1 divided by epsa 2 plus 2 epsa 1 a cube delta of E naught r 0 square, so this is something which you have for F DEP. Now, you have to find out some expression for these these electric field in this case.

What ideally you should be doing is? This electric field think of it, you have a planner electrode and a point electrode, this is simulated by considering, so here it is here it is phi is equal to plus delta v; and here phi is equal to 0 instead of that, what you do is, you do not consider this planar electrode, instead you consider a mirror electrode.

So, if the if this is at z is equal to 0, if this is at z is equal to h, this is at z is equal to 2 h and here it is minus of delta v, it is plus of delta v and here it is minus of delta v and think and and look at the electric field lines, how would they be operating in this case, they would be like this (Refer Slide Time: 50:25); and this is same as I mean whether you consider this a mirror electrode whether you consider a mirror electrode here, the effect would be same as considering a planar electrode here.

So, instead of considering a planar electrode, you can work with two point electrodes 2 h distance apart and phi is equal plus delta v here and phi is equal to minus delta v here. So, that is also another possibility and using this method you can find out, how this is, so this is a concept of inhomogeneous electric field. And in that case, you can have the phi to be written as r 0 divided by r delta v minus r 0 divided by r minus 2 h e z into delta v. So, this is the definition of phi r in this case.

You can you can you can check it yourself say, at r is equal to h e z, r is equal to h e z; that means where this plane, at r is equal to h e z check what you have, for phi? This should be equal to 0, so this is this is this is what the phi that you would be working with. So from this, you can get some idea of what should be the electric field what should be the electric field here and then impose that electric field in this expression and take the derivative of it and then, you can get an, get the final expression for F DEP.

The idea is to equate this F DEP with our original expression of or or this inequality F DEP with F drag and come up with the expression for v 0. So, what we have done is, first we have used for unperturbed electric field. So, if I if I quickly summarize, we have used this is the original expression for DEP p dot delta E we we stuck to this, we remain true to this. Only thing is we said that, if we if there is a particle inside an unperturbed electric field was E 0; that means, particle was not there, just the electric field was there value E 0.

And we said that, that is the that is the case at infinity that is the case at infinity and we solved we we have we have defined two sets, phi 1 and phi 2 for the two, one is within the sphere and the other is outside the sphere the potential. And this is the governing equation that was solved I mean we did not solve, I had said these are the steps that has been solved by the researchers to come up with an expression for p which is here (Refer Slide Time: 53:32), and then we put this expression into this.

However here, instead of the electric field we use the unperturbed electric field, but by that we made an approximation we we should have if you if we expand in Taylor series form then, this should be should have been expanded that was not considered and you unperturbed electric field and continued that way.

And there was this identity that was utilized, this identity could be utilized, since this del cross E is equal to 0 and that identity has gone into find this E dot delta E instead of this, we this this allowed us to write this as delta of this thing square; and then we are going to put we are going to put we are obtain a we are going to obtain the potential. We are going to put this electric field here and take the derivative of it to come up with the expression for F DEP, this we will equate with F drag to find out what should be the v naught we can tolerate. So, that is that is that is what the essence of this. So, I will continue with this in the next class that is all I have for today.