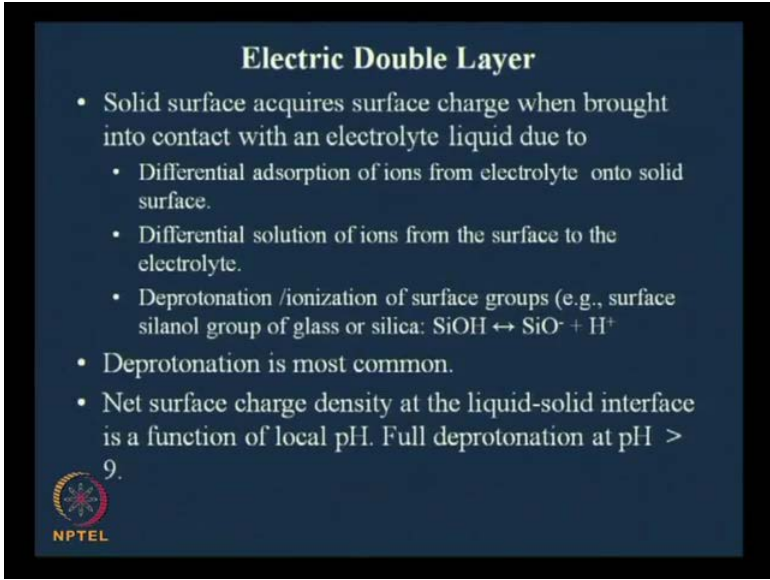


Microscale Transport Processes
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Lecture No # 22
Electro Osmosis (Contd.)


I welcome you to this lecture of Microscale Transport Process, what we have been discussing in the last class were electric double layer.

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Electric Double Layer

- Solid surface acquires surface charge when brought into contact with an electrolyte liquid due to
 - Differential adsorption of ions from electrolyte onto solid surface.
 - Differential solution of ions from the surface to the electrolyte.
 - Deprotonation /ionization of surface groups (e.g., surface silanol group of glass or silica: $\text{SiOH} \leftrightarrow \text{SiO}^- + \text{H}^+$)
- Deprotonation is most common.
- Net surface charge density at the liquid-solid interface is a function of local pH. Full deprotonation at $\text{pH} > 9$.


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Let me give you a quick briefing on what we have already discussed, since there was a gap in between. This solid surface acquires surface charge, when brought into contact with an electrolyte liquid due to several reasons. Those reasons are given there, deprotonation or ionization of surface groups that is very common.

So, just because a solid surface and there is an electrolyte in the proximity, there would be some surface charge developing. And net surface charge density at the liquid solid interface is a function of local pH, because you know what pH is, it is basically hydrogen, ion concentration log of that. So, it is **it is it is it is** very much related to the pH of the system, full deprotonation at pH greater than 9.

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Electric Double Layercontd.

- Ions of opposite charge in the electrolyte are attracted and the like charges are repelled.
- The net excess of mobile ions near the interface with charge, opposite to that of the wall is referred as EDL.
- These excess charges can be used to impart a force on bulk fluid through ion-drag.
- Counter-ions reside in compact layer next to the wall (Stern layer) in adsorbed state.
- Counter ions also reside in more diffused layer next to the wall, where the ions are free to move.



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Now, this once there is a surface charge acquiring, **one** once these charges are acquiring on the surface or acquired on the surface, then there would be opposite charges getting attracted. And this is forming an excess of mobile ions near the interface with charge, opposite to that of the wall, and this is referred as electric double layer; these excess charges can be used to impart a force on bulk fluid through ion-drag.

So, the counter-ions reside in compact layer next to the wall which is referred as stern layer, which is in adsorbed state. The other one is a counter-ions that reside in a more diffused layer next to the wall, where the ions are free to move.

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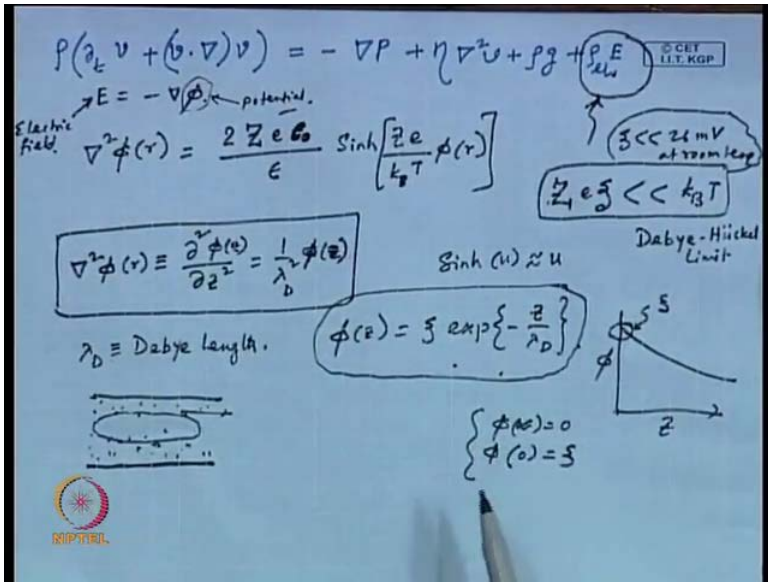
Electric Double Layercontd.

- The potential at the shear plane (the boundary of the liquid flow problem) is called zeta potential.
- Debye length
- Debye Hückel limit
- Single flat plate and parallel plate geometry
- Velocity profile under electroosmotic flow



So, what we discussed in the last class, we started discussing in the last class is that potential at the shear plane, the boundary of the liquid flow problem is called zeta potential, and there is something called Debye length, and this **this** we discussed in the **in the in the** last class. Let me **let me** point out quickly, what we had **we had we had** derived few things in the last class.

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$$\rho(\partial_t v + (v \cdot \nabla)v) = -\nabla P + \eta \nabla^2 v + \rho g + \rho_e E$$

$$E = -\nabla \phi$$

$$\nabla^2 \phi(r) = \frac{2Ze\epsilon_0}{\epsilon} \sinh\left[\frac{ze}{k_B T} \phi(r)\right]$$

$$\nabla^2 \phi(r) \equiv \frac{\partial^2 \phi(z)}{\partial z^2} = \frac{1}{\lambda_D^2} \phi(z)$$

$$\lambda_D \equiv \text{Debye length}$$

$$\phi(z) = \xi \exp\left\{-\frac{z}{\lambda_D}\right\}$$

$$\begin{cases} \phi(\infty) = 0 \\ \phi(0) = \xi \end{cases}$$

Darcy-Hückel Limit
 $\xi \ll 25 \text{ mV}$ at room temp
 $ze\xi \ll k_B T$

$\sinh(u) \approx u$

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Let me just point out the basic findings there, the basic equations there, first of all we are working **with** within the framework of Navier-Stokes equation, which appears like

this (No audio 02:44 to 03:14). What we mentioned in the mentioned previously is that these terms we are all familiar with, $\frac{d}{dt} v$ is that derivative with respect to time, and $\dot{\Delta} v$ you know, but what what we are talking about here, this is the pressure gradient term η is the viscosity, so and ρg is the component due to gravity. So, this this part you already you are familiar with and this $\rho e L E$ we introduced recently, this is the body force term arising from this ion drag, that we talked about. So, we we have this additional body force term present.

Now, what we said in the last class is that, these Δ this this $\rho e L E$ this needs to be this needs to be we need to understand this term well, and we need to see how this term is getting affected, because of this electric double layer. Now, what we have derived in the last class is, something like this that $\Delta^2 \pi r$ that is equal to $2 z e \epsilon_0 c \sinh(kr)$ $z e c$ \sinh divided by $\epsilon_0 a$, this is $c \sinh$ divided by $\epsilon_0 a$, this is $c \sinh$ divided by $\epsilon_0 a$, this is an expression that we had obtained in the last class.

Now, here we said that for the limit for the limit $z e \zeta \ll k b T$ when when this is happening, this is this is this is referred as Debye Huckel limit, when this is when this is happening when when this is when this is happening in that case, this \sinh can be, you can you can make an approximation here, that $\sinh u$ is equal to u , when u is small. So, that is exactly what you are doing here.

So, one with this, what we have found in the last class is, this this is this is coming from Taylor series expansion, and ignoring the other higher order terms, what you end what we ended up with in the last class is, $\Delta^2 \pi$ which is for one-dimensional case, this would be equal to $\Delta^2 \pi z \Delta z$ that is equal to 1 by $\lambda_D^2 \pi z$ all right 1 by $\lambda_D^2 \pi z$ all right. Now, this is an expression that we had, and this expression we said that, here this λ_D is referred as Debye length.

And what is this π now, π was basically electric field, this this π is nothing but, E is equal to minus $\Delta \pi$, where this E is, this E that is appearing in the Navier-Stokes equation (Refer Slide Time: 07:07). So, E is equal to minus $\Delta \pi$, what is E , E is the electric field E is the electric field, and this π is the potential this π is the potential. So, that is that is how this π and E they are related, and with the π we have this expression where λ_D is equal to the Debye length.

This is the case, so this is very straightforward and once you solve this equation solution of this equation would be pretty straightforward, it is referred as, **I can** I can write this as πz is equal to $\zeta \exp(-z/\lambda D)$. What you can say is, **I mean** this **this** limit that we put here, basically this $z e \pi$ has to be much less than $k b T$ now, but ϕ at what point that would be the question people will ask, **I mean** what π at what **what** z we are referring to.

So, instead of putting π at any arbitrary value we put it as ζ , ζ is the value of π at z is equal to 0 that means, if you plot π versus z , then it should be dropping and the value here, at z equal to 0, that is referred as ζ , commonly referred as ζ potential, π is the potential. And for with distance from the wall this π is decreasing, so at the bulk electric double layer does not have any **any** effect, any existence, so π is decreasing.

So, now, **this decrease is** this the way this is decreasing, this is given by this function πz is equal to $\zeta \exp(-z/\lambda D)$, where z is the distance from the wall, π is the potential which is decreasing and at the wall that means, at z equal to 0, π is equal to ζ this is the expression that this gives (Refer Slide Time: 08:47). So, this is the expression we have, so this expression came assuming this Debye Huckel limit, that is assuming $z e \zeta$ much less than $k b T$.

Now, this is probably a very hypothetical kind of limit instead of that people are more familiar with limit ζ much less than 26 mV at room temperature, **that is what** that is what people are generally familiar with this sort of limit. Instead of putting **putting** things in symbols, you can put **put** things in numbers.

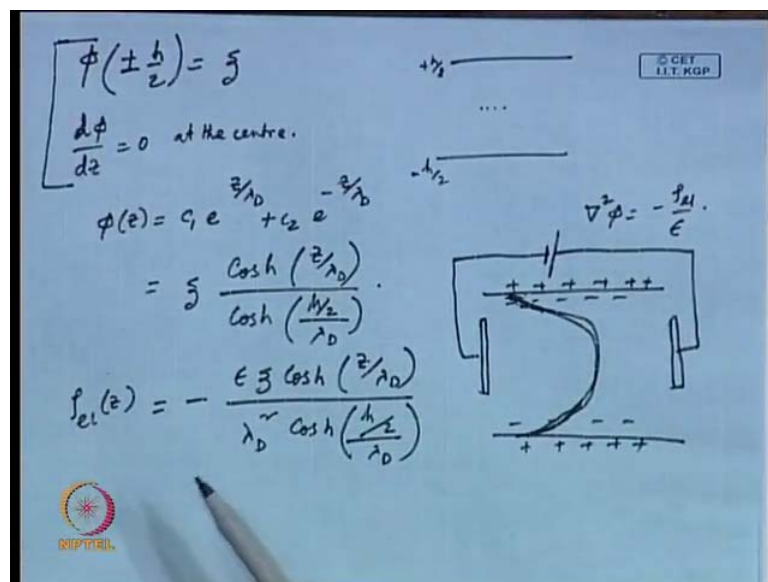
So, **this is** this is what you have, this is the expression that you have, what type of situation **this is simulating** this is simulating a wall which has acquired some charges, and the counter-ions it pulled, and this **this** expression gives you the potential as a function of distance from the wall (Refer Slide Time: 09:44). Now, **so so you** what you are assuming is that at z equal to infinity π is 0, probably **that is** that is what was one of the boundary condition you have used, you have used π infinity is equal to 0, and π 0 is equal to ζ , so these are the boundary conditions you have used.

Now, the problem in hand that we have is a channel, we are talking about a microchannel, we are not talking about the simple wall. So, if you have a microchannel this we were discussing at the end of the last class; that if you have a microchannel that

means, if you have a if you have two walls. One wall here with counter ions accumulating, another wall here with counter ions accumulating and both are pulling the bulk of the fluid.

Now, this part you could refer as bulk, this part you refer as a bulk of the fluid and this bulk of the fluid is dragged by this body force, so if the if that is the case, then you can use the same expression, only your boundary condition will change right (Refer Slide Time: 10:56). That that is that is something which I remember, I was discussing at the end of the last class, that the boundary condition will change. So, what would be the new boundary condition in this case.

(Refer Slide Time: 11:29)



The new boundary condition would be that, ϕ is equal to ϕ plus minus h by 2 that is equal to ζ , plus minus h by 2 that means, you are your your center here here your center was at the wall I mean in the earlier case, but in this case the center is if if this is the channel then the centralize here. So, at plus h by 2 and at minus h by 2 , you have this value of potential is ζ and the other thing is that at the center, there is symmetry existing, so $d\phi/dz$ is equal to 0 at the center by symmetry.

So, this is the new boundary condition you have, but the governing equation remains same, I mean we do not have any reason to believe that a governing equation should change. So, with these two boundary conditions, you remember in the last case also we came up with this expression; $\phi(z)$ is equal to $c_1 e^{z/\lambda_D} + c_2 e^{-z/\lambda_D}$

to the power minus z by λD . So, in that in the earlier case we said that, since at z equal to infinity π is π has to be finite, rather π has to be equal to 0. So, this c_1 has to be equal to 0, that was that was our c_1 to put c_1 as 0.

But, here the situation is different, it is π plus minus h by 2 is equal to ζ , so that means, you have c_1 as well as c_2 both nonzero. So, you have to work it out and what you would get in that case is equal to $\zeta \cos$ hyperbolic z by λD divided by \cos hyperbolic h by 2 divided by λD right.

So, in that case what you would have is ρe^{-lz} , what is ρe^{-lz} you remember, we have already obtained this expression, you need to go back and see how you have written it, $\Delta^2 \pi$ is equal to minus ρe^{-lz} , ρe^{-lz} by ϵ . So, this was an expression we had worked with earlier. So, accordingly this ρe^{-lz} would be equal to minus $\epsilon \zeta \cos$ hyperbolic z by λD divided by $\lambda D^2 \cos$ hyperbolic h by 2 divided by λD right. So, this is the expression you have for ρe^{-lz} .

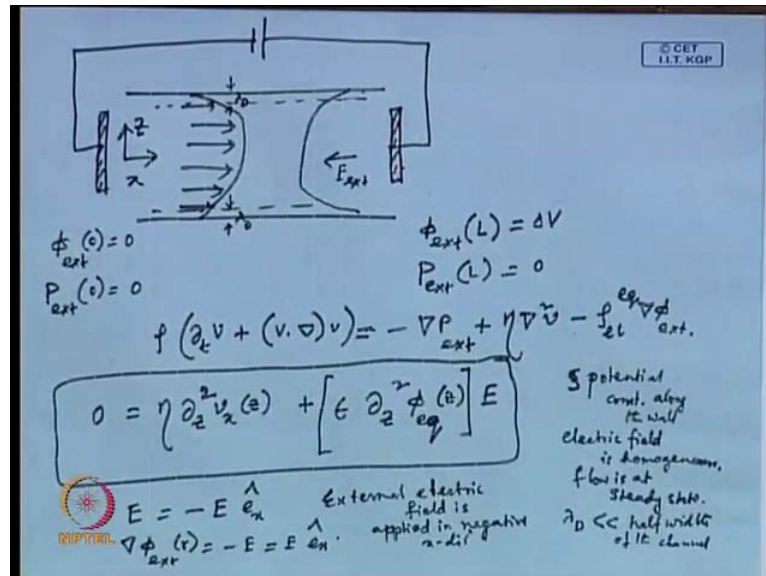
Now, if you come up with a scheme if you come up with a scheme, which is something like this, you have a channel, you have electrode here, this is the electrode and this is another electrode, they are connected (No audio from 15:17 to 15:41) (Refer Slide Time 15:15) this is the way it is. Now, if we are interested here in a velocity profile, if we would like to know what would be the velocity profile, because of this presence of this electrode, because electrodes are required to create a velocity create a potential gradient.

And because of this potential gradient we expect that a flow will we expect because of this potential gradient, we will have this term appearing ρe^{-lz} into E this becomes instrumental (Refer Slide Time: 16:21). And so there would be a velocity, even if there is no pressure gradient, suppose this Δp is not there if Δp is not there we know that there will not be any flow, but if by some way we can put this we can put a $\Delta \pi$ here, by some means we can put a $\Delta \pi$ here, some means we can make this term non zero, then we can induce a velocity.

We are interested to know what would that velocity is in the micro channel, in in the channel in the channel where we have two walls, if this is the scheme we would like to know what would be these velocity profile. So, for that we need to solve the Navier-Stokes equation with this understanding, with this being π , with this being ρ and we

have appropriate E with this we need to solve the Navier-Stokes equation. So, that is exactly what I would be taking up next (Refer Slide Time: 17:06).

(Refer Slide Time: 17:30)



So, let us let us draw this once again, this is an electrode, this is another electrode, this direction is x, this direction is z all right, this direction is z and you have pi external at z x is equal to 0 is 0, P external at x equal to 0 is equal to 0. Here, the pi external at x equal to L all right at x equal to L that is equal to delta v and p external at x equal to L is equal to 0. So, that means, there is no pressure drop, we are we are expecting the just because, we have put this potential gradient we should expect some velocity happening, because of this Navier-Stokes equation, because of that body force term. So, this is what we need to solve here.

Now, if now now there there would be this, where is this (O) layer in this case, I would I know that there would be a (O) layer, what is that lambda D, this is lambda D right, here the lambda D lies, here also there is a lambda D, this is the this this lambda D is the Debye length right. We we we have we have defined this lambda, we we have defined this Debye length earlier, so here here the charges are all accumulating, I am not drawing the charges again plus or minus, because that that would confuse.

Now, what was the equation we had at that time, we said the equation was rho del t v plus v dot delta v that is equal to minus delta p external plus eta del square v minus rho e l e q delta pi external. Now, whether it is minus or plus, that depends on what charges you

have here, and the direction of this of this field in which direction you are putting the field. So, here it needs to be minus, because of the charges that you have, because what you have in this case is, the E_{external} will be in this direction, if somebody plots the $\rho_{\text{el}} q$ it would be in this direction, and the velocity profile would be in this direction (Refer Slide Time: 20:41). So, the velocity would be in this direction, the bulk flow will be in this direction whereas; the potential would be in this direction all right. So, that is how this this sign of this is defined.

Now, if you if you do if you solve this for one-dimensional case, it would be something like this, I mean let me write the equation first and then I see then I find out what all assumptions we have taken here (No audio 21:16 to 21:54) (Refer Slide Time: 21:16). This is the equation that we have for one-dimension with the assumptions, what all the assumption what are all the assumptions, zeta potential is constant along the wall along the wall, electric field is homogeneous, flow is at steady state and Debye length is much smaller than the half width of the channel.

So, what you what you what you write here is, E here is equal to minus E_{ex} hat, because E is in negative direction, this this electric field is external electric field is applied in negative x direction, $\Delta p_{\text{external}}$ is equal to minus E that is equal to E_{ex} hat, that we already know. And other thing is that, the velocity term, v velocity is equal to v_x which changes with z_{ex} hat.

So, v at any position is, it is v_x which changes with z_{ex} that means, velocity is only in the x direction, there is no other component, and $\Delta p_{\text{external}}$ is equal to 0, that you already assumed. So, with these with these with these assumptions and with these understanding, you ended up with this expression Navier-Stokes in one-dimension.

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$$\frac{d^2}{dz^2} \left[v_x(z) + \frac{\epsilon E}{\eta} \phi_{eq}(z) \right] = 0.$$
 B.C. $v_x(\pm \frac{h}{2}) = 0.$

$$v_x(z) = \left[\zeta - \frac{\phi_{eq}(z)}{\eta} \right] \frac{\epsilon E}{\eta}$$

$$= \left[1 - \frac{\cosh(z/\lambda_D)}{\cosh(h/2/\lambda_D)} \right] \frac{\epsilon \zeta}{\eta} E.$$

So, once we know that then probably it would be easy to write this expression in a more compact manner as, $\frac{d^2}{dz^2} v_x + \frac{\epsilon \zeta E}{\eta} = 0$, that is that is exactly what we what we had in the last slide. (Refer Slide Time: 25:00) if you see this is the expression we have, so instead of writing it this way you take this $\frac{d^2}{dz^2}$, and put everything within the $\frac{d^2}{dz^2}$ all right, so it is the derivative.

So, basically $\frac{d^2}{dz^2}$, this whole thing $\frac{d^2}{dz^2}$ that is equal to 0, that is what you say and you know the boundary condition here boundary condition here is that v_x at $\pm \frac{h}{2}$ that is equal to 0, because we assumed that at the wall, whether it is upper wall or the lower wall, the stern layer is existing. So, it is all adsorbed and there is no velocity, that that is firmly attached to the wall.

So, at the wall v_x is equal to 0, this is the boundary condition you have, and this is the governing equation you have, this is the governing equation you have of course, there would be some symmetry at the center that that is always there I mean, if you need to we can use that that as boundary condition, as well or it will follow automatically (Refer Slide Time: 25:47). Now, this is the governing equation and this is the boundary condition, the solution to this equation would be $v_x(z)$ that is equal to $\zeta - \frac{\phi_{eq}(z)}{\eta}$ into $\frac{\epsilon \zeta E}{\eta}$, this is the solution to this equation solution to this governing equation with this boundary condition in place.

Now, now if you if you want to if you want to know what is what is πz what is πz , if we if we look at this πz , if we if we bringing those earlier slides, where we had just introduced this, what was the πz we had at that time you remember, we we were we were trying to find out what how would the πz look, how the potential changes right.

We have been trying to find out in the beginning of the class, how the potential changes and we said, if we have simply a wall and extending this this electric double layer simply the bulk extends to infinity, just only one wall we had this expression with λD being the d by length and π would change with z like this. Then we said, if we have two walls, then we have the bulk in the middle and we have electric double layer present here as well as there, then we said that a governing equation for π remains same in that case, and the boundary condition will change, this is the boundary condition at plus minus h by 2 this is equal to ζ , and at the center symmetry exist.

So, this is what we said was the case with π the potential, and then we we came with this expression for the potential π is equal to ζ into cos hyperbolic this divided by this. So, this is the case for a channel, this is the case for a single wall right. So, this is this is this is the case for a channel, so we said the π will follow this expression. So, now we we we intentionally we we developed this expression for π , because we know this π would be handy here, here what did we do, we put the body force term in in the Navier-Stokes equation and solved it for one-dimension solved it for one-dimension.

And by solving this for one-dimension with this boundary condition that, at the wall the velocity is 0 with this boundary condition, we came up with this expression, now we need to replace this π , because π does not this π will come from here all right (Refer Slide Time 28:59). So, this π we we have intentionally we developed this π for a channel, with two walls to be used as part of this expression, because our final aim at the very outset I said is to obtain a velocity profile first thing, and second thing is to find out how much how much flow we we have two issues here.

When it comes to electroosmotic pumping we have basically two issues, one is that how much velocity how much average velocity you can generate, suppose I put this pump in place, how much velocity it can generate, when there is absolutely no pressure gradient existing. But, still by simply by electroosmotic flow you are generating, so much of velocity that is one issue. Second issue is I mean immediately you will question is where

will you get such ideal situation that, there is no pressure drop **I mean if you** if you are pumping something then, there would be a back pressure, you would be pumping against a back pressure. So, up to what back pressure **it can it can** it can deliver the fluid, because if the back pressure is beyond the limit then there will absolutely be, there will be no flow.

So, if we have a back pressure then, how would be the velocity profile and what would be the flow rate. Now, what we are doing now is we are obtaining the velocity profile, with the assumption that pressure at x equal to 0, and pressure at x equal to 1 both are **both are** 0, p external at x equal to 0 is 0, p external at x equal to 1 is equal to 0, and p_i external this is 0 here, and this is Δv here **all right** (Refer Slide Time: 30:31).

So, with this condition we are coming, so this is the first case we are solving, later on what we will do is we instead of putting it 0, we will **put** some pressure here, put some back pressure here that means, you are pumping against the back pressure, and then we would like to know what would be the velocity profile. So, first case no pressure **we are all** we are almost we are close to getting the velocity profile, this is the velocity profile we obtained and this $p_i e q$ has to be replaced by this expression that **we have already gotten** we have already gotten for a two channel, two wall system.

For a channel system not a single wall system, for a wall system this is the expression that we already have. So, if we bring in that expression there, so then the $v_x z$ would be equal to $1 - \cos \text{hyperbolic } z \text{ by } \lambda D \text{ divided by } \cos \text{hyperbolic } h \text{ by } 2 \text{ divided by } \lambda D$, $h \text{ by } 2$ is the half aperture into $\epsilon \text{ zeta divided by } \epsilon \text{ into } E$, this is the expression you have for the velocity profile.

See, I have a z term here, this z term is changing, z is equal to 0 where, if this is the channel I am having an electroosmotic flow taking place through this channel, z is equal to 0 here, z is equal to plus $h \text{ by } 2$ here, z is equal to minus $h \text{ by } 2$ here. So, within this framework the **velocity is** velocity I have a velocity profile, and these velocity profile is expression for this velocity profile is given by this expression.

Now, if you ever get a chance to plot this, **I mean** you can plot this **you can** you can take some sample values and you can plot v_x , just the way we have plotted for a poiseuille flow, we have plotted parabolic velocity profile and all. So, if somebody wants to plot this expression, what you will end up with seeing is that the velocity profile looks like

this, (No audio from 33:12 to 33:24) (Refer Slide Time: 33:12) it is pretty much flat. And there is minor change; **I mean** it has to be 0 here, so from 0 to the final value, **that happens within a short distance** that happens within a short distance **that is** that is a trend of this **this** expression. So, this is pretty much flat and if somebody wants to know, what is this velocity he will find probably you can check with the limit, as well it will come that way, that **this the** this **this** part of the velocity this v , this is actually referred as e_o , v_o electroosmotic velocity, which is $\epsilon\psi\zeta$ divided by η into e .

So, **this is this is** this is predominantly this is the velocity over the most part of the channel, and near the wall it the velocity changes from here to 0. So, for all practical purposes, not for all practical purposes **I mean you can you can** you can if somebody wants to know, **what is the** what is electroosmotic velocity, for quick calculation you can use this expression, **this is** this is pretty much the electroosmotic velocity **which is** which is constant for most of the cross section.

Look at the look at the look at the parameters that is involved here $\epsilon\psi\zeta$, ζ is that ζ potential of that electrolyte walls that system, the you have a solution, you have ions moving around and you have developed some ζ potential on the wall, it is that ζ . η is the viscosity of the fluid, which is just simply coming because, you have that Navier-Stokes equation and there you have $\eta \nabla^2 v$ term, so this η is coming from there.

E is the electric field that we imposed, so this is the electroosmotic velocity that you have. So, if somebody wants to find out, what would be electroosmotic velocity under certain situation, so **he has** he needs to use this formula, and if he needs to know really how the profile would be, and if he is asked to draw precisely, what the profile would be then he needs to resort to this expression **all right**. So, **this is** this is what you have **as far as the** as far as this **this this** electroosmotic velocity is concerned.

Now, the issue here is that, **we we have assumed** we have assumed here that this as I said p_{external} is equal to 0, P_{external} is equal to 0, now what we will do is, this is Δp this is fine, here instead of 0 we would put this as Δp that means, we will try to find **if it is** if it has to act as a pump it has to act against a back pressure, that is the idea **right**. You have to pump the fluid through a channel; you have to pump the fluid through certain media like this, so that media or that channel will impose a back pressure, if the

outlet of the channel let us say open to atmosphere. But, to **to** get the flow going through that media, you **you** need overcome on back pressure, so instead of 0 we need to put this as ΔP . So, **so** in that case this ΔP term in Navier-Stokes equation, **we we had this** we had this ΔP term earlier, **this ΔP term was not used at all** this ΔP term was not used at all in the Navier-Stokes equation (Refer Slide Time: 37:00).

Now, what we will do is, we will use this ΔP term in a sense that, this **this this** ΔP term would be this **minus Δp external**, now we will have minus ΔP external that is equal to, then **then** it will have a value, it will be minus Δp by $L \cdot e \cdot x \cdot \hat{a}$. So, **this this would be** this is something which we would be imposing, so if we impose this, what would be the velocity profile, **that is what** that is what people would be looking at.

Now, **before I before I give you the before I give you** before I get into the expression part of it, because this again we will have several symbols, before we get into this on the side of this plot, I would like to draw how the how the velocity profile would be, **I mean** can you think about it, how **howhow** would the velocity profile be in such case, that here that there is no back pressure, but in this case there will be a back pressure. You will be surprised to see that the velocity profile in this case would be like this (No audio from: 38:21 to 38:33) (Refer Slide Time 38:21). Basically, this part you are following the electroosmotic flow, because this here you have charges and this charge is trying to drag the fluid.

Because, of this ion drag there is the bulk fluid is also drag with the ion and all these things are happening, so it would be just like this velocity near the wall however, you have a back pressure and this is acting like simple parabolic velocity profile, in the opposite direction, because you have a P there, and you have the you do not have the P on this side. So, **you have** you have this kind of trend, so this would be the velocity profile that we are looking at.

And with this velocity profile, now you need to know how much ΔP , against how much ΔP this electroosmotic pump can still work **all right**, because you have a pump in place and you need to know **you are** you are pumping it through a media and that media would give a back pressure of so many pascals, so many atmospheres, so many second **(0)**. So, you need to know whether the pump can deliver that kind of pressure or not otherwise, this pump will not work.

So, we need to solve this velocity profile, and we need to find out when this velocity is what the limiting value is, when this velocity is 0 that is the limit. So, **the when when** does that limit reach, when **when** is that limit reached and what is the corresponding back pressure. So, you know that, that is back pressure that the pump cans, that pump can **(())** pump against that back pressure, beyond that pump will fail. So, for that we need toknow the velocity.

(Refer Slide Time: 40:25)

So, with this understanding **let me let me** let me point out that, **this this** expression **would be** here we would be looking at, the Navier-Stokes equation would be something like this. The Navier-Stokes equation would be $\eta \nabla^2 v_x(z) + \epsilon \nabla^2 \phi_{eq}(z) \left(\frac{\delta V}{L} - \frac{\delta P}{L} \right)$, **this is the expression that you have all right** this is the expression that you have. And typically the solution here, **I mean** this **this this this** system of equation that allows you to may get a solution, which is basically superimposed, called superimposed velocity, the velocity that is arising from pressure and the velocity that is arising from electroosmotic flow.

So, the sum of these two, so it would be like this $1 - \frac{\cosh(z/\lambda_D)}{\cosh(h/2\lambda_D)}$ into $\frac{\epsilon \zeta}{\eta} \left(\frac{\delta V}{L} - \frac{\delta P}{L} \right) - \frac{((h/2)^2 - z^2)}{2\eta\lambda_D} \frac{\delta P}{L}$, I am not writing it as E instead of this I am writing it as δV by L, because that is easier to understand minus, this part is the pressure component $\frac{h^2}{2} - z^2$ minus z^2 $\frac{1}{2} \frac{\delta P}{L}$ **delta p by L all right**. So, this is the component which is arising

from pressure, and this is the component which is arising from electroosmotic velocity, and this is the combined $v \times z$, and if this $v \times z$ is plotted you will get the profile which we have shown here (Refer Slide Time: 42:36). In this case you will get this kind of profile.

Now, we are not very much **I mean I mean** profile is fine, we **we** intuitively also we new that there is a profile, what we are interested in at this point here is two things, that I mentioned in **in in** the beginning itself, two issues or two concerns are here. One is electroosmotic flow at 0 back pressures, and the other point is back pressure needed to exactly cancel the **E o** flow.

There are two issues here, **I mean you you** must appreciate this, one is that 0 back pressure what is the electroosmotic velocity you have, electroosmotic flow you have and the other is up to what back pressure it can still have an electroosmotic flow, beyond which the flow will stop. So, from the design point of view, if somebody is going to use this pump these **these** two factors are immensely important to him. Now, if we pick up some standard values for example, if zeta is equal to 0.1 volt, for a cylindrical channel with radius is equal to say 0.01 millimeter.

For length is equal to 0.1 millimeter, **eta is equal to 10 to the power** eta is equal to 10 to the power minus 3, **what is** what is the viscosity of water minus 3 Pascal's second, Pascal second and if the ϵ_{psa} is equal to say you take some value of say 100 ϵ_{psa} naught, you get about **this this electro osmotic flow** this electroosmotic flow would be about 0.2 nanoliter per second per volt. And this back pressure would be 5.5 Pascal per volt that means, you apply 1 volt of Δv across this electrode, you get this much of flow and you get this much of back pressure, beyond this back pressure the flow will stop.

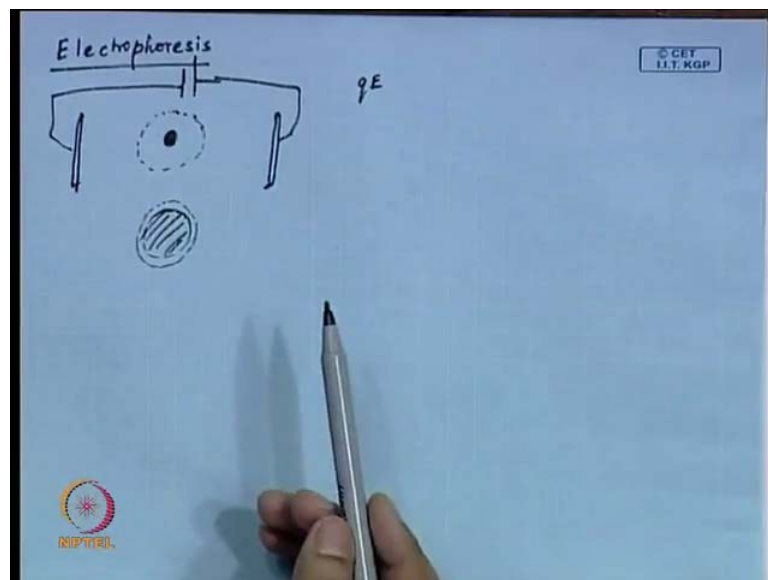
So, naturally these schemes does not, see this does not seem to be very attractive proposition, because the flow rate is pretty small and the **pressure drop** pressure also it can stand is very small, you must appreciate. So, there is the way out and people who are using this electroosmotic pump, they are not using a single channel, they have overcome this problem, and **they have** they have used this pump successfully, with the flow rate that we expect typically in a **in a** microchannel system.

What they do is, they use several channels not just single channel and by several channels **I mean you can** you can have a porous medium that means, you can have beads,

several beads packed inside the you you you have a larger channel and within the channel you have several beads. So, that you have multiple parallel channels operating simultaneously, and that can overcome this problem all right, because this would be this would be evident if you if you if you look at this this problem itself. I mean if you look at the expression (No audio from 47:30 to 47:54), if you if you have large number of large number of channels then this this this problem can be can be circumvented, this this problem you can you can get away with this problem.

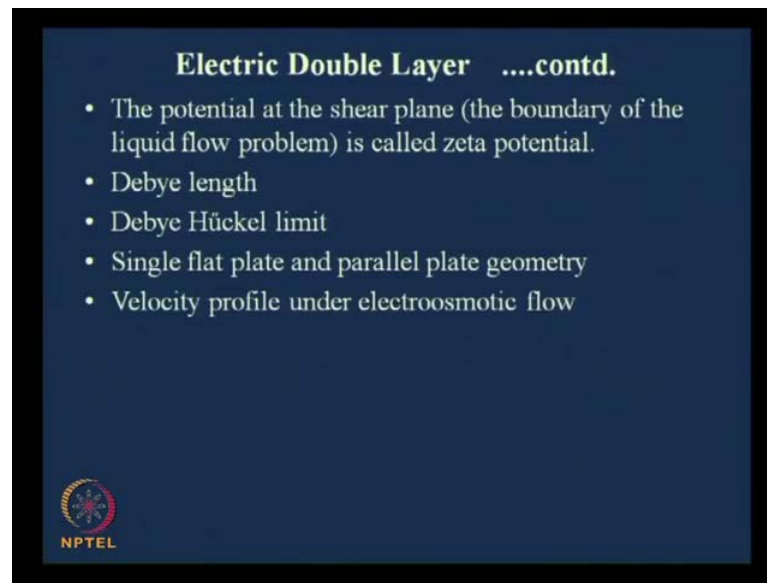
And in fact, you can have the pressure drop to the to the tune of say, then then you you can have the flow rate to the tune of microliter per second pressure drop several kilopascals. So, these are these are these are very much possible with with a channel, with either with (()) or closely packed glass spheres. So, that you can you can generate large number of parallel channels that is that that is there. So, any electroosmotic pump would become comprised of such (()) inside the channel or the channel will be packed with beads, so that you can several parallel channels operating (No audio from 48:58 to 49:15).

(Refer Slide Time: 49:27)



I will see if we if we if we have any, if we need any clarification on this issue, we can we can probably discuss this in the next class. Otherwise, the next topic that I will pick up is electrophoresis; next topic that I will be picking up is electrophoresis.

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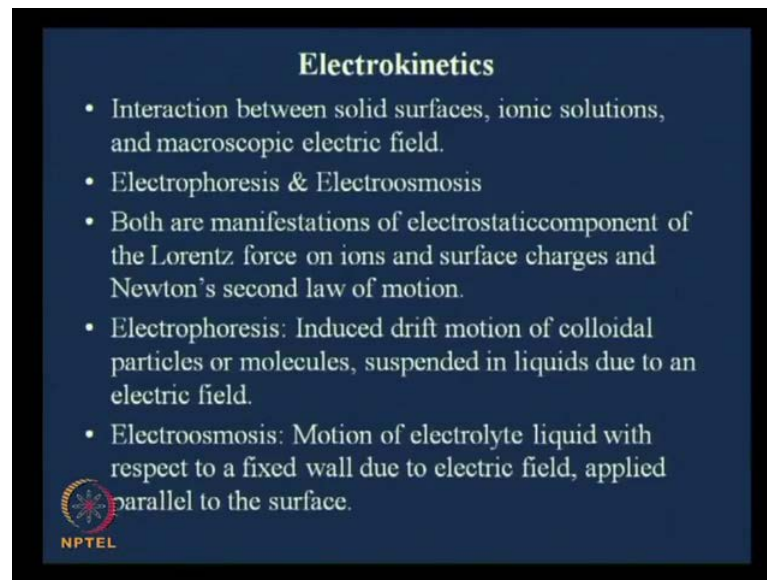


So, if I quickly look at, what we had in this in this power point slide is that, we have discussed about the potential at the shear plane, the boundary of the liquid flow problem and which is called zeta potential D by length we discussed which is λ_D , basically Debye Huckel limit, we discussed. This all our analysis is based on, the entire analysis is based on Debye Huckel limit that means, this $\sinh u$ is equal to u , that we had that Taylor series expansion, we note the higher order terms with that understanding. So, that is the Debye Huckel limit.

Single flat plate and parallel plate geometry, that we I think we understood by now, that in one case we have simply that one plate, and we have the π changing with z and in other case you have two plates at z equal to plus $h/2$ and z equal to minus $h/2$, that we have looked into and the velocity profile under electroosmotic flow, that we have already established.


The next topic that I that I that I am going to pick up is basically electrophoresis, electrophoresis once again I mean this is this is induced drift motion of colloidal particles we, I had a definition of electrophoresis at the very beginning.

(Refer Slide Time: 51:07)



Electrokinetics

- Interaction between solid surfaces, ionic solutions, and macroscopic electric field.
- Electrophoresis & Electroosmosis
- Both are manifestations of electrostatic component of the Lorentz force on ions and surface charges and Newton's second law of motion.
- Electrophoresis: Induced drift motion of colloidal particles or molecules, suspended in liquids due to an electric field.
- Electroosmosis: Motion of electrolyte liquid with respect to a fixed wall due to electric field, applied parallel to the surface.

 NPTEL

You remember, electrophoresis is induced drift motion of colloidal particles or molecules suspended in liquids due to an electric field. So, what we have is, we have an electrode here, we have another electrode here, and we have a particle we have a particle here (Refer Slide Time: 51:21). Now, this particle has certain charge in this and you have already imposed an electric field, you have imposed an electric field E and this particle has charge q .

So, if the particle is small, then the force that is exerted here, that would be equal to q into E and that force, you will be that force would be imposed on the particle, if if these if these are connected if these are connected if these are connected this that force would imposed on the particle. On the other hand, if this particle has to move, because of this force there would be drag imposed (ζ) , if it is in the liquid if this is the particle moving, so then there will be a drag force. So, there will be a balance between these two forces.

So, we we would be we would be discussing in the next class, what would be the what would be the velocity of this particle, if the particle is small small in the sense, the particle is small and it has a huge Debye length, because this particle there would be a formation of Debye length formation of an electric double layer around it or the particle is big this is the particle and you have a small double layer forming around it (Refer Slide Time: 52:45). And then this would be pooled, so what would be the velocity this particle will acquire, so if you idea of this electrophoresis at the very outset, we we mentioned is

that if **if** you have particles several such particles of different size or charge, and this particle would be attracted to the electrode by different forces. So, if you hold this, under this electric field for some time you will see that particles of same size and charge, they would be accumulating in one band particles which could not flow at that velocity, **they will not be** they will be at some other place.

So, you will be having some kind of classification **within this within this within this** within the body of this material. So, that classification **we are** we are looking at that, that is that is what we would be calling electrophoresis, now what we would be doing is we will be taking these two cases separately in the next class.

And we will see how **how** we defined force and how we defined the velocity in this case, and what would be the utility of this whole scheme; this is very good to find out, what **what** particles you have, this particle could be a cell, biological cell, this particle could be a clay particle. **(())** particles have different sizes and charges that can be classified **using this** using this technique. **I have** that is all I have for today's class, if and **and** I will try to see if there is any **any** outstanding question on this electroosmotic flow, I will try to address them in tomorrow's class, that is all I have for today, thank you.