Microscale Transport Processes

Prof. S. Ganguly
Department of Chemical Engineering
Indian Institute of Technology, Kharagpur

Module No # 01
Lecture No # 16
Dispersion

I welcome you once again to this class of micro scale transport process. In the last class we have been discussing mixing and more importantly mixing quality and mixing effectiveness. In the last class we defined a term called alpha, which changes from 0 to 1 in the mixer, alpha is equal to 0 at the inlet. When the flows are considered completely segregated and alpha is equal to alpha should be equal to one at the end of the at the outlet. So, that you can consider mixing to be complete. So, this is a term which is basically the mixing what we define this as mixing quality. So, if I have a stream and if somebody says that how good is this how good the mixing has been done so, this alpha is a good parameter for that.

However, the other term that we are interested in here other than mixing quality is what you call mixing effectiveness mixing effectiveness. That means how effective the mixer is, because you know that, if we have two streams flowing side by side. So, at some point they will all mix I mean may be it will take one centimeter may be it will take one meter length, but at some point of time it will be completely mixed. So, you can always realized alpha is equal to 1 at some length, but that is not exactly the objective, objective is to minimize the length because you have to minimize the length. And not only minimize the length you have to see how effective the mixer is in terms of pressure drop because you can have a very short length and you can achieve the mixing.

However, you may encounter a good amount of pressure drop and then that would be deterent, because you can only have so much of pressure drop. You remember when we worked with this micro fluidic devices we had those inlet and outlet fittings. We have a capillary tube which goes into the this micro fluidic device. And then how we arrange the ceiling? We arranged we put a plastic coupler; we put a press fit arrangement. We have worked with so many arrangements and some arrangements for example, the press fit one at the very outset we said that the press fit one is good, I mean you can just push it and it goes in there, but it can hold only to certain pressure beyond some pressure it will start leaking from there.

So, if you have huge pressure drop within the micro channel that means at the inlet port you have to the inlet port, would be exposed to a very high amount of pressure and you cannot afford that because then it will start leaking. I mean it is a very simple very trivial problem so; the pressure drop should be I mean you have to keep eye on the pressure drop. So, you have define mixing effectiveness such that it gives you highest amount of revenue for a given effort. So that is idea of mixing effectiveness.

So, we have this mixing effectiveness defined as revenue divided by that is what level of mixing you achieve divided by the effort you expend now there is a term called Euler number in this context this is typically, this Euler number is used in connection with vertices if you if there is vortex the amount of pressure drop that you have to expend. That you have to provide is given is defined in terms of Euler number. This Euler number is nothing but delta P if there is a vortex element involved then delta P v divided by rho w bar square. Where w bar is the average velocity (no voice04:17 to 04:24) and delta P v is the pressure loss caused by vortex creation.

So, this is the Euler number associated with the vortex and that is how you define Euler number is delta P v divided by rho w bar square. Now, you can in fact talk about specific energy dissipation(no voice 05:00 to 05:07)which is defined as delta P v the pressure drop multiplied by volumetric flow rate divided by mass into which energy is dissipated. So, then in that case you can write this as delta P v if cross section of the mixer you write this as A m so, volumetric flow rate would be A m into w bar divided by mass into which this happens so, it is rho A m and l m. l m is nothing but the length of the mixer.

So, this comes to be equal to E u v Euler number, the Euler number you can have a small subscript as v. Euler number associated with a vortex w bar cube divided by l m so, this is the specific energy dissipation. So, this is the amount of energy that is getting dissipated in this context now if you define the mixing effectiveness we said is revenue divided by effort. Now what is the revenue here, we can write that this revenue is higher if shorter mixing length at a large hydraulic diameter so, if you have a shorter mixing length it would be higher if shorter mixing length at a larger hydraulic diameter.

So, this is how we have the this is the idea that we have as far as revenue is concerned. So, in that case the revenue divided by effort, and the effort is basically the effort would be we can write this as the Euler number itself. So, this mixing effectiveness now we said revenue is higher, if shorter mixing length at larger diameter so, this revenue then in that case is equal to d h hydraulic diameter divided by mixing length. So that this revenue in the sense if you have shorter mixing length then the revenue would be higher. And if you can do it for a large amount of stream a large volume of stream that means if you have higher hydraulic diameter.

So, this is how you are defining the revenue such that revenue is higher, when you have when you achieve mixing in shorter mixing length for a large volumetric flow. So that is how you are defining revenue, now one thing you must understand this l m is nothing but w bar into t m. And A m this is of the order of d h square hydraulic diameter to the power square I mean that is that how the hydraulic diameter is define I am not saying it is equal, but it is of the order of and l m is w bar into t m. So, in that case t m is the mixing time and w bar is the average velocity.

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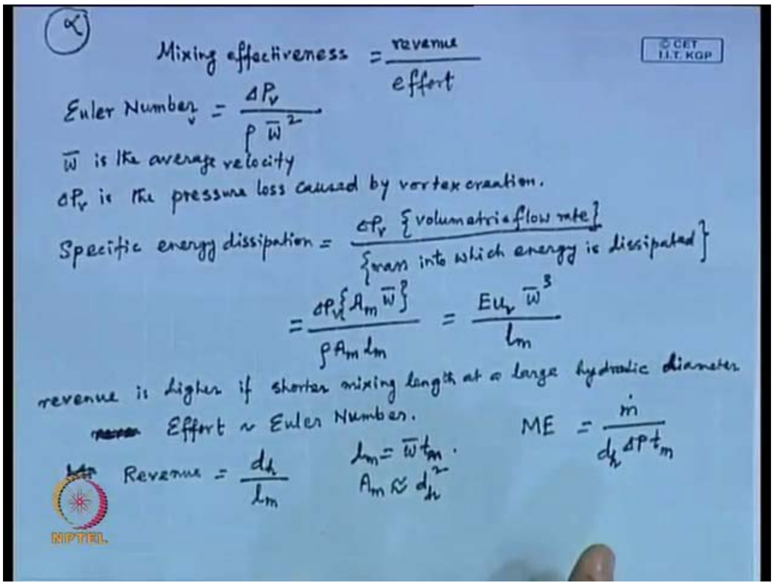

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Mixing effectiveness $= \dfrac{\text{revenue}}{\text{effort}}$

Euler Number $= \dfrac{\Delta P_v}{\rho \bar{w}^2}$

$\bar{w}$ is the average velocity

ΔP_v is the pressure loss caused by vortex creation.

$$\text{Specific energy dissipation} = \frac{\Delta P_v \{\text{volumetric flow rate}\}}{\{\text{mass into which energy is dissipated}\}}$$

$$= \frac{\Delta P_v \{A_m \bar{w}\}}{\rho A_m l_m} = \frac{Eu_v \bar{w}^3}{l_m}$$

revenue is higher if shorter mixing length at a large hydraulic diameter

Effort ~ Euler Number.

$$ME = \frac{\dot{m}}{d_h \Delta P t_m}$$

Revenue $= \dfrac{d_h}{l_m}$ $l_m = \bar{w} t_m$. $A_m \approx d_h^2$

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So, in that case mixing effectiveness this M E is defined as m dot the mass flow rate divided by hydraulic diameter delta P into t m. So, this is what you define as mixing effectiveness in some cases they multiply a Reynolds number this, but I am not getting into that let us focus on a mixing. This is standard way of thinking about mixing effectiveness, now I mean it is open to debate I mean you can add or remove some terms or you can improve the mixing effectiveness, but this is one standard way of defining mixing effectiveness.

So, what all did we define here I mean you got to understand that we have these. I mean if it if somebody sells a mixer if somebody makes a mixer and somebody wants to sell that to you what will you ask from him? One thing you may ask is that if I have this is the stream I mean I define the stream what would be the mixing quality. That index alpha at the outlet that could be a way looking at it, I mean at you can ask for it you can find out what it is Euler number or you can find out what is the mixing length l m. More importantly you can ask him what is M E, what is the mixing effectiveness so, these are some of the parameters you can ask from him or you can expect that they will provide this parameters.

A person who is making a mixer and will selling to you can expect that they will provide these factors. So that if you have to make a comparison I have a choice of say four such mixers from four suppliers which one I should purchase so, these are the parameters that will guide your decision. So, this is I mean you just perhaps some thought into it that there should be some parameters some guiding parameters to decide which mixer is more effective in your situation. And this the this is how it is done now it is open to discussion, and you can add or remove terms I mean if it is more if it is if you think it is more pertinent. However this is a framework that people have already thought about, the researchers have already thought about so, with this I am I mean with this I practically complete the discussion on mixing.

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Herringbone Micromixer

- Series of herringbone shaped grooves placed in the channel
- The hollows force the fluid to flow obliquely with respect to the direction of principal flow.
- To satisfy mass-continuity, return flows develop, resulting in helically shaped trajectory of fluid.
- Pattern changes, say every five herringbones. Accordingly, the centre of fluid helices get displaced.

Mixing quality and mixing effectiveness

The next topic that I pick up so, let me recapitulate what all we have done we have defined various kinds of mixers segmented flow, sequential injection, then having herringbone type mixers. In fact that description is on the slide itself we have discussed about having grooves on the floor and then we have cross flow, we are inducing cross flow and then we are intentionally shifting the center of the helix from left to right. So, that you have the cross flow very effective, if you have the cross flow helping you accomplish the mixing.

We have talked about various mixing strategy at the very outset, we said that if we have two streams flowing side by side then also we can expect some mixing because of diffusion. But that will take a very long channel I mean the channel has to be really long it is one centimeter to 100 centimeters. It is that way we have already shown the using some dimensionless numbers so, naturally that is not the way to do it so, you have to induce some amount of cross flow and that cross flow can be induced. This way or it could be that you have an on off type situation solute, then solvent, then solute, then solvent so, you can have that kind of situation as well.

And more or less we are governed by the mass balance equation over a differential element using diffusion coefficient. That is that has been the governing equation however we used it in different context. And I showed that you can I mean if it is a completely unknown mixer which do you do not have any guiding principle you can probably give some give some this you can give it. I mean you can define a stretching function and you can probably I mean I have shown that there would be two limits. One is the peclet number and the log of peclet number and you will be in between them, whether you are in linear stretching or elongation stretching. So, you have some way of characterizing a mixing process or characterizing a mixer so, by now you should have fair idea how to get this done.

Next topic that I pick up now is known as Taylor dispersion, this is you remember when we were talking about that on off mode of mixing there we have instead of a diffusion coefficient I mentioned dispersion coefficient. And I instead of d I have written d star there is a reason to it I mean this when we talked about spot of tracer diffusing into an infinite medium we restricted ourselves to diffusion coefficient. However when there is a flow as well as diffusion then we cannot we have to take some additional terms and then we call this process dispersion rather than diffusion. See the first if you look at in the look at the slide this.

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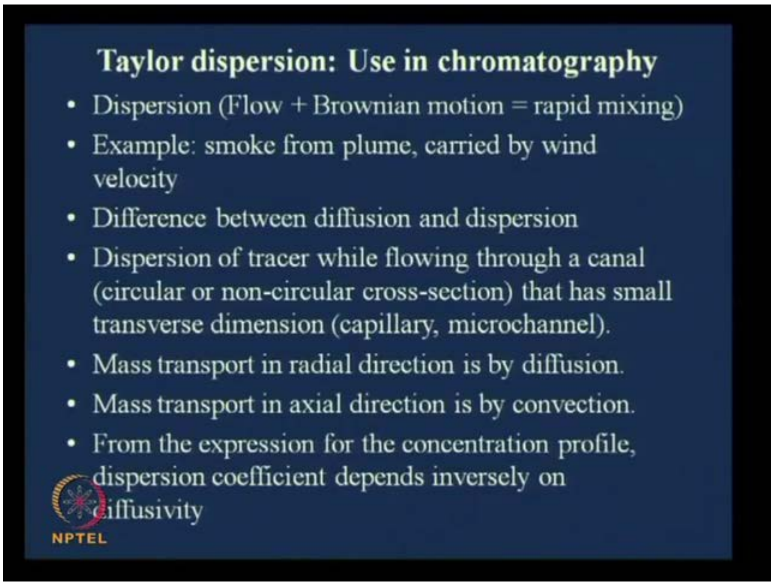

Taylor dispersion: Use in chromatography

- Dispersion (Flow + Brownian motion = rapid mixing)
- Example: smoke from plume, carried by wind velocity
- Difference between diffusion and dispersion
- Dispersion of tracer while flowing through a canal (circular or non-circular cross-section) that has small transverse dimension (capillary, microchannel).
- Mass transport in radial direction is by diffusion.
- Mass transport in axial direction is by convection.
- From the expression for the concentration profile, dispersion coefficient depends inversely on diffusivity

Taylor dispersion there the first topic is dispersion is flow plus Brownian motion which is equal to rapid mixing. So, diffusion is basically the Brownian motion only the in by molecular interactions. The some particle is diffusing into the medium if you have a flow associated with it then you call this a dispersion. A good example would be smoke from plume carried by wind velocity so, what you have in a plume I mean you have smoke coming out and then that smoke has been carried by the wind velocity. So, the smoke is diffusing I mean if there was not any wind velocity if just the smoke has been released there so, smoke would be diffusing gradually. That you know, but since there is a wind velocity so, smoke is further carried downstream.

So, you are having a flow as well as diffusion and this is a classical problem of dispersion. So, you must appreciate the difference between diffusion and a dispersion that is the third point. Now dispersion of tracer while flowing through a canal whether circular or non circular cross section that has small transverse dimension that is what we are interested in and that is referred as Taylor dispersion. What is that now suppose you have a flow going on all you have a flow going on and you have a parabolic velocity profile.

So, this Taylor dispersion mind it is important in a micro channel when the transverse dimension is small. If it is a big channel I mean large diameter channel absolutely that is not at all a problem because there you will have a potential flow I mean the effect of viscosity would be less it would be a potential flow at the center. So, there this problem may not be important at, suppose this is a parabolic velocity profile. And you have give a pulse so, suppose this is the pulse, this pulse is injected you understand how a pulse is injected, I am little shifted here you understand how a pulse is injected?

That means I have a tube through which water is flow in gat constant velocity that the dimension of the tube is very small it is a very thin capillary. You understand what a capillary must be it is a advanced level course, I mean the capillary ideal. Example of a capillary would be what you see inside a thermometer that thin tube through which the mercury rises ideally that is called a capillary. It is a very thin tube, through that some water is flowing and it has to be a parabolic velocity because the transverse dimension is really small. If the transverse dimension is like big channel, I mean it if it is one meter wide channel then probably you cannot expect. I mean you may not expect the flow to be fully developed and parabolic and all this however this is the case for a small capillary.

Now, you are introducing a small amount of die for a short while so, it is just a pulse of pressure. That means you introduce suppose you put a put an injection syringe take an injection syringe and pick up little bit of color colored say ink from a ink bottle, you pick up a small amount of ink and you introduce it at the inlet just push the piston within a short time and then remove the piston. When water flow continues so, there by what you give is a pulse this is what you have introduced in the flow now this flow as it goes through the tube it has to take a parabolic shape.

So, what would be the shape of this as it travel downstream so, it will take a shape like this, (no voice 19:19 to 19:32) do you agree with this that this will take a shape like this has to because it is it would be so. So, what I am essentially saying is that the central part of this block is travelling faster and apart of the block which is next to the wall is practically having zero velocity. So, that is how it is, that is how it is flow in, now what did you expect if I mean if it would have been just simply diffusing, how did we work with that, we said that you remember I mean let us look at this as a Dirac function.

We had a Dirac function and then we said that this would be next one would be this one then next one would be this one. Like you remember that is how the concentration will decrease so, it will be diffusing out, but now the process is completely different. Because diffusion will take place from here to here, here to here, here to here, here to here. So, the diffusion process is completely different now it is not I mean you expected that diffusion to take place in this direction. And in this direction the way it is for a spot of tracer, but that is not the way it is happening this way.

Now, if you pick up from the outlet if you pick up the concentration I mean if you find what is the concentration is now within a capillary. I do not expect that you will find a concentration distribution in this direction because this dimension is very small. So, I do not expect you to find the concentration distribution here rather at the outlet what you get is you pick up a slice measure the concentration. You pick up the slice next to one next that you receive then you pick up the slice that you receive next and measure the concentration, which is basically the average of these average over the entire cross section.

So, whatever you receive here that is the average over the entire cross section this is called a mixing cup concentration I mean it is like you take that slice mix it and get the average concentration. That is how you can this is a thin capillary and at most what you can have is a outlet it connected to a detector. So, detector would be detecting, but detector would be detecting what concentration the average concentration of this entire cross section. That is all it can detect, now pick up an average concentration. What do you get here? Suppose I pick an average concentration of this cross section I will get some amount of this which is not the solute at all some amount of solute and some amount of the other stream which is not solute at all.

So, that is what we will see if I pick up this one what do we see same thing some part is solute some part is not solute. So, think of it that suppose I do not that diffusion coefficient is negligible close to zero still if I look at so, I have given a pulse here. Pulse is actually not a Dirac function, pulse is this is a pulse so I have given a pulse here so, what do you expect here. Even if the diffusion coefficient is zero still I would expect this would be what we get so, what is this I mean do you understand what that axis is? Axis is basically time and this y axis is concentration detected.

So, do you understand what is what we are trying to say here this pulse or I mean if I mean what was our expectation our expectation was that if there is no diffusion or diffusion is negligible the we would have got ten this pulse the same there. If you put something in the water and that is having negligible diffusion to the medium around. So, that would remain the same remain there that way that is what you expect so, that is what you expect here. I mean if this would have been the if there is a pulse like this you would have expected what is coming out this, pulse means this is the concentration versus time. This is the concentration and this is the time so, concentration versus time there is pulse.

So, at the outlet also you should have received the concentration versus time as a pulse, but instead of that you are getting broaden peak. And this broadening is taking place because of this velocity profile this parabolic velocity profile even if the diffusion coefficient is not significant. So, these process this is what is referred as Taylor dispersion. And this is something which you have to be very careful with when you are working with a laminar flow in a channel which has a very small transpose direction.

Now let me give you the final result, what you get as a concentration profile, I mean I will try to come up with an come with a governing equation and boundary conditions and everything, but I would first I would like to give you the final expression. The final expression looks like this (no audio 25:04 to 25:44) here the M is total solute in the phase sorry total solute in the pulse, R zero is equal to tubes radius, z is equal to distance along the length of the tube, v naught is equal to fluid velocity, t is equal to time and e z is referred as dispersion coefficient.

So, this is the same thing what we have referred as d star in earlier and this is the dispersion coefficient. And this dispersion coefficient is equal to r naught v naught whole square divided by 48 D this is also referred as at time as peclet numbers for this dispersion this is also referred as peclet number for this dispersion. Now, what do we have here I mean if we look at this m divided by pi R zero square do you does this ring a bell M is the total solute in the pulse and pi R zero square r zero is that radius of the tube. So, pi R zero square is the area over which the diffusion takes place so, you remember when we talked about the diffusion of a spot of tracer in an infinite medium there also we had m by a term in the numerator.

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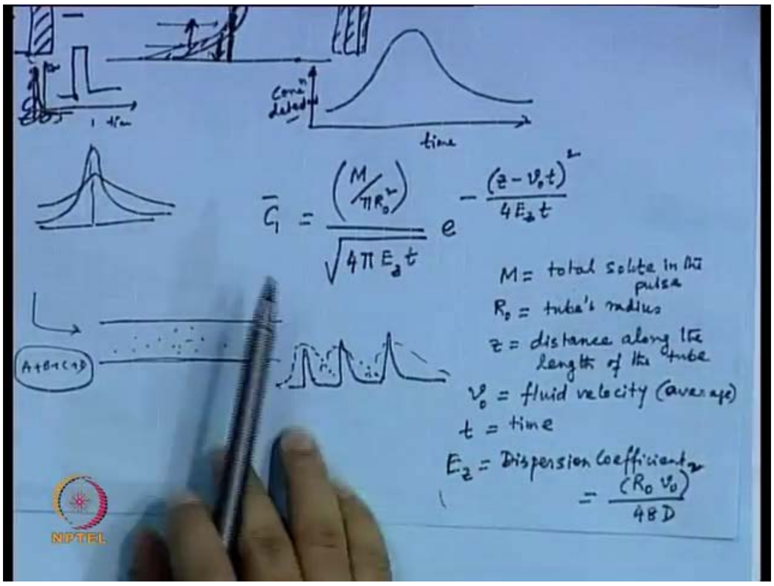

$$\bar{C}_1 = \frac{\left(M/\pi R_0^2\right)}{\sqrt{4\pi E_z t}} e^{-\frac{(z - v_0 t)^2}{4 E_z t}}$$

M = total solute in the pulse

R_0 = tube's radius

z = distance along the length of the tube

v_0 = fluid velocity (average)

t = time

E_z = Dispersion Coefficient $= \frac{(R_0 v_0)^2}{48 D}$

So, this M by pi R 0 square is that M by a term that is one thing we I want you to appreciate the other thing I point out here is this z minus v 0 t term does this ring a bell z minus v 0 t z is the distance z is the distance along the length of the tube or in other words distance from the inlet. So, the tube runs like this and from the inlet you have you calculate z and v naught t v naught is the average velocity of the v naught is the average fluid velocity means that is basically the average. So, because since we have a parabolic velocity profile so, you consider this to be average.

So, v naught t is basically the place where it is I mean where the pulse would have been if it is traveling at average velocity over time t. So, z minus v naught t is since there is a movement of the pulse I mean diffusion plus flow this flow part you can separate. If you define a coordinate system which is z minus v naught t, do you does this make some sense v naught t would have been the shift of that center of the pulse because of the flow and z you are calculating from the inlet your reference. So, z minus v naught t is another coordinate system where you are this is sort of a moving coordinate, the moving coordinate with the reference to the center of the pulse or the place where the pulse would have move by average velocity so, z minus v naught t is this term.

The other factor that I must point out here is that this dispersion coefficient. Here we have E z this expression is very similar to the expression that we had for the spot of tracer moving into the infinite medium. But instead of E z at that time we had D I think diffusion coefficient, but now we have E z, but the most important thing here is that this E z this dispersion coefficient. These expressions look at this expression R naught v naught square divided by 48 D. So, D is in the denominator an earlier you had E z this was equivalent to D so, that time D was in the numerator.

So, that means D has an inverse effect whatever D had on the diffusion of a spot of tracer infinite medium whatever the D was doing the diffusion coefficient is doing just the opposite to it. Do you appreciate because D is in the denominator if D increases the process is going other way and there if D increases the process goes in some other way. So, this effective D is completely different and another factor is that D E z which equivalent to D that is dependent on v zero square. So, that means these the factor that is responsible for the spreading of solute, these spreading I am talking about the factor that is responsible for this spreading of solute.

That factor is basically that is the spreading is defined by v zero, not only v zero v zero square that means if you have a higher average velocity you can expect more spreading. If you have so, so these are these are some of the things that you need to similarly, if you have a higher R zero you would have more spreading. So, if somebody say that I as a matter of fact this Taylor dispersion is very important in connection with chromatography. Do you understand what a chromatography? How chromatography process works? I mean we have these h p l c and g c gas chromatography. And this h p l c these are typically you have some experiments in masters level you do some experimental with these instruments.

These instruments here you have suppose you have A plus B plus C plus D and you would like to identify which component you have suppose you have a sample which as ABCD. And you do not know what is A B C Ds are so, what you have is you generally in conventional chromatography you flow this solution or you have a carrier stream going in and these A plus B plus C plus D which is the sample. You introduce this in the carrier stream as a pulse just the way you have done it here in a similar manner. You introduce them as a pulse these A plus B plus C plus D and then you have these adsorbent I mean it in a conventional chromatographic unit you have a packed bed. And this packed this packing on the those beads basically packed bed means it is a bed filled with beads.

So, there you flow can takes place through the side by beads that is idea of a packed bed. Now, this wall is coated it has a special coating and this wall adsorbs. So, when these A plus B plus C plus D when they are flowing now they have some preferential absorption. May be this A is absorb more B is absorb less it is that way so, when this sample is passing through it would be immediately absorb and then it will be released at a later time. So, what you see here is at outlet if you have a good detector you would be seeing peaks like this because one would be eluting at sometime, the other one would be eluting later. It depends on absorption desorption phenomena.

As far as this component one of this component is concerned with the absorbent that is put there. So, that is how it is it happens and you see these peaks now in a micro scale process you expect you do not expect the beads to be there two ways it is a accomplished. One is you have a micro channel with the wall coated with absorbent so, the wall acts as absorbent or it could be that you have monolith because that small bed is not very common.

You have a monolith structure that means you have a structure where I mean it is more like a honeycomb I mean it is already inbuilt. I mean this entire passages if everything is built in an integrated structure it is not like that you take a tube and then fill take pack of beds and then fill it there it is it a good example of beads would be and filling of beads would be. If you have seen these water purifier that we have in house hold water purifier they have a resin bed and they every six months probably they suppose to take that bed off and put new ones. So, those are the small beds, but in a micro scale you cannot afford to have a tube and then fill it with beads and all this you have a monolith structure like a cartage you have that inbuilt channels inside like a honeycomb.

So, you these chromatographic units are very helpful in separating this components and then detecting. And you have to of course, you have to first calibrate it you have to send known samples and see where the how the peak appears and then review that with reference to what you got for a real sample. And from there you can come up with what all components are there. Now, you should be expecting you should be getting these peaks and each peak is refereeing to A B C D. Now, if you have this problem going on if you have this dispersion going on what will happen to these peaks? This will not be a sharp peak rather this will be diffuse peak like this would be another diffuse peak like this.

There would be another diffuse peak like because there would be it would broaden the peak would be broaden because of these effect. Now, if because of this broadening you miss out a peak it could be possible that two peaks get merge and you do not get a peak at all. So, when you work with this process it is absolutely important that if these look at this dispersion coefficient E z D you do u naught have any control over D u do not have any control over you have control over R zero you can make it as small as possible. So, that E z is as small as possible so, you have control over R zero so, you can reduce the dimension you take this dimension as small as possible.

So, more important that you have a micro scale structure and you have to have v zero small. So, these are some of the factors by which you can restrict the broadening of the peak so, these are some of the issues. I mean why we are bringing this Taylor dispersion because there are several things in nature which are very interesting to study, but this is hurting us. So, that is why we are force to study so, this is the problem. Now, let us now let us get in to the actual let us see if you can define a governing equation for this.

So, I have this is this is the final result and I should I mean may not give you I may not be able to give you the exact steps all the way, but at least I will try to identify the governing equation boundary condition. That reach to this form, but this equation this is the very classical equation which is true for a diffusion of spot of tracer. And it is true for a Taylor dispersion as well so, you probably can have some feel for this term I mean which is important I mean I do not expect you to remember the entire formula memories the formula. But at least you should develop some feeling for the terms that that is all that is all I expect as far as this part is concerned.

So, let us see what would be the governing equation in this case what we need to do here is if you look at this power point slide what we see here is this so, let me recapitulate quickly what all we talked about Taylor dispersion and use in chromatography. So, it that is what we are interested in here dispersion is flow plus Brownian motion is equal to rapid mixing. Example smoke from plume carried by wind velocity. Difference between diffusion and dispersion that you know already, by now dispersion of tracer. While flowing through a canal circular or non circular cross section micro channels are pretty non circular. I mean if you have if you see the how the etching is done it is we aspect there would be an expect ratio and there would be an angle. Because that is how the etching is takes place.

So, you will end up with non circular cross section that now this Taylor dispersion is important when you have small transverse dimension that means either a capillary or a micro channel. Now what you take into account while defining the governing equation is mass transport in radial direction is by diffusion radial direction is by diffusion. And mass transport in axial direction is by convection because mass transport see in the axial direction there is a flow axial direction. The flow is taking place and in the radial direction you are assuming there is no flow practically.

So, in the radial direction diffusion will take place so that is what you take I mean these are components that you take to get that final result. And what we are going to see is that from the expression for the concentration profile dispersion coefficient depends inversely on diffusivity that means if a if something is more prone to diffuse. What this means is? Suppose you have a species which is very which is likely to diffuse very fast it is likely then in that case one thing you know for sure, is that this dispersion of would be less if it diffuses fast then the dispersion would be less.

Because while doing the dispersion it is basically spreading like a parabolic profile it is spreading, but before it spreads itself I mean as it travels down it is spreading more and more. But before it could spread itself if it diffuses so that is what is happening in this case where it is the diffusivity is more. So, and on the other hand for zero diffusion the pulse will continue to distort unabated if there is a zero diffusion or diffusion which is negligible zero diffusion is not possible. Then we would not be studying dispersion, but if the diffusion is negligible if diffusion is in significant then the pulse will continue to distort unabated. Distort means it is first it was a pulse distort pulse then it is taking a shape then it is going further.

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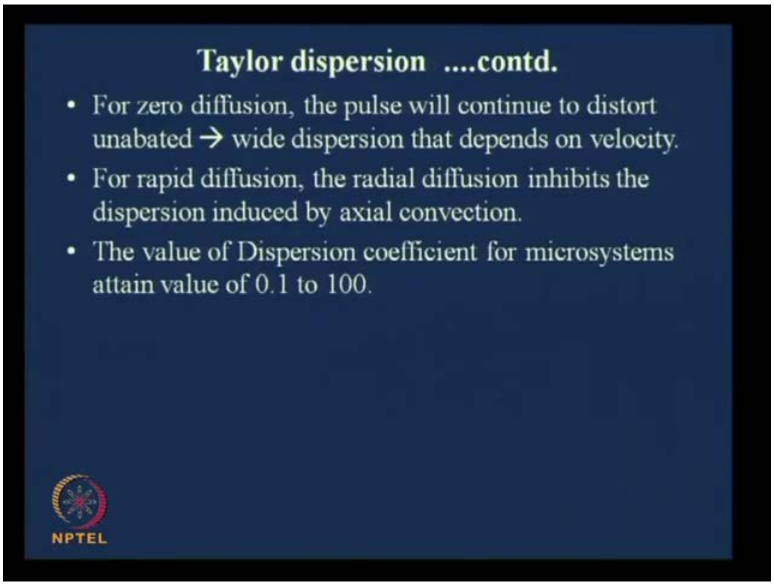

Taylor dispersion ....contd.

- For zero diffusion, the pulse will continue to distort unabated → wide dispersion that depends on velocity.
- For rapid diffusion, the radial diffusion inhibits the dispersion induced by axial convection.
- The value of Dispersion coefficient for microsystems attain value of 0.1 to 100.

So, as it travels it will continue to distort unabated and you expect a wide dispersion that depends on velocity so, this depends on velocity. If you have a higher velocity you expect this distortion to be more for rapid diffusion, the radial diffusion inhibits the dispersion induced by axial convection. So, if there is a rapid diffusion then the radial diffusion that takes control over and the dispersion that is caused by a axial convection. That is not taking major role, the value of dispersion coefficient for micro systems attain a value of 0.1 to 100 that is just a fact.

Now let us look at the look at the governing equation at least we try to write down the governing equation we may not be able to complete this today. In this class I write this here as this is the z axis and of course, we have to work with the cylindrical coordinate, cylindrical geometry. Because that would help us (no audio 42:11 to 42:41)

So, this is what you have as r and this is delta R and of course, the diameter of the tube that is defined as R zero or something which is the capital r zero which is which is different. So, we are picking up an annular element it is like a (( )) element it is an annular element which is of radial thickness delta R. And this part is say delta z so, if you have this conservation of species with no generation and you are only interested in diffusion in one axis. And axial convection in the other then you would be writing it as del c del t plus u as a function of r del c del z that is equal to d del square c. That is a typical governing equation that you would be following so, we talked about the axial convection and the radial diffusion.

So, if we extend this form of governing equation to this case what we get here is del c 1 del t that is equal to minus 1 by r del r of r gone minus del z of c 1S v z. Where j one is equal to minus d del c 1 del r c 1 is the we are talking about species one we are talking about species one and concentration of species one. So, this is the accumulation term on the left hand side this is the radial diffusion term where g one is given here. And this is the axial convection term so, that is what you would be getting so, we are picking up a differential element and if you do a mass balance we will end up with these equation.

Now so, we can write further this del c 1 del t that is equal to if we this minus sign will cancel here we have a minus sign already inside. So, this would be the first term would be simpler d by r del r of r del cone del r. And here we have a v z term velocity in z direction and that there we have to introduce this parabolic velocity profile. So, what we will write here is we will write these as 2 v naught why 2 v naught because you remember v max in parabolic velocity profile, that v max is 2 into v average here. The v average is referred as v zero so, 2 v zero 2 v naught into 1 minus small r, small r is the any radial point and capital r 0 that is what we said is the radius of the capillary.

So, this whole thing whole square so, this gives you the v z and since these we have already considered a fully developed flow. So, these does not have any functionality of z so, we have taken this out here from this term we have taken the v z out because this is the v z we know. So, this into del c 1 del z so, this is what we have here as the this is the equation. Now, if we look at what the boundary conditions are we are talking about a spot tracer here so, at initial condition is at t is equal to 0 for all z for z c 1 is equal to that Dirac function. That is m by a is nothing but pi r 0 square m by a delta z that is at time t equal to 0 for all z c 1 is equal to this is acceptable at time t equal to 0 only you are introducing a Dirac function type pulse at the inlet.

So, and at t greater than 0 see this z star z is equal to 0 is that is the place where the z starts. So, when you write this delta z that means that says

At t is equal to 0 for all r for all r you want to call it Dirac function, Dirac function is referred as for all z because it is it Dirac function exists only at z equal to zero. So, probably for all z if I write we can write and for all r you want to say m by pi r 0 square delta z for all z whether you write or whether you do not write it does not matter. Because delta z exist only at z equal to 0 and elsewhere it is 0. So, for all z it does not matter you want to say you want to call this for all r I for r and for or t greater than 0 at t greater than 0 for r at t greater than 0 r is equal to 0 and r is equal to r 0. Can you tell me then in that case what would be the conditions here at let me tell you because since we are in this mode r is equal to 0 you have del c 1 del r is equal to 0.

And here also we have del c 1 del r is equal to 0, but these are completely by different for different reasons this 1 is 0 because of symmetry at r is equal to 0 this del c 0 del r is equal to 0. Because of symmetry because whatever is in the upper half is same as the lower half and this r is equal to capital r 0 there is no solute flowing out this is basically impermeable medium. So that is what it says diffusive flux at r is equal to r 0 that means at the wall the diffusive flux is equal to 0 so, these are completely by different condition here it is by symmetry and here it is by no flow.

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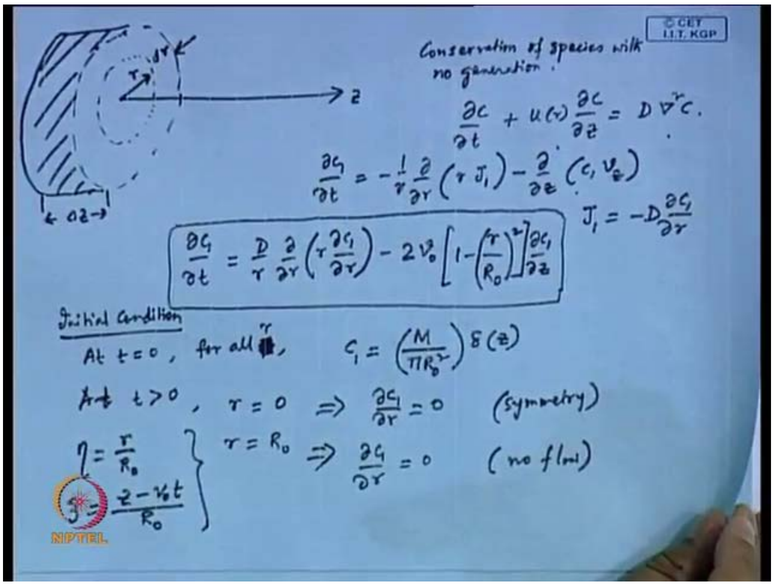

Conservation of species with no generation.

$$\frac{\partial c}{\partial t} + u(r)\frac{\partial c}{\partial z} = D\nabla^2 c.$$

$$\frac{\partial c_1}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}(r J_1) - \frac{\partial}{\partial z}(c_1 v_z)$$

$$J_1 = -D\frac{\partial c_1}{\partial r}$$

$$\boxed{\frac{\partial c_1}{\partial t} = \frac{D}{r}\frac{\partial}{\partial r}\left(r\frac{\partial c_1}{\partial r}\right) - 2v_0\left[1-\left(\frac{r}{R_0}\right)^2\right]\frac{\partial c_1}{\partial z}}$$

Initial Condition

At $t = 0$, for all r, $c_1 = \left(\frac{M}{\pi R_0^2}\right)\delta(z)$

At $t > 0$, $r = 0 \Rightarrow \frac{\partial c_1}{\partial r} = 0$ (symmetry)

$r = R_0 \Rightarrow \frac{\partial c_1}{\partial r} = 0$ (no flow)

$$\left.\begin{array}{l} \eta = \frac{r}{R_0} \\ \xi = \frac{z - v_0 t}{R_0} \end{array}\right\}$$

So, what you would do in this case is you have to define a new variable the new variable would be defined here like this you define eta as r divided by R 0 it is a very obvious choice. I mean why do we leave r as r rather make r dimensionless everybody I mean everybody does it I mean it is in very common in fluid flow with a circular cross section. And the zeta you remember we have already said we have already said that this zeta would be equal to zeta, would be equal to z minus v 0 t and it is very obvious I mean you know the meaning of this numerator. You are working with a moving coordinate and this divided by r 0 because you have to make it dimensionless.

So, r 0 is the standard dimension that you have in hand so, these are the two things that you would be incorporating in this so, these are the two things that you would be incorporating in this. And once you incorporate them then the equation that you get if you replace this eta and zeta the equation that you get upon replacement is basically D by eta del eta D by eta del eta of eta del c 1 del eta. That is equal to z v 0 r 0 half minus eta square del c 1 del zeta. That is what you will end up with that is if you substitute this eta and zeta if you substitute this two terms r and z by this eta and zeta. And a solution of this above equation that satisfies I mean solution of that this above equation which satisfies the boundary conditions.

Which you have already specified is given as c 1 is equal to c 1 at eta is equal to 0 plus one-fourth v naught r naught by d del c 1 del zeta at eta is equal to 0 into eta square minus half eta to the power 4. This is considered a solution to this equation that satisfies the boundary conditions described before and then you calculate the average concentration. Because this c 1 as a function of r and z does not make any sense because you are not going to measure it with reference to r neither you are going to express it. So, what you are interested in is something called a c 1 bar z only.

So, which is averaged over the entire cross section r and that would be equal to one by pi r 0 square integration 0 to r 0 2 pi r c 1 r z c 1 as a function r z d r. So, basically you are picking up an annular element and that annular element will have an area 2 pi r d r. And you are multiplying each such annular element with that corresponding concentration c 1 r z and integrate it between 0 to r 0 and finally.

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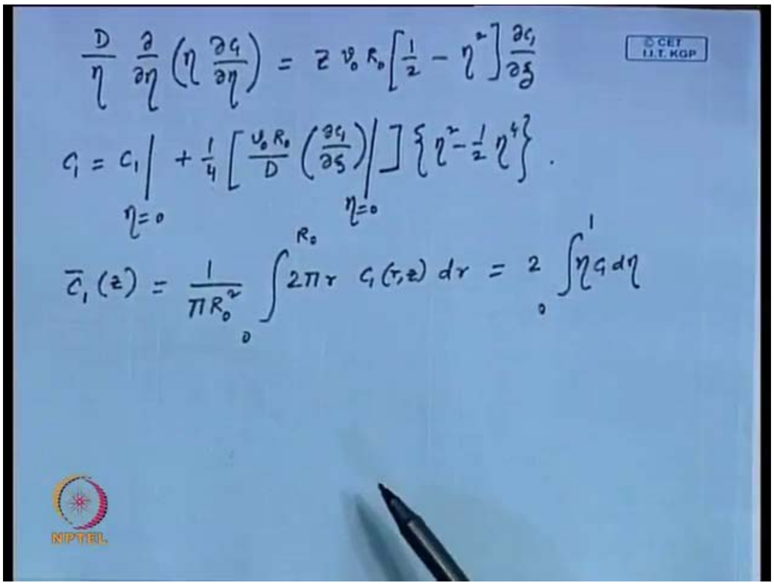

$$\frac{D}{\eta}\frac{\partial}{\partial\eta}\left(\eta\frac{\partial c_1}{\partial\eta}\right) = 2 v_0 R_0\left[\frac{1}{2}-\eta^2\right]\frac{\partial c_1}{\partial\xi}$$

$$c_1 = c_1\Big|_{\eta=0} + \frac{1}{4}\left[\frac{v_0 R_0}{D}\left(\frac{\partial c_1}{\partial\xi}\right)\Big|_{\eta=0}\right]\left\{\eta^2-\frac{1}{2}\eta^4\right\}.$$

$$\bar{c}_1(z) = \frac{1}{\pi R_0^2}\int_0^{R_0} 2\pi r\, c_1(r,z)\, dr = 2\int_0^1 \eta\, c_1\, d\eta$$

You divide this area pi r 0 square to get an average concentration over the entire at the concentration averaged over the entire cross section. So, this is a very common way of I mean you have already found out average velocity using this method you remember when you work with this parabolic velocity profile if you are give a parabolic velocity profile. And if you are asked to find out what is the velocity this is exactly what you do 2 pi r d r into u and integrate it. So, this is exactly what you are doing here so, this is equal to you can you can see lot of things are canceling out.

And you would be ending up with 2 integration 0 to 1 eta c 1 D eta this is what is the c 1 bar z that is equal to c 1 bar z. I have some amount of work left probably it will take some five or 10 minutes maximum and then we will be done with this derivation. I mean at least the essentially I wilt down all the steps that would lead to that would lead to this final form, but we have here because our aim is to get to this final form. This is a final form only thing is this is a form we are already familiar with for a spot of tracer diffusing into an infinite medium. Only thing is we are not familiar with e z term and how we ended up with this e z term that is what we are curious about

Of course this is you have to keep in mind that this e z is r 0 v 0 square by 48 D now if you have a non circular channel you will have a different such formula. There it would be a very similar 1 square divided by some number instead of 48 you will have some other number and D. So that that also we will touch upon so, probably I will take five to

10 minutes of next class first part of the next class to wrap this up this Taylor dispersion once. This Taylor dispersion and I will also discuss little bit of I mean if you bring in the chromatographic effect because we are this derivation that we are working with is basically just the radial diffusion.

And axial convection we have not talked about any chromatographic effect here, but if you introduce a chromatographic effect if you introduce a chromatographic effect by which some solute is held up at the wall by adsorption and then it is released. So, what would be the I mean that I cannot derive that is beyond the scope of this class, but I will give you the final expression and I expect that you will appreciate the individual terms and once that is done next topic that we pick up is electro kinetics. We will be talking about this electro osmosis and electro osmotic flow and all this. So, that would be probably by end of next class or next to next class we should be in electro osmosis we will be done with mixing and dispersion that is all I have for today’s class thank you very much.