

Microscale Transport Processes
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Lecture No. # 15
Mixing (Contd.)

Welcome you once again to this class of Microscale Transport Process. The topic that we have been working on was mixing, more importantly chaotic mixing and what we did in the last class is we defined something called a stretching function.

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Chaotic Mixing

$\frac{dx}{dt} = u(x, t)$ relates position $x(t)$ of the fluid particle.

A solution is considered chaotic if the property "sensitivity to initial condition" is satisfied. \rightarrow two neighbouring particles diverge on average at an exponential rate.

In local coordinate system of striation
 Normalized diffusion eqn. would be $\frac{\partial c^*}{\partial t^*} = \frac{\partial^2 c^*}{\partial x^{*2}}$

$D \sim \frac{\delta^2}{t}$

$\frac{\delta}{W(t)} \approx [t^*]^{1/2}$

$\frac{\delta}{W e^{-\alpha t}} \approx \left[\frac{D}{W^2 \alpha} (e^{\alpha t} - 1) \right]^{1/2}$

$t^* = \frac{t}{\rho} \left(\frac{D}{W e^{-\alpha t}} \right)^2 dt^*$

Striation Thickness band space

Wegand time = $\int \frac{t}{\rho} \left(\frac{D}{W e^{-\alpha t}} \right)^2 dt^*$

$t = \left(\frac{t}{W e^{-\alpha t}} \right)^2 dt^*$

SPATIAL Segmentation.

We defined a stretching function alpha, we defined a this is the what we have written in the last class is that we have a we first said that this $D \times D t$ is a function is equal to u as a function of x and t . This relates position x of the fluid particle; solution is considered chaotic if so and so. Basically, two neighbouring particles diverge on average at an exponential rate. Then what we did is, we shifted to something called a local coordinate. We said that local coordinate system of striation. What we did basically in this local coordinate system is that we have worked on. We picked up certain striation thickness and then we tried to find out how much would penetrate. How much the diffusion would take place inside this striation thickness?

Now, this striation thickness w is undergoing change. You have already seen that this has been chopped a part of this w is chopped off and then it continued. So, w is continuously changing. This w , the width of this striation that is continuously changing on the and on top of this continuously changing within this continuously changing width, the diffusion is taking place. So, what we did last time is we have done this x^* is equal to x divided by w , that is how we defined x^* , which we called striation thickness best space. So, the denominator here is not the width at inlet rather this denominator itself is changing with time, and that is how we making x dimensionless.

The other thing is we are making t dimensionless and look at how it is done. Here, you are integrating between 0 to t . The integration is between 0 to t and the inside what we have? Inside the integral we have D divided by w prime whole square D prime. So, that means you know that this w is changing with time and you are trying to find out how much of diffusion is taking place with the changing w . So, that is how you are defining this t^* that means you are working with a local coordinate system, you are picking up a strip and you are following that strip as the width of the strip keeps changing as the w keeps changing. So, that is how you have defined x^* and t^* . And you know that this is the equation that is the governing equation $\frac{\partial c^*}{\partial t^*} = \frac{\partial^2 c^*}{\partial x^{*2}}$ this is the equation, that is the governing equation.

And then what you said is that this $\frac{\Delta}{w}$, this originates from the understanding that the diffusivity scales to the penetration thickness to the power 2 divided by time, then unit of diffusivity is $\text{cm}^2/\text{second}$. So, you know that this would be the penetration thickness to the power 2 divided by time so, that is how diffusivity scales. So, if that is so then you have then you have written $\frac{\Delta}{w}$ if you simplify this, if you check the units of this t^* you it will be it will become clear to you. What you are writing is $\frac{\Delta}{w}$ scales to t^* to the power half and w is nothing but w into e to the power minus alpha t , w is changing with time and that changing is defined by this stretching function alpha in this manner.

So, this is $\frac{\Delta}{w}$ to the power minus alpha t and this is closed to this scales to D whatever we have inside. If we simplify, that means what was our t^* ? If you look at the expression for t^* , the expression for t^* is like this. Integration 0 to t D divided by w prime whole square that means, w prime is nothing but w into e to the power minus alpha t whole square d prime. So, this is what you have and now you go

ahead and integrate it between 0 to t . See what you get? You will get what is inside this t star D divided by $w^2 \alpha e$ to the power $2 \alpha t$ minus 1. So, this is what you get after integration. And so, what you are writing is Δ divided by $w e$ to the power minus αt scales to this whole thing to the power half.

Now, what we do with this expression? What we do with this expression is, then what you say is that at the end I mean once this strip enters into the mixer and then at the outlet you call that time to be t_f . So, suppose L is the length of the mixer, if L is the length of the mixer and t_f is the time that is taken, final time. So, up to time t_f this all these mixing, all this stretching folding everything is taking place over this time t_f . So, then you know that this L divided by t_f is nothing but the velocity u . So, that is how this L and t_f they work with, they come up that is how you define them. So, what you get here is basically Δ so, when this term at the outlet I mean at the outlet, this term and this term, this Δ and this w , this should be equal to 1.

That means the entire thickness has been penetrated at the outlet α . If you write this as αt_f that is the final time. So, at the outlet the entire thickness that is available, entire thickness of the strip has been penetrated by the diffusion has gone all the way into it. So, this ratio has to be equal to 1. So, if this ratio becomes 1, then you can write this quantity will scale to 1 if this you write as αt_f . So, that means at the outlet you would be writing D divided by $w^2 \alpha$ into e to the power $2 \alpha t_f$ minus 1 to the power half. This whole thing should be closed to 1 at the outlet. So, this is what you have as a condition.

Now, if we have this understanding that this e to the power $2 \alpha t_f$ is much greater than 1 number 1. And, if it would have been a linear stretching I mean this we are talking about this α has been the elongational stretching. Now, if it would have been a linear stretching, then you would have written α as equal to I mean if you we should have been a linear stretching, you would have called this you would have written this as u divided by w . That is what? That u divided by w gives you the u divided by w gives you the rate at which the two points adjacent to each other will stretch under linear deformation. That means, I have a fixed plate and I have a moving plate. I have a point on the fixed plate and I am trying to find out.

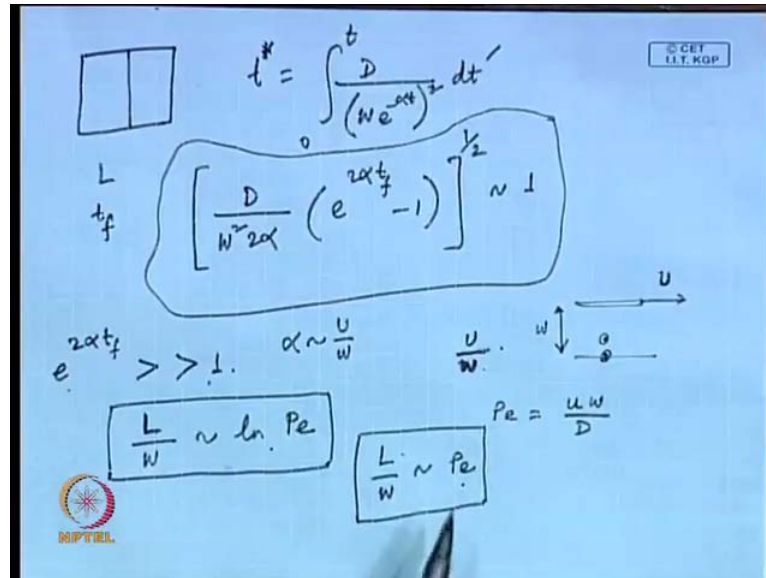
If I have neighbouring point and I am trying to find out at what rate these two points are going away from each other and that rate would be given by u divided by w . If w is the distance between the plate and this upper plate is pulled at a velocity u and you have a linear velocity profile. So, this is something which defines the rate at which these two points, they move away from each other. So, what you can I mean which is a fact in fact, that this α is not very much away from u divided by w . If you check the this is probably, this can be verified experimentally that this quantity would be much higher than 1.

And if α scales to u divided by w then what you get here is that L divided by w is this scales to ln of pecllet number. What is pecllet number? Peclet number you remember it is basically $u w$ divided by D so, this is the pecllet number. So, this and how this $u L$ and $t f$ they related? u is nothing but L by $t f$ that you know. So, at the end, at the outlet, the entire thickness whatever that thickness is, that time the thickness is probably w into the power minus $\alpha t f$ that is the thickness of the strip. But, the entire strip is already flooded by the defusing species. So, this L by w is basically scales to ln of pecllet number so, this is one of the outcome of this stretching function.

If you would have continued with a linear stretching, you would have ended up with L by w scaling to pecllet number, just pecllet number. If you would have considered, this form of stretching that means one layer is just shared against the other and the distance that you that distance that these two point distance between these two points evolving with time. If that is how you are stretching it then this L by w scales to pecllet number that can be shown. I mean probably we have done much more complex analysis to show, this L by w is scales to ln of pecllet number. So, this is the advantage you get L is what? L is the length. So, you call it the mixing length that means the length of the mixer that you should have to ensure that the entire constituent, all the constituents that are coming out they are well mixed. So, that is the L . So, L you can call, L as the mixing length that is the length required. If you want to have say another some length left, that is up to you. I mean you can have extra length, but this is the minimum length that you have to provide, if you want to ensure the mixing is complete so, that is L . So, if you have a chaotic mixing, if you have the stretching function something similar to α , the way we have defined you will get L by w scaling to ln pecllet number.

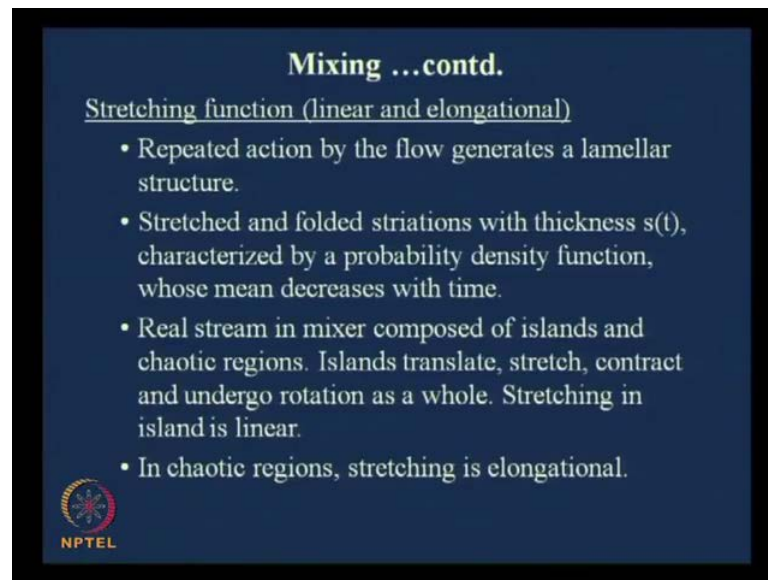
On the other hand, if you are looking at the linear stretching the two points just moving one away from the other by shear then you get L by w scaling to pecllet number. This L is pecllet number and pecllet number that is the advantage.

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So, basically you will get much shorter mixing length. Basically, by doing this chaotic mixing you get much shorter mixing length. But, at the very outset I said that there would be islands, where this linear stretching is taking place and there would be places where these elongational deformation taking place, this is a mixture of both. So, you will end up somewhere in between, you this is one extreme L by 2 , L by w scaling to \ln of pecllet number, this is 1 extreme. This we can achieve nothing like it. But, that is an ideal world. Similarly, L by w scaling to just pecllet number that is also another I mean simple two layers sliding one against the other, but that is not the case that is a too trivial situation which we do not want to work with.


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Mixing ...contd.

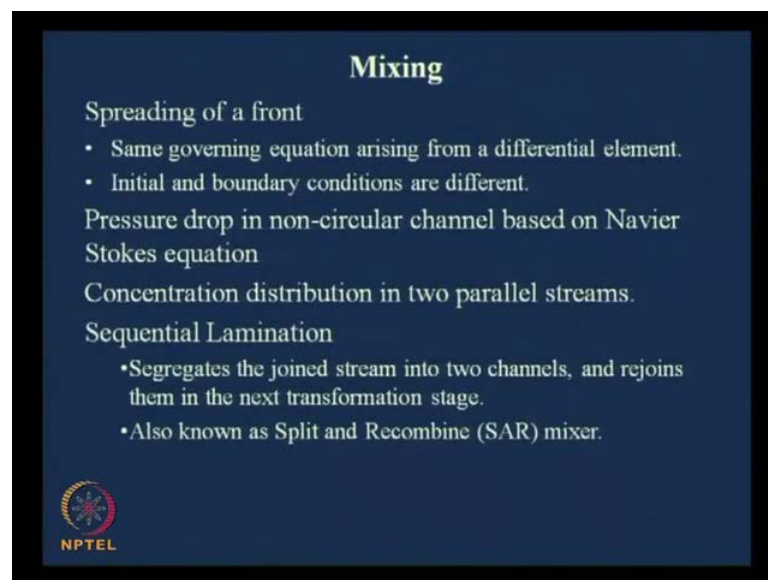
Stretching function (linear and elongational)

- Repeated action by the flow generates a lamellar structure.
- Stretched and folded striations with thickness $s(t)$, characterized by a probability density function, whose mean decreases with time.
- Real stream in mixer composed of islands and chaotic regions. Islands translate, stretch, contract and undergo rotation as a whole. Stretching in island is linear.
- In chaotic regions, stretching is elongational.

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So, you are somewhere in between that is what I would like to point out. And I think the by now, I should have made my point clear.

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Mixing

Spreading of a front


- Same governing equation arising from a differential element.
- Initial and boundary conditions are different.

Pressure drop in non-circular channel based on Navier Stokes equation

Concentration distribution in two parallel streams.

Sequential Lamination

- Segregates the joined stream into two channels, and rejoins them in the next transformation stage.
- Also known as Split and Recombine (SAR) mixer.

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So, now if I see what I have in the slides, I see that this and we see what I had earlier. We were talking about this sequential lamination segregates, the joined stream into two channels and rejoins them in the next transformation stage also known as split and recombine mixer. Then we said there are two types of stretching function, one is linear and another is elongational. Linear is basically the stretching that takes place because of

simple shear one layer sliding against the other. And elongational is where the two points adjacent to each other on a average move away from each other at an exponential rate.

So, what we what I write here is repeated action by the flow generates a lamellar structure. Stretched and folded striations with thickness s_t . Probably, this s_t is similar to w_t . We worked with w_t in these papers in the when I worked here. Now, so, this w_t is equivalent to s_t , now this with thickness s_t characterized by a probability density function whose mean decreases with time. Basically, the w_t the mean of w_t decreases with time that is what it says. Real stream in mixer composed of islands and chaotic regions. Islands, translates, stretch, contract and undergo rotation as a whole stretching in island is linear. In chaotic region stretching is elongational.

So, this is so, you have a mixture of both. And this I pointed out on this paper when I have written here, I pointed out these mixing length divided by the width. These scales to $1/n$ of peclet number in case of chaotic, in case of elongational stretching and in case of linear stretching this mixing length scales to simple peclet number and you are somewhere in between.

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Mixing ...contd.

Sequential Segmentation
Solvent and solute streams are broken up into segments along the axial direction by alternate switching of inlet flows.


Difference between chaos promoted by advection and that by turbulence

At low Re number

- No wake or recirculation after the pillars
- Pillars induce shear and hence Taylor Aris dispersion

At high Re number

- Turbulence induced by pillars
- Very effective mixing
- Not used in microsystems

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Next topic that I pick up is sequential segmentation, where solvents and solute streams are broken up into segments along the axial direction by alternate switching of inlet flows. This I am going to discuss very soon, but before that I would like to point out the second thing that I have here which is difference between chaos promoted by advection

and that by turbulence. one thing you I mean we have talked about this chaotic mixing, but one thing you must understand is that, here there is no turbulence. So, how it is, how we will look at it here? How do you promote turbulence? For example, in membrane separation process, if you have taken this course or if you have studied this in mass part or mass transfer.

In membrane separation process one established method is putting some pillars on the membrane. So, that when you induce a cross flow over the membrane surface, those pillars acts as turbulence generator. I mean around those pillars there would be turbulence; there would be cross A d's forming. And that would help mixing on the membrane surface, because what you have to ensure is that there is not any boundary or the formation of boundary layer is minimized. So, by putting those pillars you ensure that there forms these cross A d's which can only arise from turbulence. So, you can have such that is that is one established method that you put pillars and you put across flow to induce turbulence.

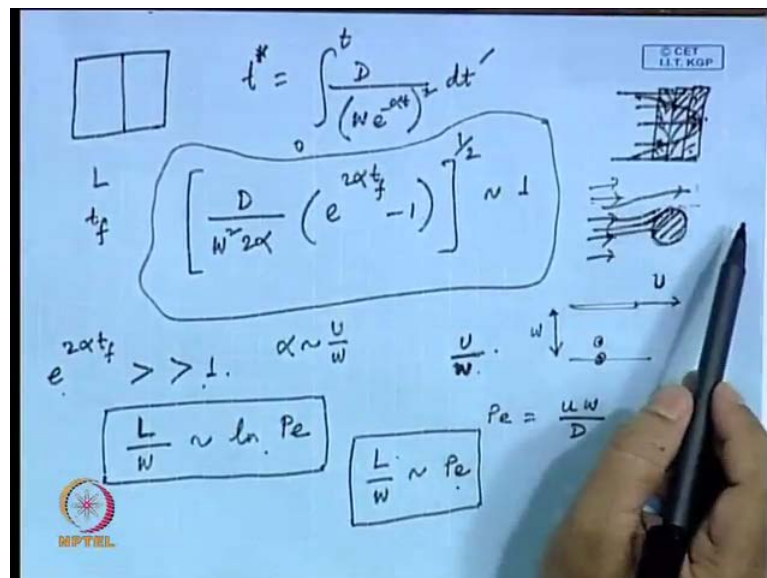
However, in this case here also you want mixing, but if you put pillars there the mixing would be of a completely different type. What you would have is there you know that the Reynolds number I at the very outset I said the Reynolds number came about 1 so, Reynolds number is very low. So, you know way you can have turbulence there, however so, at low Reynolds number no wake or recirculation after the pillars. So, that is absolutely not possible, however, pillars induced shear and hence Taylor-Aris dispersion. We this Taylor-Aris dispersion, we will discuss after this may be in this class itself we will talk about it.

Taylor-Aris dispersion is that when you have a I am writing it on this paper here when you have a parabolic velocity profile, I am writing it on this paper when you have a parabolic velocity profile (No audio from 18:12 to 18:19), that time you would be one layer is moving at a higher velocity than the other layer. So, or in other words I had suppose, I had a potential flow means free stream flow, all the layers are flowing in mass. There is not any sliding one of it is sliding against the other that is not the case. Then you have a pillar here so, because of this pillar this one probably this one will go and hit the wall, hit the wall of the pillar and this would be a stagnation point.

So, the streamline next to this, probably this would have the minimum velocity or may be a zero velocity at the wall itself. And the streamline away from it that will have little higher velocity and the one further away from it that will have somewhat higher velocity. So, when you collect the stream at the outlet, you will find at some point of time when you collect the stream at the outlet. You will find the mixture of various streams, because various streams they will be coming at different time. So, just by putting this pillar, you are inducing velocity gradient. So, when it is coming out it is not coming out in mass rather it is coming out one layer is coming say the one that is next to the wall that is coming much later whereas, the other layer is coming fast.

So, that way you are the one that the fluid suppose think of this parabolic velocity profile, what will happen? If you chop this part you will get something. If you then next one is this one, then next one is this one. So, these will be coming so, what is this? What is this all about? What do you have in this block? In this block you have some of the fluid that is here whereas, some of the fluid that was remnant here, some of the fluid that is remaining there. So, if you collect a plug out of it and measure the average concentration of this plug, what you get is a mixture of everything.

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Similarly, you have mixture of everything so, you get some amount of mixing just because of this velocity gradient. So, that kind of mixing is possible if you put a pillar, does this make sense? This is a completely different kind of mixing. The stream that you

collected the outlet, that is an average of all the cross sections that you pick up. Now, if all the cross sections are not flowing at the same velocity so then what you get is and if you take all the cross sections and then you mix them and that is that you consider as a concentration. So, what you get there that layer itself is a mixture of various concentrations.

So, that is a kind of mixing that you can possibly have which goes by the name Taylor-Aris dispersion so, if you if I go back to the PowerPoint slide I see here is that no wake or recirculation after the pillar. So, in case of turbulence you could have expected some recirculation after the pillar so those basically the what you have the boundary layer separation and all kinds of things that is not happening here. Because, Reynolds number is extremely low, however pillars induce shear at the wall of the pillar the velocity is zero away from this little higher velocity and further away free stream velocity. So, pillars induce shear and hence Taylor-Aris dispersion.

However, at high Reynolds number, you can expect turbulence induced by pillars that is a very effective mixing, because there you are basically creating cross A d's and that is helping you in mixing and that is why you put this on a put this, that is how, that is why you put this pillars on the I mean wherever you need the mixing in heat exchanger, in membrane separation unit. However, this is not used in micro system, because this Reynolds number is not in that range all. So, the next topic I mean that I pick up here is sequential segmentation. I did not describe much this sequential segmentation so, that I should be working with here the topic is sequential segmentation. (No audio from 23:21 to 23:28).

In sequential segmentation, you have.

(No audio from 23:31 to 23:56)

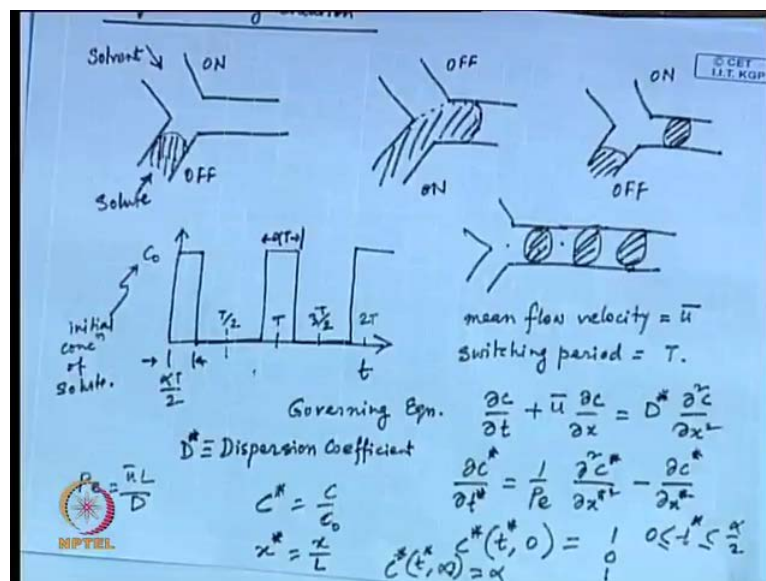
Solute coming from this side and solvent coming from this side. So, this is suppose this is say solute. Solute is coming from this side and solvent is coming from that side. And first you have solvent on solute off. Solvent is flowing in through this channel; at the next moment what you have is, solvent off and solute on. Solute started flowing into it so, it is preceded by solvent. So, you have this next you have solvent on and solute off so, what do you have? (No audio from 24:52 to 25:01) Another plug of solvent is coming going in there. So, if you have this on, off arrangement what you have at the end is you

will have like this and so this is solute and this is solvent. So, that is what you end up with, what we refer here as sequential segmentation. If we try to plot the concentration versus time what we get here?

(No audio from 25:41 to 26:01)

It would go like this and this is c_0 , the concentration of solute this is T by 2, this is T , this is $3T$ by 2 that is $1.5T$ and continue like this. This say $2T$ it goes like this and this distance is αT whereas, this distance is αT by 2. So, with a mean flow velocity \bar{u} of both fluids in the mixing channel and the switching period of capital T mean flow velocity is equal to \bar{u} for both fluids in the mixing channel and a switching period T . (No audio from 27:17 to 27:23) You can write the governing equation as, governing equation for mass transfer would be $\frac{\partial c}{\partial t} + \bar{u} \frac{\partial c}{\partial x}$ that is equal to D we call it D^* $\frac{\partial^2 c}{\partial x^2}$. What is D^* ? D^* here since it is flow through a tube we are calling this D^* as dispersion coefficient.

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What is a diffusion between, what is a difference between dispersion coefficient and diffusion coefficient? We would be taking it up I the next probably later part of this lecture. Let us just accept it that this is little different from diffusion coefficient we call this a dispersion coefficient. So, this is the governing equation and c_0 this c_0 is basically the initial concentration of solute. So, this is basically the how the concentration changes at the inlet so, this is this is probably if somebody asks you what

kind of boundary condition you have? So, this is what you have as the boundary condition.

So, if we try to write this further this boundary condition the way we will write it is c at sometime t at the inlet x equal to 0 that is equal to c_0 and again c_0 when is it equal to c_0 , when time is between 0 and αT by 2 and when is it 0 when time is between αT by 2 and T minus αT by 2. And when it is c_0 again T minus αT by 2 and T . So, this is the condition at the inlet that you are following so, that is generating this plot. This should be clear to everybody. So, this is the concentration for all the time at equal to 0 that means at the inlet.

Now, you can write this governing equation that we have pointed out, we have a governing equation and that governing equation we will we can write in terms of dimensionless quantity. We can write this c say we write this as c^* that is equal to c by c_0 . So, we can make the concentration dimensionless similarly, you can write x^* which is equal to nothing but x divided by L and Peclet number is equal to u bar L divided by D . In that case you would be writing this governing equation, there is another boundary condition this is the initial condition you have for the concentration. There would be another condition which is applicable for all time at x equal to infinity and can you guess what would be the condition?

That would be all mixed up that would be the mixed up concentration and that would be equal to αc_0 why? Because, think of it how much solvent is going in and how much solute is going in? So, that ratio is basically the defines the α so, it is αc_0 that is the other condition. So, this is at inlet and at x equal to infinity this is the condition for all T . And if you write this governing equation in terms of this dimensionless quantity I mean you can always do that, I mean this governing equation can be written in terms of dimensionless quantities.

(No audio from 32:06 to 32:21)

And then you would be writing c^* as t^* 0 this will take the value instead of c_0 this will take value of 1 0 and 1 depending on where this t^* is, whether it is 0 or α by 2 it is like this see you will have a similar. Similarly, you will have the other boundary condition c^* t^* infinity that would be equal to just simple α . You can write this in terms of dimensionless quantities and probably solve this. Ideally, they should be done

numerically there is analytical solution also existing. So, there is analytical solution available, but then numerically probably it would be better solve this numerical, because it is a long expression if I start writing the analytical form.

So, that is what you have a sequential segmentation I mean you understand the essence of it, that you are playing you are switching on off I mean you are working on off mode and you are giving pulses of solute and then helping that solute to diffuse into the adjoining solvent streams solvent blocks basically. So, that is how this sequential segmentation works.

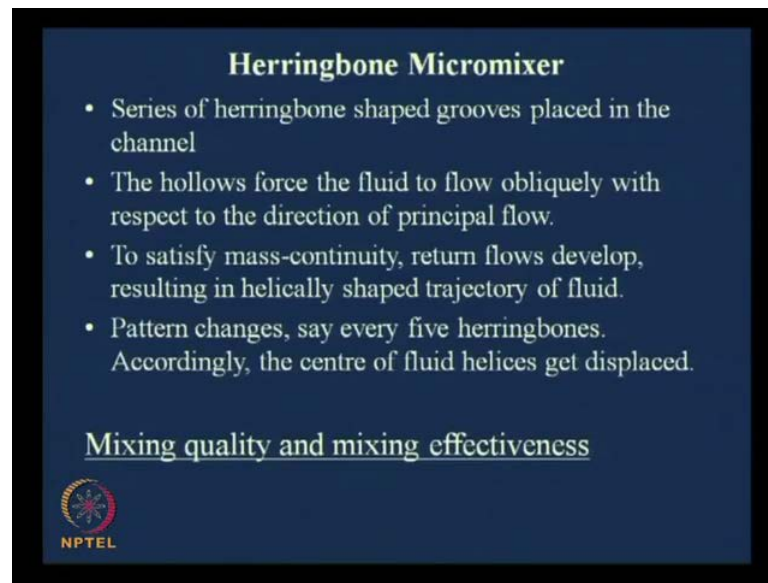
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$$C(t, x=0) = \begin{cases} C_0 & 0 \leq t \leq \frac{\alpha T}{2} \\ 0 & \frac{\alpha T}{2} < t \leq T - \frac{\alpha T}{2} \\ C_0 & T - \frac{\alpha T}{2} < t \leq T \end{cases}$$

$$C(t, x=\infty) = \alpha C_0$$

So, I am trying to this is one form of mixing that is there so, what all you have covered so far we have we have talked about basic the mix, basic mixing strategies.


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Herringbone Micromixer

- Series of herringbone shaped grooves placed in the channel
- The hollows force the fluid to flow obliquely with respect to the direction of principal flow.
- To satisfy mass-continuity, return flows develop, resulting in helically shaped trajectory of fluid.
- Pattern changes, say every five herringbones. Accordingly, the centre of fluid helices get displaced.

Mixing quality and mixing effectiveness



We have mentioned that this mass balance equation is governing equation. It is basically a mass balance equation, it is basically a mass balance over a differential element and we have written it in terms of diffusion coefficient. We started with spot of tracer diffusing into an infinite medium and then we have we worked with two streams flowing side by side. And if you have mix, if you have a stretching function by which you are taking 1.0 away from the other to (()) points you are taking one point away from the other as per certain functions. Then what would be the result, how the mixing length would depend? That we have already established.

I would like to point out here I mean before we go away from these various mixers in this p p t slide, I would like to point out this herringbone micro mixer this we have already discussed before. If you remember I have when I was introducing these lab-on-a-chip device I showed that on the floor you have groups embedded by which you are ensuring cross flow. That means you are having two streams flowing side by side and then you have groups on the floor. So, there is preferential flow through the groups and the flow that is taking place through the group is attracting the layer above it by this viscous drag. So, you have the cross flow going on and then you are changing the arrangement, after say every five such groups you are change the arrangement.

Such that you are ensuring a cross flow all over the cross section. So, these herringbone mixers we have discussed this and I have shown some simulation results as well when

we introduce this passive mixing in a lab-on-a-chip device. I just quickly point out that is herringbone micro mixer is how it works here, because that is complete the discussion on mixing. Series of herringbone shaped grooves placed in the channel, the hollows it is basically a hollow you have a channel and on the floor, floor is not intact there are hollows. There are hollows by which you are inducing cross flow; these hollows force the fluid to flow obliquely with respect to the direction of principle flow to satisfy mass continuity.

Return flows develop, because whatever is whatever goes in there has to be return flow. So, that is there so resulting in helically shaped trajectory of fluid. So, you have a cross flow going on so, it there is a cross flow going on and then it hits the wall and then it has to come back. So, what eventually what you are having is a helical shape, helically the cross flow is getting generated. So, it to satisfy mass continuity, return flows develop resulting in helically shaped trajectory of fluid. And pattern changes every five herringbones say so, these helices basically accordingly the centre of fluid helices get displaced. So, in some place the grooves start, grooves never starts from the centre.

Grooves is always off centred, some grooves are off centred, this way some are that way. So, that the centre of helices, centre of fluid helices that get though centres get displaced the two centres that you have so, they get displaced so, by that way you are ensuring that there is sufficient cross flow and there is mixing. So, there are there are various mixing strategies so, we discussed about various mixing strategies. On one hand you have sequential segmentation where you are introducing a solute pulse and then you have another one, then followed by a solvent. So, this pulse can now diffuse into the solvent slowly that is one way or the other way is you can induced cross flow by means of these putting grooves putting hollows on the floor.

So, that is one method and this how to handle these as far as theory is concerned either you have to take it in rigorous manner as we have done for two streams flowing side by side. Two streams flowing parallelly and we have showed how the diffusion take place what is governing equation, how delta is there and delta would be more at the centre and less would be, delta would be more at the wall, near the wall, and less at the centre and those things we have discussed. And then how do you theories if you have a completely chaotic advection. If you have two streams they are moving away from each other, that is

the maximum you can expect. And how to handle this, what are the essential theories to handle this kind of mixing?

So, we are more or less touched upon the issues that you have as far as this passive mixing is concerned in a micro channel that that I can say. Before we bring this to a close we need to talk about something called mixing quality and mixing effectiveness. I mean you can have a mixer I mean suppose you somebody manufactures a micro mixer and you sells a micro mixer how do you, first of all what is the mixing quality of a stream? I mean I have two streams flowing side by side into a passive mixer and then after say one centimetre length I get the something at the outlet. What is the mixing quality? I mean do you consider that to be well mixed, I mean at the inlet two streams are flowing side by side.

So, in 1 stream solute is c_0 or in dimensionless form the solute concentration is one and in the other stream the solute concentration is 0 so, it is 1 and 0 in dimensionless form. So, one and 0 they are going in half of it is one, rest half is 0 that is what is going in. And you will call it least mix, it is not at all mixed. But, as it progresses this 1 and 0 they are getting mixed. So, in some places next to the I mean next to the interface you will find it is 0.9 0.8 and things like that and faraway at corner you will still see point 0 1 instead of 0. So little bit of solute has diffused so, this is semi mixed, this is not fully mixed.

But, as it progresses your objective is to see that at the outlet of the mixer the entire stream is well mixed. That means what should be the concentration at if it is half half. If it is half half, then the concentration should be 0.5 so, you expect this to be 0.5. But, whatever instrument you use you will have something between say 0.48 and 0.52 in some places it is still 0.5 to in some places it is still say 0.48. So, how would you define a mixing quality? I mean you have a stream and you want that stream to be well mixed, but in a real life a probably it is 0.499 and 0.501. So, how do you define these this quality of the mixing that is important.

So, when it comes to the quality of the mixing suppose, I have a cross section suppose this is the cross section. So, you need to find out what is mean square deviation. Mean square deviation of concentration profile of component I say in a cross section what would that be? That would be basically if I write this sigma square, that would be integral over these entire area, what is this area? Area is basically this is continuous flow

over. This entire area A , you pick up a point some arbitrary point and that has a concentration c_i . So, $c_i - \bar{c}_i$ whole square dA . So, this area is dA and that area has a concentration c_i . So, you have to sum up all such dA 's and they will have different concentrations this one has c , this one has c_i .

So, say these dA will have another concentration these dA have will have another concentration and what is \bar{c}_i ? If you are having half half then \bar{c}_i would be 0.5. So, how far is this concentration from 0.5 it could be on positive side or it could be on negative side. So, that is why you are taking the square of it. So, you are taking the square of these deviation and that you are integrating over these entire area A and that gives you a feel for what you what kind of mixing? But, that is not the end of it, what you do is at the same time you calculate something called a sigma max square. Sigma max square is A into \bar{c}_i into $c_i \max - \bar{c}_i$. This is the maximum mean square deviation you can have in this micro channel.

And where will these maximum occur? This maximum will occur at the inlet. Because, what will happen at the inlet (No audio from 44:17 to 44:28) what would be this quantity $c_i - \bar{c}_i$ whole square. At the inlet, this will be $c_i - \bar{c}_i$ whole square so, it is suppose this is $c_i \max$ so, $c_i \max$ at the inlet it would be either 0 or $c_i \max$. If it is 0, then it is 0 minus \bar{c}_i or it is $c_i \max - \bar{c}_i$. So, you get there basically so, this is the maximum deviation that you can have. Now, you are writing this something called mixing quality as α_m . This there is another name some places this is referred as normalized segregation intensity.

So, this α_m is equal to $1 - \sqrt{\sigma^2 / \sigma_{\max}^2}$ that is equal to $1 - \sigma / \sigma_{\max}$. α_m is equal to $1 - \sigma / \sigma_{\max}$. So, this is called mixing quality or there is another name to it normalized segregation intensity. So, if α_m is equal to 0, you will conclude that it is completely segregated this means it is completely segregated whereas, if α_m is equal to 1, then you will consider this to be completely mixed. That means α_m is equal to 1 means this σ is equal to 0. That means there is absolutely no standard deviation all points are basically \bar{c}_i .

So, α_m is equal to 1 so, you will consider that to be completely mixed and whereas, if you have α_m is equal to 0 that means this quantity is 1. That means you have this

could be the highest or this could be the highest value of this quantity and that means this could be the lowest value of alpha m and that you are.

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$$C(t, x=0) = \begin{cases} c_0 & 0 \leq t \leq \frac{\alpha T}{2} \\ 0 & \frac{\alpha T}{2} < t \leq T - \frac{\alpha T}{2} \\ c_0 & T - \frac{\alpha T}{2} < t \leq T \end{cases}$$

$$C(t, x=\infty) = \alpha c_0$$

$$\sigma^2 = \int (c_i - \bar{c}_i)^2 dA$$

$$\sigma_{max} = A \bar{c}_i (c_{i,max} - \bar{c}_i)$$

Mean Square deviation of concentration profile of component 'i' in a cross-section.

Mixing quality (Normalized segregation intensity)

$$\alpha_m = 1 - \sqrt{\frac{\sigma^2}{\sigma_{max}^2}}$$

$\alpha_m = 0$ completely segregated

$\alpha_m = 1$ completely mixed

So, basically what you are doing is, you are by doing this exercise I mean even if you do not want to get into this, how you got this? You are putting this scale I mean we lot of times I mean if you say I feel hot or I feel cold or I feel good then lot of people ask tell me in 0 to tens 0 to 1 scale. So, tell me mixing in 0 to 1 scale quickly so, that is what you have. So you have this alpha m which is between 0 to 1 scale that is I mean 0 is basically completely segregated and 1 is completely mixed. Now, there is a catch to it this is not the end of the story, catch is something like this that here you are putting equal weight to all the cross sections. However, you know since you are strongly in laminar flow and a velocity profile would be parabolic and I mean one layer sliding against the other.

So, me stream is flowing at a higher velocity and some is flowing at a lower velocity. And so, when you do this averaging you should it should be weighted average, because you should not put equal emphasize to all the streams which are flowing by completely different velocities. So, there is another development, where you instead of writing this you write this you call this velocity weighted mixing quality. (No audio from 49:05 to 49:12) Velocity weighted mixing quality which is defined as 1 minus square root of sigma v dot. See, I mean I am just giving you some feel for it I mean these are all open for further research I mean you can have a better way of representing it if you want.

I can only give you some links some directions basically this you will have this time this thing $\sigma v \cdot c^2$ this would be equal to 1 divided by $A m w$ bar integration over $A m c$ minus integration $a m c w d A$ divided by integration $a m w d A$. This whole things square $w d A$ where, w is the velocity so, w bar is the average velocity. (No audio from 50:37 to 50:45) So, these are these are these are some of the things which you can I mean they are people have already thought about, that is velocity and there should be a velocity weight age given if you are really trying to find out a mixing quality.

But, essentially what you are doing here in this process is when you get a stream out of this micro channel, when you get a stream you try to find out how good is the mixing in a 0 to 1 scale and that is the mixing, that is what is defined as mixing quality. Now, you can have a more comprehensive definition of mixing quality which is open to discussion open for discussion. Now, what I basically I said at if I look at this PowerPoint slide, we said this mixing quality and mixing effectiveness. Now, I have tried to define what is a mixing quality? And I think this is good enough of a definition I mean if you look at any the people who are doing research I mean they are all relying on similar expression, I mean everywhere.

That this is what a definition of the mixing quality would be, that you are getting a stream and over this cross section you want to know how the stream is mixed and this would be the definition of mixing quality. However, mixing effectiveness is something which is different. Mixing effectiveness is how well you are achieving the mixing. That means mixing quality 1 is your final objective, that you have to get the ultimate mixing quality and so, that is your final that is when it is 1, that is what you are looking for. But, that 1 has I mean you have to pay some pay lot of effort to it. And the effort is reflected at if you look at that pressure drop. Suppose, there are couple of issues here one is the mixing length $L m$ there is something called mixing length.

What is the length of the mixer? After which you achieve an alpha is equal to 1 what would be the length of the mixer? That is 1 issue. If you can do it in a shorter length, nothing like it I mean that is good. The other issue is what how much pressure drop you are encountering. For example, if you have 2 streams flowing side by side, you do not have I mean pressure drop may not be pressure drop would be nominal. But, if you have baffles if you are continuously encouraging cross flow that means you are creating

vertices. So, what you are doing essentially is you are losing you are encouraging pressure you are expending some inlet pressure so, some pressure is lost.

So, that is the effort that you put in. So, you have to come up with some effectiveness factor mixing quality is fine, but there should you should come up with some kind of effectiveness factor that balances the revenue to the effort. That I am getting this much of revenue I am achieving alpha is equal to so and so close to 1 within that such a short length. Which is L_m the mixing length I have talked about this as L places L_m . So, within this short length we are achieving this alpha as this quality, but expending this much of pressure drop. So, mixing effectiveness is typically defined as the energy the revenue gained divided by the effort expended. So, basically you there are various say there are various suppliers who are supplying this micro mixer.

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Velocity-weighted mixing quality

$$1 - \sqrt{\frac{\sigma_v^2(c)}{Q_v^2(c)}}$$

$$\sigma_v^2(c) = \frac{1}{A_m \bar{W}} \int_{A_m} \left(c - \frac{\int_{A_m} c W dA}{\int_{A_m} W dA} \right)^2 W dA$$

W is the velocity
 \bar{W} is the average velocity.

L_m

revenue
 effort expended.

NPTEL

So, which micro mixer will you purchase? So, you can say the I should have mixing quality which is the best, but what is effectiveness of this mixer? Because, some are coming up with say on off type method, sequential some are coming up with grooves in the floor, some may be having grooves above the floor I mean that is their research, but how will you judge which one is the best. So, they have to have some parameter to satisfy and that parameter will define that whether these I mean whether this mixer is good or this mixer is not so good. So, this is something called a mixing effectiveness.

And this mixing effectiveness we will take this up in the next class, and we will complete our discussion on this mixing as far as this passive mixing is concerned. And then next topic we pick up after this discussion on mixing effectiveness. In the next lecture, we will pick up Taylor dispersion and its use in micro scale chromatography. So, we briefly test upon this Taylor dispersion, but these theories of Taylor dispersion we will get into that in the next class.

Thank you.