Microscale Transport Processes

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Lecture No. # 13
Mixing (Contd.)

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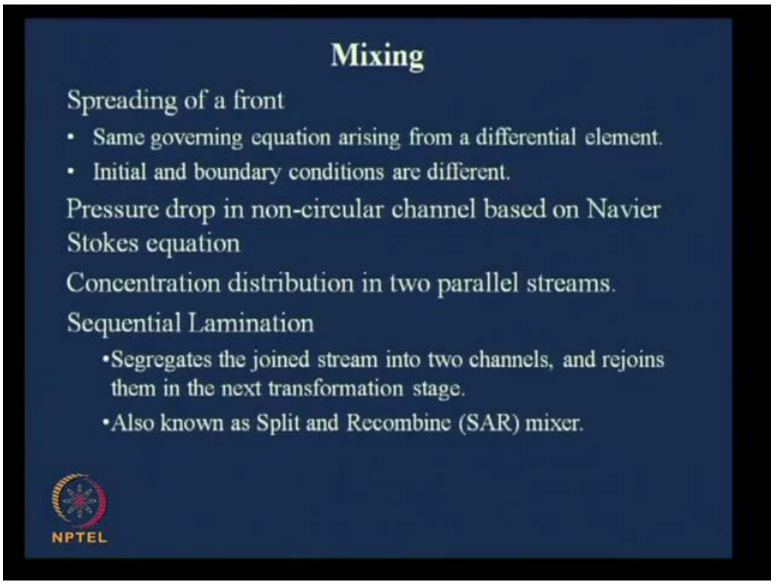

I welcome you to this lecture of microscale transport process. What we have been discussing is mixing, we have discussed about several cases; first case was a spot of tracer, which is spreading in a resting fluid.

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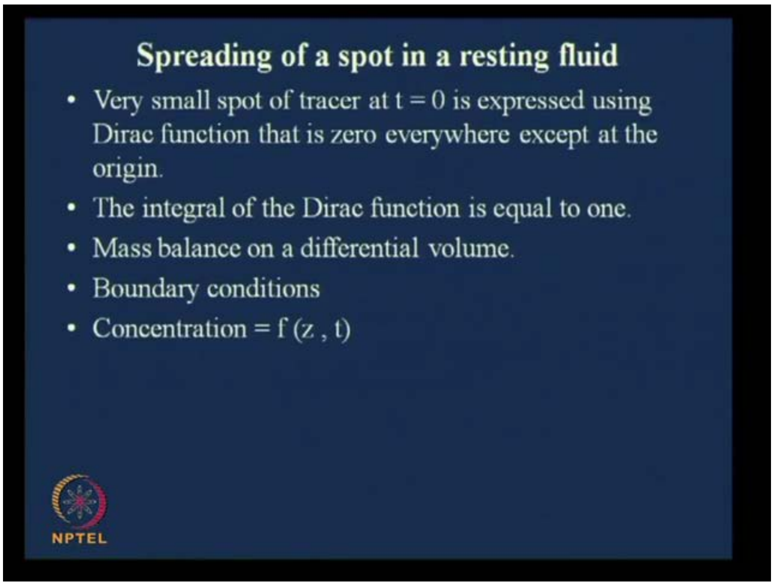

So, we, we have found out the concentration profile. We have found out the expression for concentration as a function of space, in space and time, that expression we have obtained. I have given you the analytical expression, not exactly, we have obtained, I have shown you the way and given you the, I have given you the final result.

The next, what we picked up, is the spreading of a front where instead of a spot, instead of a spot of tracer, it, this, this, there was a front, that means, there, there are two distinct, two distinct layers and one layer is held at certain concentration and other layer at time t equal to 0, the concentration is 0 or concentration is given value. And then, we have found out what would be the concentration, what would be the evolution of concentration in that other layer as a function of time and at various locations. So, these expressions I have, I have, I have given you these expressions.

And I have also shown to you the pressure drop in a noncircular channel because we are more familiar with circular channels. We are more familiar with handling how to, how to handle the, we are familiar with the procedure how to handle the pressure drop in a circular channel. But not exactly, I mean, we are, we do not encounter on everyday basis, I mean, so far, in, in your undergraduate or in your undergraduate classes you have not encountered the flow through a noncircular channel.

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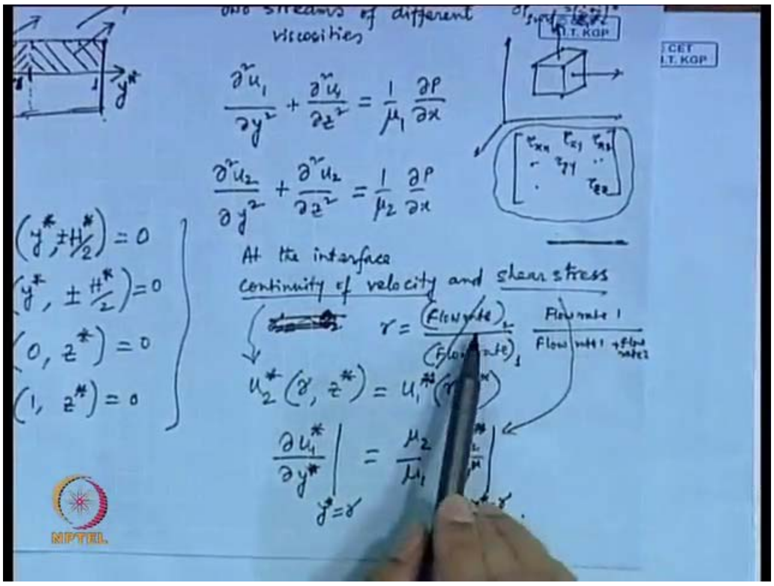

In this regard I would like to clarify couple of points that we have discussed in the last class. Number one is that this was pointed out by, by some student is that this, this, what we are doing here is, we were, we are talking about two different layers, one is of some viscosity and other layer is different viscosity and they are moving side-by-side, and we are trying to find out how would be the, how you obtain the pressure drop in such case.

Now, there was a, there was a point raised, that why we are not considering the surface tension? The fact here is that it is, first of all these two are miscible fluids. The surface tension would come into play if the two fluids were immiscible. In that case, you, you have to have, I mean, if you, if you, if you look at here, if you look at here, the, we, we are talking about continuity at the interface; we, we are talking about continuity of velocity and continuity of shear stress. On top of that you would have, you have to equate the, the sigma. Basically, this sigma is, what, what? You have this stress tensor, I mean, you understand what is stress tensor is? It is basically the tau x x tau y y tau z z tau x y tau x z. So, similarly, you will have, all this is, this is called a stress tensor. I mean, you will have all the elements filled that way. The diagonal elements, that average of these diagonal elements, that gives you the pressure, this tau x x, these are arising if you pick up a differential element.

And then, in, in, if this is the x, y and z axis, then this normal components are, they are, they are written as tau x x tau y y tau z z, and the other, the cross terms, they are basically the other elements in the matrix other than the diagonal elements. And pressure is given as the average of these three quantities, make a due of it. So, that is, that is how you, you define. So, this is the stress tensor and this stress tensor, if it would have been immiscible face, then you have to equate the normal component and, and you have to, you have to bring in, that, that Young Laplace, you have to bring in that Laplace pressure, that, that, that, that delta p surface, that is equal to the, the delta p surface, that is equal to surface tension.

And you have these 1 by r 1 plus 1 by r 2, so these, these things you have to bring in and you have to, you have to consider, that, that aspect at the interface. If, if you would have been two immiscible fluids, but since, I mean, mixing is possible only if you have 2 miscible fluids, I mean, if you, you cannot have a mixing between two miscible fluids, so I have not talked about it, I have not discussed this. However, this is not a complete equation as far as the microscale transport is concerned because there, I mean, if you, if you look at the Navier Stokes equation there is an additional term, which is typically, which, which is body force, basically the gravity or a Coulomb force.

So, there, if in fact, we will, we will discuss this down the line something called zeta potential. There you will have, you will have an additional term, you will have an additional term because on the surface, if the channel is very small what you will have is, if you have a surface and if you have an electrolyte sitting on, at that surface, there will be some charge developing on the surface because of say, deprotonation of the surface, that is, that is one reason and because of that the counter ions would be attracted to the surface.

And this is, there would be, there would be development of, development of one particular ion near the wall, so that would, that would introduce some Coulomb force on this Navier-Stokes equation, that we will discuss in connection, in connection with electro-osmotic flow, but that, that is, that is missing. So, what you can say is this equation, definitely, is not complete. This equation is a simple Navier Stokes equation that we are trying to solve in a simple, Navier Stokes equation, that we are trying to solve and definitely this does not contain all the terms.

But when it comes to the surface, the, the point, that you raised, it is, that we should have considered surface tension, but that would be appropriate if you have two, two immiscible phases, but that is not exactly the case here. But there are other case terms that need to be included in the Navier Stokes equations. The, the, I mean, that is all I want you to take from here, but later on, when, when it comes to the, when I introduce this electro-osmotic flow, that time I will discuss the other terms, other term how it, how is the, how that term is introduced in Navier Stokes equation. In fact, we will find out the pressure profile at that time, so you have to solve this equation again.

The other point, that is, that is, that is, I think is an error on my part when I have written here, this gamma is flow rate 2 by flow rate 1. I think this is not, here it would be, I mean, one of the flow rate, say I write it as flow rate 1 divided by flow rate 1 plus flow rate 2, since it scales between 0 to 1. So, I think this should be the case flow rate 1 divided by flow rate 1 plus flow rate 2, alright. So, that is something, which we have, that is something, which we have here, flow rate 1 divided by flow rate 1 plus flow rate 2. So, this is not exactly right, so make note of this. So, this is, this is, as far as the, the, the material, that we have been talking about.

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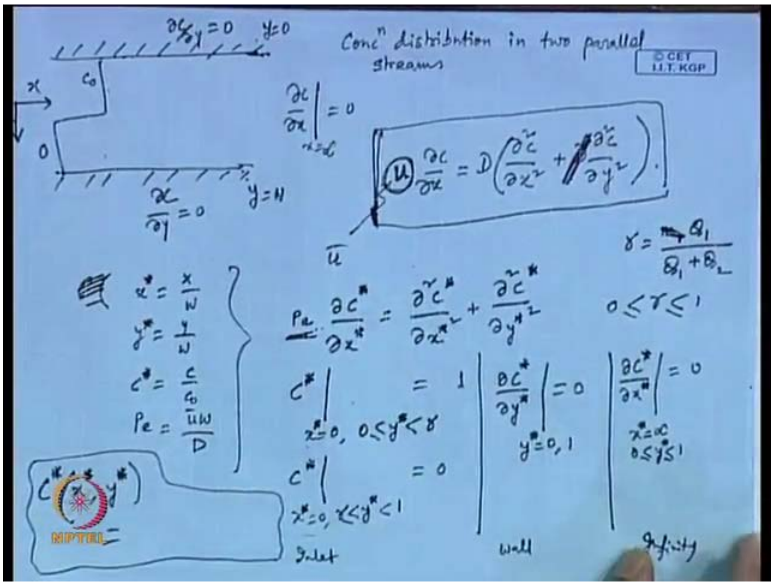

The other thing that we started last time was the concentration distribution in two parallel streams. That means, we have been talking about the, we have been talking about the flow of two fluids, two, two fluids, one is having flow of, flow of, flow of, flow of two parallel streams, one is having concentration C 0 and the other is having concentration 0 and they are flowing side by side. And I was pointing out at that time what are, what would be the boundary conditions.

Some people have, some students have argued with me later, that these del c del y equal to 0 why we are putting it here basically? What we are trying to do is that this layer is impermeable, that is all it says. That means there is no solute moving in, moving through this layer. So, that is, so it is impermeable, so flux would be given by D del c del y in y directions, so that is equal to 0. So, automatically, del c D y equal to 0. So, that means, you are treating this as an impermeable wall, this channel wall, that is impermeable, that is all. You, your, your, that is all I say here and here, I say, del c del x at x equal to infinity is equal to 0. That means, at infinity you consider the two streams to be well mixed. So, these concentrations, they stop changing. Now, this equation, that I put it, put it in the governing, as a governing equation since I put the bracket here, this D does not make sense, so later on I realized. In fact, this somebody pointed out also, so please remove D by, I have put an extra D here, it is, it is, it is, D should not be there.

Now, if we, if we try to, if we, if we look at this governing equation and we, we try to find out what the boundary conditions are, what the boundary conditions are, first we need to, at the very onset, I, I must say, that we pointed out in the last class, that this u is having a parabolic velocity distribution. I mean, if you have a laminar flow going on, so at the wall the velocity is 0 and away from the wall the velocity has, velocity would be increasing. So, this u would be changing with, u would be changing with y, alright; u would changing with y, u would be changing with z, as well. I mean, I do not know this side is not open, so there would be, this side also, in the z direction also there would be. So, this, this u is function that way, but this would not be analytically tractable. I mean, it would be analytically tractable if you, if you, if you consider u to be constant, that is the average velocity. So, that is an assumption, that is a simplification, I mean, always I mean we do some simplification. In fact, when we write this equation also, there are several assumptions we make. So, if you consider, that to be the average velocity, probably this can be done analytically.

Now, if we introduce some dimensionless variables here, say for example x star, which is equal to x by w; y star, which is equal to y by w; c star, which is equal to c by c 0 and Peclet number, which is u bar w by D. If we, if we introduce these dimensionless variables, then you can write this as Peclet number del c star del x star, that is equal to del square c star del x star square plus del square c star del y star square. So, this is what you have and then, if you, if you try to write the boundary conditions, now the boundary conditions would be c star at x star is equal to 0 and y star in between 0 and gamma. What is gamma? Now, gamma is equal to m 1 or you can write this as q 1 divided by q 1 plus q 2, that same way that we have defined earlier. So, this is between this value of, this gamma is between 0 to 1. So, this c star, this is equal to 1 and c star x star is equal to 0 gamma y star 1, this is equal to 0.

So, at the inlet the stream that is having, either it will have higher solute concentration, which is in a dimensionless form, that is equal to 1 and the one, which has the lowers or, or the 0 solute concentration, that would remain 0. So, all these two conditions are valid for x star equal to 0. So, this is at the inlet, so this is at the, this I have written is at the inlet. Then, we have another wall condition, wall condition is, that del c star del y star at y star is equal to (0, 1). See, w is in y direction, y is equal to 0 here and y is equal to w here, so w is in y direction, so it is, it is between 0 to 1. So, y star is equal to 0 or 1, both are walls and this is the wall condition.

If somebody is working with multiple streams, that means one c 0, another, another stream with 0 concentration, then there is another c 0, another 0, another c 0, another 0. If we, if we have multiple such streams, then probably by symmetry, I mean, it will not be a wall condition, but you have to, you will have a similar condition by symmetry because by symmetry it is derivative of concentration with respect to y star, would be 0. Then, other, other condition is at infinity, other, other condition is at infinity, and there you have del c star del x star at x star is equal to infinity 0 less or equal to y star less or equal to 1, this is equal to 0. So, by this time, by the time stream reaches infinity, the two streams reach infinity there, I mean everywhere, whether it is y is within 0 to gamma or gamma to 1, everywhere the concentration is well mixed. So, concentration will stop changing, so these are the boundary conditions and this is the, this is the governing equation. Now, this I, I wrote this is a Peclet number; this is on the basis, that you are writing it as u bar. If you write it as u, then probably it will not be, you, you cannot write it this way because Peclet number is defined as u bar w by D, alright. So, this is what you have.

Now, now let me tell you, that these, these c, if, if somebody wants to find out what is c star as a function of (x star, y star), this is available, I mean, you can, you can solve this using separation of variables and you can, you can come up with some kind of, it would be, it would be an analytical expression that is possible. The other way is that you consider a velocity profile here u and solve it numerically, so that is also another, another possibility. So, this is, this is something, which is I mean, I am not, I am not indicating here the exact analytical form because it is a, it is a, really it is a long expression and but just be aware, that these, these, these analytical expressions are available.

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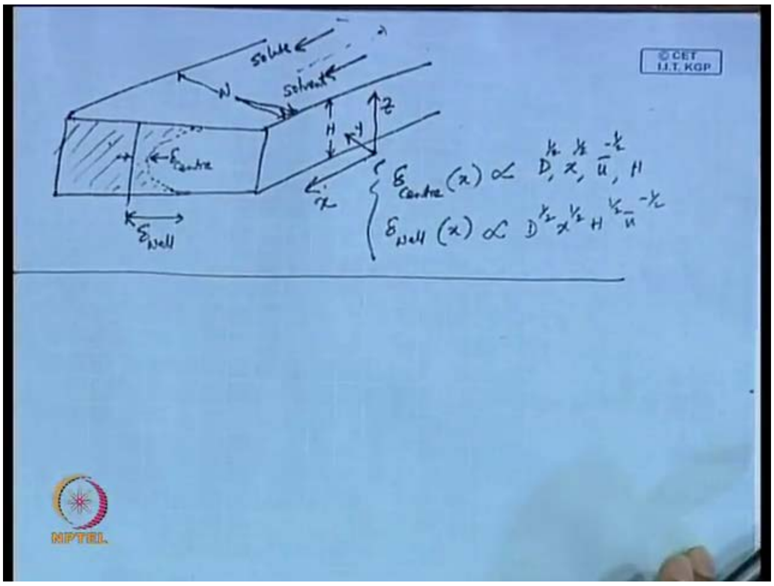

Now, what is important here is that how much, suppose you have a solute stream here, this is the solute and this is the solvent stream; solute means, solute, it is not a solid solute, it is a, it is a high concentration stream and a stream, which is, so it is basically, this is saline water, another is pure water and salt is diffusing from solute stream to the solvent stream. So, here you have, if you look at, suppose, suppose then, then in that case this is, suppose this height of course, this height does not matter as such. What I try to point out here is that you have a, you have a diffusion of solute from this layer to this layer, so as the two streams coming here, so these two stream they were, will, will separate it. But when it arrives here at this place you will find, that there would be a diffusion that has taken place. If this is the central line a diffusion, that has taken place, which suppose this is, this is the solute layer, you will find, that the these portion, that solute has penetrated and can you, so, so these, these portion you want to call, say delta wall, the, the distance by which the solute has penetrated.

Of course, you would ask me, how do you define the penetration of solute? I mean, it, it would be, anyway that would be a very continuous change. So, how you define? So, that is something like the way we define a boundary layer thickness, so it would be, I mean, it would be a similar concept. I guess, now this at the wall, the penetration is, say delta wall and at the center the penetration is, say, say, it is delta center. Now, the question could be, that why such a profile? Why it would be, it, it is taking such a shape? Why it is not straight? Why this, why this is taking such a kind of shape?

I mean, initially, you, when you started, the solute was, half of it is occupied by the solute stream; half of it is occupied by the solvent stream. When it arrived you see, that this portion, the solute has penetrated more and this portion solute by probably you might have already thought of, I mean, you must have already gotten the answer. Let me point out here, that this is the slowest moving layer because it is next to the wall. So, this is the slowest moving layer. So, this layer is moving slowly and this layer is fastest moving layer. So, how much time it is getting for the diffusion to take place? So, they depend on that. So, this is this portion, the layer, that stream is coming at this place. It is probably, it has, it has, it had least residence time inside the channel. So, that is why you do not, you, you, you cannot, you cannot expect the diffusion, diffusion of solute to penetrate far into that, that, far into the layer at that place. So, this is how it is done.

Now, what people have done is, what researchers they have done is, they have, they have, they have numerically found out, that this delta, this delta center, this delta center, they have numerically found out and this delta wall, they numerically found out, I, I mean you, you can, but to, to have this you cannot consider u to be, u to be the average, u you cannot consider u to be the average, u, u have to considered, u have to consider the actual, u you have to consider the velocity distribution and then, you solve numerically and you find out how much the penetration would be. So, from there you can, you can calculate delta center as a function of x and delta wall as a function of x.

And as a matter of fact, some, somebody has done, some people, some researchers have done experiments also on this. So, basically, this, this delta center and then they try to find out how this, how this proportionality would be with reference to D, with reference to x, with reference to u bar, with reference to H. What is H? Now, H is this height and x is this direction, y is this direction, z is this direction. So, that means if this is y, so from, from the, from the, from the old one, that we have here, there we have, if this is y, so then and this is w. So, if that is the way we have done earlier, so then this, this would be equal to w, this would be equal to this, this distance would be w, this distance would be H and this would be the x, y, z.

So, this proportionality, they try to found, they try to find out how these delta center as a function of x and delta wall as function of x, how they depend on these quantities. And what they found is, basically this is proportional to D 2 the power half, this is proportional to x to the power half and this is u bar to the power minus half because if you have average velocity, which is higher, then automatically you are giving less residence time. So, automatically, that delta center would be less, it is that way. Similarly, delta wall also they found, that this is D to the power half x to the power half H to the power half and u bar to the power minus half. So, that is, that is typically the, the trend, that has been established experimentally or numerically, alright. So, these are some of the things, that I, I, I H is this height, that delta center, delta center does not depend on H delta center. I mean, that, that is what, that is what results showed, that delta center does not depend on H, anyway. So, this is, this is what we have.

Now I, I want you to, to, I mean, all these things, these are, these are actually very, these are, these are not exactly related in any way in a sense, that what you are, what we are interested in here is, that we want to accomplish mixing and of course, if we have two streams flowing side-by-side. At the very outset I told you, that it is not possibly you require 1 meter, you require 1 meter, 100 meters, that kind of length to accomplish a mixing this way.

So, it is not at all going to, I mean, I mean, in fact the next topic, that I will pick up, that is, in fact, if you, if you look at, in the, in the slide what you have here is sequential lamination. Sequential lamination is, this segregates the joined stream into two channels and rejoins them in the next transformation stage, this is also known as Split and Recombine mixer. So, this is something, which we would be doing. This, this is something, which we would be discussing. So, in the micro channel our objective would be to split the stream and again recombine, but when, if, if the two streams happen to be flowing side by side, these are some of the aspects, that you have to be in mind, that, that you have to keep, keep, keep at the back of your mind, that these, these are, these are the few things that are going to happen.

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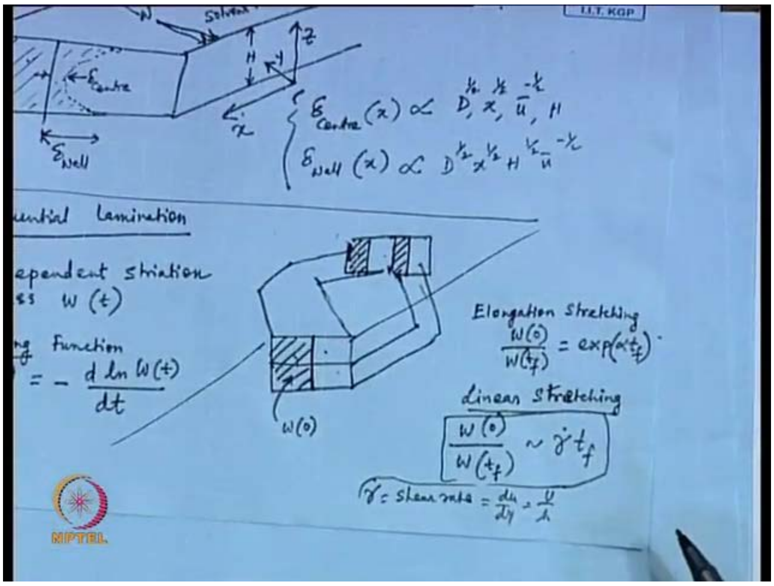

So, the next topic that I pick up now is sequential lamination. Sequential lamination, sequential lamination is that you let us see, suppose I have two streams coming here, one is your so called solute stream and the other is solvent stream. Then, what you do here is you pick up, you pick up half of it, I mean, when, when you, when you carry this, the way this lamination is done is you, you pick up half of this stream to this direction and the bottom half you are taking in this direction. So, you have the internals inside the channel such that the bottom half is completely taken through a detour and this top half is taken as a detour and then, they are delivered; then, they are delivered in one place. But this time when they are delivered, they are not put as top or bottom, they are put as side, on, on the, it is, it is like one on the side of the other, so what you get is this.

I mean, I do not know how much I could explain here, the point here is, that you started with one solute and other solvent stream pick up the top half through a detour. I mean, the, this is, this is done entirely, so this is the channel basically. This is the channel you, you do not see what is going on inside, but inside, through the internals what you are accomplishing is you are picking the upper half and delivering it and the bottom half you pick up through a separate detour and delivering it. But while delivering, you are delivering them on as side-by-side. So, that means it would be the, after delivering this is the shape it is taking. Then, you go, you repeat the step, then this would be multiplied. So, here it is one solute, one solvent; here I have two solutes and two solvent. The same thing would be repeated down the line. So, this is something, which we, we were talking about is, this is called sequential lamination and also referred as split and recombine mixer.

Now, if we try to find out and now, we try to find out the theory of it. What we see here is that you have a time dependent striation thickness, we call this, say wt, this is w 0. So, at t is equal to 0 you have a striation thickness, we call this say w 0 and this thickness is going down, it going, going, it is becoming smaller, so we call this because if it continuous with the time, so it is a time dependent striation thickness, we call this w t and w 0, w 0. You can call this half width of the micro channel, then I mean I have to start with, I mean, it depends on how you define w, it could be either half width of the micro channel or you can call this the full width of the micro channel, that, that is how you, how you refer w, but my point is that this w is going to decrease with time.

Now, how the w changes with time? So, if somebody wants to write, say I mean, what I am going to do next is I am going to define something called a stretching function, which I will call alpha t and this alpha t is equal to minus of D l n wt D t, this is what I call the stretching function, that is how this is defined. Why are we getting into this 1 n w t? There is a history to it, when it comes to stretching there are two types of stretching possible, one is called elongation stretching, one is called elongation stretching and other is called linear stretching. Now, this linear stretching is something, which we, we are familiar with. This linear stretching is the stretching, that we see in a viscous layer, the stretching that we see in a viscous layer.

What is this stretching? I have one layer next to the wall, which is having velocity 0 at under no-slip condition. Then, next layer is moving at a higher velocity, so you are doing some kind of stretching. You, if basically see, my, my objective of pulling this sequential, putting this sequential lamination is, that my objective of putting this sequential lamination is that I have to take two points, which are adjacent to each other from one place to other. I mean, suppose it is, it is like, I started, there was a, there was a, there was a point, there was a point, say here and there was another point, say here, there was another point, say here. These two points are neighboring points. Objective of this mixing is that these two neighboring points, they go away from each other, then I will consider, that mixing has taken place. If the two neighboring points remain as two neighboring points, then I, I, I do not think, then, then we, that is helping in mixing. So, this stretching function when we are introducing, we are trying to find out how these two neighboring points are stretched.

Now, when we are talking about this linear stretching, this is the stretching, that we encounter in a viscous flow. That means, one layer sliding against the other, that is also stretching because one point, which is next to the wall and the point, which is away from the wall or may be the point, one point, which is somewhere near the wall and another point is next, next door. Now, these, these two points because of this velocity gradient, they are moving at different velocity. So, I am having a stretching, I am, these two points flowing at different velocity. So, if I considered, say t f, some finite time, after some finite time I will see, that these two points are located at different places and the distance between them, I mean, we are, we are considered, we are considering, that stretching, that stretching is referred as linear stretching.

Why it is called linear stretching? Because I can write w 0 by w t f t, f is some finite time or final time, I call it. So, at the end of the event creation, thickness is w t f and at the start it is w 0, this is close to gamma dot t f. What is gamma dot? Gamma dot is that shear rate, gamma dot is equal to the shear rate, that is equal to D u D y, that is equal to, y u, u by H. These are, these are various forms we have, so this is called, this is called the linear stretching. This kind of stretching we understand, we, we do it frequently.

If we have a viscous flow going on between two layers, this stretching is possible. On the other hand, this elongation stretching is w 0 by w t f, that is equal to exponential of alpha t f. Here, the w 0 by w t f, this is not a linear function, rather it is an exponential function. Here, with time it is, it is basically linear. Here, we have an exponential function and this alpha is this coefficient. So, this, this is how this w 0 by w t f, w 0 is the striation thickness at time t equal to 0, w t f is the final striation thickness, that is equal to e a exponential of alpha t f.

Now, what is the origin of this elongation stretching, I mean, this linear stretching? I could understand, it is basically the, typically the stretching, that we encounter in viscous flow, but what is the origin of this elongation stretching? This elongation stretching arises from the process, which is referred as chaotic mixing.

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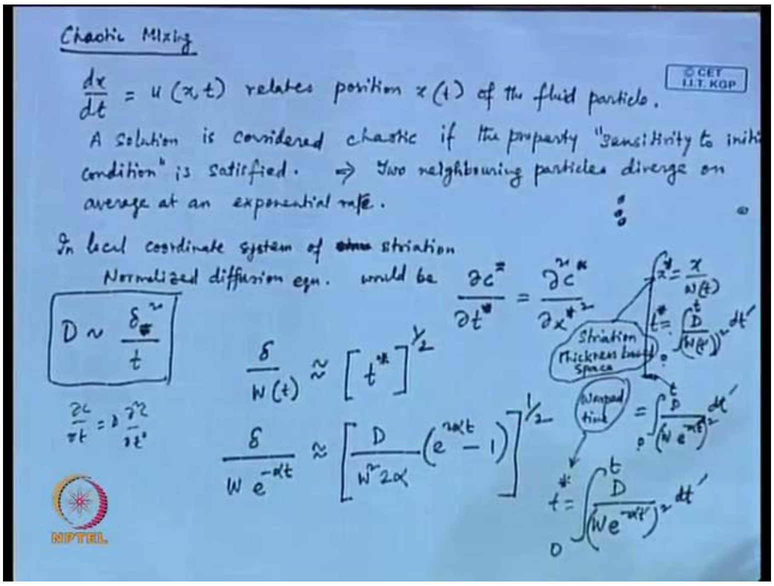

Chaotic mixing, let us say, dx dt is equal to u as a function of x and t relates position x t of the fluid particle, a solution to this equation, which is probably what you are looking at. So, the solution, a solution is, solution to this equation is considered chaotic if the property of this, I put quote and unquote sensitivity to initial condition is satisfied. What this, I mean, in the, in the, in the, in their terms what, what basically this statement means is that two neighboring particles, two neighboring particles, they diverge on average at an exponential rate, at an exponential rate. That means, two, two neighboring particles, we started with two neighboring particles. If we had a linear stretching, then I know that one layer is static, I say, may be fixed and closed to the wall and other layer is the next layer, which is moving at a somewhat higher velocity. So, after some finite time, I, I will find, that one layer has travels up to this point, whereas this layer is still static or maybe, with reference to this layer there was another layer, which was here. And then, you will see, that this layer has gone further there, so there is a, there is a linear, there is a linear stretching. So, these are, these are some other things that are happening.

Now, this, this was the linear stretching, but this here, that two, two neighboring points they are moving away from each other on average at an exponential rate. So, that is basically the elongational stretching, that we had, w 0 by w t f, that is equal to exponential of alpha t f, that is, that is what this says. So, this is referred more as the chaotic mixing. When somebody prepares a dough, prepares a dough for making bread, that there, probably if you, if you color, if you put two points, two colored points on that dough and when that person who is making that dough, when, when he is stretching it, the type of stretching, that you get is, is, is a chaotic, is, is probably this elongational stretching, that we are referring here.

Now, you cannot, ideally you cannot have, ideally you cannot have all chaotic mixing in a real mixer. In a real mixer you will have some places, which are called islands and these islands will have, these, these, these islands, as a whole they translate. So, in, in a, in a, in a real mixer, the micro fluidic mixer, that a person will fabricate, person using this micro fabrication technique, the micro fluidic mixer, that is made there, you will have so called island, which is undergoing the, undergoing translation, undergoing rotation as a whole and within that island you can expect this linear stretching to happen. And outside these islands, there would be this chaotic; there would be this elongational stretching. So, in this mixing process it is, it is a combination of this, both these stretchings and this. So, in, in, in a real micro fluidic mixer what you would expect is an island or few islands moving by translation. They are rotating and everything and within the island liner stretching is going on. Outside the island it is chaotic mixing, so that is, that is something, which is, which is going to happen.

Now, what will I do with this alpha? What is the purpose of this alpha? What I would do is, then in, what I will, what I will try to find out is that in, in local, in local coordinate system of striation, I mean, we, we try to get into this Lagrangian, I mean, we are trying to invoke the Lagrangian sense here in local coordinate system of striation. That means, I pick up this striation and I am treating this as if it is Lagrangian particle. Then, in the local coordinate system of striation the normalized diffusion equation, normalized diffusion equation would be del c star del t star, that is equal to del square c star del x star square where I mean, c star is understandable c star, you can always make it from c by c 0, that is not a problem. The way this, this diffusivity is taken into, taken in this t star, so x star here is, x star here is x divided by w t and t star here is equal to integration 0 to t D divided by w t prime whole square D t prime and this w t prime. So, you will be writing it as integration 0 to t D divided by w e to the power minus alpha t whole square D t prime.

Let me, let me point out once again, what, what we are doing here. You think about just one striation, this is this is called striation. Now, if I, if I pick up this and I try to find out how much of diffusion is taking place in this layer, if we, if we, if we pick up, if we take this one and if we try to find out how much of diffusion is taking place here, so I treat this as a, I mean, as I said, in a Lagrangian sense. So, what I do here is I write in local coordinate system. So, within this local coordinate system, see how this x is defined. I started with this as w 0 and I stated, that this is changing, this, this thickness, this, this, this striation thickness is changing, it is becoming, it is w 0 now and it is going down with time, so at some arbitrary time it would be w t. And there in that w t, using that w t I am defining here the x star; using that w t I am defining the x star. So, this x star is the local coordinate system within that striation, so I am treating that as a Lagrangian particle as it travels. So, x star is x by w t and t star, the time.

Here, I am, I am defining the time this way, that t star is equal to integration between 0 to t D divided by w t prime whole square D t prime. So, over this, between time 0 to t what would be this value, sum of D divide by w t prime whole square, that is how we are defining t star. So, now, if you bring this whole thing into x t space, this is the, this is Lagrangian space we are talking about, so we have defined this x star.

Now, if we come to the, to the actual, the, the, the, the mixer coordinates if you bring it, this would be, you know, that diffusivity scales to delta x square divided by t, where delta is the penetration. Let us, let us, let us drop this x, let us call this just delta, delta square by t, this is how diffusivity scales. That means, definition of diffusivity is that how much it penetrates, how much the (( )) penetrate the square of that divided by time. Of course, this is the, I mean, the penetration itself, you have to follow the governing equation. I mean, ideally if you know already, that if you are, if you are really solving that diffusion problem, then ideally, you need to solve del c del t is equal to D del square c del z square and all this things and solve this. It is, it is not that straightforward, that you can write D is equal to the penetration square by time, but when it comes to the scaling, this is the, this is the form, that is followed, this is, this is the scaling, that is followed. So, for, for some quick estimate, probably this is something, which, which could be handy, but definitely not a rigorous, mathematically rigorous method.

Now, D scales to delta square by t. So, what you can write here is that delta by w t is close to, is, is, is of similar order to t star to the power half and this w t, what you would be writing here is delta divided by w e to the power minus alpha t. And that is, if we, if we, instead of or you can write, continue this way, this is equal to D divided by w square 2 alpha into e to the power 2 alpha t minus 1 to the power half. How do you get that?

You bring in here this one, now t star is equal to integration 0 to t D divided by w e to the power minus alpha t whole square D t. Now, you do the integration here, I am, I am, I am, I am little bit, only thing is w e to the power alpha, here it should be alpha t prime, so you do the integration here, you do this integration. I write it clearly, 0 to t D divided by w e to the power minus alpha t prime whole square D t prime. You do this integration and then put this here as t star, so this is equal to t star.

So, this t star has to be put there and then you can, you, you will get this expression automatically. You have to go and check this, whether, whether you are getting this expression or not. All this, this t star is referred as this. This is different from times, so this t star is referred as warped, warped time and this x star, x star is defined as striation thickness, thickness best space, striation thickness best space. So, this x star is defined as striation thickness best space and this t star is referred as warped time. And if you bring in this delta by w t, then this, this would be the case.

I, I do not expect you to immediately understand all the aspects of it, you have to, what you have to do is you have to, after the class you have to go back and play with these expressions. I have given you all the threads, now you have to play with these expressions and then you have to come back to me, maybe, maybe in the next class, that what you, what you have, what, what objection you have to this expression because I have given you all the threads, now you have to assimilate it. It, it looks, it looks like, I mean, I am putting things, not, not, not I mean, I am, because this, this concept is something, which is, which is little different, so that is why I, I suggest, that you go through this once again, think about what is going on.

This is basically a local coordinate system, think in that line, that you are picking up, you are picking up this element, which started it. It started with w 0 or, or, or I, I think when I write this w into e to the power minus alpha t, automatically this w is w 0, alright. So, this, this w is w 0, but I cannot put so many things in bracket, so it is better if you put this as, put, consider this as w 0, that, that is the width at time t equal to 0. So, this is, this is, this is something, which you need to play with and see whether you get, whether you end up with this expression or not, alright.

So, one thing I must point out, that this is not the, this is, this is elongational mixing, the theory that I am discussing here because at the very outset we said it is e to the power minus alpha t. The moment e to the power minus alpha t, the moment we put this, it is elongational stretching and this elongational stretching is not everything, there are other, other forms of stretching, that is also available. And probably, in a real mixer it is, it is a mixture of both because one, one thing you must understand, that I mean, I can, I can call, I can say the two neighboring particles, they diverge on average at an exponential rate, but how long they will continue.

I mean, if you, if you look at a dough a baker is making, preparing, there you are stretching two points, but then the baker has to fold it again. So, if you, if you just, if you assume, that they will continue to expand the distance between, they will continue to expand in an exponential manner it is up to sometime and then again there would be folding. So, it is, it is basically we are trying to put together the elements of mixing. Here, this is one element of mixing, which is, which is called chaotic mixing, the other element is linear stretching and the mixing, that you would see in a real mixer is the sum of, or, or combined effect of all of them.

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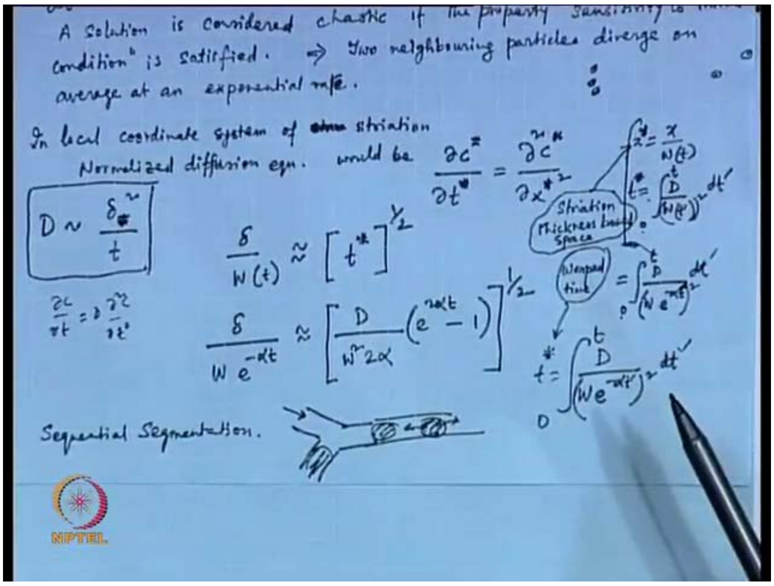

What I will do probably in the in the next class is I will, probably I will try to extract some more information from this, from this equation, that I have here, and then I will discuss a topic known as sequential segmentation. This is, this is something, which I would pick up.

Sequential segmentation, this is also another form of mixing, this is also another form of mixing where a solute stream, there, there would be two streams coming in and there would be an on-off mode. On-off mode means, suppose this is the solvent and this is the solute, so this solute goes in and then it switches off, solvent comes in, then again solute goes in. So, at the end you will see there are blobs of solute and then you allow the mixing to take place between solute and solvent. So, that is, that is sequential segmentation.

This has, this has its own theories. Basically, it would be boundary condition, which were, where the, I mean, the governing equation would be again same, that, that same equation, that we have been talking about, del c del t is equal to D del square c del z square, or in a, in a, in a two-dimensional flow. We have already discussed the, our, our basic framework is that same diffusion equation. However, we play with the boundary condition and we show you how we can get the concentration profile in this case. So, I, I suggest once again, that you go through this chaotic mixing, the expressions and probably, think over it, it, it. And then, then let me know what, what, what problem you see or what, what, what is the problem with these expressions, and then we can probably go from there, that is all I have for today's class.