Microscale Transport Processes

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Lecture No. # 12

Mixing (Contd.)

Welcome to this lecture on microscale transport process. What we have been discussing is mixing in micro channel.

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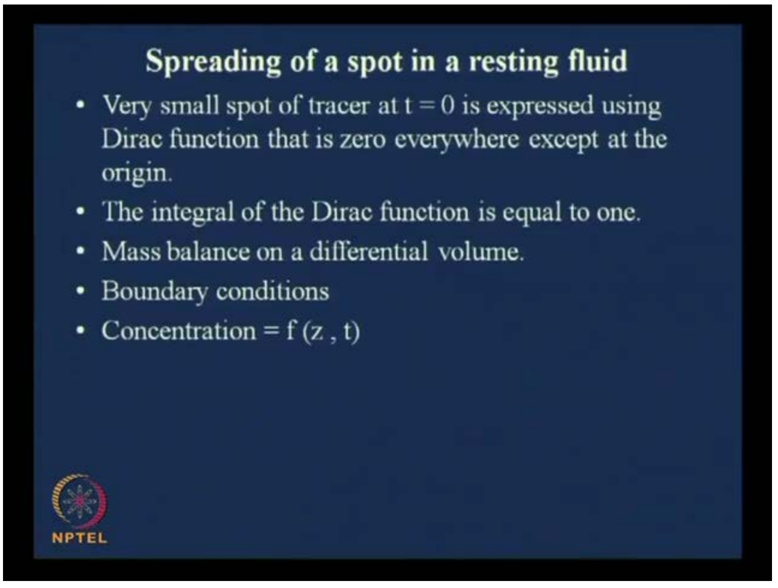

Let me give a quick a recap on what we have done so far. We have discussed about various fabrication techniques, first we discussed about various applications of micro fluidic devices and then, we talked about fabrication techniques. Next, we picked up some aspects of transport phenomena involving micro, micro fluidic devices, which is unique and we picked up first the topic of mixing. Before we get into the mixing in a micro channel, we thought it would be more appropriate to discuss about some fundamental aspects of spreading of a spot in a resting fluid, at least, look at the equations, and how a spreading takes place, how the diffusion takes place inside a micro channel device.

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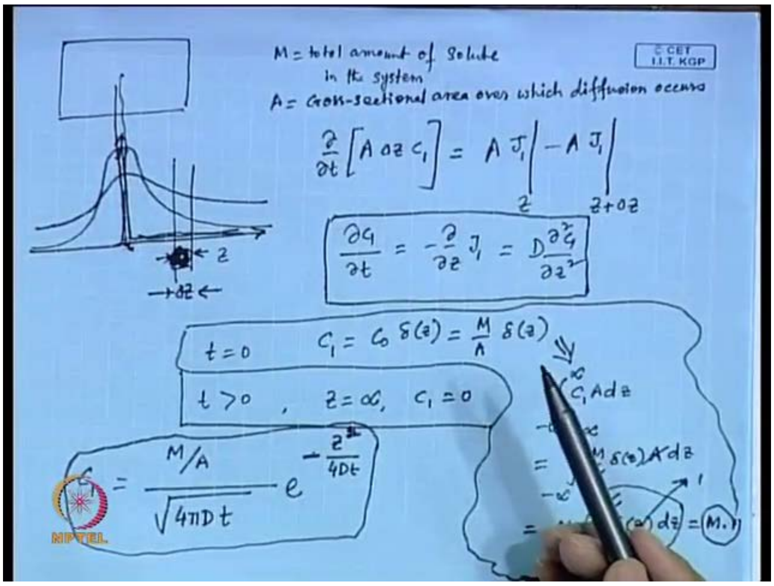

So, what we did in the last class is, we have talked about small spot of tracer, which is at z is equal to 0. This, this, this slide I picked up from last class, now this, in this slide we can see, that we have picked up, we have, we have put a small spot of tracer at t equal to 0, which is simulated by the direct function. That means, the value of this function is 0 everywhere except at the origin at z equal to 0, and integration of this Dirac function from minus infinity to plus infinity, that integration, the value of that integral is 1.

So, with that initial condition, we, we solved this governing equation, this is the governing mass balance equation, that we have looked into. For, for, for students who are coming from other disciplines, for example mechanical and all, I suggest, you go through any standard mass transfer book, or for example transport phenomena by Bird, Stewart, Lightfoot. And just like the way force balances are done, by the, by the same method a mass balance can be done. And a mass flux, if you are not familiar with this term mass flux, this mass flux, this is nothing, but we, we talk, we talk in terms of molar flux, which is basically nothing, but number of moles, crossing number of moles transporting per unit time through unit area. Just the way we discuss heat flux, just the way we, we, we describe heat flux or energy flux, by the same method we can, we can, we can talk about mass flux, which or molar flux and these, these terms J 1.

Here, J 1 at z or J 1 at z plus delta z, these are basically nothing, but that molar flux and this, just the way we have for heat transfer, flux is proportional to the temperature gradient. Similarly, for mass transfer, the molar flux, that would be proportional to the concentration gradient, just by the same token. So, if, if you, you can, you can quickly go through this, I mean, this, this is, this is, this is the mass transfer at relationship, that the flux is proportional to the concentration gradient and diffusion coefficient, which is basically the proportionality, proportionality constant. So, it, so, you, you should, now you can understand how I, how I am writing this del, del z of J 1 as D del square c 1 del z square. So, this, this you should not and if you, if you have any, if you have any doubt, you can always meet me after the class.

Now, this, so this is the governing equation, that we have and this governing equation has to be solved with the initial and boundary condition. The initial condition is that at t equal to 0, the concentration follows the direct function. That means C 0 delta z and C 0 is nothing, but M by A, M is the total amount of solute in the system and A is the cross sectional area over which diffusion occurs. So, M by A delta z and what is this delta z? That the property of this delta z is defined here, that it is minus infinity to plus infinity C 1 A D z that has to be equal to M always, because you are not adding anything from outside. This is that same spot, small spot, that is diffusing out.

So, the total solute, total amount of solute in the system, that is M, that remains constant that may get diluted over the space now, and t greater than 0 at z equal to infinity, C 1 is equal to 0. That means the concentration far away from the spot has to be equal to the original concentration, which is 0. That means, the effect of that spot, after all this is a very small spot. So, how much can you, so that boundary condition you put here is that at infinity is equal to 0, this concentration equal to 0.

Now, if you solve this equation you will get this, this, this final form C 1 is equal to M by A square root of 4 pi D t e to the power minus z square by 4 Dt. So, this is the, so basically, what, what we said at the outset, that concentration C 1 is a function of z and t. so, basically, what we are trying to say is that concentration C 1 is a function of z and t. That means, space and time, initially it was a Dirac function. As time progresses, as time progresses the next one is this one, next one is this one and at, probably at t is equal to infinity, the entire concentration would become uniform. So, with time and with space the concentration changes.

So, you need to know what is this function C 1 z t, what is a functional form for C 1 z t? And that functional form is given here, C 1 is equal to M by A square root of 4 pi Dt, so t is coming here and z is coming there and t is coming there. And of course, the diffusivity D, diffusivity and diffusion coefficient, we are just using it. It is basically a same term; we are using both, in both the forms. Now, this, so it is, it is a function of the diffusivity; it is the function of time and it is the function of space.

How you solve this equation? I, I asked, that you, you solve this by Laplace transform. Now, has anybody tried to find this using Laplace transform because this, as I said, at, at a, at a very beginning that helding this boundary condition would be pretty straight forward because this is, because that the moment, the moment you try to solve this moment, you try to solve this using Laplace transform. This will, this will take a shape of, say S, say, say, say you, you say, you define, say define C, say you define, say you write it in Laplace transform, then this would, this would appear like this, S C 1 bar minus C 1 at t is equal to 0, that is the Laplace transform of del c del t and that is equal to this D, D square c 1 bar D z square.

I can, I can write it on a fresh page, I do not have any space here. What, what we are trying to say here is that S C 1 bar minus C 1 at t is equal to 0, this is the Laplace transform of the left hand side of this governing equation. What is the governing equation? This is the governing equation we are talking about.

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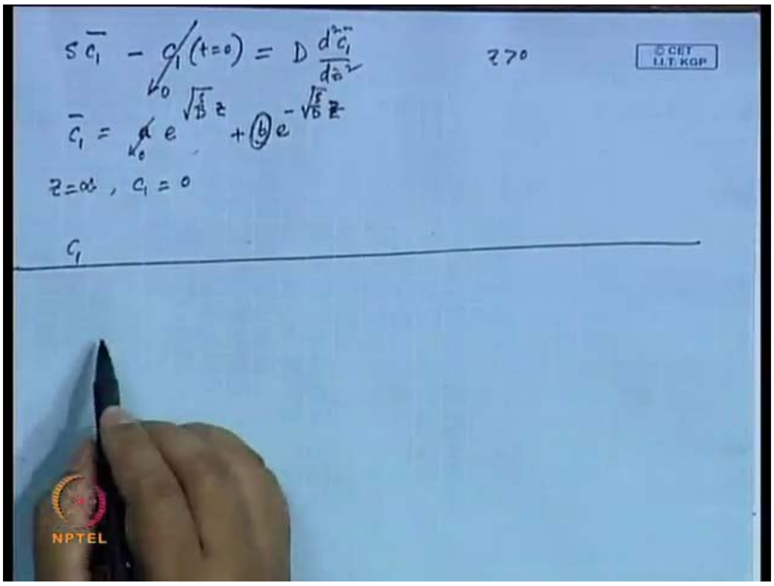

When we take the Laplace transform of the left hand side, this is what we get and what is the Laplace transform of the right hand side? That would be D, d square c 1 bar D z square. Now, since we are interested in place z greater than 0, so this term, anyway at t equal to 0 you can consider this to be 0 and we, we do not care about the center, we, we care about z greater than 0, z starting, z greater than 0, any value. So, this is the case and then, if you solve this, what you get is c 1 bar, that is equal to, let us say some constant e to the power square root of s by D into z plus some constant e to the power minus square root of s by D into z, alright.

You, you remember that general solution and all those things. So, it is, it is basically C 1 e to the power M x plus c 2 e to the power minus M x. That form, basically you can write this as, in, in that way, then you put, you try to find out what are the boundary conditions. I mean, how, how you incorporate the boundary conditions and the very outset we see, that at z is equal to infinity, c 1 is equal to finite, c 1 is equal to 0 as a matter of fact.

Now, if we do not force this A to be 0, then in that case e to the power z, if z is infinity, then e to the power infinity. Think of this term, then unless you force this guy to 0, then this, this, this would be, you, you, you will be in trouble because finally, your c 1 is equal to 0 at z equal to infinity. So, this has to be equal to 0 and you are left with only this b term and that b term has to be evaluated using the other boundary condition. The other boundary condition is what? Other boundary condition is at t equal to 0, C 1 is equal to M by A delta z.

Now, how you handle the Laplace transform of this Dirac function? I mean, this you need to look into and that is what I, I in the last class I told you to go through and find out how this transform can be taken and I, I insist, that you, you go and do it once again, see how this Laplace transform is taken, how it will handle for a Dirac function and how you can arrive at this equation.

Now, c 1 is equal to M by A square root of 4 pi Dt e to the power minus z square by 4 Dt. This equation, how you end up with this equation because this, this you will, this you will do because once you have this, you have some constant, using the other boundary condition you have to find out what is b and then you have to take an inverse Laplace transform for c. This is a C 1 bar, but actually you are looking at the, you, you, you are actually looking at this C 1. So, when you take the inverse Laplace transform, you will get this, this, this form, C 1 is equal to this quantity. So, I suggest, that you should be familiar with how it, how this, how this whole thing can be handled. So, this is one such situation.

And I, what, what I suggest, what you take from here is that you have square root of Dt type dependence here in the denominator and e to the power minus z square by 4 Dt, that is, that is a general trend. I mean, that, that you need to see, see what, what I, what I, I would say here is that if D in, if D is more, then what would be, how would be the concentration profile? So, where all D is appearing, so that kind, so you, you should have some feel for it. So, D is appearing here in the denominator, one thing and here, D is appearing in the denominator at this with this exponential term. So, these, these are some of the things, which you can, you can, you can, you can look into.

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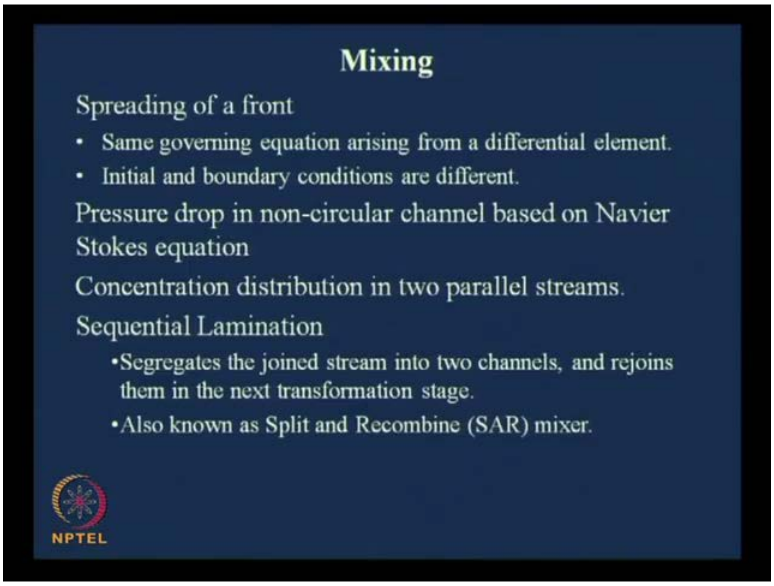

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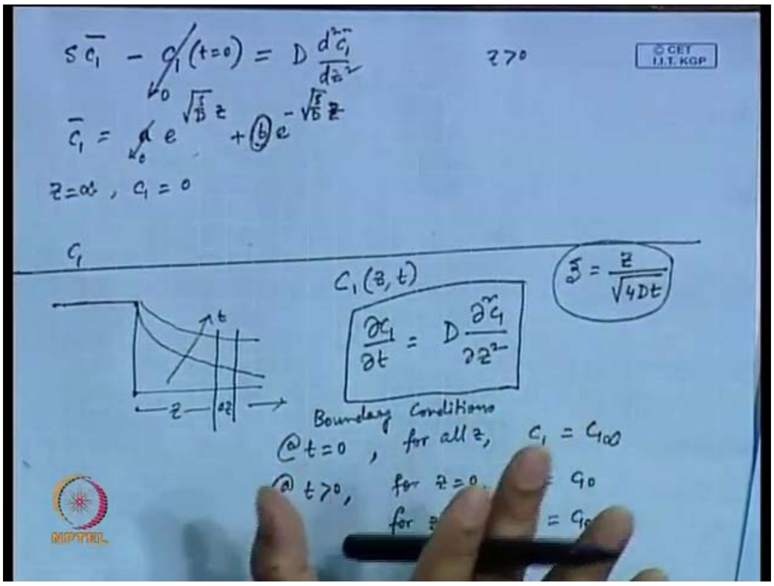

Now, suppose, we, we, suppose we look into another type of, another type of situation where instead of a spot of tracer you have spreading of a front instead of a spot of tracer. Suppose, we have a spreading of a front, so what that means is, suppose I have a front, that means I have a concentration here and I have a 0 concentration here. So, from this point onwards the concentration is 0 and from this point onwards concentration is a finite value. And now with time, how the concentration changes here? So, it would be somewhere like this, somewhere like this. So, with time the concentration will change. So, how will you, how will you solve this? How, how would you obtain a concentration profile here? Just like we said, C 1 as a function of z and t, how will you do that?

Now, at the very outset I must say, that this, when it comes to spreading of a front, we have same governing equation arising from a differential element; same governing equation arising from a differential element. That means, if you pick up a differential element here, if you pick up a differential element of delta z at a distance z from the center, at a distance z from the center if you pick up a differential element delta z and if you try to draw mass balance, just the way you have done last time, this mass balance would be valid. I mean, if you do not put the initial and the boundary condition, that is different, but this, this, this mass, this governing equation would be valid. So, that means this equation would be valid. So, you can, even closing your eyes you can write, that del C 1 del t is equal to del C 1 del t would be equal to D del square C 1 del z square. So, this governing equation would be, this governing equation would be valid.

However, when it comes to the boundary conditions they would be somewhat different; the boundary conditions would be somewhat different. And what are the boundary conditions now? Boundary conditions would be here at t is equal to 0, for all z c 1 is equal to c 1 infinity, and at t greater than 0 for z is equal to 0, c 1 is equal to c 1 0 and for z is equal to infinity, c 1 is equal to c 1 infinity. Do you, do you, can, can you relate to this? I mean, what we are saying is that this governing equation remains the same, which I, I think you must agree.

If I pick up a differential element the same process would be going on, mass flux from left hand side, mass leaving from the right hand side differences the accumulation and all this. So, this, that, that is understood. Now, when it comes to the boundary condition, earlier what was the boundary condition? That at t equal to 0 there was this Dirac function; that means, at z equal to 0 there is a spot of tracer and everywhere it is 0.

Now, here the boundary condition is that at t equal to 0, now z starts from here from this point. So, at t equal to 0, for all z C 1 is equal to C 1 infinity, that means, that is the initial concentration. I mean, we could have said, that C 1 infinity is equal to 0, that means, it is all 0, but at that we can do anytime. Anytime we can set C 1 infinity equal to 0, I mean, let us find a general form first. So, C 1 is equal to C 1 infinity at t greater than 0. That means, once the clock started and the time is rolling at t greater than 0 for z is equal to 0, now at z is equal to 0, you are holding the concentration at constant value C 1 0. So, that is what we are solving basically.

You may say how are, how are we holding, that is a different issue, but I am trying to see various, various aspects first, I mean, various cases first. Now, this case is, that you are holding this point at C, at z equal to 0 C 1 is equal to C 1 0. And of course, at z equal to infinity, far away from this place there is no effect of it. So, the concentration will remain same as it was before. So, these are the boundary conditions and these boundary conditions are much easier to handle because now one thing I must say, that I mean, I, last time I mentioned, that you would do it by Laplace transform. It is not necessary, that you have to do it Laplace transform. As a matter of fact, you can solve it using a similarity variable also.

The same equation, the similarity variable will take a shape like this z square root of 4 Dt. In fact, this term you might have seen already, see this z square by 4 Dt, that is coming there, so z divided by square root of 4 Dt, this is a similarity variable and you can use this similarity variable to simplify this equation and solve. I mean, that is also another way to do it, but let us, let us do it using, using Laplace transform. However, this, this process would be much, this, this would be simpler. The Laplace transform would be of, of this equation would be as before first.

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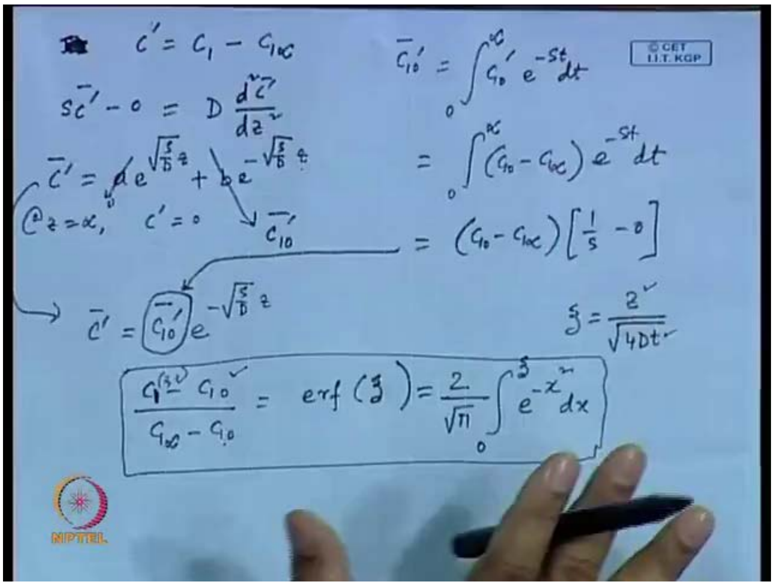

Since we have this C 1 infinity and C 1 0, we have this instead of 0 and 1. I mean, we had, since we defined these two terms, let me put a deviation variable C prime as any concentration, C 1 minus C 1 infinity. So, then, we can write this, write this governing equation in terms of the deviation variable and take the Laplace transform or write the, write the equation in Laplace transform in, in Laplace domain using this deviation variable. So, this was the equation that we are writing. So, left hand side would be as before, it would be s C 1 minus concentration at t equal to 0.

Now, that would cancel out because your C prime is a deviation variable, so C prime would be equal to 0, now got my point? So, you would be writing it as S C prime bar minus 0. This is 0 because this deviation variable will be 0, but actually at t equal to 0 it is C 1 infinity, so this would be equal to D d square C prime bar Dz square. So, as before you can write this in general, the, the solution would be C prime a e to the power square root of s by D into z plus b e to the power minus square root of s by D into z. And then, the boundary conditions are that, that boundary condition that at z is equal to infinity this c prime is equal to 0, so it is finite. So, this has to go to 0, otherwise this will blow up, so this has to be equal to 0. So, this term is gone and we, you are left with only this term, so this you can write as at z is equal to 0, at z is equal to 0 this would be what?

At z is equal to 0, this term would be equal to 1; at z is equal to 0 this term would be equal to 1. That means, C prime bar would be equal to b and what do you call C prime bar at, at z equal to 0? You would be calling this, then, then I can, this b as, I can, I can write this b as c 1 0 prime bar, alright. See, I can see, that at z equal to 0, b is equal to C prime bar and at z equal to 0, you call this C prime bar as c 1 0 prime bar. So, you can write, instead of b you can write C 1 0 prime bar. And now, if we look at what is this C 1 0 prime bar, that is nothing, but integration 0 to infinity C 1 0 prime e to the power minus s t dt, that is the definition, that is how Laplace transform is done. So, you can write this as 0 to infinity C 1 0 minus C 1 infinity, that is the definition of C 1 0 prime because C 1 0 prime, this prime is basically the deviation variable, that is how we have defined. Here, prime is basically the deviation variable, so this into e to the power minus st dt and that would be equal to C 1 0 minus C 1 infinity into 1 by s minus 0, if you simplify this further, alright.

So, what you get finally is, if you so, you have so, you have taken care of this, this, this, this boundary, the other boundary condition. So, now if you put this, so originally, what did you get from here? Originally, you had got a C prime bar as C 1 0 prime bar e to the power minus square root of s by D into z, that is what you got. A has cancelled to 0 and b, b has to be equal to C 1 0 prime, as per the nomenclature. So, that is what you have written here.

Now, instead of this C 1 0 prime, now you bring it, bring this term, bring this thing here, this C 1 0 and replace this one with this whole term, C 1 0 minus C 1 infinity by s and then, you take the inverse Laplace transform. This would be pretty straight forward, you go to the table for Laplace transform and you can see, that the, the, what you get here, I mean what is the, the final form, that you get is C 1 minus C 1 0 divided by C 1 infinity minus C 1 0, that is equal to nothing, but error function of zeta. And what is zeta? Zeta is equal to, zeta is equal to z divided by square root of 4 Dt. So, this would be equal to error function of zeta, so that, that you…

And, and if, if, if, that in, in the, in the, in the Laplace transform, in the table it is written as error function and you know, if you pick up any higher engineering mathematics or any, any such book you will find, that Laplace transform or, or, or this table for error function is also available at the end. If you pick up any engineering mathematics book, at the end this error function table is available. In fact, you can even quickly study how this error function values change.

You can see, x is changing from 0 to, say all the way infinity, but the value of error function how it is changing for sometime I think the x and error function error function of x, they go together. That means, the value of x and value of error function of x, they are close and then, as it shoots to infinity, the error, the value of this other, the value of the function stabilizes to an asymptotically, to value of 1 or something. I mean, that you, you can, you can look into the table and you can yourself get a feel for the numbers, that, that, that I can tell you and, and if you want to know exactly how what this error function is, probably you can, you can see this in any standard engineering mathematics book, it is 2 by square root of pi integration 0 to zeta e to the power minus, I do not want to call it s or should I, it does not matter probably, e to the power minus x square D x. So, this is, this is how, this, so this is the final form all, so this is the final form that you have.

So, you have defined now, this is the C 1, that is unknown, C 1 0 is known, C 1 0 is the concentration, that you have been holding, z is equal to 0 2, so this is the value of C 1 0, that is what you are holding and this is the value of C 1 infinity, that is what. So, this is the level of C 1 infinity, this is the level of C 1 0 and how the concentration changes within, that is given here, that is given here with this, in this function of form.

So C 1 0 is known, C 1 infinity is known, C 1 is the unknown and this is basically a function of z and t, so this C 1 you know is a function of z and t and that is what you want to know; that is what you want to know and that is, the functional form is error function of zeta, where zeta is z by square root of 4 dt. So, you have both, z functionality involved, time functionality involved and diffusivity or diffusion coefficient that is involved. So, these are, these are some of the analytical expressions that are available and that will quickly tell you, that if I have one front, which does not have any solute and the other front, which is loaded with solute and they are held side by side, how the concentration, how the concentration will change in the, in the place where the solute was not there?

So, these are, these are some of the things, which we need to know. I mean, that the, the, the, anyway, the, in, in micro scale process the, we are, we are relying on diffusion. We said, that we never, we are in a laminar flow region, even, even the Reynolds number is close to one, so we are in a, very things are happening in a very gentle manner and we can completely rely on diffusion coefficient or diffusion based transport. There is absolutely no presence of eddies, which can have, which can have other, other implications. So, these, these are some of the aspects of diffusion, that you need to understand.

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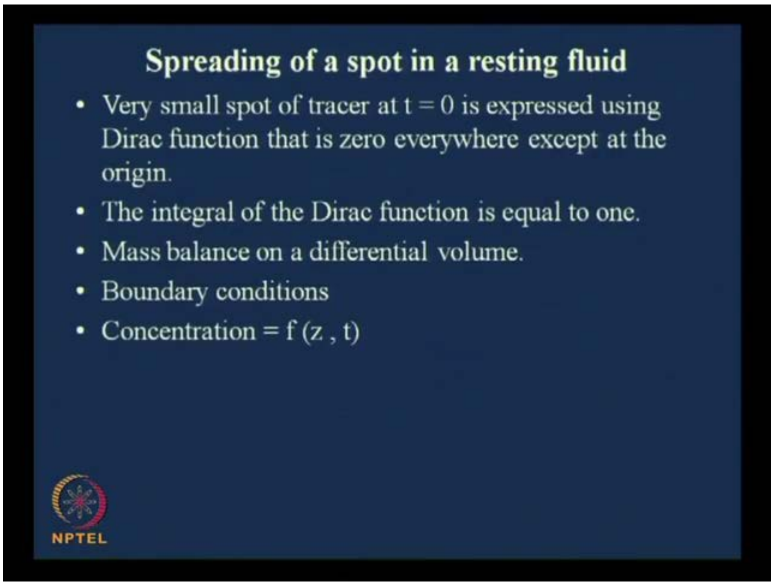

The other thing, that we have here, I mean, let me point out quickly, that when it comes to the pressure drop, in fact, that is, that is the next topic in my, so let me, let me point out quickly what all we have, we have done so far. I mean, if we try to quickly recap now, one thing is we discussed about spreading of a spot in a resting fluid. What we did is very small spot of tracer at t equal to 0 is expressed using Dirac function that is 0 everywhere, except at the origin. The integral over the entire range minus infinity to plus infinity, that is equal to 1. Mass balance is done on a differential volume. Boundary conditions are imposed and concentration is presented as a function of z and t, space and time.

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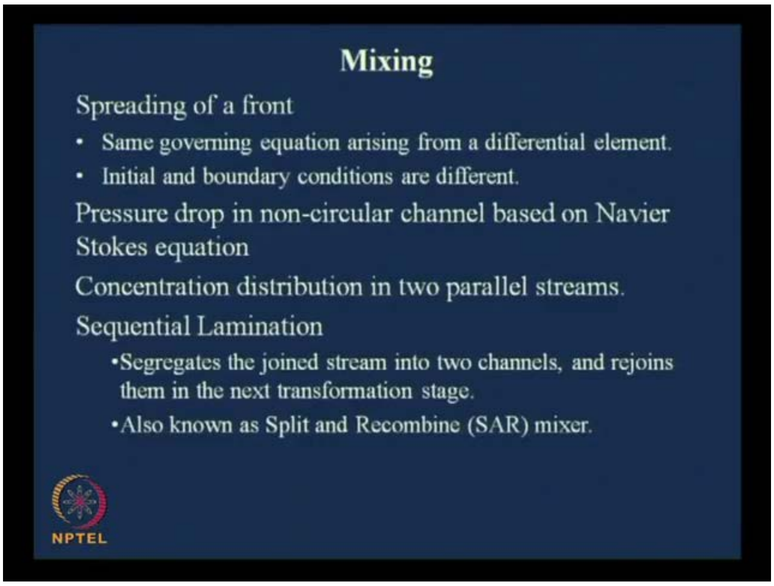

Now, next we continued with the mixing. We talked about this, we discussed about spreading of a front instead of a spot, same governing equation arising from a differential element. However, the initial and boundary conditions are different, that you must appreciate, that we have already done. And next, I, briefly I would like to touch upon the pressure drop in a non-circular channel based on Navier Stokes equation. Since we are more familiar with circular channels, I mean, Hagen Poiseuille equation and frictional pressure, friction factor and all this, we are more familiar with circular channels.

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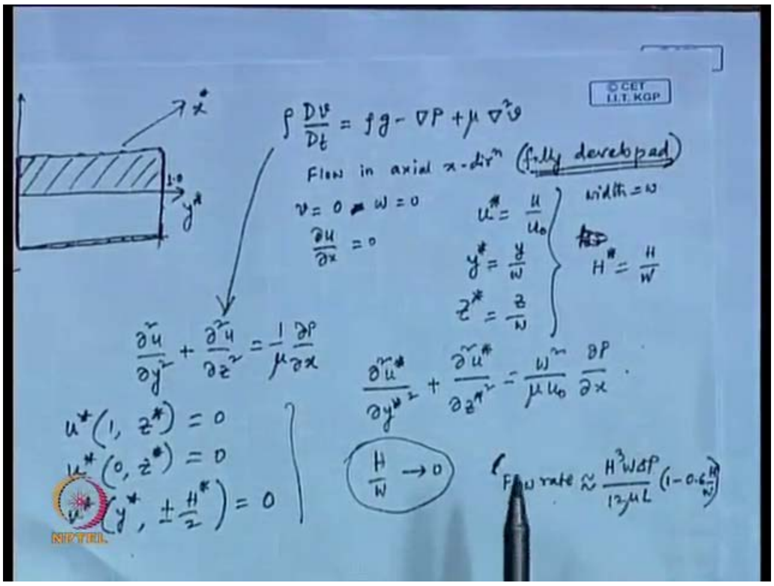

I quickly revisit the pressure drop in a, in a non, non-circular channel and what we do here is I make a drawing like this. This we call, let me, let me write down first the Navier Stokes equation, which, which would appear something like this. This is the substantial derivative, you, you are all familiar with this equation.

Now, what I want to solve here is, we want to, we want to solve flow in axial x direction, which is fully developed, that is something, which we would like to solve. And I said, that other components v is equal to, w is equal to, w equal to 0, v is equal to 0 and w is equal to 0 and del u del x, that is equal to 0. What is x and what is v, that, that we need to point out. Now, the other aspect is, I mean, I, I, let, be, before putting the drawing let me point out, that I am putting this u star, it is a dimensionless value as u by u is 0, y star as y by w, z star as z by w. So, these are the, these are, these are the, these are the dimensionless forms.

We have, now we are plotting here this direction as y star, this direction as z star and this direction as x or x star, whatever you call it. And then, the channel is, then is taking this shape. This is the channel, this is the channel, so this is, here y star is equal to 0, here y star is equal to 1, here y star is equal to 1, here y star is equal to 0. And then, because the width of the channel is w, so width of the channel is w, width is equal to w. What we define by width? This is the width, this is the width, this is the height, this is the height. So, we can write this as, if we write the height h at another dimensionless parameter, say, let us say write this as, let us say I write this as h star is equal to h by w.

So, everything we are making this dimensionless with reference to w only. w is the width in this direction, so this, this would be from 0 to 1. When it comes to, when it comes, the dimensions of the channel in dimensionless form, this would be 0 to 1. However, this part would be here, it would be plus h star by 2 to minus h star by 2, alright. I must pull this a little bit, this is plus h star by 2 and this is minus h star by 2, so this constitutes the full channel, alright. And this is the x star, this is the x direction.

So, now, if we try to solve this part and we say, that this is just the mirror image of what we have here, you can write, this equation would be, can be written as del square u del y square plus del square u del z square, that is equal to 1 by mu del p del x. We have del u del x equal to 0, what does this mean? Then del u del x is equal to 0, that means, u is not changing with x, so we had flow in axial x direction fully developed. So, there is absolutely no flow in this direction, there is no flow in this direction, flow is only taking place in x direction and that flow is also not changing with x, that means, flow is fully developed. So that is what we have.

And now, if we try to solve this equation, this is what we would end up with and then we can of course, write it in terms of the boundary conditions. That means, u star y star z star, it would be something like del square u star del y star square plus del square u star del z star square, that is equal to w square by mu u 0 del p del x. So, and the boundary conditions would be simply u star at 1 z star, 1 means, here at this wall for all values of z star, it would be, velocity would be equal to 0. If you consider no slip u star at 0, z star is equal to 0. That means, at this boundary it would be all 0 because it is a no slip and u star y star plus minus h star by 2, that is equal to 0, no slip boundary condition. This we will point out later that in a micro channel, whether you can take a no slip boundary condition or you have to consider a slip that is governed by the Knudsen number.

So, depending on what the value of Knudsen number is, it is, it is, it is decided whether you, you can take this boundary condition as no-slip boundary condition or not. Now, these Knudsen number is, is probably more important for gases, I mean, this, this we discussed probably, couple of, I mean, I think last two, last class we have discussed about this, whether the Knudsen number, Knudsen number we said, that Knudsen number is more important for gases in a micro channel. For liquids, the Knudsen number will be important if we are working at a nanometer scale of the channel, but that is not exactly, that comes, that does not come under the purview of this course. So, if it is a liquid flow, we do not have to bother that much about this no-, about this slip boundary condition we can assume no-slip boundary condition for liquids in micro channel.

There, there are other issues, there are other issues, it is not that straightforward here, I mean, I mean, let me point out at the every outset, that we are just talking about solving Navier Stokes equation in non-circular channel and nothing else. There are other issues, which we would be taking up down the line. So, these are, these are some of the boundary conditions. Then, you can solve this and ideally, I suggest that you, you solve it using numerical methods because that is, that is the common way of doing it. If somebody wants to know the, if there is any, any quick, any, any quick relation that is available, if somebody wants to figure out how, what would be the pressure drop, there is one available.

If somebody says, that h by w tends to 0; that means, this channel is having a low aspect ratio. That means, it is, it is, w is big, h is small; that means, w is big, but h is small. If this, if this is a condition that prevails, as far as the channel dimensions are concerned, then you can write the, the flow rate, the average flow rate would be given by the, the, the flow rate, the flow rate would be given by approximately h cube w delta p by 12 mu L 1 minus 0.6, 0. 6, h by w. It went out of the range; let me put this page down here. What we are talking about is flow rate is approximated as h cube w delta p by 12 mu L 1 minus 0.6 h by w. So, this is an approximate treatment you can do.

So, if, if, if the flow rate is known you can find out what is the pressure drop for a noncircular channel, but it, I, I must point out, this is entirely for Navier Stokes equation for a noncircular channel. This is the equation, but there could be other, other considerations that would be coming in here, so that would, probably that would alter the whole scenario. If you want to, if you want to get some idea quickly, that how much pressure drop you can expect, if you use a no-slip boundary condition and Navier Stokes equation to be valid, then probably you can get, you can use these equations.

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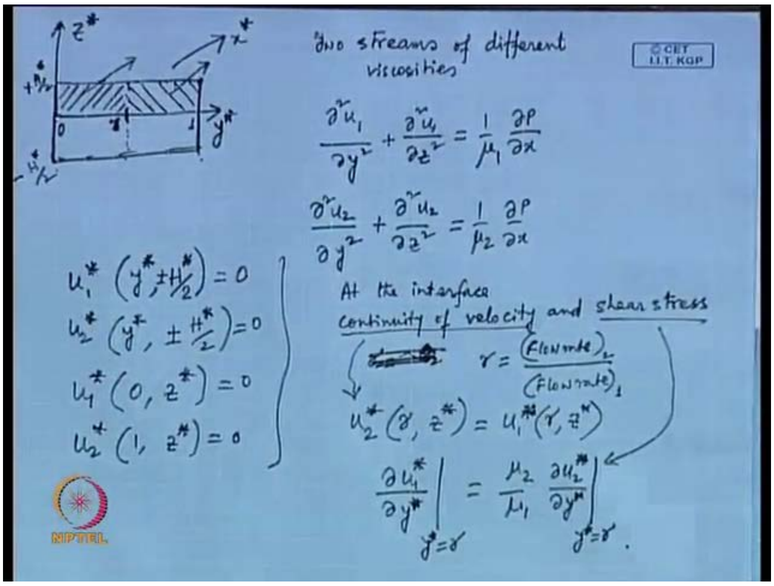

Now, I must point out, that the same, same system of equations, it could be, that you have, it could be, that you have, you have two streams of different viscosities, two streams of different viscosities, so as, before if we look at, this is the channel suppose we are looking at, and then we have this half, not, not exactly half I would say at some distance, this you have one viscosity. So, we are, we are working with the symmetric half. So, symmetric half with reference to this basically, so as before we are working with only this half. So, this is the other part and you have one viscosity applicable to this one and the other viscosity applicable to this one. And now, if you want to solve the Navier Stokes equation, how would you do that?

So, you, you are doing the same thing, but instead of having single liquid flowing through this entire channel you have a part of it flowing, part of it is of higher viscosity and the other part is of lower viscosity. How would you solve this equation? The, the method, that, or the, or the approach, that you should have here is that you, you write down the Navier Stokes equation for these two systems separately. That means, if you, if you write down, for the left one it would be del square u 1 del y square plus del square u

1 del z square, that is equal to 1 by mu del p del x, and the other 1 is del square u 2 del y square plus del square u 2 del z square is equal to 1 by, we call this as mu, say we, we call this, say, since we are writing it as 1 and 2, I write this as 1 by mu 1, this we write it as 1 by mu 2 del p del x, mind it, that the pressure remains, pressure with as a function of x, that remains same.

So, the pressure is, there is p 1 and p 2, pressure remains same, del p del x, but due to the, but this u 1 and u 2, they would be different. So, u 1 is basically, earlier we have taken only 1 u, that is flowing in x direction. Now, we have, one is this u 1, another is u 2, alright. So, you need to write these two, so there would be two governing equations and then you will have boundary conditions.

Now, the boundary conditions, that you had earlier, that means at this wall there will be no-slip velocity is equal to 0; at this wall there will be no-slip velocity would be equal to 0, how, how would it look? For example, this, this wall and this wall if we pick up, then this would be u 1 star at y star comma h star. This is what? This is plus h star by 2, this is minus h star by 2, so u 1 star y star h plus minus h star by 2, that is equal to, so, so I should, yeah, that would be equal to 0. Similarly, you can write u 2 star y star plus minus h star by 2 equal to 0. So, these are, these are the, these are the no-slip boundary condition. Similarly, you will have no-slip boundary condition here. Here, when it comes to this wall, it would be only u 1 and when it comes to this wall, it would be only u 2. So, it would be u 1 star, u 1 star would be 0 z star that is equal to 0. And u 2 star 1 z star that is equal to 0. So, these are the boundary conditions you have.

On top of this, at the interface you have to satisfy, at the interface you have to satisfy the continuity of, continuity of velocity and shear stress, how will you do that? Say, we have, first we define this, where is this position, we have not come to that point yet. Let us call this, if this is changing from 0 to 1 and we call this, say gamma is a position, let us say we call this position as gamma. So, then this gamma would be equal to, gamma would be equal to q 2 star, q 2 divided by gamma, gamma, gamma would be equal to flow rate 2 divided by flow rate 1, that will define the position gamma, and then you can write this u 2 star gamma z star, that is equal to u 1 star gamma z star.

So, this is basically the continuity of velocity, that is what is satisfied and the continuity of shear stress. To satisfy this you have to write del u 1 star by del y star at y star is equal to gamma, that is equal to mu 2 by mu 1 del u 2 star by del y star at y star is equal to gamma. So, that is how you equate the continuity of the shear stress, so that is important at the interface. So, when the two fluids are flowing through a channel, non-circular channel and one is of higher viscosity than other, then you have to include these aspects.

Now, let me point out, that an analytical solution is still available for this system of equations. I, I am not writing it here, if you, if you, if you go to the literature you can, you can see, that already these analytical solution has been, has been, is available. However, probably, you would be more comfortable solving it numerically rather than going to the analytical solution. Of course, it is the, I mean, yeah, probably going to the, going to the numerical solution would be, would be definitely better, instead of writing a big series form. So, this is, this is, as far as the, the equations are concerned for the, for the two streams flowing.

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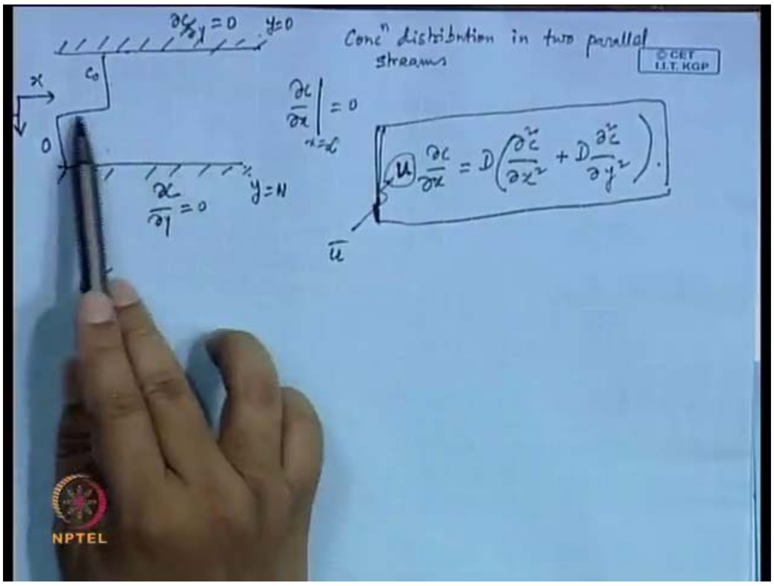

Now, next topic I would like to pick up is what I have here in this, in this power point slide, concentration distribution in two parallel streams; concentration distribution in two parallel streams. That means, if you have, if you have two streams flowing parallel, the topic here is concentration distribution in two parallel streams, here you have two streams flowing parallely and concentration looks like this, here the concentration is c 0 and here the concentration is 0.

So, this, so if, if we, this is the concentration profile, so that means, I am having two parallel streams flowing here. I, I have, I have two parallel streams flowing here, one this, two streams are flowing here side by side and they are interacting with each other by diffusion. So, one stream, the, the upper one, the upper stream has higher concentration, the concentration of a solute in the upper stream is C 0 and the concentration in this lower one, there it is 0. So, that is, that is what we have here, we are calling this C 0 and 0. I have x-axis, I have x-axis drawn here, this is, this is the x and this is the y, this is the y I have. And then, you have, here del c del y is equal to 0, that means, this is an impermeable wall here, del c del y is equal to 0, that means, this wall is impermeable, that no, no concentration exchange. And then, here you are writing on this side, del c del x at x equal to infinity, that is equal to 0. So, that means, concentration change, the concentration would be, they, they are mixing.

If it is C 0 and if it is 0, if it is saline water and if it is clean water, then salt will diffuse from here to here and then at the end of it you will see, everywhere it is C 0 by 2, the concentration will be C 0 2 down the line, upto after some length it would be the case. So, you can write, at infinity the concentration change would be 0, so there will be no longer any concentration change, but here, the concentration is changing. Here, if you look at the, if you say concentration at this point and concentration this, this point, if it is this half of the stream, then it would be going down and here it is going up. So, in x direction concentration is changing, but at infinity you would assume, that del c del x is equal to 0, and at…

So, this you call as, say y is equal to 0, let us call it and let us say call it y is equal to, say w. So, del c del y is equal to 0, at y is equal to w and del c del y is equal to 0 at y is equal to 0. So, these are some of the boundary conditions you have and the, at the inlet you have, it is 0 and C 0. Now, if you try to, if you, if you write the governing, if you write the governing equation, if you write the governing transport equation, you can, you should be writing it as u del c del x is equal to D del square c del x square plus D del square c del y square.

You have diffusion going on, what, what are taking, what all you are taking here, you are taking care of? Diffusion going on in x direction, diffusion going on in x direction means this. Whatever is coming downstream, that is interacting by diffusion mode with the upstream, so in x direction there is a diffusion going on, in y direction of course, this has to be there because that is the soul mechanism by which this mixing is taking place. So, diffusion is going on in y direction and you have convection going on. u del c del x, this is the convective transport, since the fluid is flowing in this direction, so there is a convective transport. So, if you are, if you pick up, pick up differential element here, what you are taking care of is the diffusive transport in x direction, diffusive transport in y direction and the convective transport in x direction. Of course, there is no convection in y direction, so these are the, these are the elements, that you are, you can consider at the, at the least. So, these are the, these are the elements.

Now, if somebody assumes this, this u, now if you, if you look at this u, what would be this u? If it is, if it is a parabolic, if, if it is a laminar flow, you can expect the u to have a parabolic velocity distribution for a circular channel. So, you, you will, you will be changing with y. However, if somebody can assume this u to be u bar, which is the average velocity, then probably there is an analytical solution available to this system of equation. But if you bring in the velocity profile itself with y, then probably you need to go to the numerical solution. This has, this has been, this has been studied, this, this numeric, numerical, numerical solution has been obtained of this, and, and this analytical solution with u as constant as u bar, which is, which is a very approximate one, I mean, that, that kind of treatment also has been done.

What I will do is, in the next class I will, I will get to the, get to the, I mean, I will revisit this aspect once again and try to see, if we can come up with some kind of concentration profile, some kind of concentration profile, then you know, how the concentration changes with x and y as a function of time.

Let me point out, then what all we have done. What all we have done, we, you have already, I think I have, I have stated that few times. So, we are, we are basically studying the diffusion, studying how the velocity profile is to be solved using Navier Stokes equation for a non-circular channel. What we would be picking up next is how, with this knowledge that we have how we can model a mixing where the two streams are intermingling. In fact, in the next, in the next class we will get into this process called Split and Recombine mixer, Split and Recombine mixer where the two streams will be splitted, two streams will be splitted and they will be brought back once again with the idea, that the, we can have some extended interfacial area. And then, we, we will, we will try to, try to theorize this whole process and we will try to see, we will come up with some kind of coefficient, we, which we will try to relate with the chaotic mixing.

So, so in the next class itself we will get to the actual mixing in a stream where you have (( )) and where two streams are intermingling, one stream is getting into the other stream. And then, so that, that is exactly what we are trying to say here, segregate the joined stream into two channels and rejoin them in the next transformation stage. So, how will you theorize this process if this whole thing is happening in a chaotic manner, so that is something, which we would take up next and that would, that would be, that would be a different kind of, that would not be a very, we, we our, our, our, essentially our theories will be based on the transport equation, diffusive transport, etcetera. But we will bring in the elements of chaos and other, other understanding we have here. Yeah, with that I stop here and that is all we had for today's class.

Thank you very much.