Microscale Transport Process

Prof. S. Ganguly

Department of Chemical Engineering

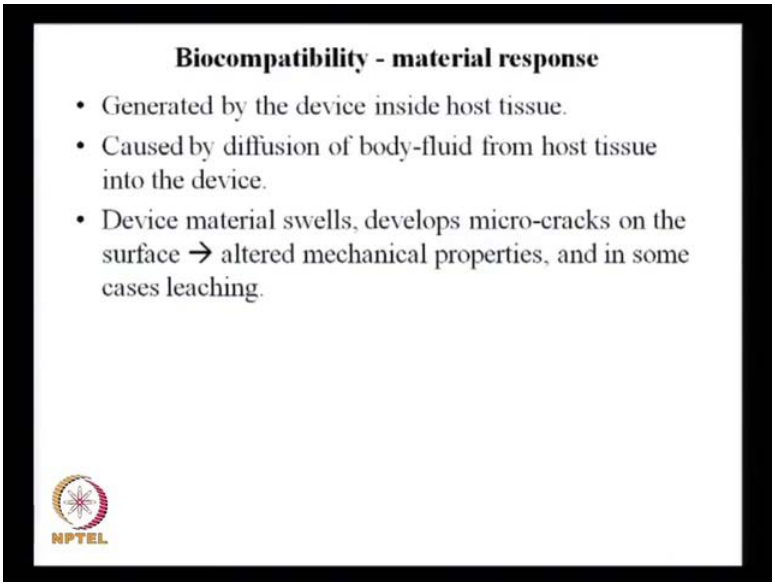
Indian Institute of Technology, Kharagpur.

Lecture No. #10

Mixing

I welcome you all to this lecture of Microscale transport process.What we have been discussing sofar is various microfabrication techniques.We have discussedhow to fabricate, discussed about how to fabricate a channel on a silicon or glass wafer,and how to cover that channel with a lid using an anodic bonder or a fusion bonder.And also we discussed varioustechniques to fabricate plastic microfluidic devices, and we discussed aboutvarious advantages and disadvantages of this devices. So, so far what we have and of course, we have discussed abouthow to put a tubular,how to put, how to insert a capillary,sothatthis microstructure device can be interfaced with the externalworld. So, we pretty much we have already done with various microfabrication techniques.

(Refer Slide Time: 01:24)



What the last issue that I am left with as far as the microfabrication is concerned isbiocompatibility.Many of this microfluidic devices, they will be used in they are supposed to be used in human body for various reasons, either toanalyze, either to release drugat some point or analyze the level of toxins or due to various reasons, you need it needs to be put human body. So, the major part of microfluid, major part of theapplications of microfluidic devices coverusing human body. So, we need to understand some issues here or at least the terms here that are important.A biocompatibility, we hear this termsso often,these material is biocompatible that mean if you insert in human body, you do not expect a adverse reaction; that is the immediateresponse that I will get, if I ask you this question, what a biocompatibility would be.

Now, the way this biocompatibility is defined here is in this slide I write, biocompatibility there are two points here; one is called material response and the other is called cellular response.One is the response that is given by the material itself; that means,you insert a microstructuredevice inside human body.Now that material will respond in certain way,I mean you have to ensure that the material remains intact; the material does not erode or the material does not swell, sothat is important.The other issue is the cellular response, the cells around that material, the human body that should not getdestroyed.

So, there are two points here, one is that microstructure device should not get destroyed by human body, because human itself contains lot of chemicals and ifthat microstructure device swells, ruptures,erodes; so, if that does not function then the purpose would be lost that is one issue.The other issue is that the material that you insert inhuman body that should not cause some adverse reaction that the cells around thatmaterial gets destroyed, then you have some other problem. So, if we get back to this material response, material response is generated by the device inside host tissue. So, this material response is generated by the device, not by the human body,it is by the device that you insert.

And this is caused by diffusion of body fluid from host tissue into the device.Diffusion of body fluid, the body fluid is there and you have inserted these material inside human body. So, this material is coming in contact with the body fluids.Now, if the body fluid diffuses into the human,diffuses into the,diffusesinto this device and then that device material swells, that is a third point that I write here; device material swells, develops

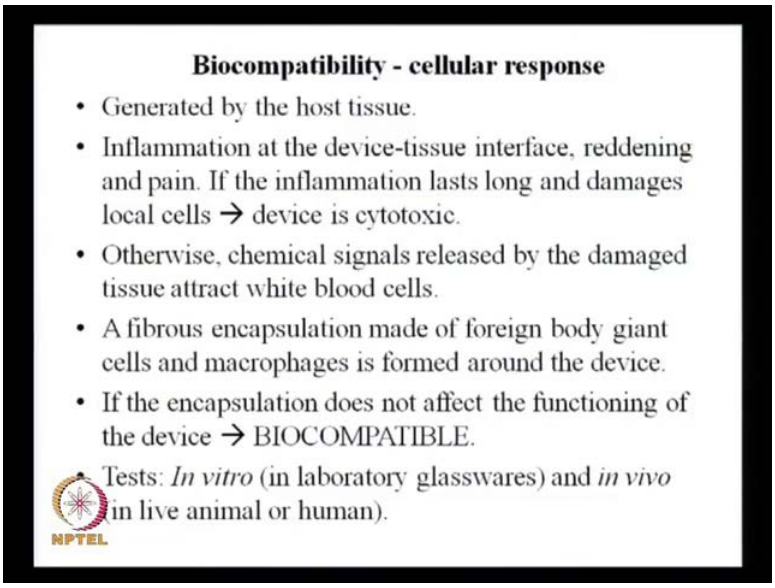
micro cracks on the surface; altered mechanical properties and in some cases leaching,so these are theproblems that would happen.

If this, if the body fluid enters into the deviceand because of that the device material swells, it develops micro crack.Swellinginvolves,I meanthe moment it starts swelling,I mean you never you do not know what is going to happen, because you are stretching it someplace it is getting stretch,someplace it is not. So, there would be stretches developed and at any point there could be a crack developing. So, swelling as I mean something which you have to be careful with develop,so develop some micro cracks o the surface and it can have altered mechanical properties.If something swells, it has to have some kind of structural integrity and that integrity would be lost,so that is a problem with the material.

And in some cases, there could be leaching; that means, the body fluid may leach out some essential part of that device.Supposeon the device, you have some coatingand the body fluid starts leaching,you know what a leaching is;so far you have in various chemical engineering applications, you understand what leaching process is.If you have not,I suggest you read any mass transfer book, say mass transfer by Coulson and Richardson or mass transfer by Treybal,you can pick up any of theany of these books in the libraryand then you can read these chapter leaching.

It is like you this body fluid will take away some material from the surface of thatmicrostructure device. So, then if such leaching happens, then a microstructure device will lose its structural integrity, it will lose its ability to perform inside human body;so, these are notwanted basically. So, this is what I refer as material response. So, this is as far as the device part is concerned.The other part is the cellular response.Once you put the device, you have to ensure that the cells around it should not get destroyed,the body cells that are around is device that should not get destroyed. So this cellular response,this is generated by the host tissue.

(Refer Slide Time: 07:05)



So, this is generated by the host tissue, inflammation at the device tissue interface. So, moment you put this microstructure device, there would be inflammation at the device tissue interface,these will cause reddening and pain.If the inflammation lasts long and damages local cells that is referred as,then the device is called cytotoxic.I mean, one thing I would like to point out at this point out here is that moment you insert a material, external material in human body, there will be inflammation, there will be reddening, there will be pain;so, these are few things which will happen invariably.And if these things happen that does not necessarily mean that the body isthat the, that microstructuredevice is not biocompatible.

You cannot say that, because anything you put this will invariably happen, but what is important here is whether that thing dies down or not.Whether the inflammation is after sometime the inflammationgoes away, after sometime the pain goes away, then only you will take that material as biocompatible, it is as straight as this.So, inflammation at the device tissue interface that will happen, invariably whether it is biocompatible or not biocompatible.This reddening and pain, all this things will happen, but these will not last long, that is the indication of a biocompatible material.If it lasts long, and damages it starts damaging local cells, then the material is not biocompatible;then the material is referred as cytotoxic, then the device is referred as cytotoxic.

On the other hand, chemical signals released by the damage tissue attract white blood cells.I mean typically, if we material which is supposed to be biocompatible, what will happen is immediately there will be inflammation, because a moment you put external material inside human body, the body will react in certain away,that is a must.If it does not happen, then it is not doing its job. So moment is happens, moment you insert a material inside human body, then there would be chemical signals released by the damage tissue.Some tissue will be damaged, otherwise you will, you cannot because it is anintegrated, it is an intact tissue.

Inside that moment you are putting a material; that means you are damaging some tissue some portion you are cutting out. So, those damage tissues they will release some chemical signals and that chemical signal will ensure that white blood cells get attracted to that place.Basically,it is a human bodysent that portion of body sends(( ))torest of the body thatwhy there is a problem here. Soimmediately this there would be signals and blood cells,this white blood cells getting attracted.And in due course, a fibrous encapsulation made of foreign body giant cell and macrophages is formed around the device; that means, this white blood cells that get attracted, it form a fibrous encapsulation made of foreign body giant cells and microphages, this is formed around the device.

So, if this fibrous encapsulation forms; that means, you are stillokay, you do not problem; the material is, it is destroying the cells rather what it has done is, the portion the tissue that you have damaged intentionally, when you put the device inside, those tissues they send a signal and attracted white blood cells, which forms an encapsulation around the material. So that part isstill the material can be biocompatible as a matter of fact I mean,I heard people who has I heard of people, who have bullet inside the body and they living, they are just they said that it is too troublesome, you have operate and take out the bullet,so let it be there.

So let it be there means it has already formed a fibrous, there is a fibrous encapsulation around that bullet and it is justsafe,I meanthere is no more fallout of that injury,so that is what is what I am referring to here.A fibrous encapsulation made of foreign body giant cells and macrophage is formed around the device.Now, one problem could be that these encapsulation that you have, these encapsulation may notallow the device to function properly; that means, suppose it is this device is a controlled release device.

Controlled release device means when you want it will release some drugat a specific location and when you do not want the drug will not be released;so that is the purpose of the device.Suppose the device that you have inserted.Now, if you have encapsulation made, and that encapsulation does not allow the material to release the drug,I mean it isdrug is release, but it cannot permit through that encapsulation, then the purpose is lost. So, if the encapsulation does not affect that is the point I have here, if the encapsulation does not affect the functioning of the device,then you call it biocompatible,I wrote it in uppercase here in the slide; that then this is called then this encapsulation then this device is called biocompatible.

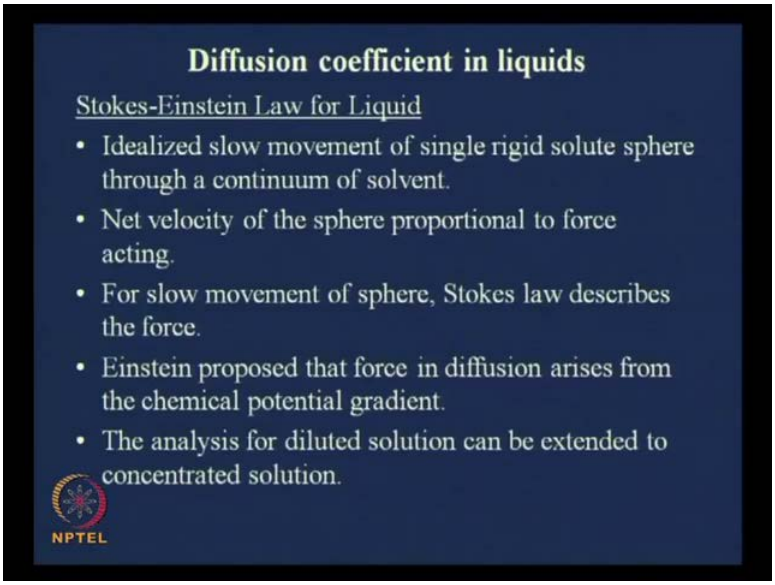
So, you havesomany checks.First of all, it should not be cytotoxic.First point, if it starts destroying the cells no way, you cannot take it that is cytotoxic,soit is left out.Then it come,then it may come that it is not destroying, but it is forming a fibrous encapsulation and that is disturbingthe performance of that device, then again you have a problem.Butif that does not even that does not happen, then you can consider this material to be biocompatible;so, this is how you define biocompatibility,I mean in simple term.

Maybethere are other issues involved, as far as this course is concerned I thought this two slides on biocompatibility would be important, at least you have this in mind that anything that if that is inserted in human body and it starts inflammation or pain that does not mean that the material is not biocompatible.It could be that it is just simply biocompatible, because any material you insert in you human body, this will end up withinflammation, pain, reddening, and all this, but this should subside this should not continue for long.Now, there are various tests done to check, whether the material is biocompatibleor not.

There are two types of tests, one is called In vitro test.In vitro, vitro you know where the origin of this word vitro,see In vitro means in laboratory glassware and In vivo, that means, in live animal or human. Sofirst, suppose you haveyou can probably do some kind of screening by In vitro test.And then, once you have identified, once you shortlistedfew items,these are items I can considered to be considered them to be biocompatible, then you can run it on some animals, whom you can scarifies,for example, rat. So that is one that is how it is done.

And once you know that it is not going to affect at least some animal body,then you will go for human trial. So, these are the ways biocompatibility is tested.So, this is one importantissue biocompatibility and this microstructure devices, they are very often used in human body for various reasons. So, one could be release controlled release of drug, other could be say there could be glucosemonitor placed inside the human body, it is continuously its measuring the level of,if the patient is diabetic, it can continuously monitor his health. So, there could be such devices inserted into human body. So, these are basically some sensor sowhich would sense how the body is performing.Or it could bethe material which can release drugs in a controlled manner.Or it could be simplymechanicalit gives some mechanical action like pacemaker ormaterials like this. So, this these are the,sothese are issues of biocompatibility thatwe must know as far as this course is concerned.

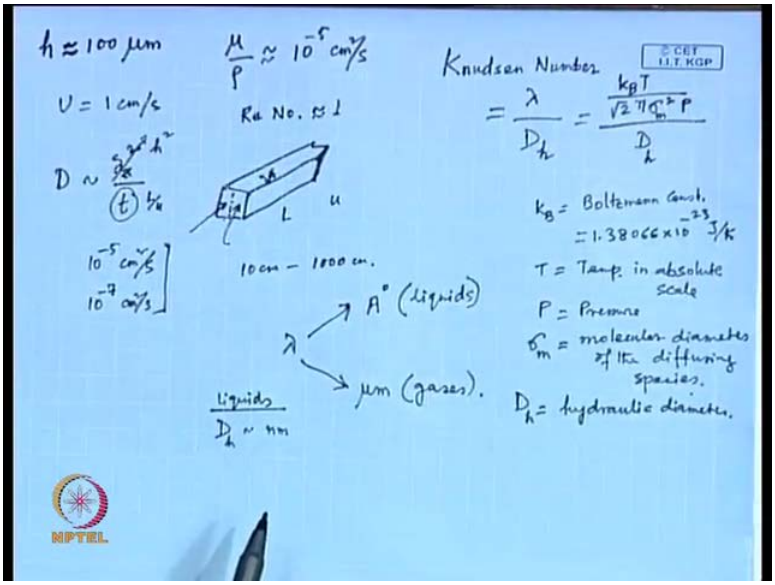
(Refer Slide Time: 16:19)



Now, this probably this ends the discussion on microfabrication, what are you do is,I will probably come back to thislater, if anysuchproblemcomes up if my if we need revisit this microfabrication issue.WhatI want to get in now is basically the theories of microfabricated devices;I mean the transport processes which wethe, our original intent was to study the transport process in microscale and that now we are getting into this.ButI thought this background of how this microfabrication is done, will give a verygood idea about how things are done in practice.

Instead,I mean you can you can always do it in theories, but first you would know where the limitations are as far as implementing things in practice; then probably your understanding would be much better,I mean what we do with theories.

(Refer Slide Time: 17:35)



Now, when it comes to the theories,what I would like to introduce first is, one thing you have to keep in mind that suppose the dimension ofmicrochannel is say 100 micrometer.And you have mu by rho, this is say I should write say channel somewhere close to 100 micrometer and mu by rho is closed to 10 to the power minus 5 centimeter square per second.I mean you can check, what is the viscosity of commonmaterial that you have, what is the density you have?And you will see probably, we come here mu by rho is around 10 to the power minus 5 centimeter square per second.

And then, if we considered a velocity of one centimeter per second,I mean which is reasonable I mean think ofI mean we are talking about a microstructure device which we are going to insert in human body. So, velocity of one centimeter per second would be considered pretty high that in that respect, one centimeter per second we are talking about. So, if we use these parameters and you somebody checks what is the Reynolds number.You will find that even with the higher level of velocity your Reynolds number come,this Reynolds number comes closed to one.Reynolds number closed to one,I mean I have studied fluid mechanics we are we talked about thousand is the transition.

So, Reynolds number of one is practicallyat the lower end of laminar flow; even I mean you can say laminar flow, even at the lower end of laminar flow;so that where we are operating, when it comes to the flow in a microstructure device. So, when we you remember inearlier classes, we talked about thesefluid flow fractionation and we are using these, if the concept of laminar flow one layer sliding against the other and then diffusion is taking place from one layer to the other; these I mean we know for sure that we are nowhere closed to turbulence.

We know for sure that then one layer is moving and then the other layer is gently sliding against the other, because we arereally at the low end of laminar flow. So that is one thing you need to keep in mind that generally we operate around this Reynoldsnumber2, 10, not more than that.The other point here is that you havethis diffusivity,diffusivity is of the order of,diffusivity can be defined as penetration of the, penetration for species divide the distance by which a species penetratessquare of that divided by the time.Time it takes for the species to go up to that distance.That is how we commonly understand diffusivity or diffusion coefficient.

The distance that it haslook at the unit of diffusivity, it is centimeter square per second. So, this is typically what we understand about diffusivity; a molecule diffusing means it will penetrate up to distance delta x in time t, then delta x square by t istypically the diffusion coefficient.Now, if that isso,then you can write suppose I have a channel,I have a certain length of a channel,suppose I have a channelI have half of it is a solute stream going in and half of it isI mean a stream which does not have a solute and they are flowing side by side.AndI expect them to mix, when it come when it comes out.If one layer is flowing against the other, if one layer is sliding against the other, then you can expect that there would be diffusion taking place from one layer to the other.

So, if one stream has the solute; that means, solute means I am talking about say salt solution; if you have salt solution and if you have pure water andif you put them together side by side, you expect salt to go into the other stream which is pure water and when you extract, when you take the entire solution at the outlet, you should expect that the entire stream would be well mixed; that means, the portion which was pure water is now salty. So, that is that is the diffusion we are referring to.

Now, if we expect that over the length L, if the length of this channel is L,the dimension of this channel is say h,if the half width of this channel is h say if the half width of this channel is hand you have the velocity as u,you have the velocity as u.In that case,you can expect,just think about the diffusivity.What is the diffusivity you have,10 to the power minus5 centimeter square per second that is the typical diffusion coefficient you have.For a commonsay NaClsolution if you have, then that is the common diffusivity.Now, if you put this diffusivity and if you expect this delta x, that is the penetration.Now thatit has to penetrate by entire half width, at the end of the channel, then only you can consider them to be well mixed.

So, if that has to happen, what should be the length?Because, you can write t as, sothis t can be written as L by u;that is the time taken for this fluid to reach the outlet. So, that is the time taken for the fluid to reach this outlet.Andat the outlet, the penetration should be complete; that means, this delta x square should be replaced by h square.If you use these parameters, what you will find isthat,now the diffusivity can be sayI am assuming 10 to the power minus 5 centimeter square per second, it could go all the way up to 10 to the power minus7 centimeter square per second; if we are talking about diffusion of hemoglobin.Instead of sodium chloride solution,I mean sodium chloride is just a hypothetical one; nobody will mix water and NaClin a microstructure device.

Now, one can mix hemoglobin that he can do. So, if you are talking about diffusion of hemoglobin, we are getting into 10 to the power minus 7 centimeter square per second. So, if this is the range of diffusivity I have to cover and if somebody asks you, what should be the length of this channel?If this is the fact, this h you know this is a typicaldimension of a microchannel is 100 micrometer. So what would be the length we are talking about here and you will find that this length varies from 10 centimeter to 1000 centimeter;if this is the range diffusivitythat has to be covered. So, it seems this diffusion is not is the slow process,I means it is it will take.

So, if you expect two streams flowing side by side and it will have complete mixing at the outlet, the length of the channel should be 10 to 1000 centimeter;1000 centimeter means we are talking about 10 meter.Typically, humanheight is 1.6,1.7 meter. So 10 meter,I mean we are talking about that kind of length. So, we are not in definitely in a microchannelthis is not a microstructure device. So, you cannot accomplish such level of mixing using two streams flowing side by side and relaying entirely on diffusion.

So, you have to have some other mechanism going on,whichby which what you do is you increase the interfacial area; that means, the two stream they are flowing side by side, if you introduce. So, called sayprobably in(( ))it would be baffles.If you introduce baffles, by whichyou introduce extra area, interfacial area over which the mask exchange can takes can take place; then probably you do notneeds such a longchannel. So, my point I mean why I introduce this diffusivity?I mean the two issues that I point out here, first of all the Reynolds number is really small in this case, we have already pointed out it would be around one.

And second point is that if it comes to mixing of two streams in a channel, you cannot expect them to flow side by side and get mixed.You have to introduce some bafflesor some other methodto arrange the mixing.And that some other method isintroducing baffles,introducing some amount of chaos in this mixing process.So, but yougot to understand here isby putting baffles, when we put baffles, one idea of putting baffles is to create some amount of turbulence. So that around the baffles there would be AD's forming,I mean in a conventional sense in a pipe, when you put baffles, you will make you will start having AD's.

AD'smeanspackets of fluid flowing from one end to the other, which is in transfer to the actual principle flow direction. So, these areand that is that is associated with turbulence.And in this microfluidic device, we are nowhere close to turbulence;we are in the low end of laminar flow. So, by putting baffles, the idea here is to create additional interfacial area.That the two streams, one is the solute stream, another is the solvent stream,they should have larger area for mass exchange. So, for that you introduce baffles. Sothat the purposes little different, we will discuss this,we will put the theories of what is,we call thischaotic mixing; this mix by putting baffles and arranging a mixing, this mixing is referred as chaotic mixing.

Now, this chaos is different from the chaos that is referred in context of turbulence, because nowhere it is turbulence.Butthis chaos is someI mean we will get it to the theories of it probably by the end of this class or may be in the next class. There is third issue,there are three issues I would like to point out, one is the issueof Reynolds number I said, another issue is the length that if you relay on diffusivityalone, then it would be a very long channel, which we cannot afford.

The third issue is there is something called a knudsen number,which is defined aslambda divided by D h. and what is lambda?Lambda is the mean free path of the molecule, which can be written asK B T divided by square root of 2 pi sigma m square pand this whole thing divided by D h.D h is,I mean hydraulic diameter, you can say or some kind some characteristic dimension of the channel.So, one is the mean free path and the other is the hydraulic diameter.Now,for now what are the various stamps,I have here K B T,etcetera;I mean you know what that is,K B is basically Boltzmannconstant,the value would be 1.38066 into 10 to the power minus 23 joule per k.T as temperaturein absolute scale.

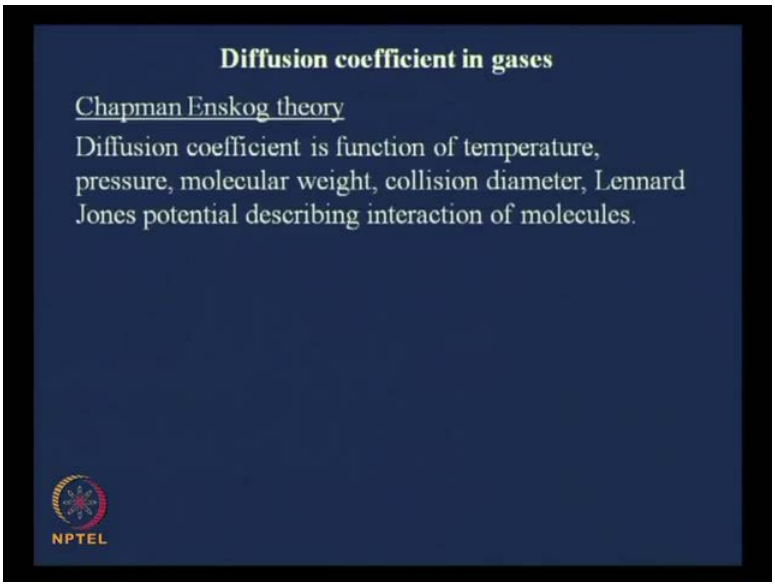
Then you have P, which is pressure and you have sigmam, which is molecular diameterof the diffusing species;andI said D h is the hydraulic diameter.Now, if you look at the values of this lambda,if you look at the value of values of this lambda, in case of a gas and in case of a liquid; for liquid, it is of the level of angstrom and this is of the level of micrometer for gases. So, what iswhat we conclude from here?If you have lambda as,lambda in the order of angstrom and lambda is of the order of micrometer in case of gases, thenthe knudsennumber would be important for what which material; it would be for a gases.For the gases, the knudsennumber would be important,I mean see we are talking about this D h.

If the knudsennumber will be important for liquids knudsennumber will be important for liquids, if the D h is of the order of nanometer.For liquids, if the D h is of the order of nanometer, then you can consider the knudsennumber to be important.Butthat is not probably that does not come under the preview of this microscale process.Possibly, if you are working withaseparation process, where you were separatingsome molecules within the liquid, then probably you would be working with that kind of range.Butfor a microstructure device,I mean our, we at the very at the very outset, we said the channel dimension is 100 micrometer. Sothat for the, that kind of size the knudsennumber is not important for liquids.

For gases,knudsennumber has some relevance;in fact, it is of the order of micrometer, sofor gases you need to consider the knudsen number.Now, when it comes to a mixing, generally the mixing we referred to the mixing of liquids,because mixing of gases is much easier; because diffusivity of gases will be much higher. So, mixing we are referring to is mixing of liquids.

So, here in this context knudsennumber is notimportant. So, these are some of the issues that we should we should have in mind.Now, when it comes to the definition of diffusion coefficient d,these are available in various handbooks, the diffusivity of various materials.Butif I get into thetheories of it,I seethat there are twoestablished theories, which arewhich define diffusion coefficient for liquid and for gas.

(Refer Slide Time: 34:31)

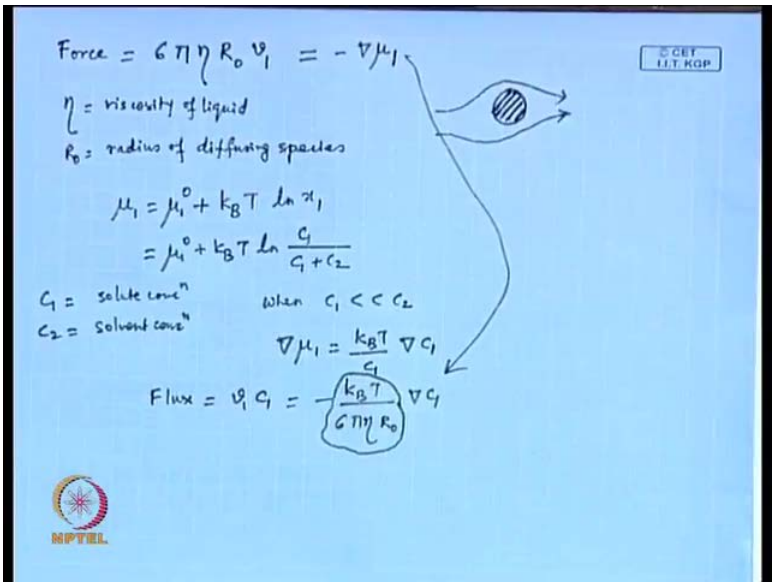


For gases,this goes by the name Chapman-Enskog theory and for liquids;it is Stokes-Einsteinlaw for liquid. These,you can refer any standard textbook on transport process;it need not have to be a microscale, because diffusion coefficient is a very general topic. So, transport phenomena by bird StewartLightfoot that would be just,I mean they have a very good discussion on this Chapman-Enskog theory as well as this Stokes-Einstein law.Briefly,I point out what we have here in Stokes-Einstein law, here we havethe assumption here is about idealized slow movement of single rigid solute sphere, through a continuum of solvent. So, while deriving this theory, they have assumedslow movement of single rigid solute sphere; sphere through a continuum of solvent.

Through a continuum of solvent, solidrigid solute sphere is moving, idealized slow movement.Net velocity of the sphere is proportional to force acting that is what the assumption there is.For slow movement of sphere,Stokes law describes the force and there has to be a driving force here.I mean you are treating the diffusing species as a rigid sphere and that is moving through the continuum of solvent.

What is the driving force?Where is the force?I mean why to move, the force is basically arising from chemical potential gradient, they that is Einstein'sproposal here.TheImean for a rigid sphere in a continuum,what would be the force, what would be the drag acting on it that is already defined from fluid mechanics.Now, in case of a diffusing species, there wasnot any force,I mean I am not pushing the molecule like a,I mean I am not using an external force to push the molecule. So, the driving force here is basically the chemical potential gradient,because diffusion happens, becauseyou know why a diffusion takes place; because of a concentration gradient and mother of concentration gradient is probably the chemical potential gradient.The and then, this analysis can be extended forfirst analysis was done for diluted solution and then it can be extended to concentrated solution.

(Refer Slide Time: 37:16)



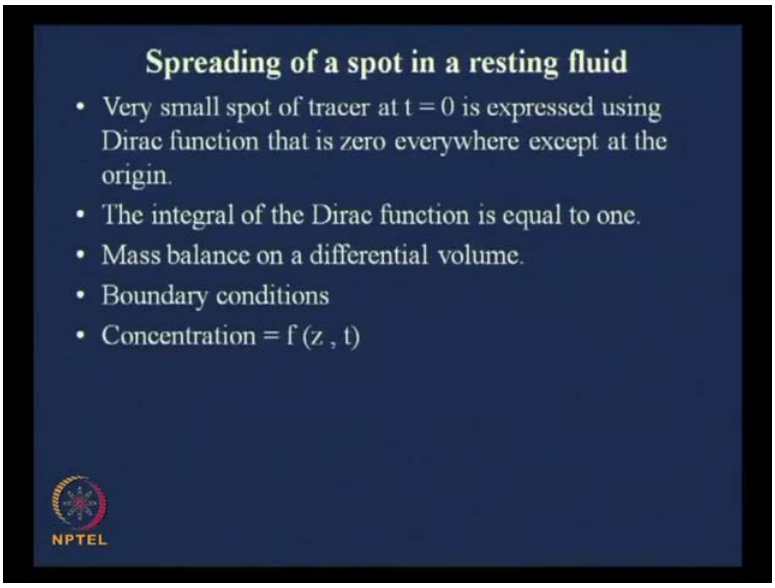
IfI write these Stokes law here, the Stokes law states that force is equal to 6 pi eta R0 into v 1.What are these now,pi is3.14; eta is the viscosity of liquid,R0 is the radius of diffusing species and v 1 is the velocity.This is originally;this equation is available wherethese formula was available for a drag force.Suppose, if you have a rigid sphereand you have fluid flowing like this,the drag force exerted on the solid that can be given by this formula; where v 1 is the velocity of thissphere. So, the drag force is defined by this 6 pi eta R0 v 1.

Now,thisthe proposal ofthe proposal here is that this is equatedwith thechemical potential gradient.Now, what is chemical potential for dilute solutions?Chemical potential is defined as some reference plus that same Boltzmann constant temperature and l n of x 1.x1 is nothing but C 1by C 1plus C2, where C 1is equal to solute concentration and C2 is equal to solvent concentration. So, when C 1is much less than C2, you can write grad of mu 1 is equal to, from this expression,you can write K B T by C l grad of C 1.

And the flux by in the context of mass transfer, the flux is equal to v 1 into C 1, solute concentration and the velocity of that solidof that rigid sphere;so that is equal to v 1 into C 1,so you can write this as minus K B T divided by 6 pi eta R0 grad of C 1, because you are invoking this formula here. So, flux can be defined as this and you know what a flux definition of flux is; flux is d into grad of C 1. So, from this you can conclude that this could be thediffusivity of the liquid. So, this is how,I mean if it is completely an unknown material. So, for a dilute solution, you can find a diffusivity.Now, for concentrated solution, there are correction factorswhich are introduced.

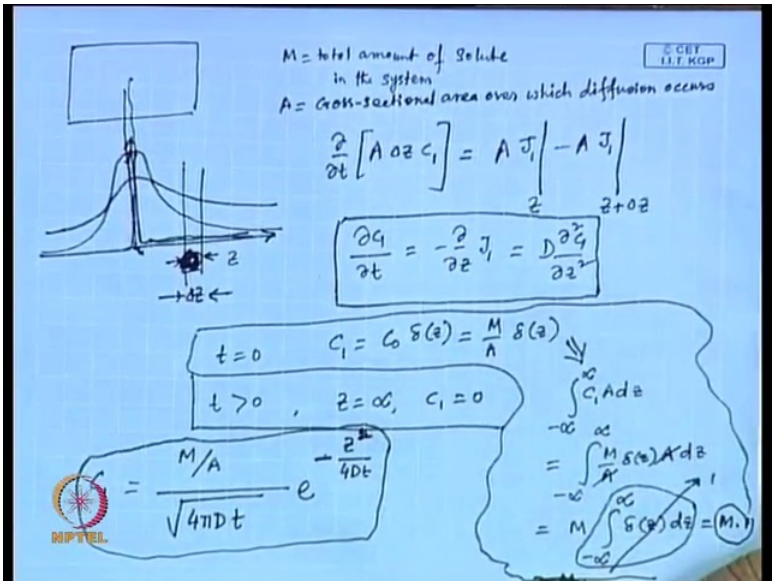
So, the analysis for dilute solution that is a last line in the slide can be extended to concentrated solution. So, if nothing is there in your hand, probably you can use a formula which already people havealready the scientists and researches they have already proposed.In case of gas there is this Chapman-Enskog theory, where diffusion coefficient is writtenas a function of temperature, pressure, molecular weight, collision diameter,LennardJones potential describing interaction of molecules.You can as I said you can refer bird Stewart Lightfoot and find the exactformula there.I mean,I just heresitting here,I can point out that you can find these diffusivity values in hand books tabulated or you can use some established theory, at least you can go close to that; if not exact, becausethe measurements are far more exact thanthese theories.These theories are based on certain assumptions and models; however, these could be handy, if you do not have any place to start with.

(Refer Slide Time: 42:27)



The next topic,I consider here in this discussion isthe spreading of a spot in a resting fluid.See,I meanmythe idea here is that we have to characterize this mixing, we should be able to theorize, how this mixing will take place in microchannel geometry, if you introducebaffles,if you introduce moresaidartifacts in the microchannel.Butfirst you need to understand, how the diffusion takes place in a simplesituation. So, we first try to find out what all equations we need to consider, if we need to find, if we need to describe spreading of a spot in a resting fluid.

(Refer Slide Time: 43:32)



Spreading of a spot in a resting fluid means this is a resting fluid, a beaker full of water,and then you put a spot a minuscule drop of a dyeink; and then immediately you see the color diffusing into this resting fluid.How will you theorize this,what kind of theory will describe this spreading of color. So, a very small spot of tracer at t equal to 0 is expressed,how will you express a very small spot of tracer.Let us see intuitively,what do we see;intuitively,if we try to find out; suppose, we are treating this as a one dimensional problem. So, if it is one-dimensional problem, suppose this is zand this is the concentration; you plot the concentration as a function of z.

So, at t equal to 0 what is the condition?At the beginning, it is a spot of tracer. So, spot of tracer,a very small of spot tracer at t equal to 0 that can be expressed using a Dirac function.TheDirac function is zero everywhere except at the origin;so, origin is this point. So, to start with you haveprofile, which looks like this. So, it is zero everywhere except at the origin.Now whatthe,what these the other thing that has to be satisfied here is that suppose you put, say M is equal to the total amount of solute,total amount of solutein the system M is equal to total amount of solute in the system.

Now, this total amount of solute in the system has to be conserved, by that what I mean is if I look at say at higher time at t equal to 0,I said it is zero, everywhere except atz is equal to zero; it has a finite it has some value.Andthe integral of this area under this curve,I mean the area under this curve that has to remain same, because as time progresses how would be the profile.This is travelling all the way uplittle higher timelike this, some more higherat a furthertime instant, it would be like this; and gradually, it would all flat end up.

Now, the area under this curve that should remain same and that should correspond to the total amount of solute in the system.Now, whether you consider Dirac,I mean so the Diracfunction has to take care of this aspect.That area under this curve even if you have a Diracfunction, if even if you have zero everywhere except at the origin, the area under that curve as well has to correspond to thistotal amount of solute in the system. So, the area under the curve remain same, because you are not importing any solute from outside; that is that original spot which is spreading out. So, the integral of the Diracfunctionis equal to one; that is because that has to otherwise these you cannot satisfy this total amount solute in the system,so that has to be satisfied.

Now, if you write the mass balance if you write the mass balance on a differential volume.How that be?you would be writing, here suppose you pick up, at some distance z, you pick up a differential length dz or say delta z.Let us say delta z; we let us develop this from scratch. So, you are picking up a distance delta z. So, you would be what you would be writingat, if you are drawing a mass balance there, you would be writing del del t of A delta z C 1that is equal toAJ1 that is a flux at z minus, the flux at z plus delta z. What isA? A is equal to cross-sectional areaover which diffusion occurs. So, this is themass balance on this differential element and then if you write this as del C 1 del t, then this would be equal to minus del del z of J1, which can be written as D del square C 1 del z square.

So, this is the governing equation.I mean you are familiar already, familiar with this governing equation, you must have seen this equation in the context of mass transfer.Only we revisit that and we ensure that for (( )) part of tracer that we arrive at this equation.Now,one thing you must understand that this C 1 would be written asc 0delta z that is equal to M by A delta z. Sothat is the condition at t is equal to 0.Andat t greater than zero,only thing we know is thatat z is equal to infinity C 1 is equal to 0 that is the other condition we have. So, at t is equal to zero concentration follows the Dirac function that is the idea of Dirac function.

And what is essentially what does this mean?This means that integration between minus infinity to plus infinity;so, it is zero everywhere except at the origin. So, what that mean see C 1A dz, that would be equal to integration minus infinity to plus infinity M by A delta z Adz.And A,A cancels out,M comes out of the integral,so A cancels outM comes out of the integral. So, basically this is M integration minus infinity to plus infinity delta z dz andI pointed out this that integral of this Dirac function is equal to 1; sothat is what I am I said that it is this is equal to M into 1. So, this is basically M.

So, material thisresult is M, because this integral the value of this integral is equal to 1. So, that is what probably I referred here in this slide as the integral of this Dirac function,basicallyI should have said from minus infinity to plus infinity. So, this is basically the condition that we have. So, one condition is at t equal to 0.We satisfy this, but that is what we have, in hand the other condition is at t greater than 0, z is equal to infinity this C 1 is equal to 0.Because add z equal to infinity that is far away from the

spot, from far away from the place where you put this spot of tracer, the there is no effect.

That is what yousay here the C 1 is equal to 0.That means, the color cannotgo all the way to the infinity, because after all you have put a very smallspot of tracer. So, these are the two boundary conditions and this is the governing equation.Now, this equationthe there is a solutionexisting. So, what may be do is,we will we can I can give you the solution right way,so that you can get,I mean when you go back you keepthis in mind how it looks.And probably, in the next class,we will briefly touch upon,basically you need to use the Laplace transform to solve this equation.

However, this handling this Diracfunction handling, this boundary condition will not be straightforward, there is trick to it;this particular boundary condition in Laplace transform.You try to do it, after you go back,after this class, you try to do it yourself,Igive you the final form.The final form is M by Asquare root of 4 pi D t e to the power minus z square divided by 4D t;this is the final form z square mind it, z square by 4 D t, this is the final form that we have.So, when you go back after this class, you try to see if how it can solved. In fact, the next problem that will be pick up is where you do not have a spot tracer,rather you havefront.

Suppose you have a front, that means, you have concentration which is all the way here. So, your boundary condition would be different at that time.This would beat z is equal to 0 you have C 1 as C0, that kind ofboundary condition.They are probably handling this boundary condition is much easier, but here this it is not straightforward.What you what I suggest you do is you go back and try to solve this using Laplace transform.And next class,we will probably spend first couple of minutes tofind outwhat are theintermediate steps. So, thank you all,that is all for today.