Biochemical Engineering

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Module No.# 01

Lecture No # 36

Multiple Interacting Microbial Population Prey-Predator Models

(( )) Biochemical engineering, this lecture as you saw, is titled Multiple interacting microbial population - prey-predator models. And, what we are going to talk about today is, something related to growth; something that we have done before but, in a new perspective. If you remember, we, when we did the growth, the chapter on the microbial growth, what we talked about was the growth of a single bacteria, or microbe, or a fungus also. What is the difference between, what we are going to do today, and what we, what we did earlier? The earlier, as I said, we had worked, or talked about just a single microbe, or bacterial growth. We had, if you remember, talked about different substrates, inhibitions and so on, but, in all these cases, we have just confined our cells to single microbial growth.

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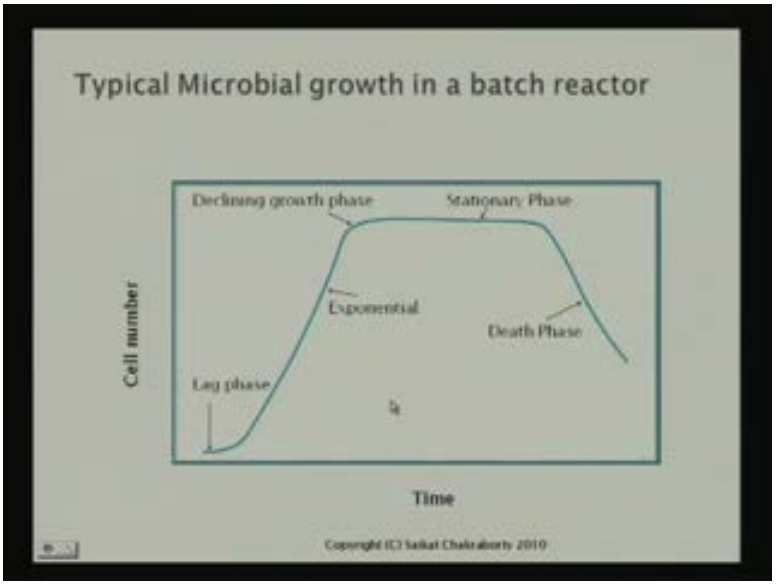

So, if I can go back, you know, to the old slide here, of a typical microbial growth in a batch reactor, this is how it looked like, if you remember. The cell number verses time, and this is the, there is the slight lag phase, for the, for the growth to start and then, there is exponential growth, and then, there is a stationary phase, and then, there is the decay. The, during the growth phase, the exponential part of the growth phase, only the growth of the bacteria occurs; during the stationary phase, what happens, the amount of bacteria that is growing, or microbes that are growing, or being incubated in the reactor, equals the amount that is dying; therefore, there is a stationary. So, there is no overall growth, but, there is, as far as the growth of bacteria, but, which equals the (( )) bacteria. And then, you have final death phase, where the death rates far overcomes the growth rates; as a result, there is a decline in the cell number. And, the reason I am showing this to you, because, is because, this is important, and we will, we will, show as, draw some more curves for you today, and then, we can compare how it looked, as compared to the single microbial growth case.

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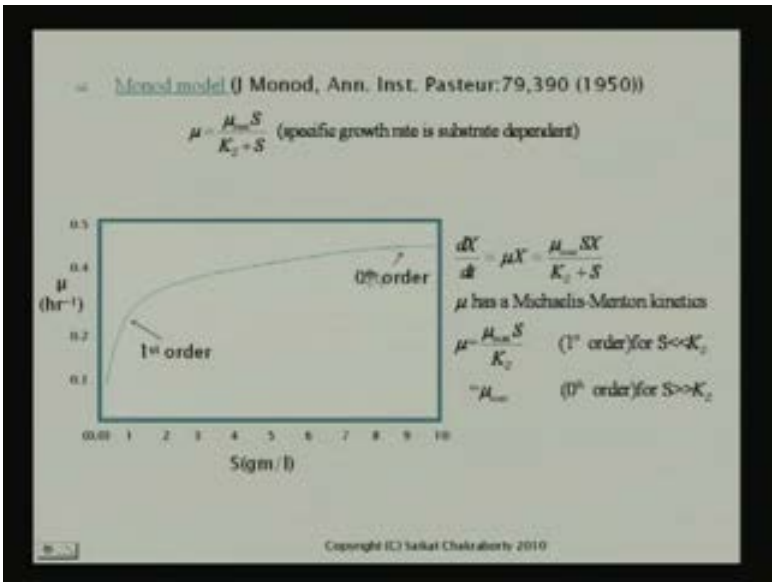

And, the most important model that we talked about is, and we used it several times, if you remember, just to recap, is the Monod growth model, which was obtained, or first derived by J Monod in 1950. And, it says that, mu being the specific growth rate, equals mu max, the maximum growth rate times substrate concentration, divided by K S plus S, S being the substrate concentration and K S being the growth constant. And, this is very similar to the Michaelis-Menten kinetics. Therefore, mu almost have the Michaelis-

Menten kinetics. And when, in the case, when substrate is present in a large amount, this mu, almost goes to mu max, where it becomes a constant; that it becomes independent of the substrate, and the reaction becomes zeroth order; when substrate is very low, then, it becomes, goes to a first order rate constant. So, here again, you know, this is, this is the plot of mu versus the substrate S, and for low concentrations of the substrate, it almost behaves like a first order; for very large concentrations in the substrate, here, it almost behaves like a zeroth order.

There are other kinds of growth models that we had talked about, but, we are not really interested in Malthusian and so on. We really had not been using these models. There is a modified growth, Monod growth model, which includes some more, you know, the rate of decrease at high values of initial concentration, and different other models; but, this is what we need to stick to, and what we need to remember, the Monod growth model. And, we are going to use that today, again. Now, apart from the simple, very simple Monod growth model, what else did we study? We studied the effects of mass transfer, which is valid here, again, and we can apply these same techniques, and we can come back and talk about this at the end of this lecture. And, we had also looked at multiple substrates here, so, where two different substrates S, and S 1 and S 2, both are used to, by the growth, both are used for the growth of the bacteria, or the microbe X. And then, we had also talked about effective inhibitory substrates; substrates that inhibit; but, an allosteric inhibition also, which is, you know, the substrate reacting with the microbe to form X plus daughter cells, and that reacting again to form XS SXS S S, that gives the daughter cell plus XS; it can again give the X.

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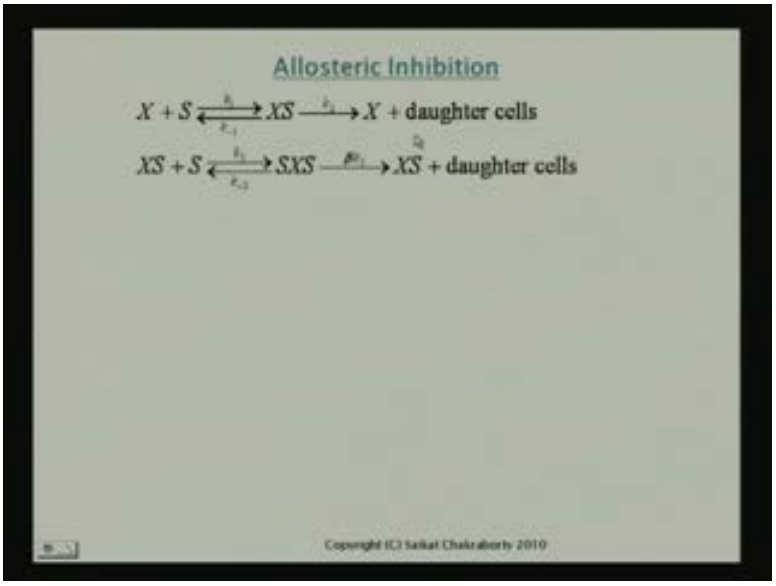

So, these are different kinds of inhibitions we have talked about. But, what is, what is important to know, it is that, even in this inhibitions, or this particular case of multiple substrates, we had always looked at, we had always looked at a single microbe, or a single bacteria.

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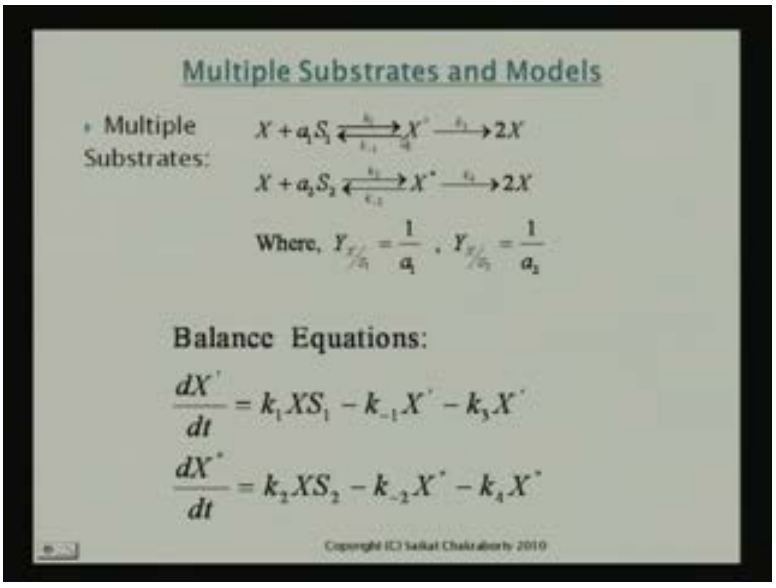

What we are going to talk about today is multiple microbes, or bacteria. Now, you might ask that, what is different when you have multiple bacteria, or multiple microbes? Well, nothing is very different, but, something still is different, and what is not different is that,

both of these microbes, or bacteria going to use up the substrate. Now, you might ask that, if that is the case, then, both these microbes or bacteria may be working independently; they might be consuming nutrients independently, and growing independently; then the, where is the problem? Well, there is no problem, if that is what happens; that is, both these microbes or bacteria grow independently. But, we are going to consider the case, when they do not work independently; or, in other words, the growth of one, affects the growth of other. And, that particular kind of growth is termed as interacting growth. So, that is why, I call multiple interacting microbial population and the, we give specific attention to the prey-predator model, and we will redo some of the old stuff, but in a much more complicated light and in a, in a more, throwing a more, I mean, looking at it from a more complex perspective.

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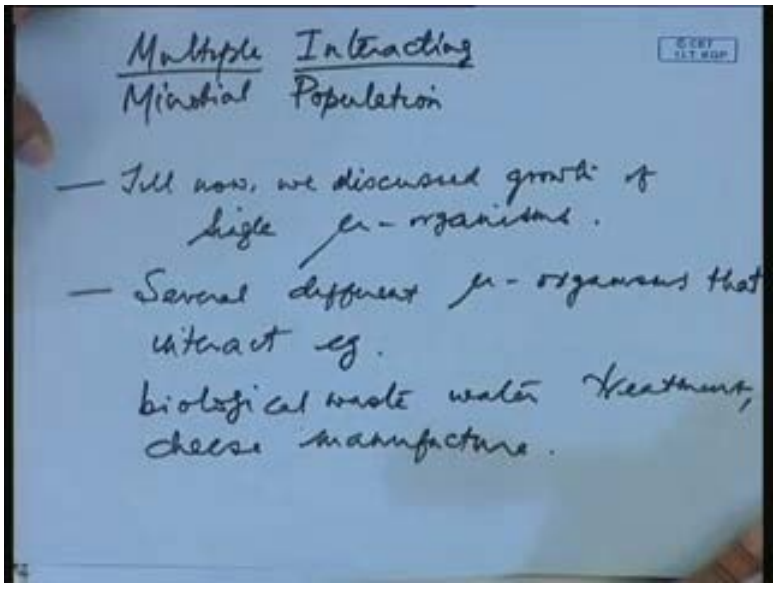

So, let us start, start working on it. So, as I said, this is multiple interacting microbial population. So, the stress is on both multiple, and interacting. So, till now, we have discussed, we discussed growth of a, of single micro organisms, alright. Now, we are going to start discussing several different microorganisms, which interact, that interact during the growth process. Example, biological waste water treatment, cheese manufacturing; these are examples from everyday life, you know, where multiple and multiple bacteria, multiple microbes work together.

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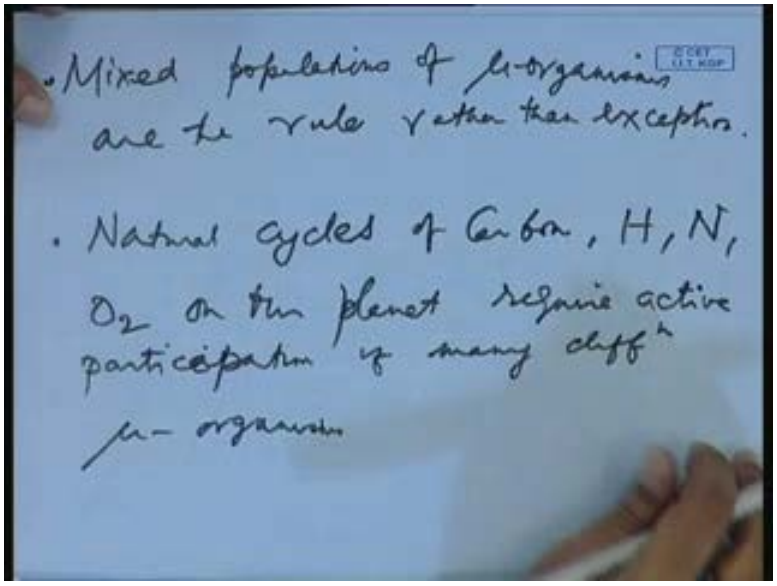

It turns out that, mixed populations of microorganisms are the rule, rather than exception. And, natural cycles, what is their role, actually? Why are these multiple interacting growth models, or multiple interacting bacteria important? So, natural cycles of carbon, hydrogen, nitrogen, oxygen required on plan, on the planet, require active participation, participation of many different microorganisms.

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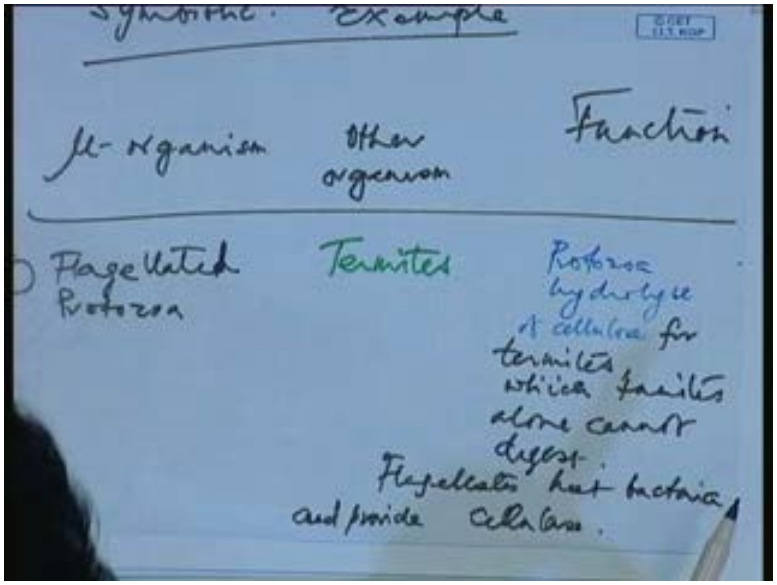

So, what about examples? Just talk about an example, which is symbiotic, symbiotic example, where the growth of one bacteria helps the growth of the other bacteria. So, one

is the microorganism, and the other one is the other organism, and the function, what is the role, how this happens. So, let us consider this example of a flagellated protozoa; it is the first example. Flagellated protozoa; this is termites, and the function is, protozoa hydrolyze, the hydrolysis of cellulose, cellulose for termites, which the termites alone cannot digest. So, they cannot digest these cellulose molecules; and so, the protozoa helps in hydrolyzing into, hydrolyzing into glucose; and in return, what is the, what, what is done is that, they, they turn, the flagellate turned in the, turn in host, they host the bacteria turned in as host, they host the bacteria to provide...So, flagellates host bacteria and provide cellulase; cellulase is the enzyme that helps in hydrolysis of cellulose. So, essentially, this is what happens. So, the protozoas hydrolyze the cellulose for the termites; they, they are the ones which can hydrolyze the, (( )) and they hydrolyze the cellulose and because the termites cannot digest and the flagellates hosts the bacteria and provides the cellulase.

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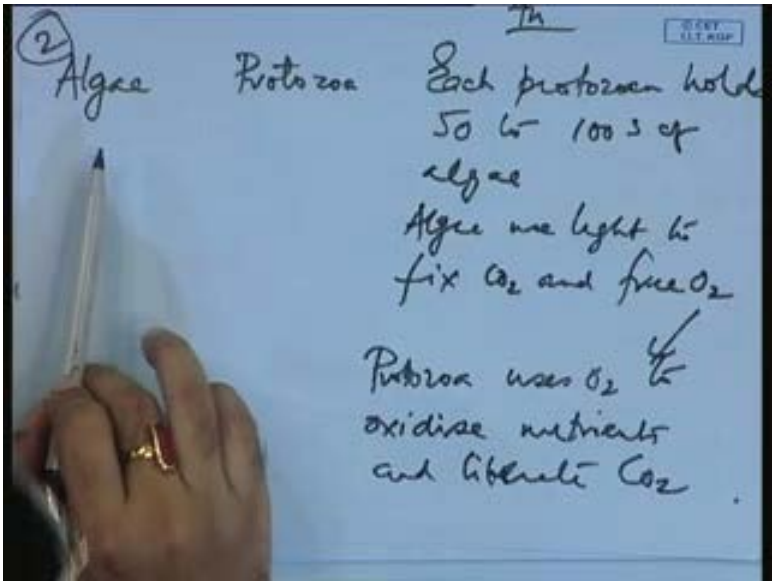

So, let us look at another example of the same symbiotic and in this case, it is a symbiosis between algae and protozoa. So, what is the function? Function is that, each protozoa holds around 50 to 100s of algae; and what does the algae do? It is well known what an algae does. What does it do? The algae uses light photosynthesis to fix the carbondioxide and free oxygen, so that, which the protozoa, this oxygen could be used; protozoa uses this, uses oxygen to oxidise nutrients and liberate C O 2. So, these are the two examples we looked at. So, first one is the flagellated protozoa and termites, and

second one is the algae and protozoa. So, as I said that, each protozoa holds around 50 to 100 of these algae and the algae essentially, what it does is, it uses light photosynthesis to convert, to fix carbon dioxide and release oxygen. And, this oxygen is taken up by the protozoa for its, you know, metabolic activities and it releases carbon dioxide, which is then, again, taken up by the algae. So, there is a symbiosis between these.

So, you, this is one set of interaction, where the growth of one, one particular organism, helps the growth of other particular organism. So, you can have different cases, where the growth of one organism helps the growth of the other; that is symbiotic. Then, you can have cases, where they just work, as you know, hosts; but the growth of one does not necessarily, unlike in the algae protozoa case, where the growth of one, helps in the growth of other, you can have the cases, where the growth of one, does not necessarily help the growth of the other. But, what it does is that, it is sort of neutral; and then, you can have competition and the Darwinian model, which is what we will look at in a few more minutes, you know.

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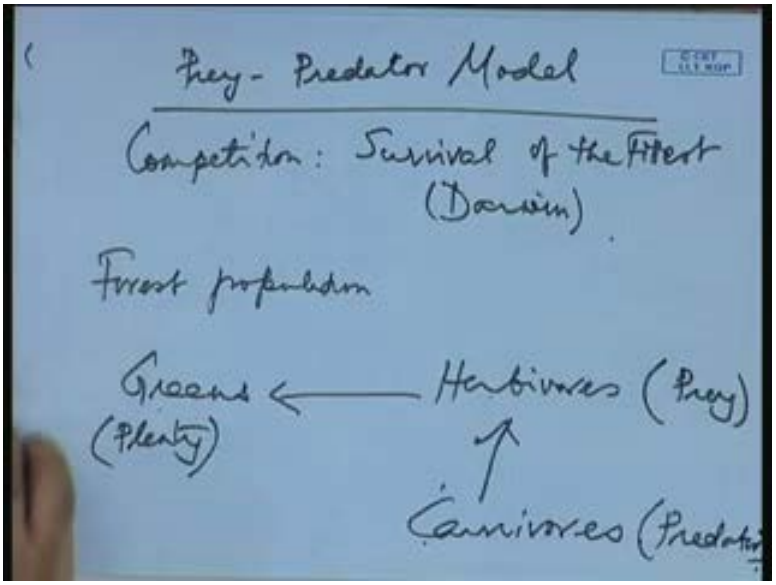

So, the next thing that we look at, is the Prey-Predator model and this is a more interesting model, because, if there is a, this, you know, growth of one organism does not hamper the growth of the other, is not as interesting. Here, what happens, there is competition and the survival of the fittest theory of Darwin comes into play. So, what essentially happens in a prey-predator model is that, let us consider a population; it is not

even considered microorganisms; let us consider a forest population and you have grass, which is eaten by the herbivores, let us say, the cows and the cattle, and then, you have, let us say, crocodiles and, and lions, and tigers, which feed on these herbivores; so, the carnivores, which feed on the herbivores. So, you have the, you have the grass, the greens, and they are eaten by the herbivores, and the, and the herbivores are eaten by the carnivores.

So, this is the, this is, this is the model for, for growth of this entire population. So, essentially, if I think of the forest population over here, you have the greens, which are eaten by the herbivores, and the herbivores are eaten by the carnivores. So, in this system, for example, as you can see, the greens are not eaten directly by the carnivores, but, this, only through the herbivores. Now, if the greens are assumed, let us, if you assumed that, the greens are in plenty, then the herbivores become the prey, and the carnivores become the predator.

Even if greens are not in plenty, you can still assume that, the greens are in, are the, herbivores are the prey, and the carnivores are the predators; but, if they are in plenty, then, it is easy to assume that, one is the prey and the other is the predator. So, how is this model going to, going to work? What will happen is that, the herbivores eat on the carnivores. Now, the carnivore population, the ironic thing about it is, the carnivore population is dependent on the herbivore population; or, in other words, the carnivores need to eat the herbivores in order to grow, but, at the same time, if all the herbivores are depleted, then, they have no access to the green. It is the carbohydrates that are, that are taken up by the herbivores; as a result, which will happen, this entire prey population, system will fall apart. So, this prey population, sorry, the prey predator chain that we have, the prey predator cycle that we have, so to say, is a very important component of the food chain; the most important component of the food chain. And, in order to keep this cycle alive, we need to keep the, in order to keep the food chain alive, we need to keep this cycle alive. That is one of the important points. So, how do we keep this cycle alive? That is what we are going to look at.

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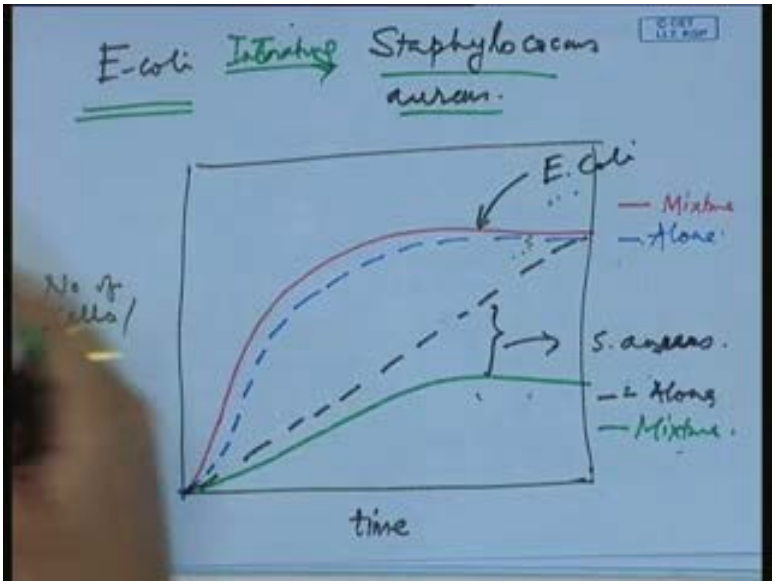

So, let us look at the first, the case of two microorganisms, which are part of the prey-predator model, in a way; one is E-coli and the other one is staphylococcus aureus. Now, experiments, if you do experiments with these systems, this is what you will see. So, let us see, this is my and this is, this is E-coli and let us see, this is staphylococcus aureus, and, this is staphylococcus aureus mixture; I will explain everything. This is the number of cells per unit volume; this is time; this, these two are E-coli and these two are staphylococcus aureus, aureus; the red line is for mixture, blue line is for alone, for E-coli; similarly, black line is for alone, for staphylococcus aureus, and green line is for mixture.

So, if you look at this curve, what is it that you find? What you find is that, when staphylococcus aureus is let, allowed to be, to grow alone, this is its growth rate, which is pretty high. Now, when you allow it to grow in a mixture with E-coli, its growth rate ,decreases substantially. Now, look at E-coli; it is more or less the same, but, still, when you allow it to grow in mixture, it grows slightly faster than staphylococcus aureus. So, why does that happen? Let us try and understand why that happens. So, reason why it happens is because, E-coli over here, that you see, has a much higher growth rate than staphylococcus aureus, and the substrate that is available, it is available to both. So, E-coli would eat up all the substrates, or much of the substrates that is available, and as a result, staphylococcus aureus cannot grow. So, this is the case, where the growth of one molecule, one microorganism, in this case E-coli, hampers the growth of the other one,

staphylococcus aureus. So, these are interacting in that way. Now, what we want to study is, now what we need to do is, come up with a model of these kind of prey-predator model.

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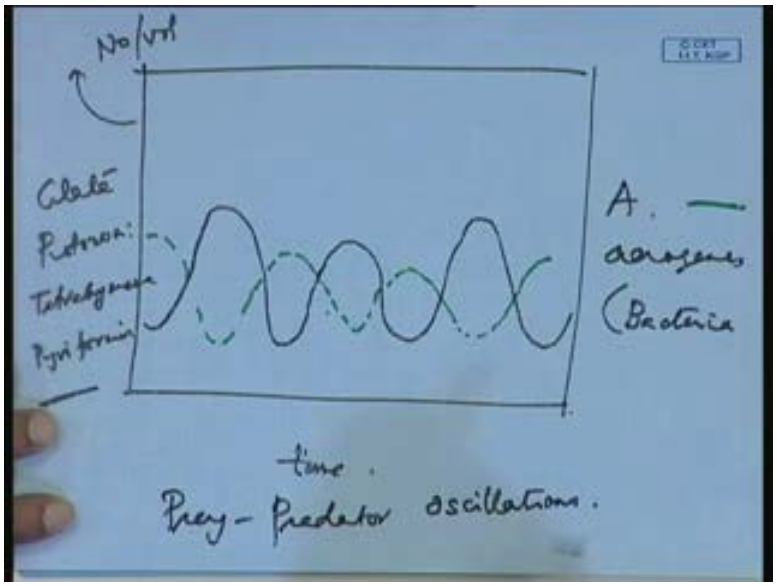

So, I want to give you another example. This is one example. The other example that I want to give you is, this protozoa versus a bacteria. So, cilates and this is a protozoa; its name is tetrahymena pyriformis and the bacteria is a aerogenous; this is the bacteria. And, the y, and the x, the y axis is essentially, number, number per unit volume; the x axis is time. So, we are trying to grow both of these together, in the same, same medium, let us say, and if I am to draw this one with black, the protozoa, if I am to draw this with black, this is how it looks like; goes up, down, up, down and up; if I am to draw the bacteria, this is how it looks like. So, this is, the green is this one and black is this one. So, what you notice over here? What you notice is that, whenever the protozoa is up, the concentration of the protozoa is up, the concentration of the bacteria is low; whenever the concentration of the bacteria is up, the concentration of the protozoa is low. So, they are going through a, some sort of a cycle, where, the concentration of these two together, is more or less a constant; but, each, when one is goes up, the other goes down; and, this is known as prey-predator oscillations. Why does this happen? Let us try and understand this qualitatively, why this happens. So, let us say that, the one is, the one is the prey and the other one is the predator.

So, when the predator population goes up, it goes up because, it is consuming the prey. As a result, the prey population has to go down, when the predator population goes up. Now, what happens, as soon as the predator population, prey population goes down, there is less food for the predator now, and as a result, the predator starts to die. And, the predator reaches a point, when this predator starts to die, and decrease in number, the prey population goes up, because there is not enough predator that is eating up the prey, at this point of time. Now, when the prey population goes up, then, the predator population, predator starts to eat up the prey again, and then, it goes up again, and as a result of the eating up, the predator, prey population goes down and this cycle continues again and again. So, let us try and understand, you know, how to model this entire system.

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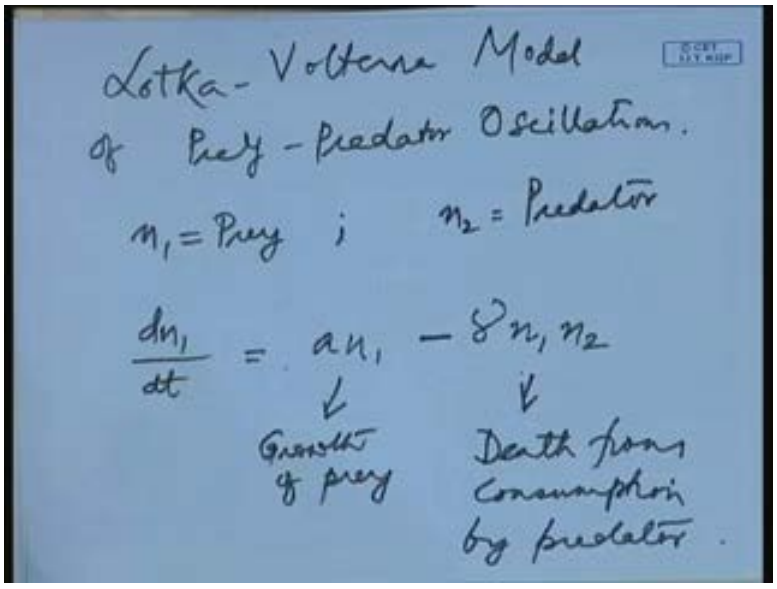

So, the model that we talk about is known as the Lotka...The model was derived by these two scientists, Lotka-Volterra model. So, it is called the Lotka-Volterra model of prey predator oscillations. So, let us say, n 1 is the prey and n 2 is the predator. If I am to write a balance for n 1, a simple, ordinary, differential balance for n 1, so, I will write d d t of n 1 equals a n 1; this is the growth of prey from the substrates that is here; could be a greens, or whatever it is eating; but remember, the prey growth is not dependant on the predator; prey is growing it, by itself, and the death of the prey; the only way, so, that is, that is with the negative sign; because this is the growth, so, there is the positive sign; the predator as I said, the death; so, that (( )) comes with the negative sign, and that is given

by gamma some constant times n 1. Why n 1? Because, more the number of prey is, the possibility of more preys dying is, of course, more and therefore, it goes as n 1 times n 2. This one depends on the predator, why, because this is death from consumption by predator. Therefore, you have the growth here, and the death here, and so, this is the first, first part, that is the balance on the prey.

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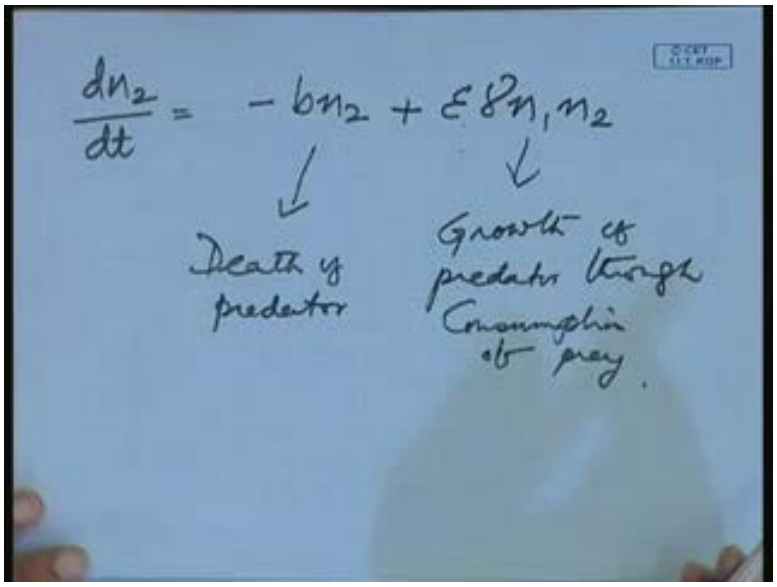

Now, if I am to do a balance on the predator, that would be d d t of n 2 equals, minus b n 2, which is death of predator; why is it independent of the prey, because the death of the predator has nothing to do with the prey, in general; it has to do with the fact that, predators are dying naturally; plus, the birth of predator; remember, we had the death of the, death of, death of the prey, death of prey from consumption by predator. Now, the rate at which prey, prey is dying, should be similar, or proportional to the rate at which predator is growing, because, this food, the prey is, is being consumed and that food essentially gives rise to the predator; you know, that gives to the growth of the predator. So, this is, therefore, proportional to gamma times n n 1 n 2, which is the death rate; but, is it equal to the death rate? No, it is not equal. Let us multiply this by a, by a factor epsilon, because, not all the food that has been consumed by the predator goes in the formation of the predator itself. So, a part of it...So, epsilon is typically less than 1. So, growth of predator, through consumption of prey.

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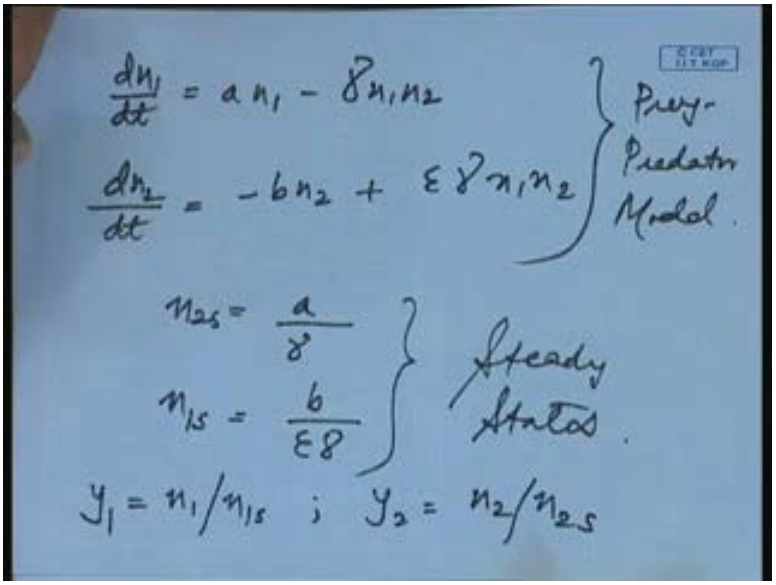

Now, if I look at these two equations together, probably, I will write them down, again, one after another, so that, it is easy. So, first one is d 1, d n 1 d t equals a n 1 minus gamma n 1 n 2, and the second one is d d t of n 2 equals minus b n 2 plus epsilon gamma n 1 n 2. So, if I look at these two equations, this is my model, prey predator model. If I look at these two equations together, what do I see? What I see is the fact that, I can get a steady state out of it. So, it is possible to have a steady state. So, if we equate the left hand side to 0, then, the first equation would give me n 2 equals a over gamma, a over gamma, that is it, n 2 s; and the second equation would give me n 1 s equals b over epsilon gamma. So, these are my steady states. Now, the next step that I want to do is, I want to convert this equation, these two equations, into dimensionless form, using these steady state values; or, in other words, this is called y 1 equals n 1 over n 1 s, and y 2 equals n 2 over n 2 s. So, this is what I am going to do.

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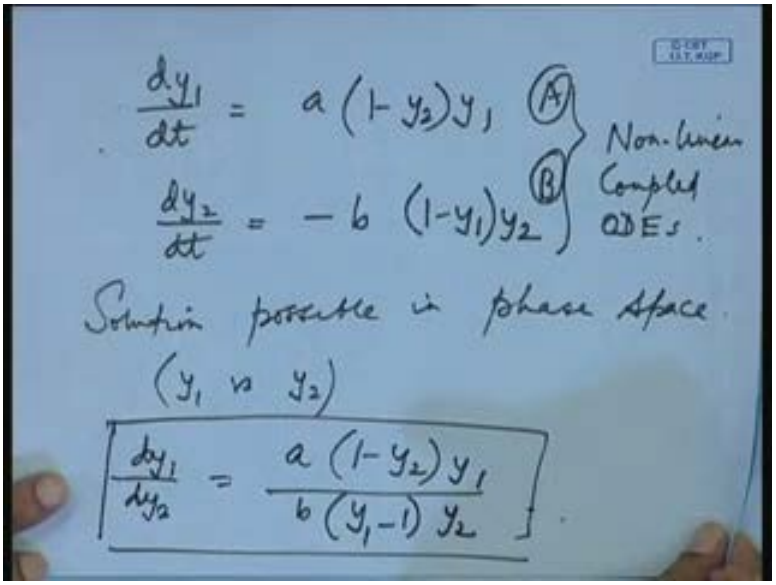

So, when I do that, the dimensionless equation that I get, is given by d y 1 d t equals a 1 minus y 2 y 1 and d y 2 d t equals minus b 1 minus y 1 y 2. So, how do I solve these equations? If you look at these two equations, both are functions of time and each of them involves their couples. So, each of them are non-linear, coupled non-linear, coupled ODEs. Can I solve them as it is? The answer is, no, I cannot solve them as it is. But, I can solve them, in what is known as solution possible in phase space; so, not in this space of the variable t, not in the time domain, but, in the phase space, that is, y 1 versus y 2 space. So, y 1 versus y 2 space, that is where,the solution is possible. How do I, how do I do that? All I need to do is, divide the equation, let us say, if this is a and this is b, divide equation a over b, and you get, d y 1 d d y 2 equals a 1 minus y 2 y 1 divided by b y 1 minus 1 y 2. So, this is what I get. Now, can I solve this equation, the one that I have over here, that I have boxed out? The answer is, yes; simply by transposing the terms that contain y 1 to this side, and the terms that contain y 2 to the other side.

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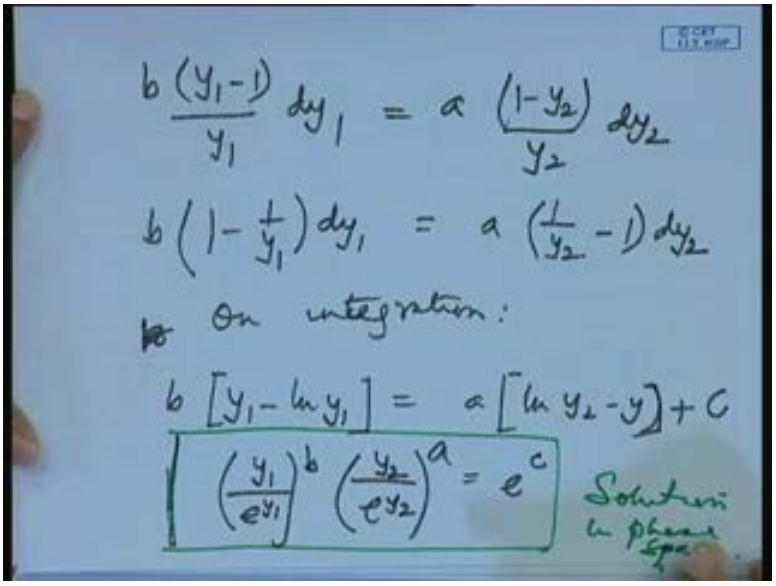

Let us do that and when we do that, I get b y 1 minus 1 over y 1 d y 1 equals a 1 minus y 2 over y 2 d y 2. So, this is what I get, and so, I can go ahead and solve it. Let us do that. 1 minus y, 1 over y 1 d y 1 equals a 1 over y 2 minus 1 d y 2. So, if I integrate this, on integration I get, b y 1 minus ln y 1 equals a ln y 2 minus y, plus c is the constant over here. Now, I can convert this into exponentials. So, y 1 to the power, y 1 over e to the power e y 1. So, this comes here, and this is, this becomes e to the power e y 1 whole to the power b, times y 2 over e to the power y 2 whole to the power a, equals e to the power c. So, this is my, this is my, this is my solution of this equation, that solution in phase space. So, it is possible to solve this, but, it is only possible to solve this in the phase space. So, when I solve this, you know, I get this equation and solve this in the phase space, what kind of plot do I get in the phase space? I can convert this back into my dimensional form, because y 1 and y 2, I know them, how they vary in the dimensional system.

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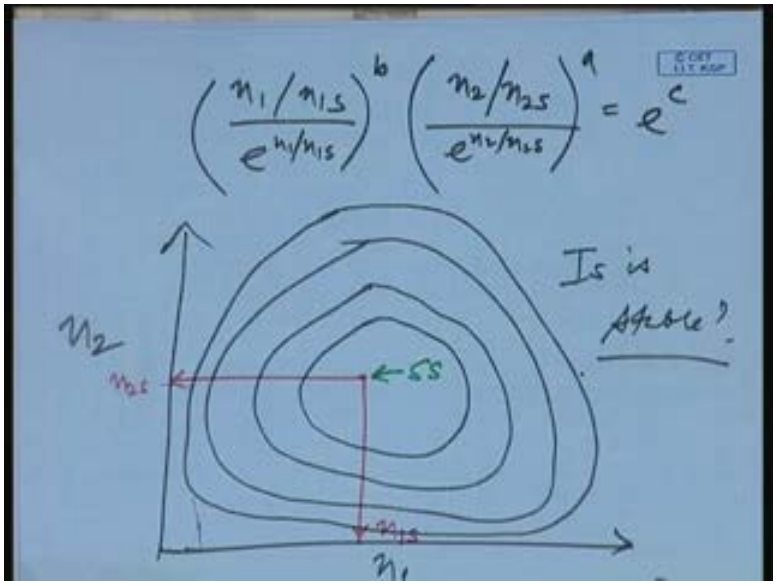

So, simply write n 1 over n 1 s divided by e to the power n 1 over n 1 s whole to the power b, and this is n 2 over n 2 s divided by e to the power n 2 over n 2 s whole to the power a, equals e to the power c. So, what is the plot in the phase space? How, how does the plot look? This is how it looks; let us draw this. This is the phase space remember. So, there is no time here; one axis is n 2 and the other axis is n 1. So, let us watch the, let us say, this is my centre; this is my centre and this is n 2 s, so, centre is, or the point, steady state; so, n 2 s and n 1 s. So, this is my, this is my steady state. Now, how would the plot look? The plot would look something like this. What are these? These are the sort of limit cycles, you know. So, they are going in cycles again and again, and if I am to go back to one of the earlier plots over here, the prey-predator oscillation, see, this was how it looked in the variable space; if I am to plot n 1 and n 2 over time, this is how it look, it would look like; and, if I am to plot them against each other, so, look at this. What happens is, when this is maximum, this is minimum, or, in other words, if you look at it from the phase space, this is how it would look. So, this is large, if this, if this is, this is, n 2 is large, n 1 is small; or, if n 2 is large, here, it could be large, but, if this is, this is a part that it forms, a part of the limit cycle. So, it goes on and on, in limit cycles. Now, so, this is a circular thing in a phase space that happens.

Now, what I have to figure out is this. Is this thing that you are looking at here, is it stable? Is it stable? And, that is a very important question that we need to answer. Why do we need to look at the stability of prey-predator models in the phase space? The

reason we need to look at the stability of prey-predator model is just because, as I said that, we want the food chain to be stable; if the prey-predator model falls apart, then, the whole ecosystem will fall apart. The food chain is a natural way the, that has, through evolution, that has happened, to preserve the ecosystem as it exists today. Now, if the food chain falls apart, then, there is, there is a problem, and this is not just confined to bacteria and microorganism that we are dealing with, but also with the general food chain, and that is why, I gave this examples of the carnivores and the herbivores and the greens, to start with. So, what, what is necessary? It is necessary that, this dynamic steady state that is, that is produced, is sort of stable.

I mean, the reason I call it a dynamic steady state is that, you see, it goes through the cycles. If it is, it is a very oxymoronic way of calling it the dynamic steady state; but still, I mean, you have a steady state, but, the dynamics of it goes around the steady state just as I showed; you have the steady state at the centre, and the, and the cycles go around it. So, the dynamics of it, in the phase space at least, go around it. And, that is why, there is a periodicity involved. We talked about the, how the wave formed, you know. I showed you the wave form of the prey and the predators. So, there is a periodicity involved, and there is...So, it is, you can, sort of, call it like a periodic steady state, or a dynamic steady state. Now, we need to understand, whether this dynamics that is there, the dynamic steady state, or periodic steady state, that is obtained, is it stable or not; because, if it is unstable, then, the whole food chain will collapse; not just because of the microorganisms thing, but, in general, and we are in trouble.

So, we have to look at and investigate the stability of this system. We have looked at stability of problems, you know, we have looked in great detail about stability of reactors, and what is that we are going to, we had done step by step. So, you can go back and look at those lectures. I will quickly summarize. What you need to do is, when you take, look at the stability of a dynamic state, essentially, you need to take the dynamic equation, linearise it, get the Jacobean and then, look at the Eigen values of the Jacobean. So, what was the necessary condition for the system to be stable? That the Eigen values have to have a negative real part. So, what were the different, different points we looked at, and you know, different, different cases we looked at; one, when the Eigen value had a positive real part, and an, and a negative imaginary part, or a imaginary part as such. So, the system will be unstable, and it will have oscillation. The other possibility is that,

the system will have a negative real part, and an imaginary part, in which case, it will have oscillations, but, it will be stable.

And then, you can have a 0 real part and an imaginary part, in which case, it will be neutrally stable, but, it will have oscillation. You can have a real part, a positive real part, with no imaginary part, in which case, it is completely unstable and you can have, with no oscillations; and then, you can have a negative real part, with a 0 imaginary part, in which case, it is going to be stable and without oscillations. And, you can have a 0 real part and a 0 imaginary part, in which case, it is going to be neutrally stable, with no oscillations. Now, as you can almost predict in this case that, what is going to happen? Is it going to have a real part, positive or negative? We have to figure out, but, what about the imaginary part? You can almost predict in this case that, the imaginary part is going to be there. Why is that because, just as I showed you, the case, you know, the diagram over here, the prey-predator model here, as you can see, it goes through these oscillations. So, there is no way it can go through these oscillations, unless the imaginary part is there. So, there has to be an imaginary part, that we can almost predict. What we have to figure out, is the real part, positive or negative; or, is the real part 0? So, at long, you know, when we look it at long time, are these oscillations, they are going to be there; but, are these oscillations going to slowly decrease in amplitude and, and then, you know, become stable, the system is going to become stable? So, that is what we need to look at. So, for that, what we need to have is that, we need to go back to the system of equation. So, this is my system of equation, and this is equation 1; let us call this f 1 and this is f 2. What we need to look at is, we need to look at the Jacobean.

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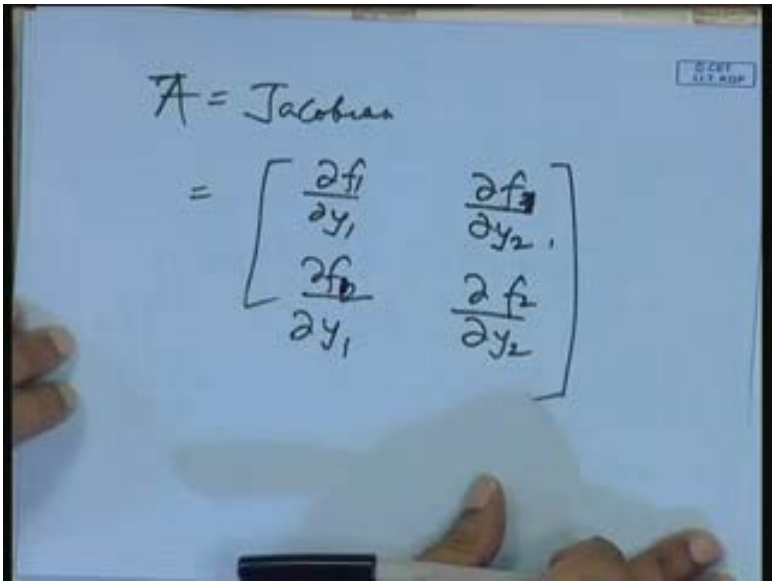

So, if you have forgotten, I will recap how the Jacobean is. So, A is a Jacobean that equals del f 1 del y 1 del f 2 del y 2 del f 1 del f 1, sorry, del del y 2 del f 2 del y 1 and del f 2 del y 2. So, this is my Jacobean; I will write it one more time.

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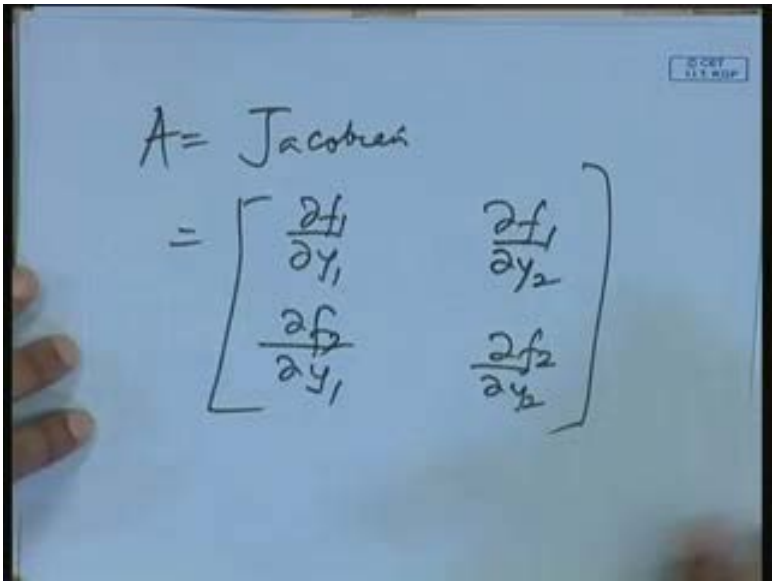

So, A equals Jacobean equals del f 1 del y 1 del f 1 del y 2 del f 2 del y 1 del f 2 del y 2. Now, for our case, which is this, these are my f 1, f 2; I need to find what my Jacobeans are. So, let us go ahead and do this now.

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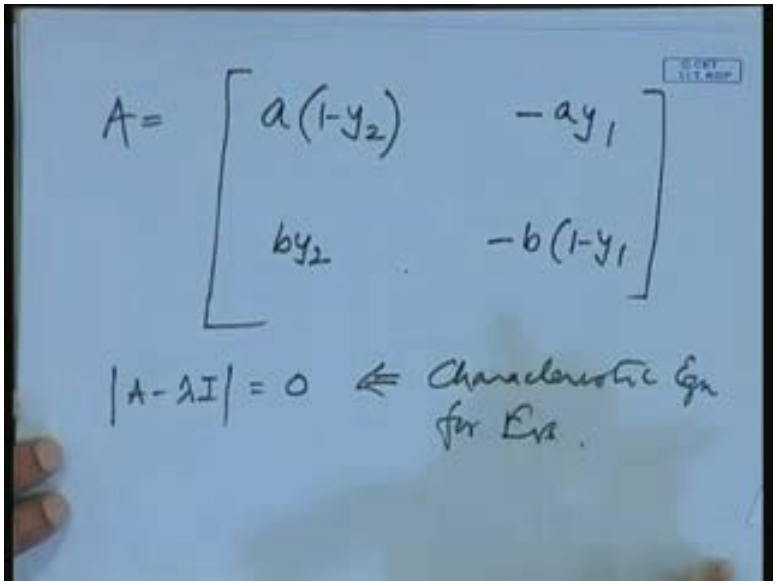

So, A would be, the first term is del f 1, del f 1 del y 1 and that is simply a times, a times 1 minus y 2; then, next term is minus...So, the, which is, del f 1 and del y 2, which is minus a y 1. Now, del f 2 del f del y 1, that is, b y 2 and del f 2 del y 2 is minus b 1 minus y 1. Now, I have to find out, what my Eigen values are. What, how do I find that out? I have to, first, I have to calculate the determinant of A minus lambda I and equate that to 0; this will give me my characteristic equation for Eigen values.

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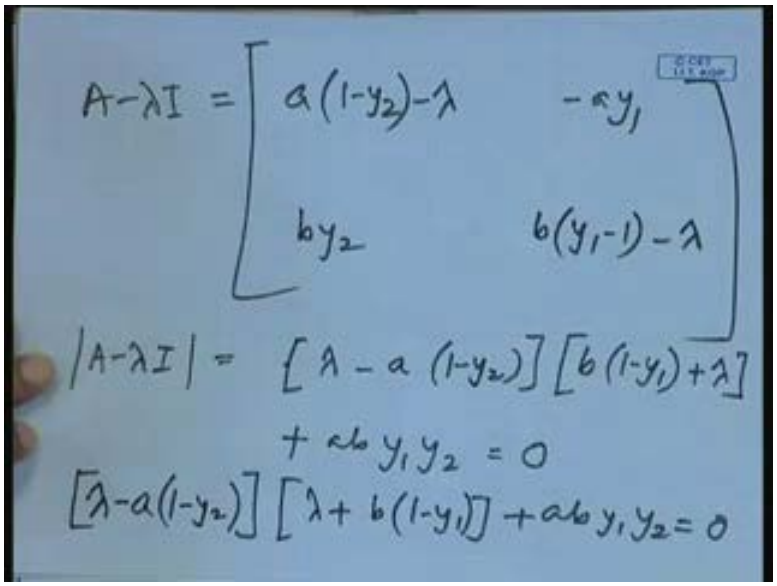

So, let us form the characteristic equation first. So, A minus lambda I would be equal to

a 1 minus y 2 minus lambda, minus a y 1, b y 2, b y 1 minus 1 minus lambda. And, the determinant of A minus lambda I equals, minus, equals lambda minus a, 1 minus y 2, times b 1 minus y 1 plus lambda, plus a b y 1 y 2, equals 0. So, the next step is lambda minus a 1 minus y 2, lambda plus b 1 minus y 1, plus a b y 1 y 2 equals 0.

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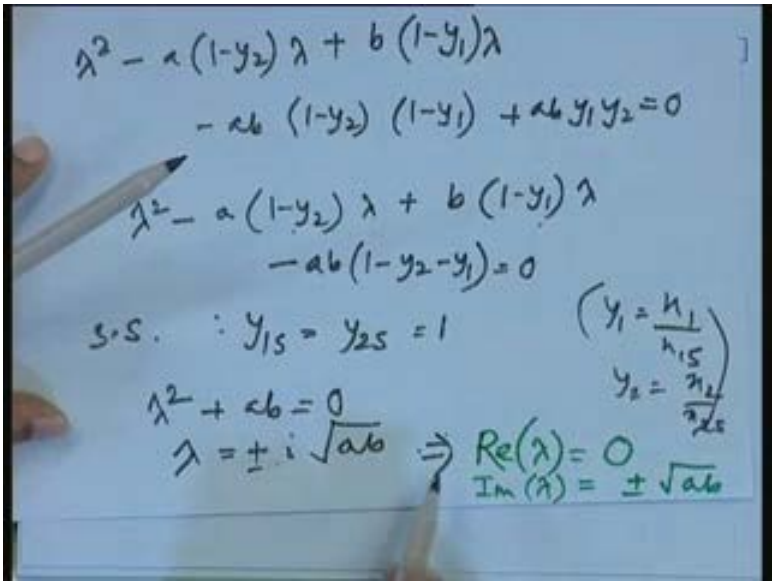

Now, so, the characteristic equation is formed as lambda square, minus a 1 minus y 2 lambda, plus b 1 minus y 1 lambda, minus a b 1 minus y 2 1 minus y 1, plus a b y 1 y 2 equals 0; that is my characteristic equation. When I simplify it, I get lambda squared, minus a 1 minus y 2 lambda, plus b 1 minus y 1 lambda, minus a b 1 minus y 1 minus y 2, equals 0. Now, I have to evaluate the lambdas at the steady state. So, steady state in my case means, y 1 s equals y 2 s equals 1. Why, because, y 1 was defined as n 1 over n 1 s and y 2 was defined as n 2 over n 2 s, right. So, obviously, at steady state, these values are going to be 1. So, if I put 1 over here, this term, this term vanishes, this term vanishes, and what I have is, lambda squared, plus a b equal 0; or, in other words, lambda equals i imaginary times a b. So, this is my answer, you know, plus minus, sorry. So, this is, this is my answer. This is what I find that, plus minus. So, two values of lambda that I get are, plus and minus i; this is a conjugate roots, i square root of a b.

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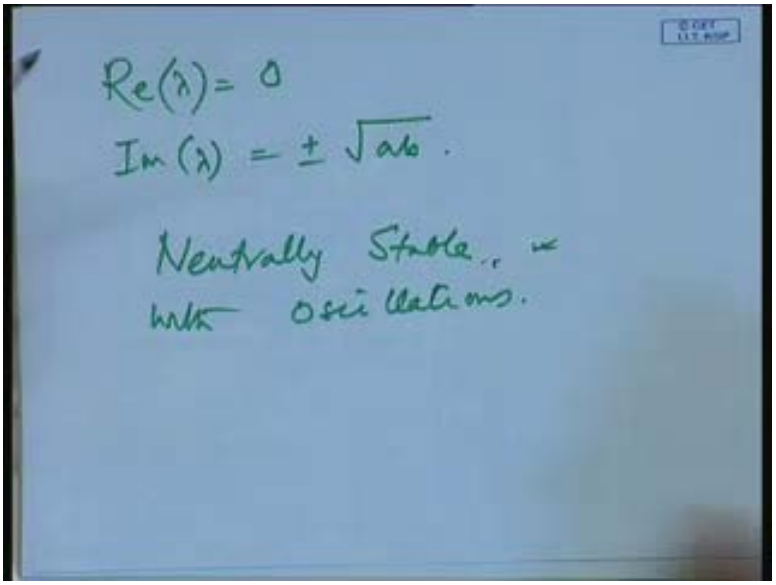

Which means that, the real part of lambda, which means that, the real part of lambda equals 0, and imaginary part of lambda equals plus minus square root of a b; which means what, that the system...So, if real part of lambda is 0 and imaginary part of lambda equals plus minus square root of a b, this means what? This means that, the system is neutrally stable, stable, with oscillations. So, the oscillations part, we had already seen, known and we had sort of predicted. And, what we find over here, the new thing that we find over here is that, the system is neutrally stable, if not asymptotically stable; it is not unstable, but, it is neutrally stable. So, it just remains as it is, and that is why, the pictures that we drew over here, this one, for example, and this one, this one. So, this one, these oscillations will continue and in the phase space also, it will keep going on in these circles.

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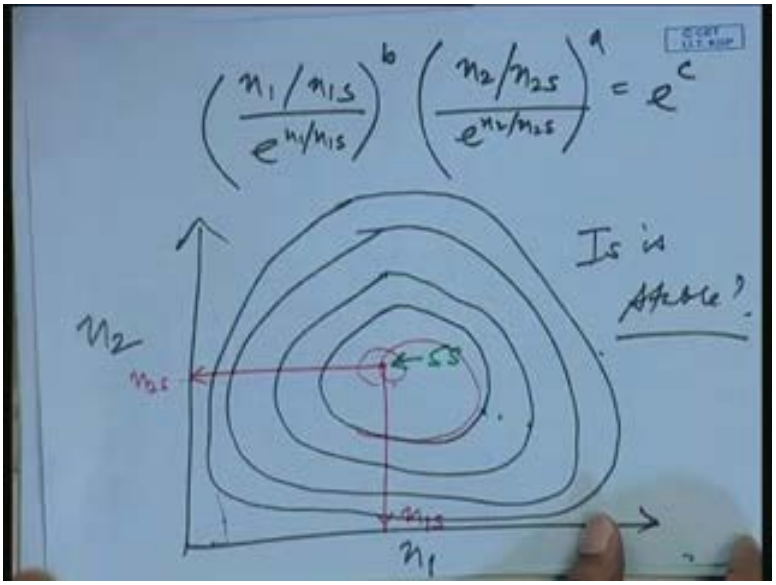

Now, you can ask me, what are these different circles about? Why do you have so many different circles? It depends on initial condition. Remember, these are dynamic equations, we are solving. So, for one certain initial condition, will be on this trajectory; another set, it could be somewhere here, and, and if you keep changing the initial condition, you could go further away from the steady state; but remember, these are dynamic systems, so, they would all depend on the initial condition. So, these circles are for a particular initial condition; but, for a given initial condition, the system will continue grow, growing on this cycle forever; it is not going to come in and you know, it is not going to happen that, it is going to come in like this; that is not going to happen; it is going to go in, on and on, on these cycles; the reason being that, the real part being 0, the system being neutrally stable. Now, what we are going to do next is, competition and selection. So, we are looking at competition here also, but, this is remember, we, what we did here, over here, is the prey-predator model, in general; but, our interest is essentially, the chemostat.

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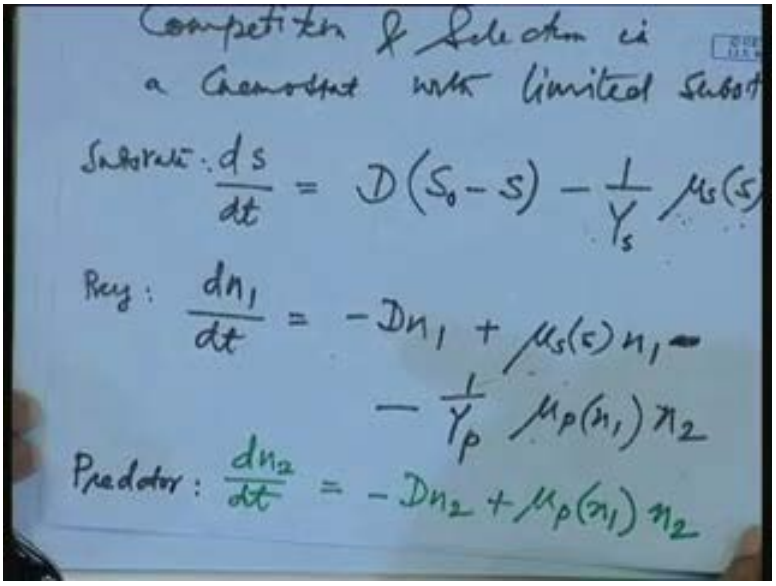

So, what we are going to do next is, and very quickly is, competition and selection in a chemostat with limited substrate; substrate is not unlimited. So, it is a limited substrate. So, what does it mean? What does it mean, when I say that substrate is limited? What it means is that, in addition to the prey and the predator, I need to write a balance now, for the substrate which will do. So, let us say, one is the prey and the other one is the predator and one again is the prey and two is predator before. So, ds dt which is the substrate, is, if you remember, we wrote this chemostat equation before dilution rate D times S 0, the initial amount of the substrate present, minus S minus 1 over Y S mu S S times n 1. So, mu S is the specific growth rate of n 1. So, what is happening, substrate is, substrate is coming in and going out and the prey here, which is exactly, now, I gave you the example of the herbivores and carnivores and the greens. So, think of the substrates as the greens and the prey is eating on the substrate and growing. As a result, the substrate is being depleted and this is leading to the growth of this, of the prey. So, the prey n 1 is growing; this is, this is proportional to the growth rate, the consumption on the substrate and because of this growth process, the substrate is being consumed. And, how much is the substrate being consumed? That depends on the specific growth rates. So, larger is mu, more substrate is being consumed. So, larger is mu, more substrate is being consumed over here, is that clear.

So, the next thing that we do over here is, write the balance for the prey. So, this is the substrate and this is the prey. So, that equals minus D n 1; this is entering the system

because of the, you know, because dilution rate; or, in other words, the entry of the, and exiting the system and entry. So, essentially, this is essentially the exit term, plus mu S times of function, as a function of S times n 1. So, this is the growth of the predator, prey from the substrate; it is eating on the substrate and growing and this is the growth rate. And, that is why, it comes with the plus sign. This one comes with the negative sign, because it is exiting the system. And then, you have minus 1 over Y p mu p n 1 times n 2. What is this? This is, comes with the negative sign, I will put the negative sign here, actually; this comes with the negative sign, which means that, this is the, not the growth, not the birth, but, the death of the prey. And, this is happening because, the predator n 2 is eating on the prey. And, at what rate it is eating on the prey? At the specific rate. This is not a growth rate anymore; it is like a death rate; but, you people can still call it as growth rate, just mu p as a function of n 1 and Y p is the, you know, selectivity ratio. And then, we have to write the model for the predator. I can, I hope you can see it here; this is the model for the predator.

Let me write this in green. So, d n 2 d t equals minus D n 2. That is, why is there no entrance? See, for the substrates, there is a entry term, D S 0; but, for the prey and the predator, there is no entry term, because no prey, or no predator is entering the system; only the substrate is entering and it is growing; prey and predator is, are growing and this is given by plus mu p n 1, mu p as a function of n 1 times n 2. So, the, these, this is the overall model that we have. Now, what we need to compute in the next few minutes, is the steady states.

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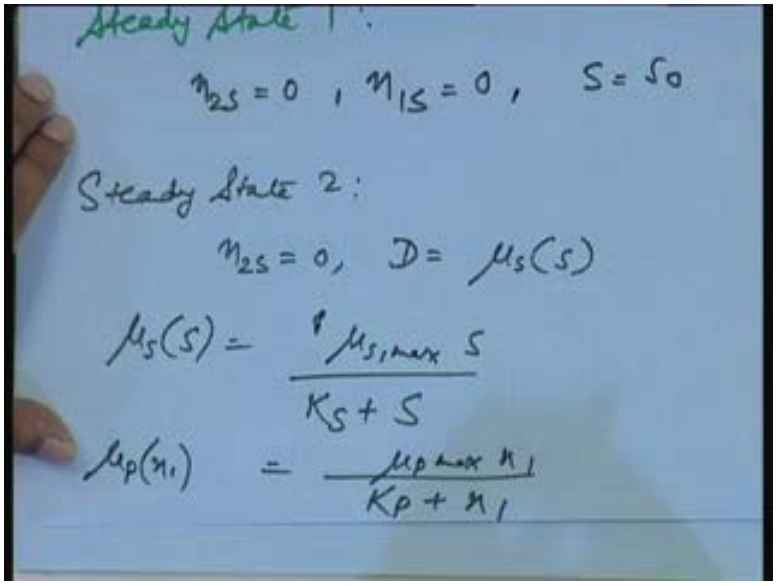

So, there are three steady states for the system. Steady state 1 is n 2 s, if you go and solve it, you will find this, you know and the solution is not very difficult, equals 0, n 1 s equals 0, and S equals S 0. Steady state 2 is n 2 s equals 0, D equals mu S of S and which means that, D and mu S of S is given as mu X max times S over K s plus S and mu p of n 1 is given as mu p max n 1 divided by K p plus n 1. So, this is one.

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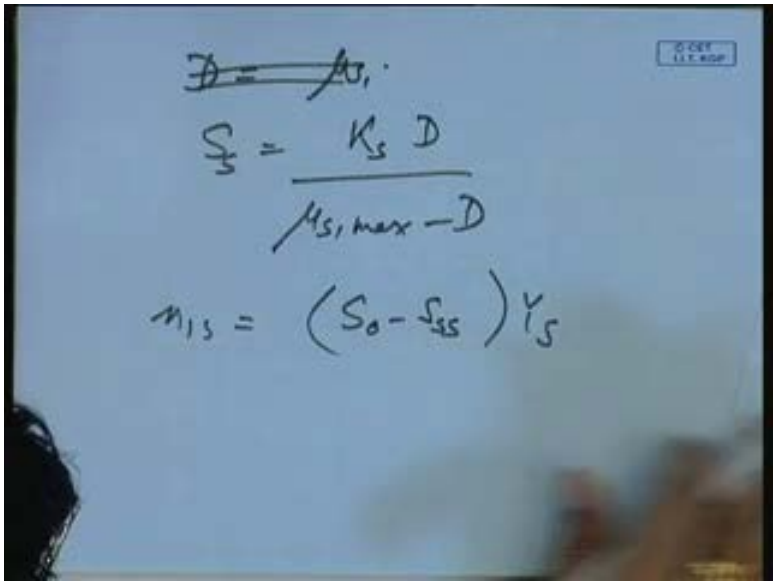

So, from this, you get steady state, I am still on steady state 2. So, this gives you mu d s as, this gives you S S as K S D over mu S max minus D and n 1 s equals S 0 minus S S

times Y S and let me write steady state 3 quickly.

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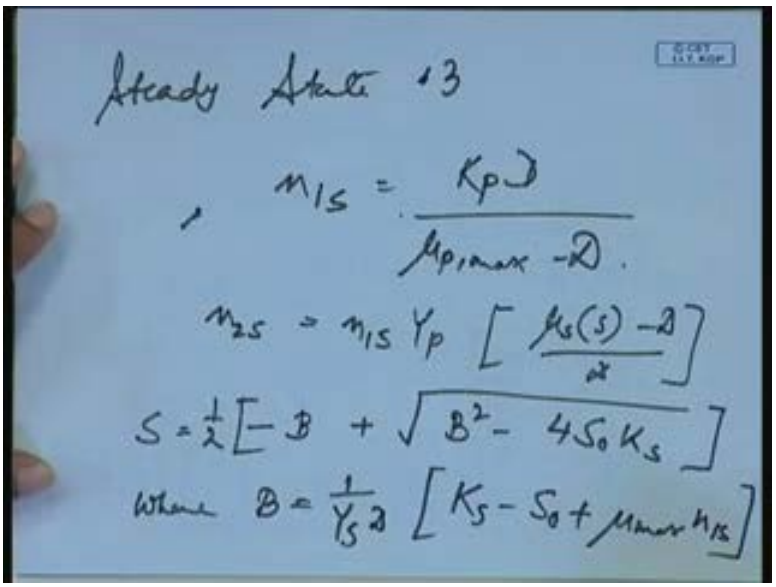

Steady state 3 is, what I get, n 1 s equals K p D mu max minus D and n 2 s equals n 1 s Y p mu S S minus D over D, and S itself comes out to be minus B, plus, there is plus minus, but, I will take only one of the roots, because the other root does not make sense; 4 S 0 K S, this is half, where B equals 1 over Y S times D times K S minus S 0 plus mu max n 1 s. So, it is a little complicated solution, the steady state 3; steady state 1 and 2 are very easy. Now, what you can do is, as we did in the previous case is, go and look at the stability of these steady states. How do you do that? You just form the Jacobean like we did before, and you have a 3 by 3 Jacobean, and you figure out how, how the Eigen values look like. And, the positive, if it is a positive real part, then, you know that, the Eigen values are, the system is unstable; if it is a negative real part, then, you know that, the system is stable; if it is 0, like we had had before, then, you know more or less that, the system is neutrally stable.

So, this sort of concludes what we are trying to do over here, but, what I will do in the next couple of minutes is, just run through what we had done in the previous, you know, lectures, you know, in all these 24 lectures put together, that we had been doing. So, we started with, initially there was a introduction given on the fundamentals of biology, and so on and then, to enzymes, and we started with the biochemistry and thermodynamics of enzymes; we looked at different kinds of inhibitions, competitive, non-competitive,

substrate inhibition, different kinds of inhibition; we looked at the effects of pH and temperature; and then, we moved on to immobilized enzymes. So, in mobilized enzymes, there were no mass transfer effects, and we moved on to immobilized enzymes; and in immobilized enzymes, we found that, there was mass transfer effects that were there; you know that, that would, that would hampering the growth of the, or the reaction kinetics of the enzymes; and so, we looked at the effect of mass transfer in immobilized enzymes and, and the effective of factor in immobilized enzymes. And, we looked at how kinetics is being, is being hampered; or, in other words, how the reaction rate is been decreased, because of the presence of immobilized enzymes, enzymes and the mass transfer rates there. And then, the next thing we looked at was microbial growth, the different phases of microbial growth, the model for microbial growth, and effects of mass transfer on microbial growth and so on. And, we also looked at the effects of multiple substrates and inhibition. And, today's lecture, we looked at the effects of, not just multiple, not multiple substrate, but, multiple microorganisms and the interaction between them. And, there is a continuity between what we did, and then, we looked at bioreactors and, and design of bioreactors and stability of bioreactors, and just today also we did.

We looked at what happens, when not just...So, we had looked before, at what happens when a microorganism grows in the bioreactor and the stability of the growth process. Today, we looked at the stability of interacting growth processes. So, two microorganisms are, are, are growing together; then, how does its interaction relate to the growth process? Does it hamper the growth of one? Does it accelerate the growth of the other? And, so on. Then, we moved, had moved on to different area altogether, and looked at receptor ligand binding; we had looked at, in great details, of the kinetics of receptor ligand binding. We had looked at a particular disease called familial hypercholesterolemia and we had tried and, tried to understand, how the receptor ligand binding, the kinetics of this, it influences this disease. And, we had studied in significant detail, the process of receptor mediator endocytosis, which is a very fundamental and important process in many, many, many physiological systems, and it and ligand receptor binding, plays a very important role.

So, this is what we have been doing in a very, sort of a quantitative approach, this part of the course. Another part of the course deals with the more qualitative approach. I hope you enjoyed the process and had learnt something in the entire process of, you know,

how to deal with biochemistry and biology, and bring in a chemical engineering perspective of transport and reaction into it, and make it more quantitative than we typically do. So, thank you.