Biochemical Engineering

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Lecture No # 27

Design of Chemostats

Design of chemostats, but before we do that we will do what I promised in the last class which is a brief review of the history of reactors, chemical reactors.

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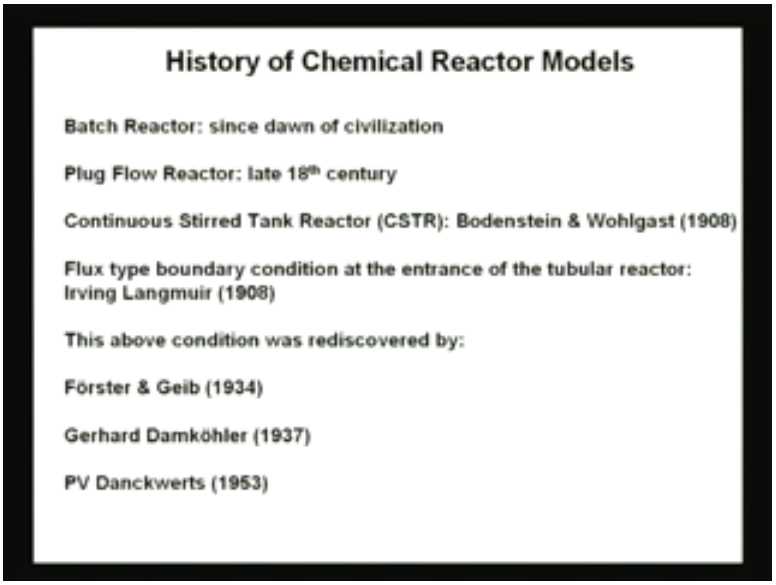

So, if you look on the screen here that the history of chemical reactors. You know it is essentially talked about this chemical reactors are of two in major kind. So the batch reactor the history of that is when we started is the dawn of human civilization with process of cooking and so on. The second one is the plug flow reactor which started in the late 18th century. Then a big conceptual leap so the plug flow reactor was being used from the late 18th century and so essentially the major difference between a batch and a plug flow is that you had continuous inflow and the outflow in the plug flow. Whereas, in the batch you just have to put the things in and wait for how long the reaction took.

So, the reaction could have taken a day or two days and still you have to wait to get the products. Where as in the plug flow reactor, you have a continuous inflow and outflow. Now a big conceptual leap came in the form of a CSTR reactor, which sort of combined the advantages of the batch reactor with that as the plug flow reactor. The advantages of the batch reactor is it is a easy portable vessel, whereas the plug flow reactor is a long tube and it is hard to carry from one place to another and stuffs like that. So, that was a advantage of the batch reactor, the advantage of the plug flow reactor is it is continuous inflow and outflow. So these are the two advantages. So, the CSTR model CSTR thing what it did is combined the advantages of the batch the portability and the factor that it requires less space of the batch reactor with the advantage of the plug flow reactor which is the continuous outflow and inflow.

And this was remembered not you know at some point of time once discovered things so this was not there before that. And this came in 1908 I said this in the last class. This was discovered by two german scientists bodenstein and wohlgast in 1908. It turns out so this was the this is a big conceptual leap as far as the whole idea of chemical reactions, and chemical reactors was concerned. Because as I said it you know it is portable it is takes less space and as at the same time it. It has continuous inflow and outflow and that kind of revolutionize the whole chemical reactor mean the whole chemical industry.

And if you now look at chemical industries all most all chemical reactors are CSTRs or continuous inflow you know continuous stirred tank reactors, only a few percentage of them you know a small percentage of them would be batch reactors. Now 1908 it turns out is a very event full year for the history of chemical engineering or chemical reaction engineering. Because you had the tubular reactor model, the plug flow reactor model rather. And in 1908 again it was irving langmuir who founded the flux type boundary condition. The which what is now was known as danckwerts boundary condition but, it is no longer known because there are difficult in problem issues there. So, as I said that langmuir was he got the noble prize later, because of his work with the kinetics langmuir kinetics adsorption kinetics. But this was another of his pad break breaking you know finings the flux type boundary condition. The reason being you know for us it might seem very mundane at this point of time to think of a flux type boundary condition. But think of it when somebody was discovering it 100 years back to come up with the concept of the fact that you know whatever is coming in through convection at one end

the equals what is going out in, because of convection and diffusion at the other end is quite a leap. The reason being that the concepts of convection and diffusion were not as properly underlined at that point of time. It was only underlined around 1937 and I come to that paper which dealt with that, but so they that is that there is a little bit of fuzziness around these concepts of convection and diffusion and to come up with the boundary condition in 1908 which is stayed for more than 100 years.

Now and nobody has proved it incorrect yet is quite a conceptual leap so that was done by langmuir in 1908. So, and he the so that that is a very important boundary condition and it came to be later known as the danckwerts boundary condition. Which it should not have been, but we did some unraveling of the real history of behind this. So, it turns out that after danckwerts this condition this danckwerts boundary condition not danckwerts let us call it the flux type boundary condition was rediscovered a few times. So, the first time it was rediscovered by forster and geib two more german scientists in 1934, and in maybe the you know it is not really rediscovered. May be because bodenstein and wohlgast wrote their paper in german so forster and geib would have read those papers most likely 1934.

Then it was again rediscovered gerhard damkohler another of the most famous chemical engineers of along with Langmuir. I think he is probably the most famous chemical engineer of his times and our times. So, he rediscovered in 1937 it was not rediscovered actually what he did was this is the land mark paper the 1937 paper of gerhard damkohler. And it was published in german. And what he did in this paper was he reviewed the like we are reviewing the whole history of chemical reaction engineering in 1937 paper he reviewed the entire history of chemical engineering including langmuirs work and forster, and geib's works and you know the whole history of bodenstein and wohlgast discovering the CSTR.

You review the entire work of chemical reaction engineering that was available till that point of time then danckwerts rediscovered. And he was truly trying rediscover you know trying to show the world that you know that he discovered it because he did it. So, there is a fundamental difference between what damkohler did and forster and geib did and danckwerts did because forstein and geib and damkohler they referred to the earlier papers of Langmuir. And showed that these are they have discovered these boundary conditions whereas, danckwerts buried everything, he buried all of the previous work

and he pretended that he had discovered the boundary condition.

And that pretension worked really well, because you know these papers of the forster and geib and damkohler were all kind of buried by the political events of the time which was the second world war. So, because there was germans and there was enormous amount of hatred gains germans at that point of time, and you know the whole world opinion against germans at that point of time. So, their works were lost nobody was reading and the german science nobody was paying any attention to them so their works were practically lost. And then danckwerts in 1953. He sort of claimed to rediscover the boundary condition and now it has been un urged much before it was not us, but around 20 years back it was been found that danckwerts did not discovered this boundary condition.

And people stopped calling it the danckwerts boundary condition. They just so and there is a enormous confusion. You know whether you want to call it the langmuir boundary condition, and forster and geib boundary condition. So, just people decided we will call it the flux type boundary condition. But if I think I wrote it in the last class but, i'll write it one more time.

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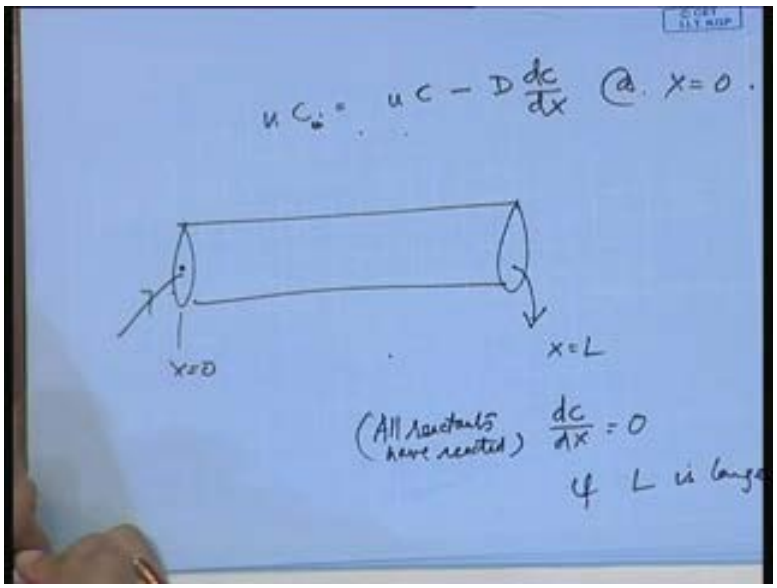

So, the boundary condition simply goes as so whatever is coming in u times C equals whatever is so u times C in equals. Whatever is going out minus D times del C del X so you have the tubular reactor over here, and you are trying to find out this is X equals 0. If

you are trying to find out what is the boundary condition at X equals 0 so this is my boundary condition so whatever is coming in at this point is because of convection u times C in. You can call it even u in times in, but u is suppose to be the same on both sides so u times C incoming in. Because of the convection because equals what goes out because of convection u times C C minus plus what goes out of because of diffusion and diffusion term is minus del C del X so this was essentially the flux type boundary condition and we still continue 100 years down the line we still continue to use this boundary condition it is a quite phenomenal impact breaking boundary condition.

So you know we if we are trying to write something like if you second order you know differential will come to that, but if you are writing a second order problems and you need a second boundary condition. So, what would be the boundary condition at the other end? So, X equals 0 this is my boundary condition. What would be then my boundary condition at X equals L? It would be typically it would be del C del X equals 0 why if L is large as compared to say the radius of the reactors do if the, if the length is very large what does it mean what means is that all the reactants has reacted and there is no reactant left at the end to leave. This means that all reactants have reacted that there is no reactant left so these are the boundary condition anyhow so let us go back to the history.

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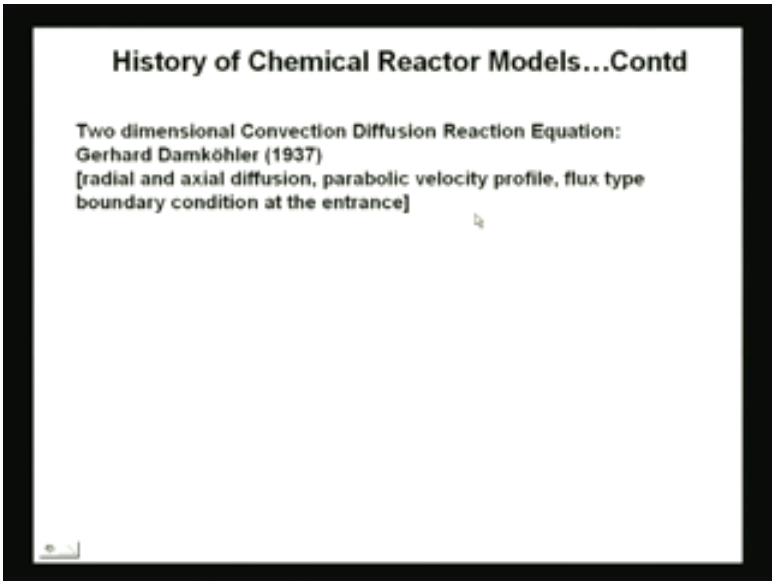

So, then again as I said the 1937 paper of gerhard damkohler that was the one of the most

important papers in the history of chemical reaction engineering and it is a two dimensional convection diffusion reaction equation so we for the first time wrote the two dimensional convection diffusion reaction equation and this as paper is written in 1937 in german. And then it was later translated into English. And what are other things it did and let us recount, because so in this paper. He introduced the idea of radial and axial diffusion so just as I said you know the whole ideas of different kinds of diffusion and convection were not very clearly outlined of way back in those days.

And it was damkohler who was who first introduced the idea of radial diffusion and convection axial diffusion and how they are different. So, it is not at really understood at that point of time how the radial and axial diffusions are different, than he introduced a parabolic velocity profile, till then with a plug flow reactor right plug flow reactor model, p f r model. What is the p f r model? Intel it means that, the velocity profile is flat within the within the reactor. And the but the velocity profile is not really flat because when you have flow inside the tube the velocity profile is parabolic. But if you understand that that in a plug flow reactor there is no way I mean there is difficulty at least to introduce a plug flow radial velocity profile.

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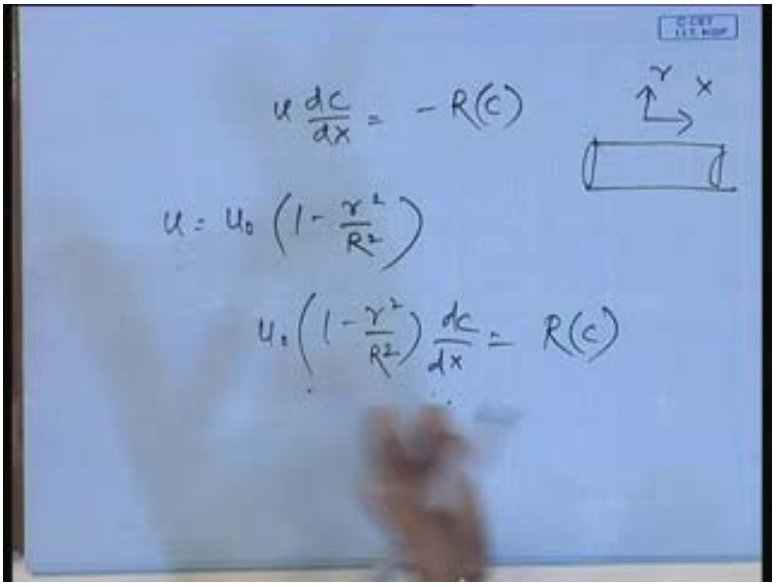

Let me show you why because what you have is u del C del X equals minus R of C, this is your plug flow reactor model. Now this is in X so this is my reactor, and this is the X direction, and this is the r direction. Now if u equals u naught 1 minus r over R square

then it is, how do you solve it ? You know so if I put it back over here 1 minus r square over R square so this R of C how do you solve it you see what I am trying to say the difficulty of solving it why, because the differential in the X direction and then you have a term in the radial direction. So, how do you solve it? There is no way to solve it so the only way of being able to incorporate a radial velocity profile is to actually is to actually.

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Actually go for a two dimensional description so which is what damkohler, did you know he went for a two dimensional description and as a result of which he could incorporate the parabolic velocity profile. He included radial and axial diffusion and he included the flux type boundary condition. So it was the first time somebody looked at the chemical reactor in a very comprehensive way incorporating convection radial, and axial diffusion parabolic velocity profile, as well as flux type boundary condition which includes both convection and diffusion effects as the at the entrance.

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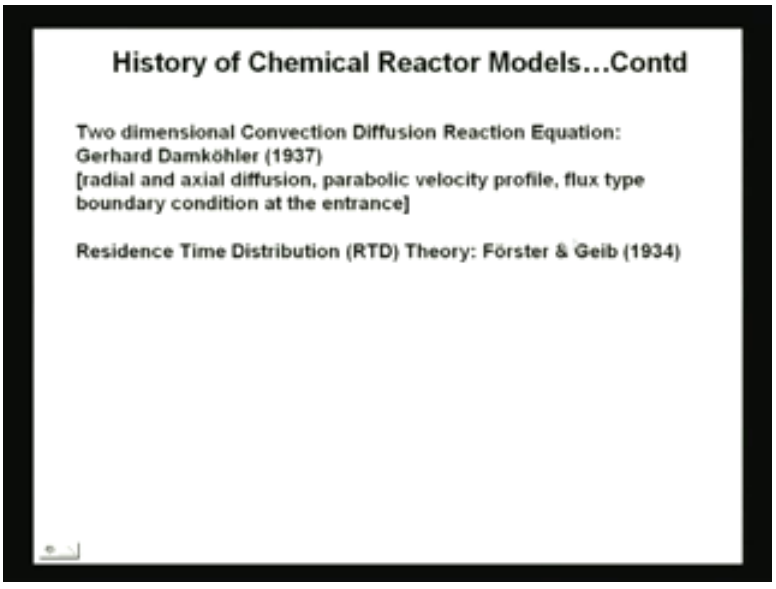

So this was the 1937 paper and then the whole idea you were aware of the idea of residence time distribution theory. So, and surface renewal in residence time distribution theory so they came around like the same time from danckwerts you know paper. But again it turns out that the forster and geib was actually the first one to discover the RTD theory. The residence time distribution theory the whole idea that you know that some fluid the different fluid elements in a reactor will have different residence times. You

know so you can say that the reactor residence time is tau, but that does mean that the entire fluid in the reactor spend the amount of tau in the reactor.

What does the residence time mean? You know when I say that the residence time what is the residence time mean the amount of time you spend here so the residence time of the B tech student here is 4 years or the residence time of a Phd student is 4 years something like that so which means that the amount of time that you spend from input inlet to outlet. But when I say that the residence time in a CSTR is tau strictly speaking the not so this was it was discovered essentially by forster and geib that strictly speaking. It is not that all every element of fluid in the reactor is actually spending the tau amount of time.

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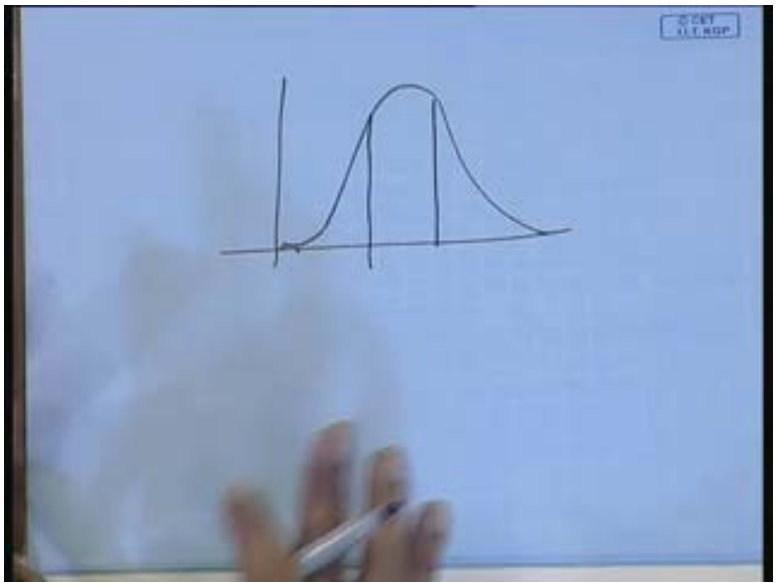

So, typically the residence time distribution is you now goes something like this so this is how residence time changes. So, the average of this you know if you take the whole of it and divide it then the average of it is tau, but some elements as you see her[e]- over here some elements of fluid the so this is the amount of elements of the fluid, some elements of the fluids spend very small time again. You know some very small so the bell shaped curve and only a fraction says spend a very large amount of time in the reactor. So the when I say that reactor is the something you have to remember, when I say there is a reactor residence time is tau. It does not mean that everything in the reactors spends tau

amount of time it means that the average amount of time spent within in the reactor by a fluid element is tau.

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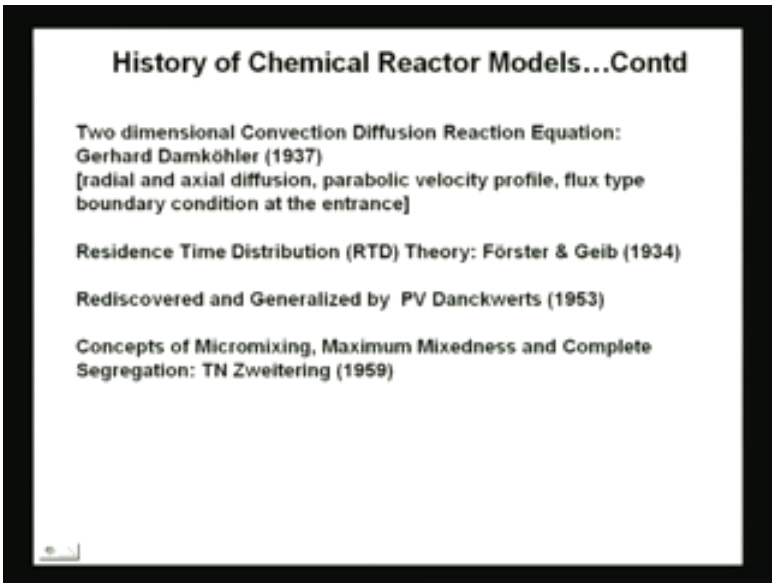

So, this whole concept was actually started by forster and geib in 1934, and then it was again you know danckwerts is a great rediscoverer so he was he rediscovered and this theory in 1953. But he had some inputs also which is he generalized, that it was in a forster and geib's paper in 1934 which I got translated through somebody, and read it and it was a very localized concept. But danckwertz generalized it for the whole reactor thing and it really you know the his presentation was good. Then last important things that came in the whole CSTR business was a concept of micromixing, which is mixing at the molecular scale. Because reaction takes place at the molecular scale, and if reaction has to take place in the molecular scale. Then there has to be mixing at the molecular scale right because if a and b two reactants are there and unless they are mixed with the molecular scale they cannot react.

Is that clear? Because the reaction has to take in the molecular scale. So, the concepts of micromixing maximum mixedness and complete segregation. Complete segregation when there is no mixing at all between the two components. So, these were propounded by as zweitering in 1959. So, I think that concludes our brief review of the history of chemical reactor models what we will do now is that we will so much of this theory as you see is related to CSTR because as I said CSTR was a big conceptual leap.

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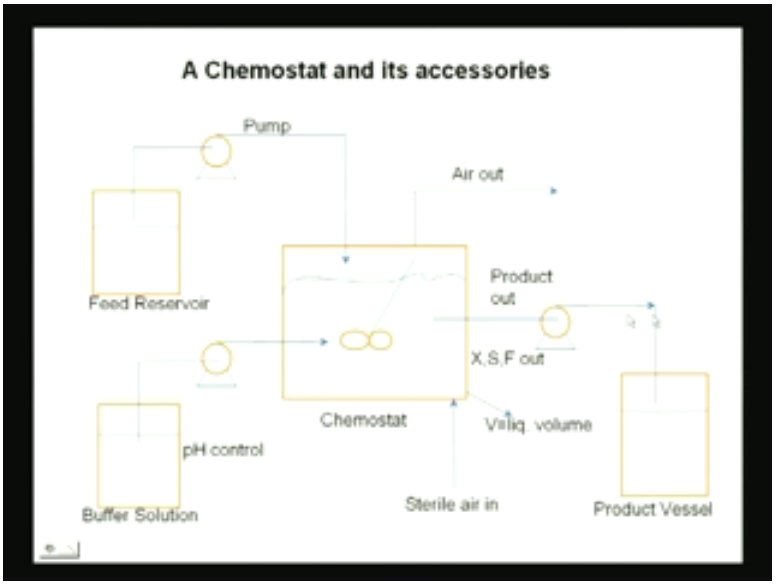

So what I will do now we will start try and look at the CSTR for a bioreactions. So, the CSTR for bioreactors is called chemostats, and this is let me show you the picture and try and explain. So, this is the central thing that you have over here is a chemostat. So, it is a CSTR which means it is a continuous stirred tank reactor, which has two parts with one is continuous another is stirred and for four parts. Actually continuous stirred tank reactors. So, the continuous means there is a continuous inflow of feed and a continuous outflow of product, stirred means it is being stirred by a a stirred over here, tanks so it is a tank and then it is a reactor so the four elements come together.

So as you see that the way it works for the chemostat is that, it is the feed reservoir over here from which feed is being continuously pumped into the reactor. So, then you have a buffer solution which is also being pumped, because these are being bioreactions you have to maintain the p H at the certain values could be 7 could be 5 could be 6 whatever. but you have to maintain the p H at a certain value so you have a p H controller and a buffer solution being sent in. You have you know air being taken out and replenished so those kind of things we have stopped about this you know you need a certain amount of oxygen into the system and so on.

So, air being taken out and you know replenished in stuffs like that here and then the product. So, the air taken out here and replenished from here just a while air comes in here just for the oxygen you know that you need to supply and you know the nutrient that

is the carbon, hydrogen, nitrogen in liquid form are a part of the feed reservoir in the feed reservoir itself. So, the product is taken out now the product will include these 3 X S and F what is X? X is the cell are growing. S is the substrate and feed is you know everything else apart from the substrate and the cell.

So, that part of the feed which is not either substrate nor cells. V is at any point of time the liquid volume in the reactor, why is V important not the volume of the actual reactors? Beause the residence time of the reactor the average residence time as we call it now depends on the volume of the reactor and not the V. In volume of the liquid sorry in the reactor and not the actual volume of the tank. Because the rest of the tank that has air in it is not of any use. So, V here and then the product is taken out and pumped out and take put in a product vessel. So, this is a whole flow chart of this system, and is there any question or do you want me to stop up for anything here or shall we proceed? We should proceed.

So, the next question that comes in is that we have to in order to understand the dynamics of this process we have to write balances for this. So, the balance would be straight forward similar to the CSTR balance that you have written except that it is for two components here. So, what are the two components you are going to write the balances for? One there will be let us go slowly. So, how many balances do you think that you will have to write? The first balance would be the overall balance of volume which is the volume accumulated or volume held back in the reactor, and volume coming in and volume going out so that is a overall balance then what else we have.

Component Balances.

So, one would be the substrate balance. So, the two major components as you can see in the picture over here, that X S and F so one would be the substrate balance S then would be the cell balance X so essentially three of these.

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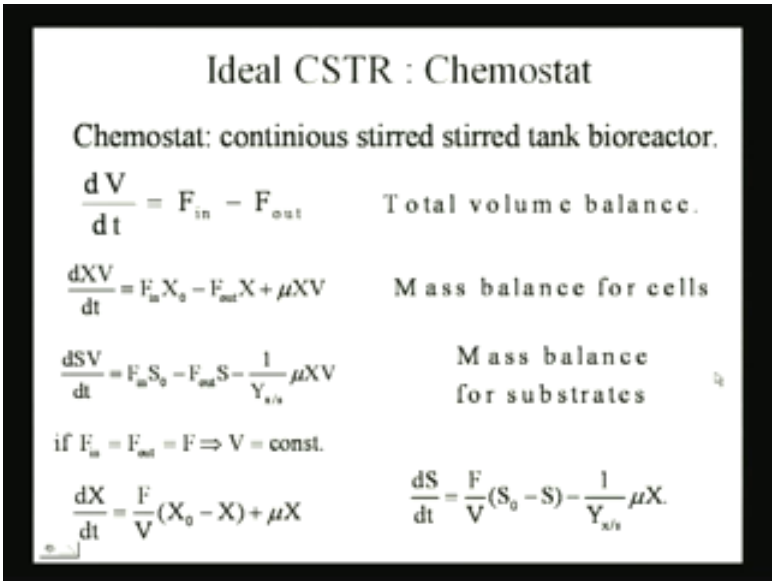

So, let us go and do it so the chemostat as you can see over here is an ideal CSTR. Because it is stirred and you know of everything and continuous, and so on. So, the first you need to do a total volume balance. So, the total volume balance is very straight forward d V d t equals F in minus F out feed coming in minus feed going out. So, the next balance we do is the mass balance for the cells and mass balance of the cells is simply if you take this. For example, it you have to multiply it with the cell concentration so d d t of X V this and these two plus.

They are the generation of cells so that is the only difference between the other component, and the cell is that there is a generation. I mean overall volume is not been generated where cells are being generated so d t of X V equals F n times X naught is the amount of cell that is coming in the feed. So, the feed itself can have some cells minus F out times X is the amount of cells that is in the outlet products, and mu times X is a specific. You know mu is a specific growth rate per unit cell mu times X is the total for the total amount of cells times, the volume you know so because the volume of the liquid should come in over there.

Sir the communications death of cells.

Equation is.

Death of cells. Death of cells death of cells has not been included, but you can add another term minus beta X that we did last time. So, you can just add minus so you instead of mu you can, you can replace mu by mu hat where mu hat is mu minus beta it is fine. So, it is straight forward so the last thing that we need to do is the mass balance for substrates fine so which is d V t of S V equals F n times S naught minus F out times S minus mu X V over Y. Because I explained this several times in the last couple of lectures Y is just the el ratio of X over S. Now typically F you know if you want to typically in CSTR what you would like is that the inlet is equal equals outlet and if that is the case F in equals F out.

Then d V d t is 0 so as a result of which the V is a constant fine and you can do a lot of simplifications in these equations. Because V is a constant you can take V out of this equation you can take V out of this equation fine, and you can get rid of b in some other places because the one of the over V will come in here. So, when you do the simplifications you get d X d t equals F over V X naught minus X plus mu X it looks like a lot simpler equation and just as he said that if you want to include the death of cells replace mu by mu hat where mu hat is mu minus beta being the specific death rate of cells. So, and the last thing would be the substrate balance equation for the substrate d S d t equals F over V S naught minus S 1 over Y mu X fine.

So what would be our step now so once we have done this. So, what would you like we will like to find out how it would changed? But what would be your step? What would you do you think we should do now. So, if F is a constant and known then how many variables do we have just two variables, and two equations so what we would intend to do is essentially solve this, but how do we solve this? I mean you know what kind of solution do you want first let me ask you that.

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What is that.

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Possible for what for every thing.

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Well I do not think analytical solution is possible one of the things, you can try to do is get an invariance. I taught you have to get invariance in the last class, you can get an invariance, and then you can get an analytical solution for combination of X and S naught for X or X separately. But what we want to do first is the easy step out which is up in the steady state solution of it.

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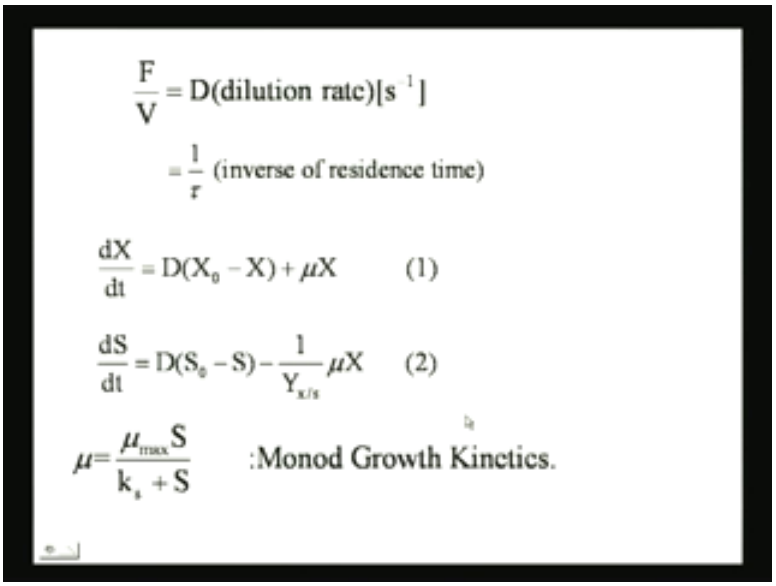

So F over V is called the so we do that. So, let us define some quantities first F over V is called the dilution rate of the cells F over V is a dilution rate of the cells, and what is the unit of Sover V F over V?

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Hm what.

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Inverse of time which means it is inverse of the residence time of the system so F over V is 1 over tau, which is the inverse of the residence time of the system. So, when you do that? You can put here d t and d t of X equals dilution rate d e you know instead of 1 over tau. In this chapter or in this course we will use the dilution rate d instead of the residence time. So, D is 1 over tau X naught minus X plus mu X and d s d t is a d e S naught minus S minus mu X over Y fine. Now mu is we are going to assume a Monod growth kinetics as I said it is a most popular and most useful growth kinetics there are

plenty others as. I showed you in someone of the earlier lectures but, this is the one that is that we are going to try and use for now.

Now as I said that you know sometimes I might ask you in the exams for example, to do the similar kind of thing with other growth kinetics. I can give you more complicated growth kinetics or I can give you growth kinetics where inhibition is involved or all other kinds of things possible. But then but you have to be aware of how this is the process so what we're going to do from now on todays class and next class is slightly hard and you should pay attention to the process, what is really happening and how to do it? Because then you should be able to do it for other kinds of systems so that is fine.

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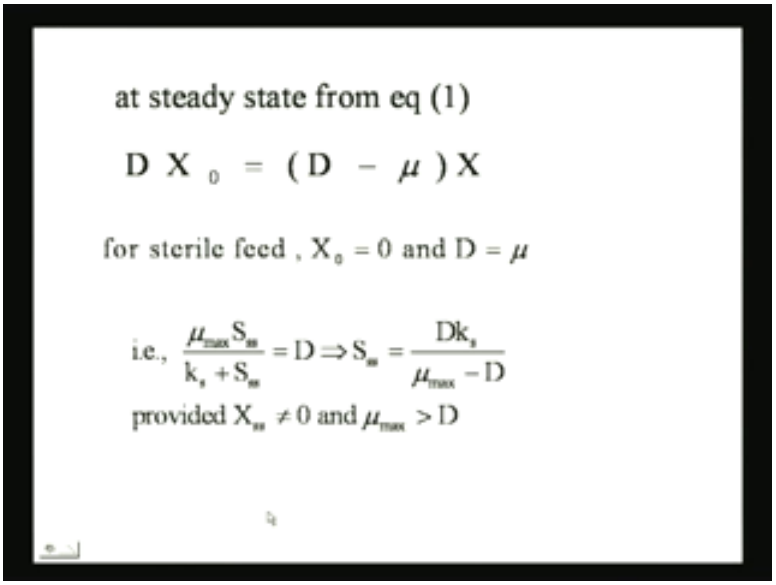

So we put the monod growth kinetics in and so then we go for the steady state solution. So, at steady state this is your basic equation. So, d X naught so if you look at this here so we put a steady state out here so D X naught equals D minus mu times X for the X equation and then similarly, for the substrate also you will have another one. Now the and the first approximation that we can, we make assumption that we make is that it is a sterile feed. It is we are solving it for a sterile feed sterile feed means that there is no cell in the feed that is coming in. It is an assumption of coarse because typically there are cells, but we are just trying to solve it for this system.

We will solve it for other systems also without sterile feed but, let us try and solve it for with sterile feed is it clear the sterile feed means, that X naught there are no cells in the

in the incoming feed so when you do that you simply get D equals mu the dilution rate equals mu. Now mu is this specific growth rate remember so it goes as mu max times S over K S plus S which equals the dilution rate. So, if you have a fix dilution rate. So, what this means is? For a sterile feed for the system to have any sort of meaning full solution, if you have a dilution rate that is fixed then yours steady state substrate concentration has to be this. You see what I am saying why is that? Because see if X naught is 0 then d minus mu times X is 0 what is the solution of this equation.

The two solutions of this equation one is that X naught it is X itself is 0 then the other one is that D equals mu so if D is not equals mu, then the other solution is X naught X equals 0 which means that no cells are being produced which is worth less because I am running the whole reactor to produce cells. So, the only feasible solution in this place is d equals mu. Now when you put mu max X over F plus s equals D then the steady state substrate concentration is D K s over mu max minus D fine. Provided that X s s is not 0 I mean of coarse and mu max is greater than D mu max has to be greater than D because you see why, because mu if mu is greater than equals D then this fraction over here is always less than one so mu max obviously has to be greater than D.

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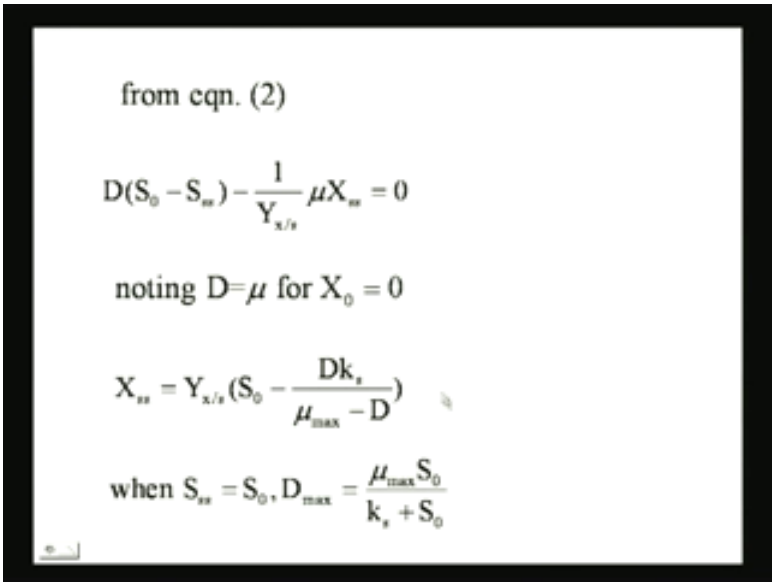

So, from the second equation that is the equation for the substrate this is this is what we get. Now the substrate concentration and you know D equals mu for X naught equals 0 so you can substitute back over here D equals mu in this equation. So, you know over

here and then you can get your X that is the steady state concentration X s s in terms of these. As this because we wrote our S s s before if you remember in the last slide I will just wait for a few seconds because some of you are writing so and then I will show you if you need to so if we wrote the S s s in the last slide.

So you can substitute your S s s from the last equation that we wrote before in to this and is it clear to all of you or do you want me to go through this steps. So, all you need to do is let me just quickly see show. So, all you need to do S s s is given here as D K s over mu max minus D all you need to do is substitute for S s s in from that equation into this equation. And you put D equals mu over here, and then you will get X s s that steady state concentration of cells in the system goes as if we forget the Y let us you know ignore the y then it goes as s naught minus D K s over mu max minus D and.

You know this we already did D max equals mu max D equals mu max S naught over K s plus S naught D equals mu max S over K s plus X so D max would be the maximum value that the D can take is mu max S naught over K s plus s naught. Why because the maximum values that the S can take is s naught for the feed concentration clear to everybody, and if there is any point you need to stop me please stop me and i'll explain one more time .

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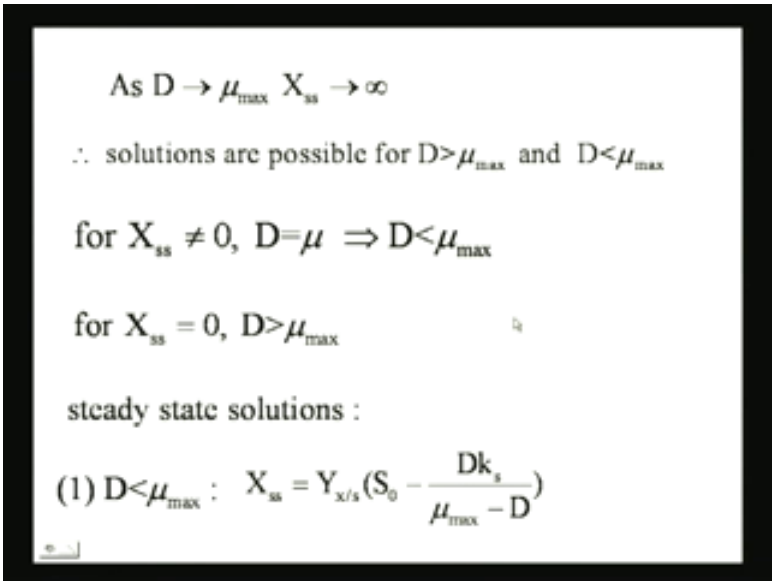

So if that is clear then as in this equation as mu max goes to D you see over here. Then what will happen this is goes to 0 the denominator here goes to 0 and this goes to infinity

as a result solutions are possible for mu D greater than mu max and D less than mu max is it clear see. What was what did we get from our initial assumption of sterile feed? We got that D equals mu fine and one of the things we said is that D is typically if D equals mu then D would be less than mu max, because mu max is always greater than mu. So we are actually working in this space of D less than mu max, but just to make sure that you know to make sure that this a there's a feasible range.

So, D equals mu max there is no solution because this whole thing blows up the solution is only possible in the D less than mu max, D greater than mu max. But obviously we are working in the space of D less than mu max clear no confusion. So, this is what I said that we are working in the space of D, D less than mu max because it is not possible for D if D equals mu max mu. Then it is not possible for D to be greater than mu max because mu max is obviously greater than mu .

Now for X s s equals 0 D greater than mu max which is not in the you know which is not in the it is in the feasible range, but it is not of any interest to us if no cells are being produced. it is of no interest to us so this is a just another possibility so for X s naught equals 0 D has to be less than mu max clear. From not necessarily from the mathematics but, the physics of the problem. V has to be less than mu max because V is D equals mu and if D greater than mu max and X s s is 0. So, my steady state solution is that for D less than mu max I have my steady state solution as this and for D greater than mu max it is 0. So, what does this mean? what it means is that? Why am I why am? I you know trying to answer this question? Why am I trying to a kind of boil everything down to d and mu max? Both of these two are parameters of the system d and mu max but, what is the difference between these parameters.

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Absolutely yeah, see the mu max is a parameter that cannot be changed it cannot be manipulated whereas, D is a parameter which can be manipulated D is one over the residence time of the reactor. How can I change the residence time of the reactor? Just flow rate yeah just basically by changing the flow rate or the volume that of the liquid in the system. So, what this is trying to say is that here so what this is trying to say over here is that these two cases, that if you want if you want cells to be produced your d has

to be less than your mu max or in other words your dilution rate has to be less than the maximum specific growth rate of the cell.

If your max dilution rate is greater than the maximum specific growth rate of the cell then there are no cells which were have going to be produced. What does this physically mean for you what it physically means is? That if dilution is very large then it will sweep this as the way you know it would. It would take the things away with it so if D is very large for example, then what it means what it means is the residence time in the reactor is very low fine because D goes as one over tau. If the residence time of the reactor is very low what does it mean it means that the cells are not getting enough time to react for and grow. So, this is the physical interpretation so in bioreactor system in biochemical engineering, they say that dilution of the system dilution rate being very large means that the system is being excessively diluted and as a result.

Cells cannot be grown but, if you look at it from a chemical engineering point of view ,then we convert it into the residence time. So, D very large means residence times very time very small which means that the cells are not getting enough time in the reactor to grow.

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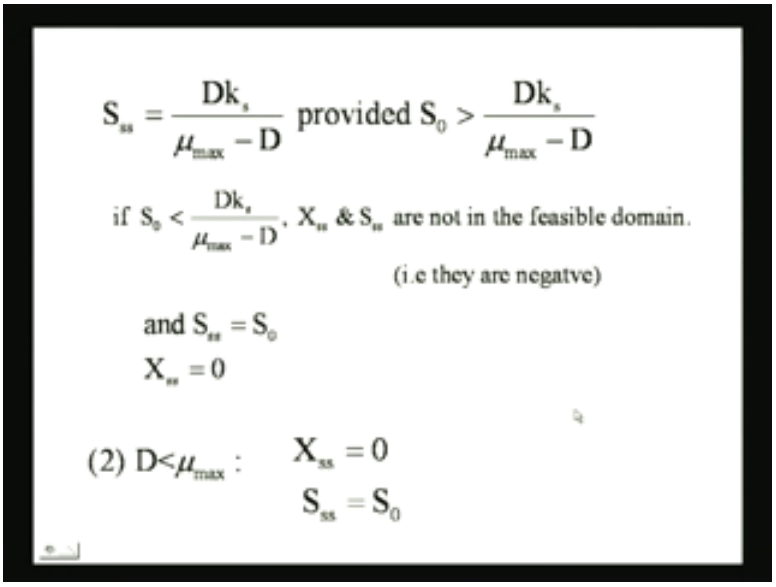

So, that is why it happens? From mathematically let us look a little bit of the mathematical space of it. So, if this is the case that is D is less than mu max, then your steady state X s s I just gave you here. This is the steady state solution for the cell and the

first thing here is the steady state solution for the substrate. Do you need to write the steady state solution for cell? I think you wrote it already, but if you want to it will give you a few seconds so the steady state solution for the substrate now is given by D K S over mu max minus D. So, this has to be satisfied S naught greater than D K S over mu max minus D so because mu max is greater than D so this denominator is positive so the second criteria that we get.

So there are two criteria that we have to satisfy one is that the residence time of the system or dilution rate of the system. One is the residence time is one over dilution rate dilution rate of the system has to be greater than mu max. And the second criteria we get is given here S naught, which is the initial concentration of substrate in the feed has to be greater than this quantity D K S over mu max minus D. So, both of these two criteria have to be satisfied in order to be able to produce cells so that your X s s is greater than 0 both of these two criteria to be satisfied fine is it clear.

So, both these straight two criteria have to be satisfied, in order for it be, if they are not satisfied then X s s and S s s are in the not in the feasible domain. So, what happened was that? If you remember the that how do we get the first criteria we got the first criteria by showing that if mu max is less than d, then your X s s is 0 or negative. We got the second criteria by showing that if your S naught is not greater than this, then your S s s would be negative or 0. Because see what happens is that both the substrate and the cell that are coming out have to be positive numbers both of them. Both the substrate so for for the positivity of the cell you need to ensure that D is greater than D is less than mu max for the positivity of the substrate you need to ensure this quantity.

That is the initial amount of substrate that you to put into the system is greater than D K S over mu max minus D S and so and the second regime is D no. I do not know this is a case of X s s equals 0 and S equals at S naught so what this means is the nothing is happening in the reactor for the same case. But nothing is happening in the reactor. So which means that you start with cells this is what kind of solution is this trivial solution. So, this is known as a trivial solution whereas, X s s is 0 that is you put in cells is or input solution is free of cells when you are putting it in now no cells. In no cells out and as a result whatever substrate you put in is whatever substrate that leaves the system so this is a trivial solution.

Why am I going through? This is because you know one of the things you kind of forget like let us go back, and show you this equation here. For example so a system like this you know guys, you know very calmly you go and write the one single solution that is there possible. But when you are doing a mathematical analysis you have to take into account all kinds of solutions. So, one of the solution is a trivial solution and the trivial solution as you will see a little later may be in next lecture is important also. So, why we are doing this is trying to look at the entire space composed of all different kinds of solutions. So, the first one we discussed is the non trivial solution the second one is the trivial solution.

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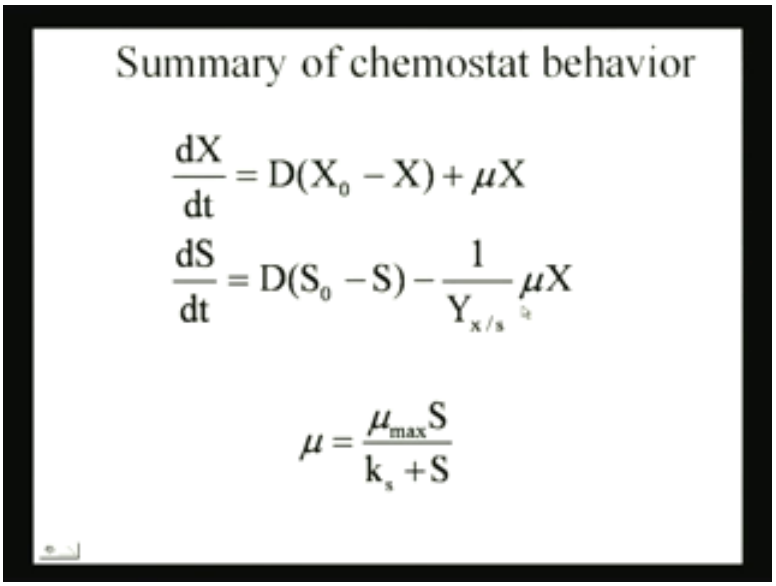

And this is summary of the behavior of the chemo stats. So, these are my equations which I am sure you had already written down. So, there is no point spending any more time on this so d X d t equals this and d S d t equals this and mu equals mu max S over k s plus S and what I will next do is summarize the results also the ones we looked at.

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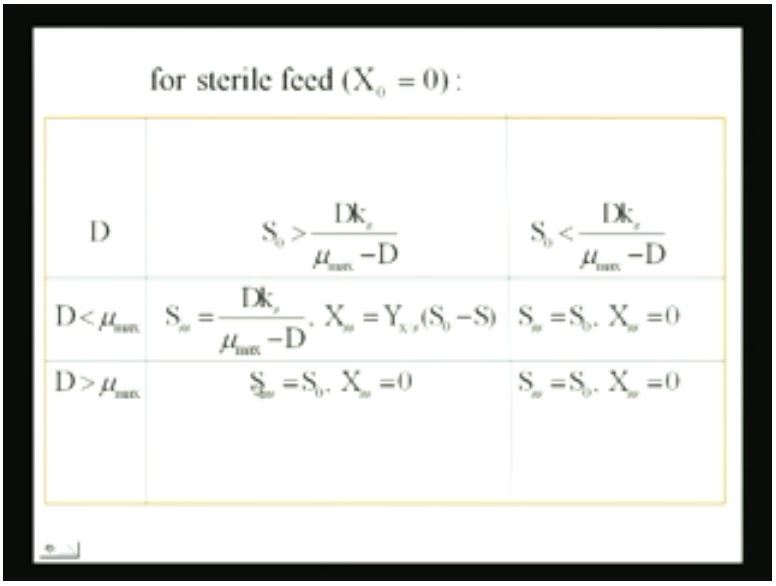

So this is a feasible range this is the feasible range of solution here the rest as we discussed that there are two constraint for this things to happen that feasible range means both positive results for X s s and S s s that is the substrate and the and the cells going out of the reactor are both positive numbers. So, the two constraints for this one is that the dilution rate has to be less than mu max. The second one is the initial concentration of the cell is greater than this number. Now if any of these two constraints are violated then what you end up having is a trivial solution as you have over here all these are trivial solutions.

Because X s s is 0 and the steady state concentration of the substrate is same as, what you put in so what you need to remember? you know the best way to remember this is that these two constraints have to be satisfied in they are then in this space where both these constraints are satisfied and intersect You get the solution anything else you get trivial solution. So this is essentially the steady state analysis of the chemostat. Now what we are going to do is? Look at so when you in a chemical reactor or any kind of you know continuous reactor, you want to run these reactors at steady states fine. You would like to why do you want to run these reactors as steady states because.

Control is easy. Control is easier because whenever you have dynamics of the system coming in control gets harder, but what happens in a real system? you know we are running it at the pilot scale or even forget plant scale even at the pilot when you are

running and even in the lab scale, what happens? You are trying to run it at the steady state but, you are probably not able to run it at the steady state, why does that happen? You can have your control as an everything and you will see that, when you do even experiments in the control lab you will see that you you're trying to manipulate your control variables. So, that the system is at steady state the level of the fluid in a in a you tube or anything for that matter and you will see that it keeps oscillating it is very hard to retain it at the study state. Why is that?

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What.

Disturbance will be there.

What kind of disturbances that is , what kind of disturbances?

(( )) fluctuations.

The key word here is as he said fluctuations or what we call in mathematical balance is perturbation there are these natural noises and natural perturbations that are there in the system. And why do these natural noises and perturbations coming every angle? You know you think that this table probably has no vibration but, it is full of vibrations. So , where ever you putting in your reactor as lots of vibrations in there, and these vibrations lead to perturbation or fluctuations in the system. And as it turns out that you know these system many of these reaction systems are pretty sensitive. The reason these are sensitive is that, I talked about I do not remember. I think I did talk about the case for the autocatalytic reaction in this class itself at some point of time. So, that one is example of why these systems are very sensitive? The reason these systems are very sensitive is because they are non-linear so when they are non-linear.

So a small perturbation in one of the variables can lead to large perturbations in the entire system, because of non-linearity this is a multiplying effect. So, small perturbation in one you know leads to another perturbation in other. And these two effects kind of multiply, and then the wholes the whole kind of whole amount of perturbation that is generated in the system is multiplied and kind of exaggerated. So, this you know there is

a whole analysis for that and that analysis is known as stability analysis so we will that is that is an extent we are going to look at.

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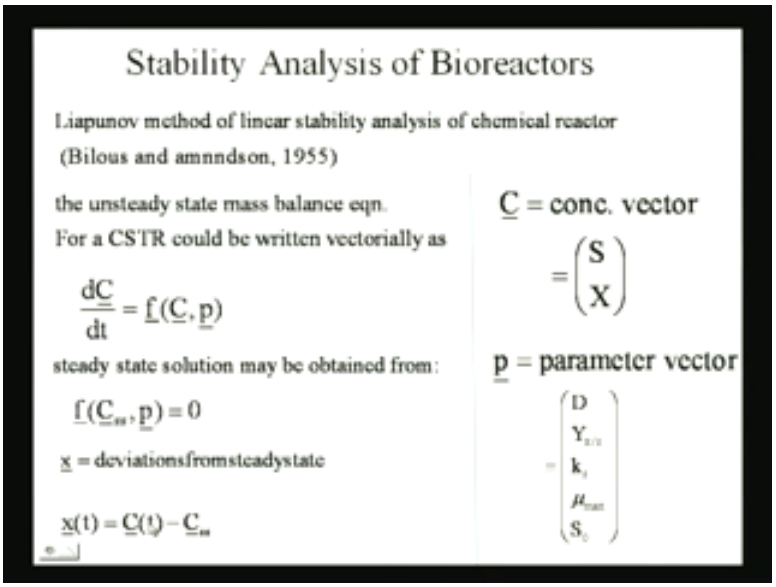

So what we are going to do is stability analysis of bioreactors and you know we will start today, because it is a complicated thing and we would not finish today and we will continue into the next lecture. So, stability analysis is that analysis of the system when you apply perturbation to a study state. So, if a system you have done this partially. I believe in the control class process dynamics and control and we are going to use some of those things. That you had learnt over there so what is the idea of a stability analysis the ideas of a stability analysis. You first obtain the steady state of the system then you perturb the steady state slightly. And you see how does a system react?

How this how this system react? There are two possibilities two major possibilities three actually but, let us say two major possibilities, one is that the system will go back. Go back to the steady state stabilize the other is, system will move away from the steady state which is destabilize. And the third possibility is the minor possibility is that the system remains exactly as you put it you know you perturb it. And the perturbation continuous that little perturbation you gave continuous like that it does not grow, does not decay. But that s that is a minor possibility because that does not happen. The two major possibilities. One is the perturbation that you give decays as a result of which

remember the perturbation that you give decays as a result of which the system returns to the steady state.

And the second one is the perturbation that you gave grows as a result of which the system destabilizes so how do we mathematically analyze. And understand this so this is what we will go we are going to do over the next lecture. So, the method that we use as a liapunov's here the liapunov's method of linear stability analysis for a chemical reactor, and you probably used it in your control class. It was found the and amnndson just type over here m u n d Amundsen this should not be n not n n m u n d amundson 1955 amundson it turns out that he is known as the father of modern chemical engineering as we know it .

You know the statement and so the chemical engineering department which is the best in the world he was the chair of that for 20 20 5 5 years. And then the chemical engineering department at university of minnesota is known as a Amundsen hall. And the major most important award of chemical reaction engineering is also named after Amundsen. So, he is the father of modern chemical engineering, why I am calling it modern chemical engineering is that? It is with amundson he is the one who introduced along with birds to attain light food of coarse, but he is the one who introduced mathematics or applied mathematics to chemical engineering. And as a result of which chemical engineering has a form that it has today so birds to attain light food are responsible for introducing mathematics.

Applied mathematics to transport phenomena. So, the way you we talk about heat transfer mass transfer and momentum transfer is, because of with the changes that were in you know a brought in by birds to attain light food in 1950s. And the very famous book that they have. So, we do not look at heat transfer just is and the terms of a heat transfer equipment or mass transfer as a distillation column. But we understand the basics of the system, and we understand that everything that follows be it distillation be it adsorption or absorption heat exchange in a heat exchanger.

Everything is a product of the very basics of heat mass and momentum transfer and if as you have done in your transport phenomena lecture. There is a tremendous analogy between heat mass and momentum transfer so if you describe one set of systems the momentum transfer equations. You can use the same systems a boundary layer and so on

to describe heat transfer and mass transfer so this was the it is a concept that was introduced by birds to attain light food to chemical engineering and we owe it you know we owe it to them. Similarly, we owe it to amundson neal amundson for introducing the whole of this similar kind of approach a very mathematical way of looking and a very basic way of looking to chemical engineering.

Before that chemical engineering is also just about using chemical reactors and getting products out. But then he introduces whole approach and one of the major things did he did was looking at stability. He also did introduce bifurcation and analysis, but so these two major contribution is stability analysis, and bifurcation analysis. And what we study in todays lecture in next lecture is essentially a contribution of Amundsen. The linear stability analysis and 1955. So, what we do is that? We have to write the CSTR equation in an unsteady state form now so we looked at the steady state we had the unsteady state equations we wrote it before, but we did not solve it at that point of time .

Now either are we are going to solve the unsteady state equation now but we are going to analyze the stability of the steady state using the unsteady form. Now I want to give you a generalized way of looking at it and therefore, what I do is I use a victorial analysis a victorial form of it. So, if you remember we had two components X and the F the cell and the substrate now we write it in the victorial form d C d t equals f C and C is a concentration vector which is composed of S and X fine now similarly, just because C is a is a vector f also has to be a vector .

So, f is vector of functions and P over here is a vector of parameters. So, P also involves lot of these parameters and let us chat them out So, it includes the dilution rate it includes the Y the el ratio the k S michaelis the monod growth kinetic constant mu max the maximum growth rate and the initial substrate concentration fine. And So the solution of the steady state solution would be if I wanted to write in mathematical form what would it be here.

(( )).

If S steady state just f S it is so we will write it like this f C s s so f remains the same C s s comma P equals 0. So, let me ask you this you know so what we are trying to? Let me try and explain first what we are trying to do is, not obtain the unsteady state solution of the problem as I said what we are trying to do is, give a little perturbation to the system

and try to see how the system functions. So, what we are trying to look at is, how it functions in the vicinity of the steady state, or in the neighborhood of the steady state. So, as soon as I say that so what rings a you know brings a bell? So I am trying I have the steady state and I am trying to look at what happens in the vicinity or the neighborhood of the steady state so what should I do mathematically.

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How.

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How.

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No what is the theory? What is the theory that we are going to use? What is the theory you use to look at deviations from anything small deviations from anything? So, F over here if you look at the screen over here f at unsteady states slight for small perturbations at the unsteady state is slightly deviated from F at the steady state how do we quantify that deviation what is the theory we use. What is the theory? We use you you know this you know you should and you I believe you know all this tailor series expansion. So, simple when I say it so for tailor series expansion we need to figure out what this deviation is?

So, the deviation is a small deviation about the steady state is that clear about the steady state, that has to be very clear first we obtain the steady state solution and then we give a small perturbation. And then look at deviations around the steady state so X is my deviation around the steady state, which is given as C t minus C s s C s s being the steady state and C t being the current one in the presence of deviation. So, what we want to study is? We gave a small deviation to the variables around the steady state we want to figure out that. how is this system going to behave is it going to blow up is it at the deviations is going to grow with time or the deviations are not going to grow with time.

And we will do a linear stability analysis which means that linear stability analysis would mean that this is a non-linear system, what we have to do?.

Linearise.

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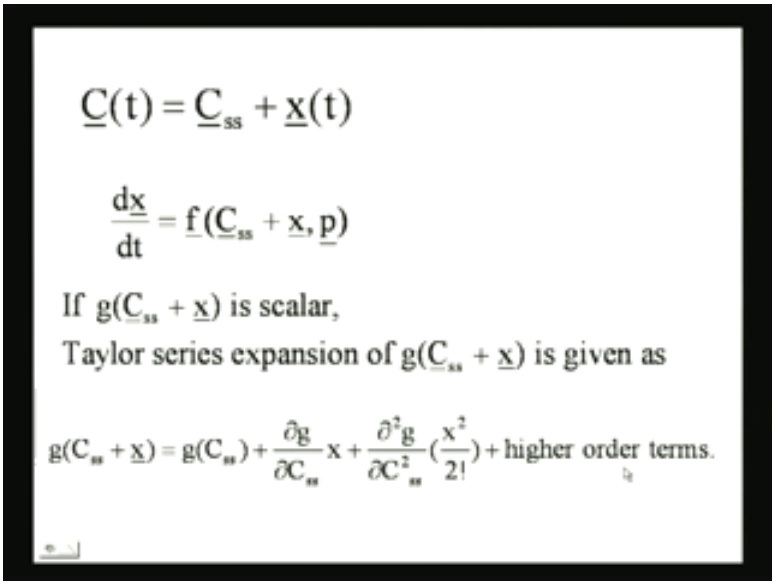

Linearise yes So, because I wrote my X as like this now C t could be written as C s s plus X t why am I writing in terms of X t just. Because I want to do a tailor series expansion fine. So, my d X d t would be written as f s C ss plus X comma P fine why do I write it, because I want to you know so what would be the tailor series expansion of this about C s it would be f C s comma P plus the rest of the term. So, if this I am now showing for any term so if g which is the function of C s s plus X is a scalar then, and I am going to go to the vector form of that so let us i'm giving you the simpler thing.

Today which is just tailor series expansion in a scalar form so and then tailor series expansion of g s s is given as this. So, g and g C s s plus X is simply g g at C s plus del g del C s times X del two g del C s square this is not remember. You know one thing I want to tell you in sometimes people make mistake it is not del g del C s s it is del g del C evaluated at C s s. Similarly, it is del two g del C square evaluated at C s s and X's square over two plus higher order terms, now if am to do quickly tell me if I am to do a linear stability analysis how many terms should I take in the system.

First two of the terms. So, I have to stop at X if I want to do a linear stability as soon as I do take X's square. I do a get an non-linear tailor series. So, I do not want to do that the I the reason. I want to do a tailor series expansion is, because it is easy to handle you know

if I wanted to solve a non-linear equation, I would have actually gone and solved the full equation why would I do a tailor series expansion about it.

So a linear analysis is easy to handle and it would give me ,what I want is? I just need to figure out the earlier initial trend like you know, whether the election people are always interested in the initial trends because initial trend show. Basically most of the time what is going to happen? So, similarly, here also I mean really interested at the initial trends and as soon as I get the initial trend I will have a ballpark idea of whether the system is going to decay or grow.

So, I will stop here today and from the next class next class, what we have to start with is? How to convert? Tthis remember our system is victorial, and this is a victorial form out here. And if you look at the screen so if you so my question is that, how to convert this tailor series expansion in the first thing. We will do is in from the scalar form to the vector form, and then we will continue from this so let us stop here and we will continue from here tomorrow morning.