Chemical Engineering Thermodynamics Professor Jayant K. Singh Department of Chemical Engineering Indian Institute of Technology Kanpur Lecture 43 Activity coefficient of Mixtures

Welcome back. In the last lecture we discuss about reference for liquids just like the way we have taken the reference for the gas phases as ideal gas, we said that well we can extend this analogy and the properties of the ideal mixtures we define it in the last lecture.

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Where essentially it is in line with what we have done for ideal gas mixtures. So, ideal mixtures or ideal solution properties can be written in a similar sense that Delta u, Delta V and Delta H is going to be 0 for the mixture and the partial molar properties are going to be molar properties that is something which we are extending it and they here essentially this is for the pure.

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And then we use the similar kind of expressions which we have derived for ideal gas mixture, to the ideal mixtures or ideal solution which we are written for Delta S and Delta G. Then what we talked about is that you know how do you obtain the chemical potential difference between the pure component and that in the mixture and we showed that we can relate this to RTInxi.

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And using the relation of chemical potential to fugacity, we showed that well RTInxi is nothing but RTIn fi of ideal mixtures divided by fugacity of the reference phase here for the pure component. Now, this is where I put fi is zero.

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Using these two relations we said well fugacity of component I in an ideal mixture is proportional to the mole fraction with a constant here which is that of the fugacity of the reference state. Now, reference state we said well this we can take in the two ways depending on whether I is basically a solvent or a solid. For the solvent where all possible x are present, we will be considering

something like the Lewis-Randall rule where this fi0 would be that of the pure phase that means it will be a fi pure.

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So, this was the case here fa ideal corresponding to mixtures is going to be xa that is a mole fraction multiplied by the reference state or a fi0 which is now in this case is fa pure because all xa exist from 0 to 1.

However, there would be some cases where the which would be relevant for solute which possibly may not exist in all composition from 0 to 1 in such case what we are going to consider is fi ideal is proportional to xa multiplied by something we call it H which is we call it Henry's constant and this is valid at extremely small concentration where xa tends towards is 0.

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So, that will be the case of the dilute condition and it is relevant for the solute, where the solute will be present in a very small amount in concentration. So, there are two cases, one is when xa is approaches 1 then we will be considering the reference the fa is 0 here as simply fa pure that will be your Lewis-Randall rule and other reference is the one which is relevant at the very low concentration of the component.

So, we do use this also sometimes where basically for example salt and some solute which are not even present in the complete range in such cases we prefer to use it. We will be trying to do some examples to illustrate this the differences in this reference in order to get more clarity but as of now we are going to just put it like this and then we will move on to other aspects of that of this fugacity and the derivative of fugacity with respect to x and relating it to the pure component or Henry's constant. But as far as the clarities in terms of examples we will take it up in the subsequent lectures.

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Edit View Insert Actions Tools Help Binany soln Z Xi d Gi => >> X, du, +x, du => a+ T, P $\begin{array}{rcl} \chi_{1} & d \ln f_{1} + \chi_{2} & d \ln f_{2} & = 0 \\ d x_{1} & d \ln f_{1} & + \chi_{2} & d \ln f_{2} & = 0 \\ d x_{1} & d x_{1} & + \chi_{2} & d \ln f_{2} & = 0 \\ d x_{1} & d x_{1} & d x_{1} & a + T_{1} \end{array}$

So, let me first consider in this today's lecture case where it is a binary solution and essentially, we know for the binary solution in general that Gibbs-Duhem relation or in general Gibbs-Duhem relation is going to be valid. So, essentially, I can write like Gibbs-Duhem relation as xidGi bar or this is nothing but of course mu.

Binary soln. $\sum x_i d\overline{G_i} = 0 \quad or, \qquad x_1 d\mu_1 + x_2 d\mu_2 = 0 \quad at \ T, P$ $d\mu_i = RT dln \ f_i$ $x_1 dln \ f_1 + x_2 dln \ f_2 = 0$ $divide \ by \ dx_1: \ \frac{x_1 dln f_1}{dx_1} + \frac{x_2 dln f_2}{dx_1} = 0 \quad at \ T, P$ $x_1 + x_2 = 1 \quad or \quad dx_2 = -dx_1$

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File Edit View Insert Actions Tools Help xy dlufi + x2 dlufz $\chi_{1} \frac{d \ln f_{1}}{d x_{1}} + \chi_{2} \frac{d \ln f_{2}}{d x_{1}} = 0$ $\chi_{1} + \chi_{2} = 2 \quad \text{or} \quad d \chi_{2} = -d \chi_{1}$ clividu by dr,

Now, considering is a binary mixture that means x1 plus x2 is equal to 1 or dx2 is minus dx1. So, in that case we can write this expression here as x1dln x1 therefore dx1 is x2dlnf2 divided by dx 2. So, this is something which I can write because if I bring take this term to the right-hand side it is going to be minus of that and since dx1 is nothing but minus dx2.

So, this minus will cancel out and what we get is this expression. So, let me rearrange this expression now. So, if I rearrange this expression, I can write this as because d lnf1 is nothing but 1 by f1 df and then you have this x 1 and x 2. So, I can take this expression and this will be your f1 df 1 by dx1. Similarly, this is going to be 1 by f 2df 2 by dx 2.

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$$\frac{cl fi/dx_{1}}{f_{1}/x_{1}} = \frac{d fr(dx_{2})}{f_{1}/x_{1}}$$

$$In the dimit - 5f x_{2} \rightarrow 2, x_{2} \rightarrow 5$$

$$\lim_{n \to 2} \frac{d fr(dx_{1})}{f_{1}/x_{1}} = \lim_{n \to 5} \frac{d fr(dx_{2})}{f_{2}/x_{2}}$$

$$\lim_{n \to 2} \frac{d fr(dx_{1})}{f_{1}/x_{1}} = \lim_{n \to 5} \frac{d fr(dx_{2})}{f_{2}/x_{2}}$$

$$\lim_{n \to 5} \frac{d fr(dx_{1})}{f_{2}/x_{2}} = 1$$

$$\lim_{n \to 5} \frac{d fr(dx_{1})}{f_{2}/x_{2}} = 1$$

$$\lim_{n \to 5} \frac{d fr(dx_{2})}{f_{2}/x_{2}}$$

$$\frac{\frac{df_1}{dx_1}}{\frac{f_1}{x_1}} = \frac{\frac{df_2}{dx_2}}{\frac{f_2}{x_2}}$$

In the limit of $x_1 \to 1$, $x_2 \to 0$

$$\lim_{x_1 \to 1} \frac{\frac{df_1}{dx_1}}{\frac{f_1}{x_1}} = \lim_{x_2 \to 0} \frac{\frac{df_2}{dx_2}}{\frac{f_2}{x_2}}$$
$$\frac{1}{f_{1,pure}} \left(\frac{df_1}{dx_1}\right)|_{x_1=1} = 1$$
$$\frac{df_1}{dx_1}|_{x_1=1} = f_{1,pure}$$

Expression of Lewis – Randall rule: $f_i^{Ideal} = x_1 f_{1,pure}$



So, using this let me now get the expression here this would be your df1 by dx1 divided by f1 by I take this here to the denominator x 1 this is going to be equal to df 2 by dx 2 f2 by x2. So, where we are heading by doing this exercise. So, let me try to provide this information a bit here. So, if you look at let us go back again to look at a bit ahead.

So, what was this here. We showed that well the slope here is equal to fa pure. Essentially that is a slope here or in other word you can also show that the slope here is essentially the Henry's constant and here is this fa pure. So, how do you prove that? So, while this is something which we relate it in this form but we can also use this Gibbs-Duhem relation to get these expressions and that is what we are trying to do here.

So, we got this expression here from here how do you go to the next stage. So let us say now we put some limits. So, this is a expression which is true for all conditions for the binary mixture and in the limit of x1 tending towards 1 or x2 which tends towards zero if we put that limit to this expression then essentially I am going get a limit x1 tending towards 1 df 1 by dx 1 divided by f 1 by x1.

And this is going to be limit x 2 because this is all x 2 df 2 by dx 2 f 2 by x2, x2 goes to 0. In the limit of x 1 goes to f 1, f1 is going to be simply f pure and x1 is of course 1. So, on the left-hand side what I am going to get is 1 by f1 pure and then you have this df1 by dx1 in the limit of x1 equal to 1.

And what about the right-hand side? Now, right hand side if you use this rule so if you differentiate this what you are going to get is the same term here and this will cancel each other and this is will lead to 1 here.



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So, from this expression what we are going get is this and essentially this all this implies that your the slope of f1 with respect to x1 at x1 equal to 1 is nothing but f1 pure and that is something which we could clearly see from the ideal expression. That for the ideal solution, if you take the derivative of this with respect to xa is nothing but f a pure.

And that is essentially the case of if you take it the real system and apply Gibbs-Duhem relation you obtain that relation particularly at x equal to 1. So, this is the basically the expression of Lewis-Randall. Where we have said that you know fi ideal is nothing but x1fi pure. So, this is what exactly get and that means that basically when the composition x1 tends towards 1 and when it approaches towards 1 the solution itself behaves like an ideal solution.

And that what is very clearly shown that because for an ideal solution the fugacity has to be proportional to composition and with the proportionality constant fa pure so for the case of x1 tending towards 1, fi is nothing but the slope is nothing but therefore pure and that is what we could see that this is only feasible when the expression holds and that means at x equal to 1 this is nothing but idle solution. So, we could prove it using that Gibbs-Duhem relations.

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Now, having said that I also made a remark that essentially fugacity is something which we often use for the gas phase and because its reference is usually the ideal gas, so what would be the right kind of parameter for the case of the solution. So, from that thoughts what we try to do is we try to define an another variable because as I mentioned here again the fugacity is more relevant for gases and mixtures.

So, with instead of this using fugacity for the liquid we still define a term which we use the same expression we said well is a mu i T, P x that difference of the chemical potential of a component I with respect to mu i T, P for the pure case and we define this as RT ln ai by ai0. So, instead of fugacity we say activity. So, here ai is nothing but the activity of a component in a mixture.

$$RTln\frac{a_i}{a_i^0} = \mu_i(T, P, \{x\}) - \mu_i(T, P, pure) \quad a_i \to activity of a component in a mixture$$

$$RTln a_i = \mu_i(T, P, \{x\}) - \mu_i(T, P, pure)$$

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$$\frac{\partial \Delta G_{mix}^{IM}}{\partial N_i}|_{T,P,N_{j\neq i}} = \overline{G}_i - \underline{G}_i = \mu_i^{IM}(T,P,\{x\}) - \mu_i(T,P,pure)$$

$$RT \ln x_i = \mu_i^{IM}(T, P, \{x\}) - \mu_i(T, P, pure i) = RT \ln \frac{f_i^{IM}}{f_i^o}$$
$$a_i^{IM} = x_i$$

So, unlike fugacity we usually put ai0 is set to pure number. That means we are saying that a0 usually we set to 1 which is basically for the case of a pure component limit xi tending towards 1 which is what we are taking this as a reference. So, this would be the case where xi tends towards 1.

So, that is a reference here which corresponds to the pure case here. So, thus we often write instead of what I have written earlier we often write RT ln ai is simply mu i T, P x minus mu i T, P pure. Now, this is the definition here which we are going to use that.

Now, if you compare this particular definition of activity with respect to that what we have defined in for the case of ideal mixture then essentially the ideal mixture if you look at it again you could see that the chemical potential difference was connected to RT ln xi. So, this is the same thing but it is the ln x i. So, essentially for the ideal mixture the activity coefficient should be equal to composition.

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So, therefore for ideal mixture ai for ideal mixture should be equal to the composition. But note that ai is not a measure of a composition it is a measure of a chemical potential, because is the by definition we are relating to the chemical potential. Now, similar to the analogy of a fugacity coefficient which we have used for when we discussed the gas phase is we are going to define the activity coefficient for a component in a mixture.

So, we are going to define the activity coefficient for a component in a mixture. The way we are going to define is that we will use the symbol called gamma i and this is going to be ai divided by xi. In the same way as we have done for fugacity coefficient where we said well fugacity coefficient is nothing but is the ratio of the pressure and the partial pressures. So, remember that phi i was fi by pi or pi was nothing but yip. So, in the same way we are going to define the gamma i as ai which is the activity coefficient and the ratio of ai with though that of ideal mixture.

So, in other word this is nothing but ai divided by ai ideal mixture correct. And so, say with the same way we have done that, here fugacity is fi by fi ideal which is nothing but yip. So, it tells you the deviation from the ideal mixture in some way because for aIm, ideal mixture aIm is nothing but xi. So, which means that for ideal mixture gamma i has to be equal to 1 for all composition, for all components. So, for ideal mixture here gamma i is 1 for all composition and for all component.

activity coefficient for a component in a mixture

$$\gamma_i = \frac{a_i}{x_i} = \frac{a_i}{a_i^{IM}}$$

IM: $\gamma_i \rightarrow 1~$ for all composition and for all components

 $a^o = 1; a_i \rightarrow 1 at x_i \rightarrow 1$

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$$RT In a: = Mi (T_1 P_1 Exs) - Mi (T_1 P_1 purci)$$

$$\begin{bmatrix} RT In fi (T_1 P_1 Sx) & - \begin{bmatrix} RT In fi (T_1 P_1 fx) \\ P m fi \\ + ui T_6 (T_1 P_1 purci) \end{bmatrix} + M_u^{T_1} (T_1 P_1 fx)$$

$$\begin{bmatrix} RT In fi (T_1 P_1 Sx) & - \begin{bmatrix} RT In fi (T_1 P_1 fx) \\ P m fi \\ + ui T_6 (T_1 P_1 purci) \end{bmatrix} + M_u^{T_1} (T_1 P_1 fx)$$

$$\begin{bmatrix} RT In fi = Mi (L) - Mi^{T_1} (T_1 P_1 fx) \\ P m fi \\ RT In fi (T_1 P_1 fx) \end{bmatrix} - RT In fi (T_1 P_1 fx)$$

$$RT In ai = RT In fi (T_1 P_1 fx) - RT In fi (T_1 P_1 fx) \\ P m fi \\ RT In fi \end{pmatrix}$$

Now, for the ideal mixture remember that since we have considered a zero is one which essentially means that if we are considering gamma equal to i is 1 then at xi tending towards 1, I should get my because gamma is tends towards 1, ai is xi and xi also tends towards 1 and this condition then ai should also tends towards 1 when you have this such a condition.

So, that is a definition of ai which we talked about now how do you relate ai to fugacity let us look at down because we have been talking about fugacity all the time when it related to the difference in the chemical potential even for the solution phase. Now, we had defined the activity coefficient and activity we would like to also relate this to the fugacity for the sake of completion of understanding. So, what is the relation between ai and fi.

So, let us look at it again by definition V is RT Ln ai. So, that is going to be mu i TP xi minus mu i TP pure i. Now, what I have to do is I have to use this expression and replace this in terms of the fugacity. Now, recall the definition of fugacity, fugacity was RT Ln fi was equal to mu i minus that of the ideal gas that means the reference state there was an ideal gas so essentially this if you think about this, this was RT Ln fi t PX plus mu i ideal gas TP sum zero pure i.

Because by definition, let me write it down the definition of RT lnf was so this is the definition RT lnf and considering f 0 is 1 was nothing but mu I, minus mu i ideal gas. TP sum 0 purifies where p0 and f0 was connected. So, here this I am writing this as this because we are replacing mu by in terms of fugacity so it is going to be RT ln fugacity plus this term.

So, this is the first term which we get and this the other term I also can use the same thing because now this is for the pure phase that means pure component in a liquid phase. So, I mean that is what we expect that is it has to be liquid. Now, let me write it here like this. So, this I can put it in the complete bracket and this again I can write it like this RT Ln fi this is a pure phase so it is going to be TP pure then you have this mu i ideal gas again TP 0 pure.

Now, this is there is a negative sign and you have these four terms one, two and then you have negative sign there is another term and this term and this is within the bracket. Now, this term and this term can be cancelled, so what you have now is RT Ln ai is RT Ln fi TP x minus RT Ln fi, TP pure of i.

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So, this further can be simplified by taking this in the denominator. So, this is going to be the ratio of the fugacity of component i in the mixture and with respect to that in the pure phase. So, this is now RT ln ai. Now, if you use this if you consider the fact that ai is by definition ai is nothing but xi gamma i this is by definition.

Then I can relate this as because now this RT Ln ai is this essentially this also means that ai is this ratio fi TP x, fi TP pure i and this is nothing but gamma i xi. So, now we have a complete relation of activity or activity coefficient with this fugacity. So, this is basically the relation between the fugacity and activity.

Relationship between a_i and f_i

$$RT \ln a_i = \mu_i(T, P, \{x\}) - \mu_i(T, P, pure i)$$
$$RT \ln a_i = RT \ln f_i(T, P, \{x\}) - RT \ln f_i(T, P, pure i)$$

$$RT \ln a_i = RT \ln \frac{f_i(T, P, \{x\})}{f_i(T, P, pure i)}$$
$$a_i = \frac{f_i(T, P, \{x\})}{f_i(T, P, pure i)} = \gamma_i x_i$$

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So, in words we can write this as that activity is the ratio of fugacity of a component in a mixture to that of fugacity of the pure component at the same temperature and pressure. Basically, it means the same aggregation whatever the state was we are considering the same conditions.

So, this way we could relate now the activity which is more appropriate for the solution phase to the fugacities but remember that many components in the solution may not exit in the pure liquid at temperature and pressure. So, for example they could be gases which are dissolved in solution and the gases may not be condensed at that particular room temperature for example if they are in a supercritical state.

So, in such case we usually do not arbitrate take this activity to such a condition, so we consider something called dilute solution and so forth and those kind of concept we are going to describe it

little more when we get a little bit of more understanding of the usage of the activity coefficient, activity and similarly for the Henry's law, Henry's constant.

So, something which we have to now spend little more time and that I am going to do that in a couple of more lectures today's lecture we are going to just focus on this which we just covered that and tomorrow's or next lecture we are going to extend this exercise and define another term which we call it access function and the reason is that now we will be talking about appropriate model for the liquid phase.

So, if you think about it the gas phase, we have a Van Der Wall equational state, we have other equational state which was very valuable or is valuable for obtaining such as fugacity coefficient. If you have information of volumetric data or the pressure explicit equational state you can use that.

However, for the case of the liquid phase we do not often use this equational state because those equational states are not valid except for low pressure region. So, what would be the appropriate model in order to use for liquid phases, if you want to find out something like activity or activity coefficient. So, that is something which we are going to spend little more time in the next few lectures, hope to see you next time. That's it for today.