

**Chemical Engineering Thermodynamics**  
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**Maximum work and entropy of ideal gas**

Welcome back. In today's lecture we are going to discuss maximum work and entropy of ideal gas and with some certain examples to illustrate our understanding and our, rather to test our understanding.

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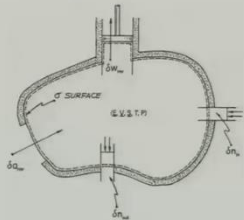
**The Combined First and Second Laws**

**Open, Single-Phase, Simple Systems**

- Consider an internally reversible, quasi-static process with mass entering and leaving a system as shown to the right
- Under these conditions, all intensive properties must remain the same  
 $T_{in} = T_{out} = T, P_{in} = P_{out} = P, \text{ etc.}$
- A mass balance requires,  

$$dN = \delta n_{in} - \delta n_{out}$$
- Application of the First Law gives  

$$dE = dU = \delta Q_{rev} - PdV + (U + PV)dN$$



So, let us look at this combined First and Second Law on this Single Phase and Simple Systems. So, what we have is a similar kind of geometry which we have discussed earlier. So, you have the insulated regions which is in the shaded form, piston cylinder geometry and this is a flow system and this is a non-insulated hence heat can be transferred here. So, there is inlet and as well as there is an outlet in this case. So, this is a very generic open system.

Now what we are considering is that these are, this is internally reversible, quasi-static process with mass entering and leaving the system. Under this condition,  $T_{in} = T_{out} = T; P_{in} = P_{out} = P$

If we do a mass balance:  $dN = \delta n_{in} - \delta n_{out}$

Applying First Law,  $dE = dU = \delta Q_{rev} - PdV + (U + PV)dN$

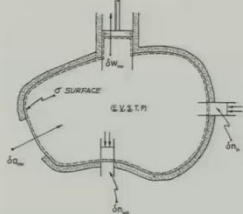
So that is the basic, the First Law for the open system. Now this, now what you can do is we can try to relate this delta Q reversible to entropy and how do we do that?

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**The Combined First and Second Laws**

- An entropy change in the system:
 
$$dS = \frac{\delta Q_{rev}}{T} + \underline{S}dN$$
- Combining the previous two expressions
 
$$dU = TdS - PdV + (\underline{U} + P\underline{V} - T\underline{S})dN$$
- Where  $\mu$  is the chemical potential for a single component system
 
$$\mu \equiv \underline{G} = \underline{U} + P\underline{V} - T\underline{S} = \underline{H} - T\underline{S}$$

$$dU = TdS - PdV + \mu dN$$



So, let us first look at what is the changes in the entropy of the system that is your  $dS$ , So there will be two contribution, one due to the heat, exchange or heat interaction that would be simply your  $\delta Q$  reversible by  $T$ ,  $T$  is constant here. So the second part is the, is the contribution due to the mass flow and that is why it is written as specific entropy multiplied by the differential change in the moles or if it is molar entropy that would be multiplied by differential change in moles or if it is specific entropy then you have to consider this as mass.

$$dS = \frac{\delta Q_{rev}}{T} + \underline{S}dN$$

Combining previous two expressions,  $dU = TdS - PdV + (\underline{U} + P\underline{V} - T\underline{S})dN$

*$\mu$  is the chemical potential for a single component system*

$$\mu = \underline{G} = \underline{U} + P\underline{V} - T\underline{S} = \underline{H} - T\underline{S}$$

$$dU = TdS - PdV + \mu dN$$

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### The Combined First and Second Laws

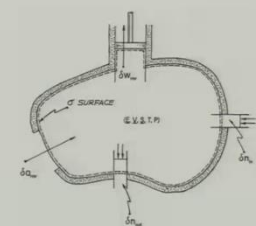
Open, Single-Phase, Simple Systems

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- Under these conditions, all intensive properties must remain the same  
 $T_{in} = T_{out} = T, P_{in} = P_{out} = P$ , etc.
- A mass balance requires,  

$$dN = \delta n_{in} - \delta n_{out}$$
- Application of the First Law gives  

$$dE = dU = \delta Q_{rev} - PdV + (U + PV)dN$$

$\downarrow$   
 $TdS - STdN$



Now what we can do is we can combine this expression with the previous one. We can rewrite, we can take this here and redo this, rearrange this expression and plug in this expression of del Q in this expression and, so this would be your T d S minus S T d N right, so this is nothing but del Q reversible.

$$\delta Q_{rev} = TdS - STdN$$

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### The Combined First and Second Laws

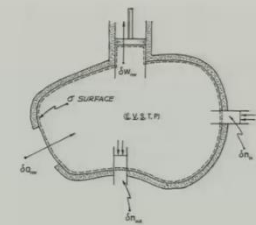
- An entropy change in the system:  

$$dS = \frac{\delta Q_{rev}}{T} + SdN$$
- Combining the previous two expressions  

$$dU = TdS - PdV + (\underline{U} + \underline{PV} - \underline{TS})dN$$
- Where  $\mu$  is the chemical potential for a single component system  

$$\mu \equiv \underline{G} = \underline{U} + \underline{PV} - \underline{TS} = \underline{H} - \underline{TS}$$

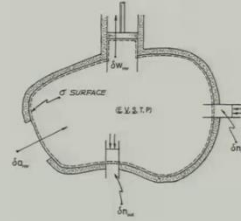
$$dU = TdS - PdV + \mu dN$$



## The Combined First and Second Laws

### Open, Single-Phase, Simple Systems

- Consider an internally reversible, quasi-static process with mass entering and leaving a system as shown to the right



- Under these conditions, all intensive properties must remain the same

$$T_{in} = T_{out} = T, P_{in} = P_{out} = P, \text{ etc.}$$

- A mass balance requires,

$$dN = \delta n_{in} - \delta n_{out}$$

- Application of the First Law gives

$$dE = dU = \delta Q_{rev} - PdV + (U + PV)dN$$

$$\delta Q_{rev} = Tds - STdN$$

So, if you plug in this the  $T ds$  comes here and we can take the minus  $T S dN$  in this, in this part okay. That is what is done here.

$$\delta Q_{rev} = Tds - STdN$$

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**The Combined First and Second Laws**

- An entropy change in the system:
 
$$dS = \frac{\delta Q_{rev}}{T} + S dN$$
- Combining the previous two expressions
 
$$dU = TdS - PdV + (\underline{U + PV - TS})dN$$

$\mu \equiv G = \underline{U + PV - TS}$   
 $\downarrow \underline{G} = \underline{g}$
- Where  $\mu$  is the chemical potential for a single component system
 
$$\mu \equiv \underline{G} = \underline{U + PV - TS} = \underline{H - TS}$$

$$dU = TdS - PdV + \mu dN$$

So, this combined expression is now  $T d S$  minus  $T d V$  plus this molar internal energy plus  $P V$  bar again, molar volume and then you have this molar entropy here multiplied by  $T$  and then the whole expression is multiplied by  $d N$ . Now we will define this  $\mu$ . This is something which we will learn more about this expression or this particular variable  $\mu$ . That is something very valuable in the later part of this course.

So,  $\mu$ , this is nothing but basically  $\mu$  because this is by default  $G$  something which we have already discussed in the first week of this course. So  $G$  bar which is a molar  $G$ , sometimes we also write  $g$  like this. So  $\mu$  is nothing but  $G$ , and this is nothing but  $\mu U$  bar plus  $P V$  minus  $T S$  and this can also be written as  $H$  minus  $T S$  by definition, this would be  $\mu$ . So you plug in  $\mu$  here. So this is going to be  $\mu d N$ .

So the final expression comes out to be this and this is something which is true for open systems. So this is not open system of course so you can keep earlier, this will go to 0 and that case you will get back the original expression which is nothing but  $d U$  is equal to  $T d S$  minus  $P d V$  for quasi-static and slow process, an equilibrium process. So, this will be true for closed system. So, what we have this expression is for only open system, or you can consider this to be a generic expression.

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**The Combined First and Second Laws**

Open, Single-Phase, Simple Systems

- Generalization to a multicomponent system requires summation of the  $\mu dN$  over all  $n$  components

$$dU = TdS - PdV + \sum_i^n \mu_i dN_i$$

- The variable  $\mu_i$  refers to the chemical potential of component  $i$  in a multicomponent mixture
- Note that  $U$  is an explicit function of  $n + 2$  variables ( $S, V, N_1, \dots, N_n$ )

So, if you have many components in the system then you have to sum it up,  $\mu_i$  multiplied by  $dN_i$  so this is the generalized expression of  $dU$  for open system, with multiple component system.  $U$  is indeed from this  $U$  would become a function of  $S, V$  and  $N_1, N_2$  and so for. That is something which we have discussed in the first week also.

$$dU = TdS - PdV + \sum_i^n \mu_i dN_i$$

So that is something we can clearly see that you have also proved it from the basic analysis of your First Law of Thermodynamics considering the relation of the entropy with the heat interactions and the mass flow and basically considering the basic definition of the chemical potential, you obtain the same expression as we have discussed earlier.

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**Reversible Work of Expansion or Compression in Flow Systems**

**Minimum/Maximum Work**

- Engineers are often interested in the minimum amount of work required to operate a device or the maximum amount of work that can be generated from a device
- E.g., minimum amount of work required to operate a compressor
- E.g., maximum amount of work a turbine can produce
- One can deduce that the minimum/maximum amount of differential work for a given process is determined as follows

$$\delta W = \delta n \int_{p_1}^{p_2} V dP$$

- For a device operating at steady state, we find

$$\dot{W}_s = \dot{n} \int_{p_1}^{p_2} V dP$$

- Where,  $\dot{W}_s$  and  $\dot{n}$  are the power input/output and molar flow rate of the working fluid, respectively

So, let us move forward and talk about minimum and maximum work or the reversible work of expansion or compression in flow systems. Often engineers are interested in finding out what is the minimum amount of energy which is required for operating a device, or rather minimum amount of work required to operate a device or the maximum amount of work that can be generated from a device.

So, the relevant devices would be, for a compressor we would be interested in minimal amount of work required to operate and for turbine the maximum amount of work which we can produce. So, these are the two important usual questions which comes in the mind of engineer because he would like to minimize the energy required, overall cost is associated with energy and we would like to maximize the energy and produce of course more value out of it.

So, one can deduce the minimum energy, minimum and maximum amount of differential work for a given process in the following way.

$$\delta W = \delta n \int_{p_1}^{p_2} V dP$$

For device operating at a steady state,  $\dot{W}_s = \dot{n} \int_{p_1}^{p_2} V dP$

Now this is something which you can prove it also and if you have taken the earlier class of Engineering Thermodynamics you would see this expression and the derivation as well. So where this one is nothing but the power input, and  $\dot{n}$  is nothing but the molar flow rate of the working fluid, which could be gas, which could be steam and so forth. So, we will do some examples on that later.

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**The Maximum Work Theorem (from Callen)**

**Overview**

- Consider a system that is to be taken from a specified initial state to a specified final state
- Two auxiliary systems are available: into one work can be transferred and into the other heat can be transferred

**Maximum Work Theorem**

*For all processes leading from the specified initial state to the specified final state of the primary system, the delivery of work is maximum (and the delivery of heat is minimum) for a reversible process. Furthermore, the delivery of work (and of heat) is identical for every reversible process*

But let us take different ways to understand this concept of maximum work and this is something which is described by Callen. So, consider a system that is to be taken from a specified initial state to a final state, so there are two ways we can work in. You can actually interact with the reversible heat store, heat source or you can also interact with the reversible work source W R W S and the change in the internal energy is given in this way.

So, what the Maximum Work Theorem says that for all processes leading from the specified initial state to the specified final state of the primary system the delivery of work, which is in this case is maximum for a reversible process. Furthermore, the delivery of work and of heat is identical for any, for every reversible process. Now this is something very interesting. So let us try to make use of this concept and let us look at this a bit more in detail.



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**The Maximum Work Theorem (from Callen)**

**Reversible Work Sources**

- Systems enclosed by adiabatic impermeable walls characterized by relaxation times sufficiently short that all processes within them are essentially quasi-static

**Reversible Heat Sources**

- Systems enclosed by rigid impermeable walls characterized by relaxation times sufficiently short that all processes within them are essentially quasi-static

So, by definition Callen says that reversible work sources are defined as systems enclosed by adiabatic impermeable walls characterized by relaxation time of the system sufficiently short that all processes within them are essentially quasi-static. So, what he says that whatever you are doing associated with the devices, the equilibrium is reached so fast that they can be considered at each moment you are moving from one point to another, you are basically undergoing a quasi-static process.

Similarly, the reversible heat sources can be defined as the following. Systems enclosed by rigid impermeable walls characterized by relaxation time sufficiently short that all processes within them are essentially quasi-static. So, one deals with the adiabatic impermeable wall, there is no heat interaction as well as the work as far as device is concerned, the other talks about rigid internal wall that means there is no movements of the boundaries when it comes to heat sources. But, both of them should have relaxation time sufficiently short so that they can be considered as quasi-static.

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**The Maximum Work Theorem (from Callen)**

**Mathematical Relationships**

- Application of the first law gives
 
$$dU = \delta Q + \delta W$$

$$dU + \delta Q_{RHS} + \delta W_{RWS} = 0 \quad *$$
- For a totally reversible process  $\Delta S_{total} = 0$
- The total entropy change is given by changes in the entropy of the system and those in the reversible heat source
 
$$dS_{total} = dS + \frac{\delta Q_{RHS}}{T_{RHS}} \quad **$$
- Combining the expressions above gives
 
$$\delta W_{RWS} = T_{RHS}dS - dU$$

Now let us look at the mathematical relation of this diagrammatic way of representation of interactions of the system in, with the reversible heat source and reversible work source so let us look at the First Law. So, you have, by definition you know this.

$$dU = \delta Q + \delta W$$

$$dU + \delta Q_{RHS} + \delta W_{RWS} = 0$$

Now what we assume that the work is being done by the system and hence this is going to be negative. So, you bring it here.

Similarly, you are also interacting in the way that heat is being given out from the system so essentially this is the expression of the First Law. Now for totally reversible process

$$\Delta S_{total} = 0$$

Now if you look at delta S what are the changes associated with this system.

$$dS_{total} = dS + \frac{\delta Q_{RHS}}{T_{RHS}}$$

So, combining the expression

$$\delta W_{RWS} = T_{RHS}dS - dU$$

So, if you combine these expressions, we are going to get this expression.

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**The Maximum Work Theorem (from Callen)**

**Overview**

- Consider a system that is to be taken from a specified initial state to a specified final state
- Two auxiliary systems are available: into one work can be transferred and into the other heat can be transferred

**Maximum Work Theorem**

*For all processes leading from the specified initial state to the specified final state of the primary system, the delivery of work is maximum (and the delivery of heat is minimum) for a reversible process. Furthermore, the delivery of work (and of heat) is identical for every reversible process*

Now further we can expand this and try to come up with the integral form. So, if the reversible heat source is the thermal source then the previous expression can be integrated. That means there is no changes in the properties of this thermal reservoir over time, within the time of the interest of the process. So that means it is so large that heat transfer can be, does not alter the temperature of the thermal reservoir and  $T$  can be considered as a constant here,  $T_{RHS}$ ,  $T_{RHS}$  so if you integrate this you are going to get this. This is from the first equation. This is from the second equation and this is from the third equation.

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**The Maximum Work Theorem (from Callen)**

**Mathematical Relationships**

- If the reversible heat source is a thermal reservoir (a reversible heat source that is so large that any heat transfer of interest does not alter the temperature of the thermal reservoir), then the previous expressions can be integrated to give

$$\Delta U_{system} + Q_{RHS} + W_{RHS} = 0$$

$$\Delta S_{total} = \Delta S_{system} + \frac{Q_{RHS}}{T_{RHS}}$$

$$W_{RHS} = T_{RHS} \Delta S_{system} - \Delta U_{system}$$

So, this is from the first equation, this is second equation, this is third equation. So that is the integral form of this.

$$\Delta U_{system} + Q_{RHS} + W_{RHS} = 0$$

$$\Delta S_{total} = \Delta S_{system} + \frac{Q_{RHS}}{T_{RHS}}$$

$$W_{RHS} = T_{RHS} \Delta S_{system} - \Delta U_{system}$$

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**Example**

A 1-m<sup>3</sup> tank that initially contains air at 1 bar and 300 K is to be evacuated by pumping out the contents. The tank contents are maintained at 300 K throughout the operation by heat transfer through the walls. The compressor discharges the air at 1 bar and is operated isothermally at 300 K. What is the total work done by the compressor? Assume that the compressor operates reversibly and that air is an ideal gas.

The diagram shows a cylindrical tank on the left labeled '1m³ TANK' with 'INITIAL T₀ = 300 K' and 'P₀ = 1 Bar' written below it. A pipe connects the tank to a compressor on the right labeled 'ISOTHERMAL COMPRESSOR'. The compressor has a vertical discharge pipe labeled '1 Bar'. A red circle highlights the compressor.

So now let us look at some examples to make use of our understanding. So, this is the first question. This is 1 meter cube tank that initially contains air at 1 bar and 300 Kelvin is to be evacuated by pumping out the content. The tank contents are maintained at 300 K throughout the operation by heat transfer though the walls. So, it is maintained at 300 Kelvin.

The only thing it loses out is basically the air. The compressor discharges the air at 1 bar and is operated isothermally at 300 Kelvin. So, this is 1 bar but this is operating at 300 Kelvin. So, what is the total work done by the compressor when you take the whole amount of the gas and pumped out from the or discharge it to the air? So, what you have to do is the other thing which you have to assume is the compressor operates reversibly and air is an ideal gas.

So this is very gross approximation but this will ease out the problem to solve. So we will just simply jump with the fact that this is reversible process and essentially for this, the work associated with this reversible process is nothing but integral of V d P.

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$$\begin{aligned} \delta W_s &= \delta n \int_{P_{\text{tank}} = P_t}^{P_{\text{discharge}} = P_d} v dp = \delta n \int \frac{RT}{P} dp \\ &= \delta n RT \ln \frac{P_d}{P_t} \\ &= \frac{V_t}{RT} RT \ln \frac{P_d}{P_t} \\ \delta W_s &= \frac{V_t}{RT} RT \ln \frac{P_d}{P_t} dP_t \\ W_s &= V_t \left[ P_t \ln \frac{P_t}{P_d} - P_t \right]_1^0 = V_t \\ &= 10^5 \text{ J} \end{aligned}$$

So, let us consider that this is the differential work to move a pump a small amount of air delta N from the pressure, tank pressure to discharge pressure which is nothing but 1 bar and this is V d P.

$$\delta W_s = \delta n \int_{P_{\text{tank}}}^{P_{\text{discharge}}} V dP = \delta n \int \frac{RT}{P} dp = \delta n RT \ln \frac{P_d}{P_t}$$

$$\delta n = \frac{V_t}{RT} dp_t$$

$$\delta W_s = V_t \ln \frac{P_d}{P_t} dP_t$$

$$W_s = V_t \left[ P_t \ln \frac{P_t}{P_d} - P_t \right]_1^0 = V_t = 10^5 \text{ J}$$

So, this is the reversible work done by the compressor which is nothing but 10 to the power 5 Joules from the values which is here.

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**Entropy change of an ideal gas**

10 mol of an ideal gas with  $C_v=20.8 \text{ J/mol}$  at  $T_0=300 \text{ K}$  and  $P_0=0.3 \text{ Mpa}$  occupy the left of an insulated vessel. The other half is evacuated.

At time  $t=0$ , a 1 kW electrical heating element is turned on. After 30s, the partition dividing the vessel ruptures and the heating element is turned off. Calculate :

a) The final temperature and pressure of the gas in the vessel and  
 b) the entropy change of the gas during this process.

$$\Delta U = \overset{0}{Q} + W = NC_v \Delta T = NC_v (T_f - T_0)$$

$$T_f = T_0 + \frac{W}{NC_v} = 444 \text{ K}$$

So, let us now try to work on example related to entropy change in ideal gas. So, this is the question, the 10 moles of an ideal gas with heat capacity at temperature 300 Kelvin and pressure point 3 mega Pascal occupy the left of an insulated vessel. The other half is evacuated. So you can consider something like this, that you have insulated vessel, and this is separated by a partition and now this contains the ideal gas, okay with that 300 Kelvin and .3 mega Pascal. So this is your partition. Now at time  $t$  equal to 0, 1 kilo Watt electrical heat element is turned on.

So, you can consider that you have a heating element where it is turned on at  $t$  equal to 0 to let us say 30 seconds which is being set here. The particle dividing the vessel ruptures at, after 30 seconds and the heating element is turned off. Now we need to find out the basically the final temperature and pressure of the gas in the vessel and so forth. So we will assume that this partition does not as such volume, it is a 0 volume and hence when it ruptures basically the ideal gas occupies the whole volume. So now let us try to solve this problem.

One of the things which we can do is we can first find out First Law of Thermodynamics over this system here so:  $\Delta U = Q + W = NC_v \Delta T = NC_v (T_f - T_0)$

$$T_f = T_0 + \frac{W}{NC_v}$$

$W$  in this case would be your 1 kilo Watt multiplied by 1 kilo Watt is 1000 Joules per second multiplied by 30 seconds. So, this is the work, electrical work which is done on the system. System is defined by this here so we can consider this to be system. So, what will be the delta

T here? Delta T is  $T_f$  final minus  $T_0$ , right so I can obtain the  $T_f$  would be your  $T_0$  plus  $W$   
 $N C_V$ , and what about  $N$ ?  $N$  is, we know number of moles are 10 moles.

$C_V$  we know are 20.8 Joules per mole and  $W$  of course we know.  $T$  initial is given as 300 Kelvin. So when we plug in these values,  $N$  is 10 moles,  $C_V$  is 20.8, the value comes out to be 444 Kelvin. So that is the final temperature of the gas in the system. So now let us find out what is the pressure of the gas in the final state.

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Final pressure,  $N = \text{constant}$   $= \frac{PV}{RT}$   
 $N_f = N_o$

$$\frac{P_f V_f}{RT_f} = \frac{P_o V_o}{RT_o}$$

$$\Rightarrow P_f = P_o \frac{T_f}{T_o} \left( \frac{V_o}{V_f} \right)$$

So to obtain the pressure, that means what is the final pressure, we have, we can consider that since the number of machines.

$$N = \text{constant} = \frac{PV}{RT}$$

$$N_f = N_o$$

$$\frac{P_f V_f}{RT_f} = \frac{P_o V_o}{RT_o}$$

$$P_f = \frac{P_o T_f V_o}{T_o V_f}$$

So, let us look at again this.

(Refer Slide Time: 21:12)

**Entropy change of an ideal gas**

10 mol of an ideal gas with  $C_v = 20.8 \text{ J/mol}$  at  $T_o = 300 \text{ K}$  and  $P_o = 0.3 \text{ Mpa}$  occupy the left of an insulated vessel. The other half is evacuated.

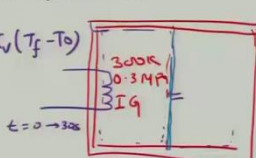
At time  $t=0$ , a 1 kW electrical heating element is turned on. After 30s, the partition dividing the vessel ruptures and the heating element is turned off. Calculate :

- The final temperature and pressure of the gas in the vessel and
- the entropy change of the gas during this process.

$\Delta U = \overset{0}{Q} + W = N C_v \Delta T = N C_v (T_f - T_o)$

$\downarrow$   
 $1000 \text{ J/s} \times 30 \text{ s}$

$T_f = T_o + \frac{W}{N C_v} = 444 \text{ K}$





So essentially what I am going to consider is that initial volume which is occupied by the gas is actually half of the final volume. So essentially my system is not going to this but rather this, so what I am going to do is I am going to erase this part, because essentially I am going to consider this which expands so actually I am looking at the volume initial which is this half part, the first compartment and then the final one is the first and second compartment.

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Final pressure,  $N = \text{const}$   
 $N_f = N_0 = \frac{P_f V_f}{RT_f} = \frac{P_0 V_0}{RT_0}$

$$\frac{P_f V_f}{RT_f} = \frac{P_0 V_0}{RT_0}$$

$$\Rightarrow P_f = P_0 \frac{T_f}{T_0} \left( \frac{V_0}{V_f} \right)^{\frac{1}{2}}$$

$$= 0.222 \text{ MPa}$$

$\Delta S = \int \frac{\delta Q}{T}$  not a reversible

So that is the system which basically has a non-rigid boundary and it can expand and if that is the case we can consider this to be 1 by 2, and  $T_f$  we know,  $T_0$  we know,  $P_0$  we know, so if you plug in these values which are available from the question it comes out to be 2.2 mega Pascal. This is something which you can try and try to solve this problem at home.

Now second part of the question is the entropy change of the gas. We can try to think that why can not we use this simply  $\Delta S = \int \frac{\delta Q}{T}$ ? Well we cannot because this process we know, this is not a reversible process. So how do you solve and obtain  $\Delta S$ ?

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To obtain entropy change

$$dU = Tds - PdV$$

$$ds = \frac{NC_V dT}{T} + \left(\frac{P}{T}\right) dV \rightarrow \frac{NR}{V}$$

$$ds = \frac{NC_V dT}{T} + \frac{NR}{V} dV$$

$$\int \frac{ds}{N} = \frac{\Delta S}{N} = C_V \int \frac{dT}{T} + R \int \frac{dV}{V}$$

$$\frac{\Delta S}{N} = C_V \ln \frac{T_f}{T_0} + R \ln \frac{V_f}{V_0}$$

$$= 13.9 \text{ J/mol}\cdot\text{K}$$

$$\Delta S = 10 \times 13.9 \text{ J/K} > 0$$

( $\Delta S/N = C_p \ln T_f/T_0 - R \ln P_f/P_0$ ) process is irreversible

So, obtain entropy change we will revert back to the thermodynamic equation, the fundamental equation so that is going to be:

$$dU = TdS - PdV$$

$$dS = \frac{NC_V}{T} dT + \frac{P}{T} dV$$

$$dS = \frac{NC_V dT}{T} + \frac{NR}{V} dV$$

$$\int \frac{dS}{N} = \frac{\Delta S}{N} = C_V \int \frac{dT}{T} + R \int \frac{dV}{V}$$

$$\frac{\Delta S}{N} = C_V \ln \frac{T_f}{T_0} + R \ln \frac{V_f}{V_0}$$

So, when we plug in this value this comes out to be 13.9 Joules per mole Kelvin.

So, we can multiply with the number of moles to get the total change in entropy that is going to be 10 into 13.9 Joules per Kelvin and this is certainly greater than 0. This is also indicative of that the system or the process is irreversible. So that is the part which to solve this particular problem. Now one can also come with an expression of delta S with P f and V f also. So that is something you can also come up with. I can just write it down for you.

$$\frac{\Delta S}{N} = C_p \ln \frac{T_f}{T_0} - R \ln \frac{P_f}{P_0}$$

This is another expression you can evaluate for your delta S of ideal gas. So, in that case if you have the pressure information you will be using pressure information. The one which you have used is the volume information.

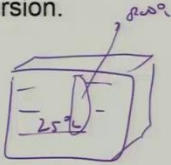
Both yield the same expression because it is a state property and hence the final and initial states are the same for a given process then this expression  $\Delta S$  whether you use this expression or the other one both of them should yield the same value.

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**Example 2**

Consider a common problem in Forging of immersing hot steel in water for hardening. Consider a process in which a 1 kg steel blade at  $T_1=800^\circ\text{C}$  is immersed in a large vessel filled with water at  $T_2=25^\circ\text{C}$  and immediately quenched. Assuming that steel has heat capacity  $C_v=460\text{ J}/(\text{kg K})$ , and independent of temperature, calculate the entropy changes during this process of a) The steel blade, b) the water, and c) the universe. Assume that there is enough water in the vessel so that the water temperature doesn't change upon immersion.

*Ignore the change in vol. of the steel.*



Okay so now I can move to the other problem. This is the problem which is related to the forging. Consider a common problem of forging, of immersing hot steel in water for hardening. Consider a process in which 1 kg steel blade at  $T=800$  degree Celsius is immersed in a large vessel filled with water at 25 degree Celsius and immediately quenched. So, you have a large vessel and the steel blade at very high temperature.

So large vessel is there maybe, so essentially large vessel and then this is filled with water at 25 degree Celsius and then you have steel blade which could be considered like this and this is at 800 degree Celsius and immediately it quenched at  $T$  equal to 0 basically you put into this water and it will get quenched. So the question is what is the change in the entropy during the process for the steel blade and the water and the universe.

Consider that the steel has heat capacity of 460 Joules per K g Kelvin and it is independent of temperature. That is one thing which we can consider. Also assume that there is enough water in the vessel so that the water temperature does not change upon immersion. So this remains at 25 degree Celsius. So what we saw in this problem? So one of the things we can do is we can ignore the change in volume of the steel. So, this is something which you can consider for metals for this temperature range.

(Refer Slide Time: 28:05)

For steel blade  
 $dU = Tds - PdV \Rightarrow ds = \frac{1}{T} dU$   
 $= \frac{1}{T} Nc_v dT$

$$\Delta S_{\text{blade}} = \frac{Nc_v \ln \frac{T_f}{T_0}}$$

Let us look at the steel blade and apply this First Law. So for steel blade:

$$dU = TdS - PdV$$

$$dS = \frac{1}{T} dU = \frac{1}{T} Nc_v dT$$

$$\Delta S_{\text{blade}} = Nc_v \ln \frac{T_f}{T_0}$$

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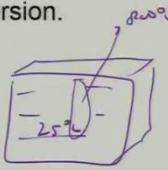
**Example 2**

Consider a common problem in Forging of immersing hot steel in water for hardening. Consider a process in which a 1 kg steel blade at  $T_1=800^\circ\text{C}$  is immersed in a large vessel filled with water at  $T_2=25^\circ\text{C}$  and immediately quenched. Assuming that steel has heat capacity  $C_v=460\text{ J/(kg K)}$ , and independent of temperature, calculate the entropy changes during this process of

a) The steel blade, b) the water, and c) the universe.

Assume that there is enough water in the vessel so that the water temperature doesn't change upon immersion.

Ignore the change in vol. of the steel.



So essentially if you look at, this is in Joules per k g.

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### Entropy change of an ideal gas

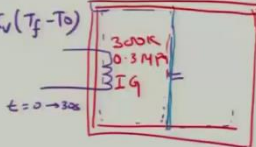
10 mol of an ideal gas with  $C_v=20.8 \text{ J/mol}$  at  $T_0=300 \text{ K}$  and  $P_0=0.3 \text{ Mpa}$  occupy the left of an insulated vessel. The other half is evacuated.

At time  $t=0$ , a 1 kW electrical heating element is turned on. After 30s, the partition dividing the vessel ruptures and the heating element is turned off. Calculate :

- The final temperature and pressure of the gas in the vessel and
- the entropy change of the gas during this process.

$$\Delta U = \cancel{Q} + W = N C_v \Delta T = N C_v (T_f - T_0)$$

$\downarrow$   
 $1000 \text{ J/s} \times 30 \text{ s}$

$$T_f = T_0 + \frac{W}{N C_v} = 444 \text{ K}$$


So this is different from this, which was in moles, remember that so unit does make a lot of difference. So one has to look at the units of the heat capacity in order to appropriately use this N here.

(Refer Slide Time: 29:18)

For steel blade

$$dU = T ds - P dV \Rightarrow ds = \frac{1}{T} dU$$

$$= \frac{1}{T} N c_v dT$$

$$\Delta S_{\text{blade}} = \frac{N c_v \ln \frac{T_f}{T_0}}{1}$$

$$= 1 \text{ kg} \cdot 460 \ln \frac{273+25}{273+800} = -589 \text{ J/K}$$

So N in this case would be in kg, so this would be like say 1 kg, that is what the given to us and  $C_v$  is 460 and then I put the value of  $T_f$ .  $T_0$  is 800 degrees Celsius for steel blade and eventually at equilibrium this is going to be 25 degree Celsius, okay. So this turns out to be minus 589 Joules per Kelvin. Now you might think about why it is negative, because again the blade is the subpart of the overall process.

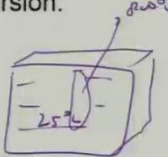
But even for irreversible process you can have a  $\Delta S$  of subsystem can be negative. That is something which is evident from this, even if it is irreversible process. Now let us try to get more information. The second part was for the water.

(Refer Slide Time: 30:12)

**Example 2**

Consider a common problem in Forging of immersing hot steel in water for hardening. Consider a process in which a 1 kg steel blade at  $T_1=800^\circ\text{C}$  is immersed in a large vessel filled with water at  $T_2=25^\circ\text{C}$  and immediately quenched. Assuming that steel has heat capacity  $C_v=460\text{ J/(kg K)}$ , and independent of temperature, calculate the entropy changes during this process of a) The steel blade, b) the water, and c) the universe. Assume that there is enough water in the vessel so that the water temperature doesn't change upon immersion.

*Ignore the change in vol. of the steel.*



What is the delta S for the water?

(Refer Slide Time: 30:16)

For steel blade

$$dU = T ds - P dV \Rightarrow ds = \frac{1}{T} dU = \frac{1}{T} N c_v dT$$
$$\Delta S_{\text{blade}} = \frac{N c_v \ln \frac{T_f}{T_0}}{1} = 1 \text{ kg} \cdot 460 \ln \frac{273+25}{273+800} = -509 \text{ J/K}$$

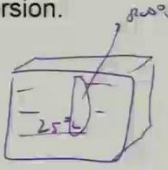
Now this is a rather tricky part. So in order to obtain this we need to have some information for the water.

(Refer Slide Time: 30:23)

**Example 2**

Consider a common problem in Forging of immersing hot steel in water for hardening. Consider a process in which a 1 kg steel blade at  $T_1=800^\circ\text{C}$  is immersed in a large vessel filled with water at  $T_2=25^\circ\text{C}$  and immediately quenched. Assuming that steel has heat capacity  $C_v=460\text{ J/(kg K)}$ , and independent of temperature, calculate the entropy changes during this process of a) The steel blade, b) the water, and c) the universe. Assume that there is enough water in the vessel so that the water temperature doesn't change upon immersion.

*Ignore the change in vol. of the steel.*



But of course in the problem, nothing is given as far as the water is concerned besides only the temperature.

(Refer Slide Time: 30:30)

For steel blade

$$dU = T ds - p dV \Rightarrow ds = \frac{1}{T} dU = \frac{1}{T} N c_v dT$$
$$\Delta S_{\text{blade}} = \frac{N c_v \ln \frac{T_f}{T_0}}{1} = 1 \text{ kg} \cdot 460 \ln \frac{273+25}{273+800} = -509 \text{ J/K}$$
$$ds = \int \frac{dQ_{\text{rev}}}{T}$$

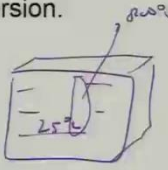
Then we cannot use also directly simply this  $\Delta Q$  reversible by  $T$ , for that we need to find out this but the process is irreversible. So, what we should do that is the important question. So how do we do that? Now one can think of, because the temperature difference here for the water, so this is your water here.

(Refer Slide Time: 30:52)

**Example 2**

Consider a common problem in Forging of immersing hot steel in water for hardening. Consider a process in which a 1 kg steel blade at  $T_1=800^\circ\text{C}$  is immersed in a large vessel filled with water at  $T_2=25^\circ\text{C}$  and immediately quenched. Assuming that steel has heat capacity  $C_v=460\text{ J/(kg K)}$ , and independent of temperature, calculate the entropy changes during this process of a) The steel blade, b) the water, and c) the universe. Assume that there is enough water in the vessel so that the water temperature doesn't change upon immersion.

*Ignore the change in vol. of the steel.*



So the temperature difference here around this and the water is extremely large. So  $\Delta T$  is quite large right. So what you can think is a kind of a thought process where basically we can devise a thought process in which  $Q$  transfer to the water is done reversibly. So eventually we know what is the amount of the heat which is transferred from the steel to water because essentially, we know that the change in the  $\Delta U$  of the blade and since there is no work associated with it. So essentially the  $\Delta Q$  is nothing but  $\Delta U$ .

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For steel blade

$$dU = T ds - P dV \Rightarrow ds = \frac{1}{T} dU = \frac{1}{T} N c_v dT$$
$$\Delta S_{\text{blade}} = \frac{N c_v \ln \frac{T_f}{T_0}}{T}$$
$$= 1 \text{ kg} \cdot 460 \ln \frac{273+25}{273+800} = -509 \text{ J/K}$$
$$ds = \int \frac{dQ_{\text{rev}}}{T}$$
$$Q = -\Delta U_{\text{blade}} = -N c_v \Delta T = 356 \text{ kJ}$$

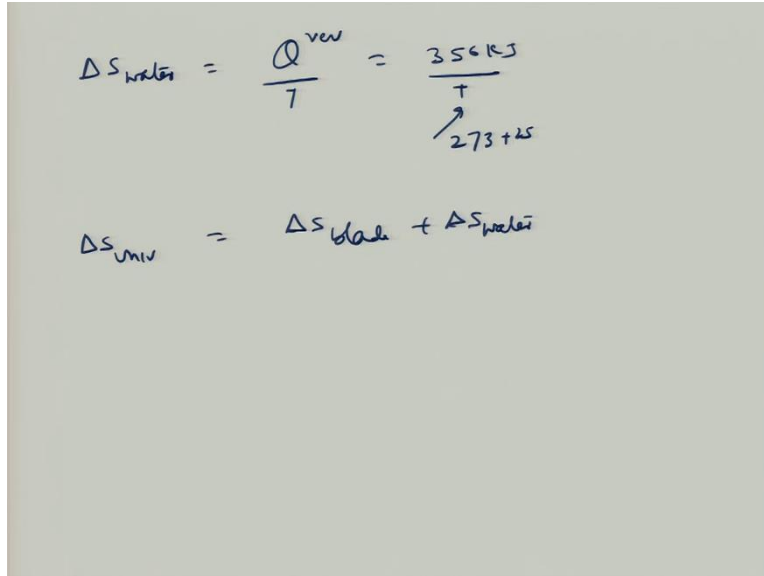
Transfer to water is done in a reversible manner

So  $Q$  is nothing but minus of  $\Delta U$  blade. So this we know, so and then we can plug in this value to get the value of the 356 kilo Joules. So this is the amount of the heat which is transferred from the blade to the water. Now this is done of course irreversibly but what you can do is now to calculate the entropy change of the water we can come up with the way to



find out a thought experiment where we can device this amount is transferred in a reversible way. So in that case we can write, so this is something which is a thought experiment and this case Q transfer to water is done in a reversible manner.

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The image shows a slide with handwritten mathematical equations. The first equation is  $\Delta S_{\text{water}} = \frac{Q^{\text{rev}}}{T} = \frac{356 \text{ kJ}}{273 + 25}$ . The second equation is  $\Delta S_{\text{univ}} = \Delta S_{\text{blade}} + \Delta S_{\text{water}}$ .

So with this we can find out the delta S for the water, that is going to be simply the Q reversible by T and that is what we can find it out. So this is one way of doing it. This T is 273 plus 25, and then we can find out the delta S of the universe which is delta S of blade plus delta S of water. But, yeah so this is one thought experiment based delta S calculation, because we are not being given any information.

$$\Delta S_{\text{water}} = \frac{Q^{\text{rev}}}{T}$$

$$\Delta S_{\text{univ}} = \Delta S_{\text{blade}} + \Delta S_{\text{water}}$$

(Refer Slide Time: 33:02)

For steel blade  
 $dU = T ds - P dv \Rightarrow ds = \frac{1}{T} dU$   
 $= \frac{1}{T} N C_v dT$

$$\Delta S_{\text{blade}} = \frac{N C_v \ln \frac{T_f}{T_0}}{1}$$
$$= 1 \text{ kg} \cdot 460 \ln \frac{273+25}{273+800} = -509 \text{ J/K}$$
$$ds = \int \frac{dQ^{\text{rev}}}{T}$$
$$Q = -\Delta U_{\text{blade}} = -N C_v \Delta T$$
$$= 356 \text{ kJ}$$

Work done on water is done in a reversible manner

Otherwise if you consider this there is no change in the volume and there is no, you know, if the other properties are not being affected then one can also find out using the First Law Analysis also one can try to do that but I am trying to avoid that part in this problem. But it is something you can try different ways to solve such a problem.

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$$\Delta S_{\text{water}} = \frac{Q^{\text{rev}}}{T} = \frac{356 \text{ kJ}}{273+25}$$
$$\Delta S_{\text{univ}} = \Delta S_{\text{blade}} + \Delta S_{\text{water}}$$

Okay so this is one way of approaching it. Okay so with this I think we will stop and then in the next day class which will be the last lecture of this Engineering Thermodynamic aspects in the particular course. I will be covering Power Cycles. So, I will see you in the next class.