Chemical Engineering Thermodynamics
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Carnot cycle and thermodynamic temperature

Welcome back, In this today's, lecture we are going to cover Carnot Cycle and thermodynamic temperature.

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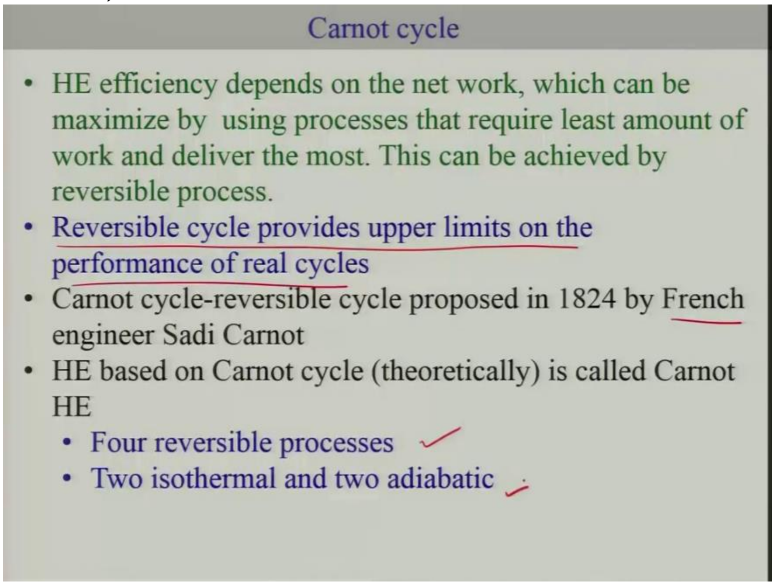

We have already discussed about heat engine and the definition of efficiency, right. It's cyclic device. So, it is very clear from the last discussion that heat engine or heat engine efficiency depends on the net work, and of course you have to throw out certain amount of heat. So what matters is both the thing, okay, how can you minimize the loss as well as how can you maximize the work?

So mainly the focus is to find out a way to enhance or maximize the net work out of the heat engine. So, saying heat engine efficiency depends on the work, net work done and hence the interest is to find out way to maximize the net work. So that can, that means that you would like to consider a case where basically the all other losses are minimized or rather 0. So, one of the easiest ways to make the processes involved in the cycle is reversible so that means that reversible cycle provides upper limit on the performance of the real cycle.

Now this is something which was realized by Carnot, and Carnot cycle is nothing but a reversible cycle which was first proposed by French engineer Sadi Carnot. Now in the case of the Carnot cycle which is of course theoretical, it cannot be realized because of the fact that is all the processes involved is reversible, it has 4 reversible processes and the 2 of them are isothermal and the 2 of them are adiabatic.

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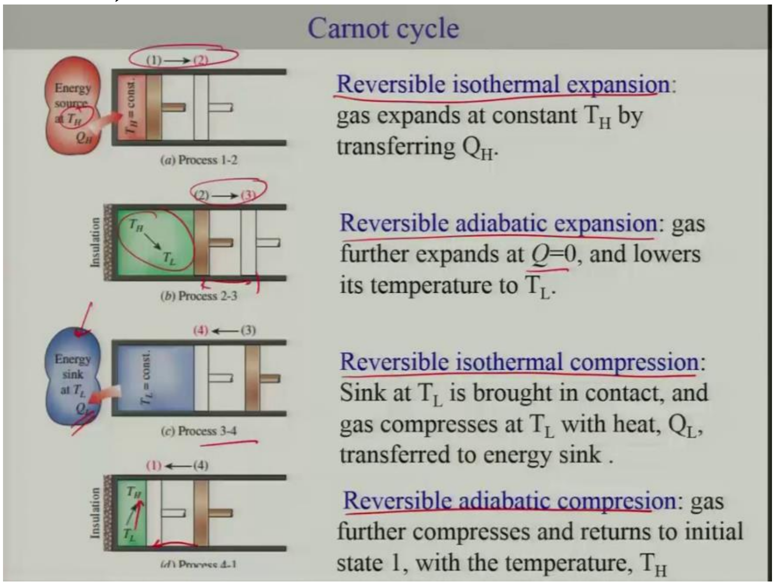

So, I will just describe very quickly this Carnot cycle processes, so using the piston cylinder concept. So, in the case of, or the first process is nothing but reversible isothermal expansion. So, if you consider this piston and cylinder, energy is being provided a constant T H that will be Q H and at a constant T H which is isothermal process it expands.

Now the second process is a reversible adiabatic expansion. Once the T H, once the piston or cylinder has been expanded then essentially it further expands from here to here, from here to here, but in the process of expansion it loses the temperature and allows the process adiabatic which means basically what we are doing is we are insulating the system here. So that is why it is a reversible adiabatic process. Gas was expanded at Q equal to 0 and lowers the temperature to T L.

The third is trying to bring back, so, this has been expanded now. We are trying to compress it because we would like to get back to the original state. In the process what we do is we use a reversible isothermal compression where it is being compressed but at a constant temperature T L, okay and it is being supplied as heat Q L at a sink T L. Of course, by considering this, you can imagine that the differential temperature is negligible and hence it is very slow process and that is why it is reversible.

So, this is your third process, process 3 to 4, and the last process, which enhances the temperature is reversible adiabatic compression. So, you have insulation and it further compresses in the process from here, it enhances the temperature, brings back to the state conditions of initial T H. So, this is reversible adiabatic compression where gas returns to the original state with temperature T H.

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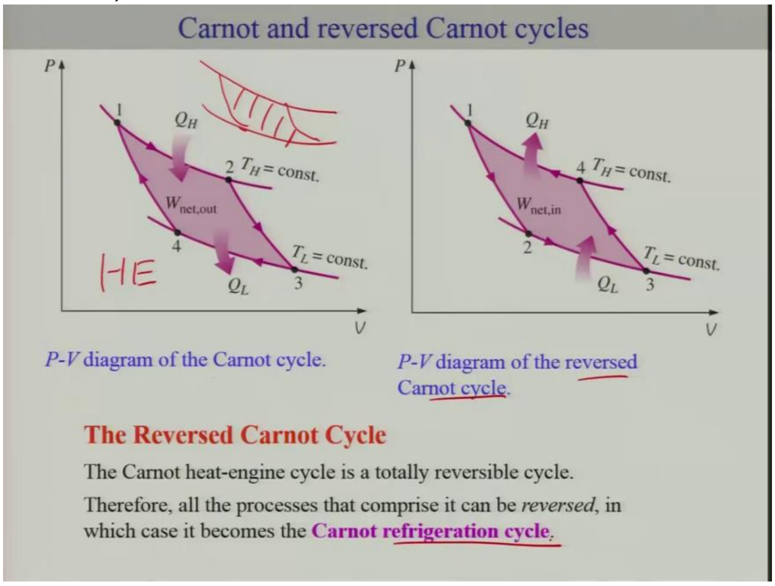

So, if you consider a P V diagram, the first one was isothermal expansion, where the heat was supplied. Then you have at a constant T H and you have adiabatic compression and then subsequently you have this isothermal expansion, adiabatic expansion and then you have this isothermal compression followed by adiabatic compression. So, it follows these two lines.

So, like this, if you consider P V diagram for gas, in this case we will be considering, let us say ideal gas, so essentially you have two lines which is nothing but isothermal lines and then you are connecting with these adiabatic lines. That is why you have this work effectively which is within this enclosed vision. So that is the case for heat engine. If you reverse this cycle direction then you are going to get reverse Carnot Cycle which is nothing but refrigeration cycle. So, this is something which you must have seen it earlier bit.

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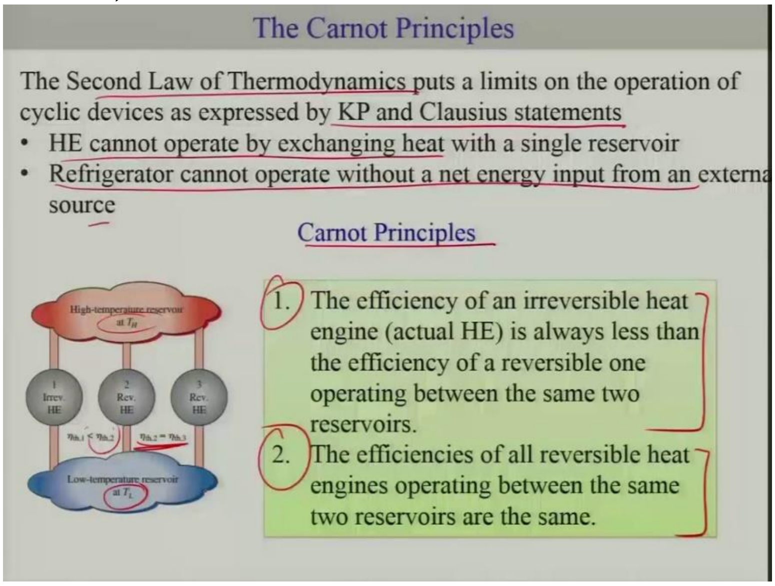

Now the Carnot cycle can be used to understand many things. So, one of the things which Carnot came up with is the Carnot principle. It is nothing but the same kind of definition which we have used or the principles or the laws which we have considered earlier in the more formal ways.

So, the Carnot principle says that, you know as we have already seen earlier, that the efficiency of an irreversible heat engine is always less than the efficiency of the reversible one operating between same two reservoir and efficiency of all reversible heat engine operating between the same two reservoirs are the same. So, these are the two important aspects of the Carnot principle.

So earlier, we also discussed about the Second Law of Thermodynamics putting a limit on the operational device which is usually expressed in terms of the Kelvin Planck and Clausius statement which is nothing but that the heat engine cannot be operated by just exchanging heat with a single reservoir and similarly refrigerator cannot operate without net energy input from an external source. So, these are the two principles and that is what Carnot considered in providing these two statements here.

So, it tells you from a simple exercise here that if you consider this high temperature reservoir at temperature T H and you have a low temperature reservoir T L and we have considered 1, 2 and 3 heat engines. 1 is irreversible, and 2 is reversible. So of course, the efficiency of reversible is going to be more as the losses are less. So, your net output, work output is going to be more and hence the efficiency is more.

But what it also tells you the following that irrespective of this, when you consider the reversible heat engine, particularly this, considering the Carnot cycle, you are going to get the same efficiency irrespective of how you develop. So, if it is a reversible heat engine and the sources are same that means the high temperature and the low temperature sources are same, the efficiencies are going to be same.

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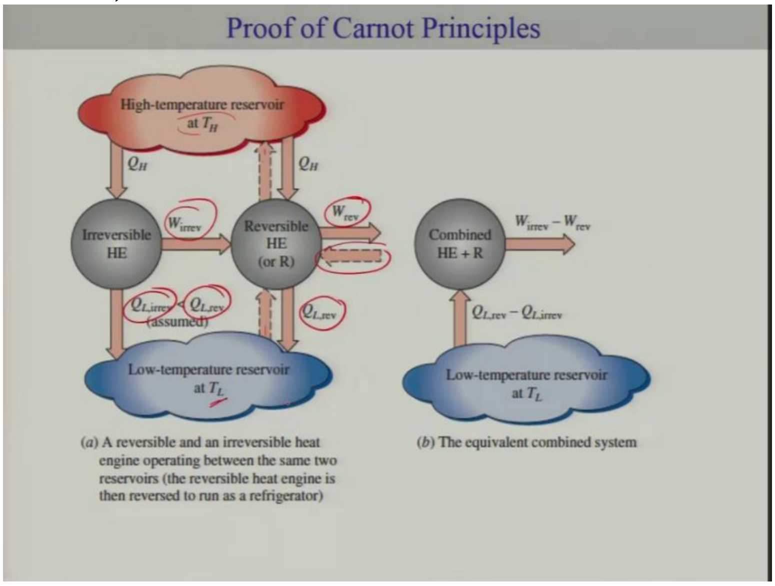

You can consider this statement and you come up with a kind of proof Carnot's principles which is very similar to what we have talked about, the Kelvin Planck and Clausius. So, let us redo this exercise again. You have this temperature, high temperature reservoir at T H. This is the heat supplied to the irreversible and this is the Q L which is wasted as given to the low temperature reservoir T L and this is the work which is of course irreversible.

So, it means it is going to be, because of the fact that it is irreversible heat engine, hence it is irreversible. It is supposed to be less than the reversible. But let us consider the case that you have assumed that Q L is going to be more than Q L irreversible. If you assume that, then we can come with the ways to combine them. But let me also first define this. So, this is a reversible heat engine.

Or we can consider R, refrigerator. R is the one, the directions are considered this. The solid directions are in the case of the heat engine. So, Q H is taken and Q L, reversible is thrown and you generate, and this reversible heat engine generates the W reversible. Now for the case of the refrigerator the directions are going to be opposite. So, this is assumed that Q L reversible is more than Q L irreversible for the same operation here.

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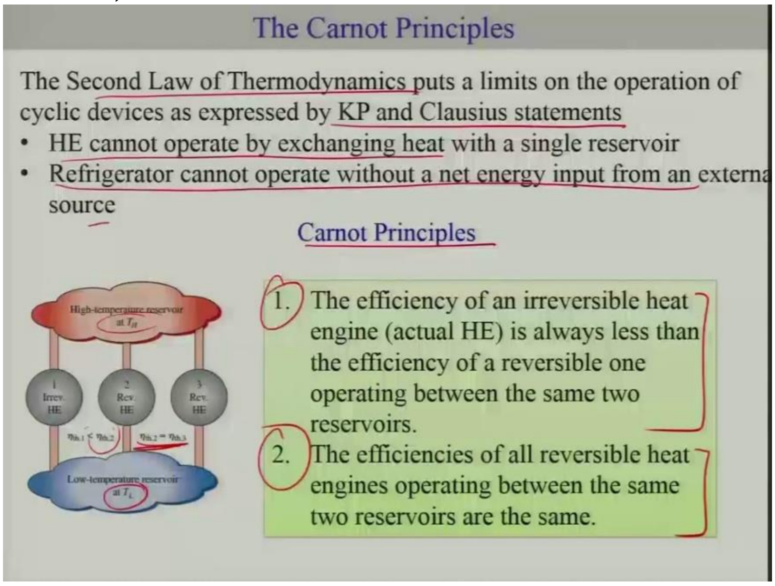

And of course, we have to say that efficiency of that is going to be more for the reversible compared to irreversible. So essentially, what we are saying is this is not true.

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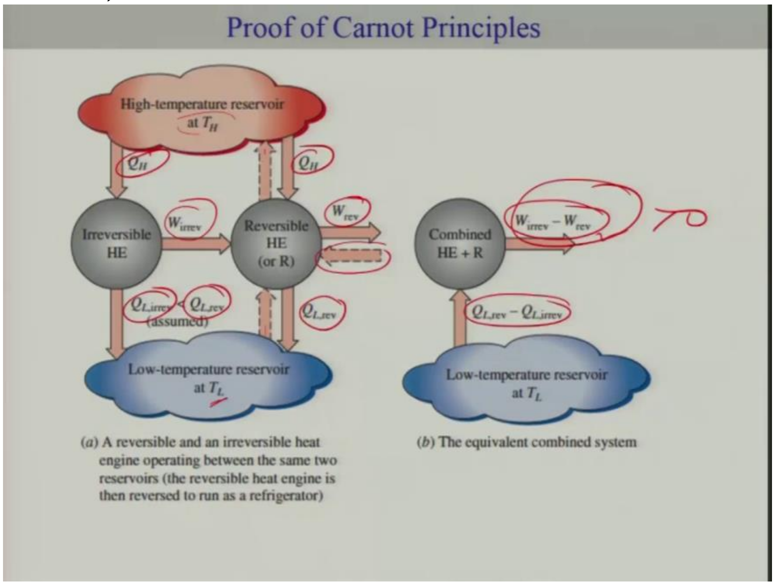

If this is not true what we can do? So, let us say we consider the, combine the irreversible heat engine and the refrigerator and if we combine, we are going to get this because these have the same source. So, we are going to get this and we are going to get this also, Q L reversible minus

Q L irreversible. So, what it tells you that it is basically violating the Clausius-Clapeyron or Clausius statement which says that you cannot extract the heat and generate the heat from the low temperature reservoir and generate the work. So, this is what it says.

So, this is impossible. You cannot have this also greater than 0 and this also greater than 0. So, this is something which tells you that this is not violated which means that this W irreversible cannot be more than W reversible and hence our earlier assumption is wrong. So, this is one way of proving these principles but let me just now take this idea and come up with the thermodynamic temperature based on the Carnot cycle.

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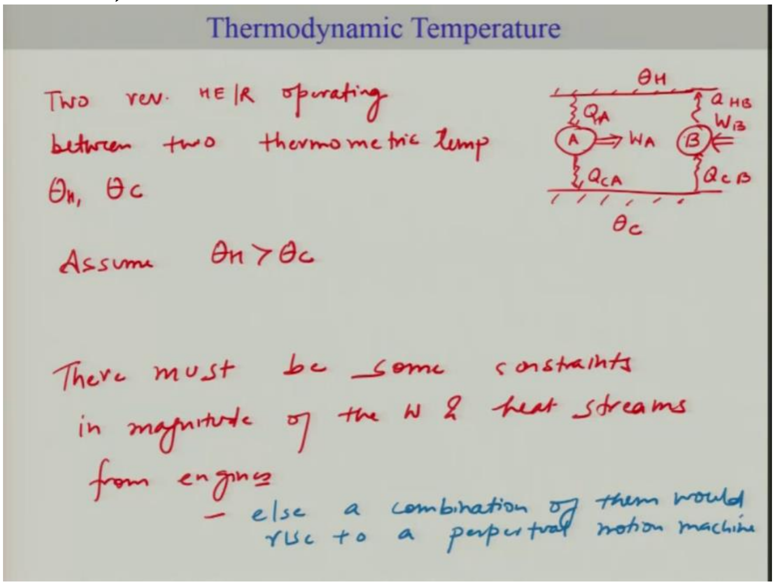

So, let me start with 2 heat engines here which basically uses the reservoirs, so for the sake of differentiating with the other examples, I am going to make use of theta as a temperature of the high temperature reservoir. So, let us say this is high temperature reservoir and this is your heat engine A which takes the heat Q of A and generates work A but it also throws out Q C A to the low temperature reservoir which temperature is theta C.

So, we are going to use theta in the form of temperature later on but for the sake of usual textbook which we use, you will find similar kind of notation but soon to distinguish from usual temperature notion that this is nothing but some kind of a scale from where we are going to get this value and we are coming with a some kind of a scale using this Carnot cycle based analysis. So, this is the second reservoir, second heat engine and, in this case what we are saying is that…

Well this is second cyclic device and here what we are saying is, that well we are going to consider work which is being provided. So essentially in a way this is nothing but a refrigerator and we are throwing it here Q H B so we are going to differentiate between A, this is cold, C that is what means, this is nothing but high temperature and then we are saying A corresponds to this particular device. So, this is by definition. We are going to define this.

Now, what we are saying is that these are two reversible heat engines operating between 2 thermometric temperature and that is why we have kept it as theta H and theta C. So, let us assume that theta H is greater than theta C. Now, this is of course, the heat engine which removes the heat from the reservoir and dumps some of the cold reservoir and produces some work whereas B removes the heat from the cold reservoir while consuming some work and dumps into the hot reservoir.

So, this is nothing but basically the refrigerator, okay. But still we are saying that is the heat engine. So, if you look carefully, both of them are feasible, nothing is violated. That means not the First Law and Second Law so which means that basically both the processes are feasible if you look at in this way, but they must be, so what we are saying is that well, while this process seems feasible from the perspective of First Law and Second Law, I mean nothing is violated.

But there must be some constraints, okay in the magnitude of the work and heat streams from the engine. So, what we are saying is well, I mean you can look at this and say well, this is fine because this is what we have learnt. But the question is that, if that is the case then I can come with the combination of both which can lead to perpetual motion and this is something which is important.

So, that means there must be some constraints in terms of Q, in terms of W okay and that is something which we are going to now first analyze. So, this statement means that, just to complete here, else a combination of them would give rise to a perpetual motion machine, of course, which will violate Second Law.

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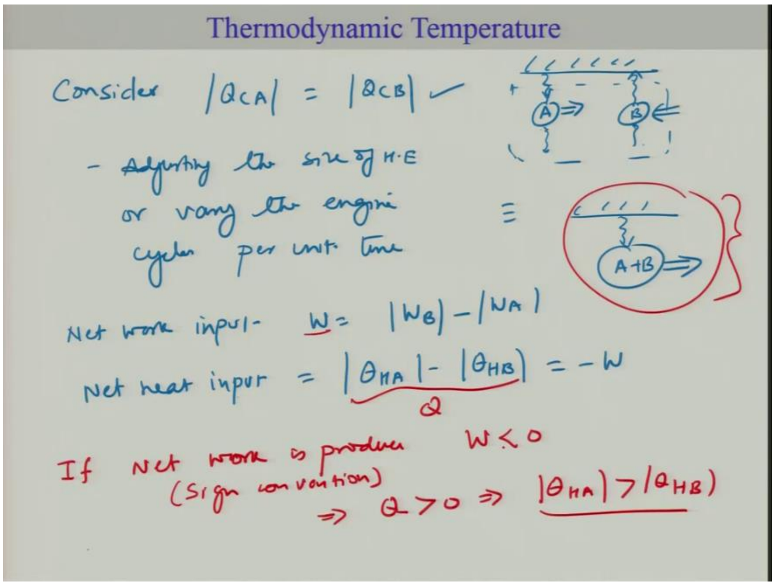

So, let us first understand, the statement we can understand by considering this following, the same thing. So, you can consider that if, consider that both the heat are same, right. This and this are same. So, considering the directions we can take the absolute value. So if they are same then I can actually come up with the combination of this, so I can come up with the combination by just considering this and this is equivalent to saying that, well okay because this would be same so essentially I can get rid of that and what we have is this.

$$|Q_{CA}| = |Q_{CB}|$$

This which is nothing but it is basically violating the Kelvin Planck statement here, right. Now I can adjust the size of the devices such that the heats are similar, Q C A this can be done by just adjusting the couple of things. So, we can do that by adjusting the size of heat engine or vary the cycle per unit time right and then you may get similar kind of heat. So, essentially you can come up with a mechanism in which you can get these values same, alright. So, you can come up with this thing.

$$Net\ work\ input, W = |W_B| - |W_A|$$

$$Net\ heat\ input = |\theta_{HA}| - |\theta_{HB}| = -W$$

$$If, W < 0, Q > 0 => |\theta_{HA}| > |\theta_{HB}|$$

Now, this is something which comes out from basic analysis from this, that if you are considering the net input is W and net heat input is Q, so, if net work is produced then W has to be less than 0 which essentially means Q must be greater than 0 which essentially means this. So, this is just based on simple analysis. Now, it is very clear that this violates the Second Law. In general, it violates the Kelvin Planck statement and this is nothing but a perpetual machine.

So, whatever the heat comes in, it goes out. There is no loss in the energy and of course this violates which means this statement or this expression cannot be true, okay and similarly this cannot be true. So, W cannot be less than 0 because it violates that.

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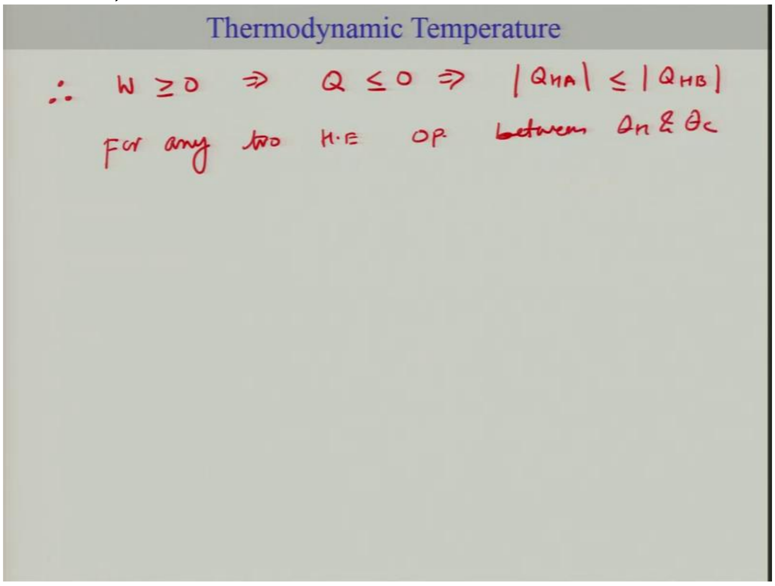

So, that leads to the statement or this means that W has to be greater than equal to 0, which means that Q has to be less than equal to 0 and this implies your Q H A should be less than equal to Q H B. This is for any two heat engines operating between theta H and theta C. So, this is what we can come up with this exercise.

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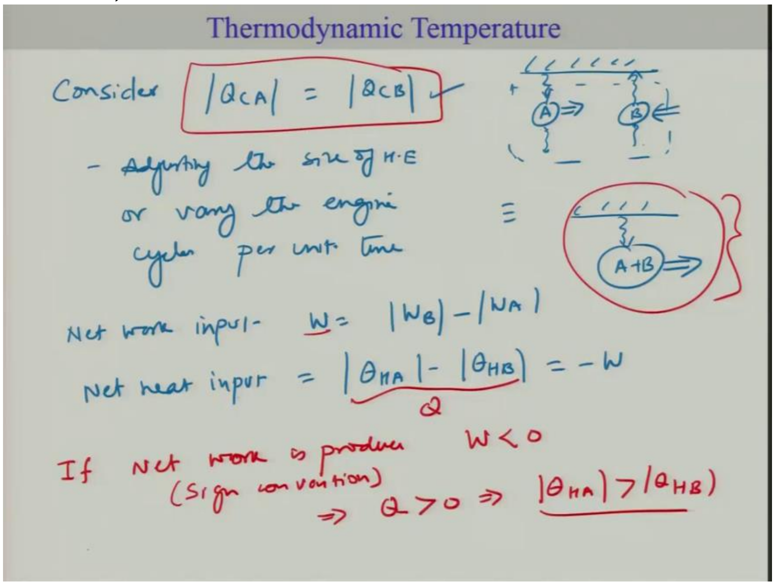

So we consider two heat engine such that one represents engine, other one represents the refrigerator but when we did the analysis we observed that if we consider this such that this is true, this will lead to that, which essentially would also mean that it is perpetual machine, which essentially also implies that the W cannot be less than 1. It cannot just produce, hence W has to be greater than equal to 0 and that is what we coming over with there.

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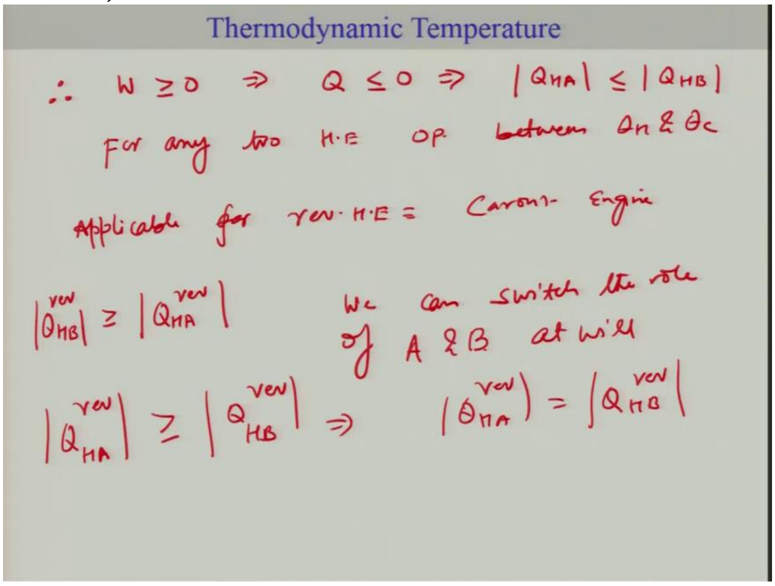

$$W \geq 0 => Q \leq 0 => |Q_{HA}| \leq |Q_{HB}|$$

$$|Q_{HB}^{rev}| \geq |Q_{HA}^{rev}|$$

$$|Q_{HA}^{rev}| \geq |Q_{HB}^{rev}| => |Q_{HA}^{rev}| = |Q_{HB}^{rev}|$$

So, this brings the constraint to these variables for any two heat engines operating between theta H and theta C, which is thermometric temperature. Now this is between any two, right. We are not saying it is reversible but of course it also depends on that, it is applicable also for reversible, for reversible heat engine also, which essentially means Carnot engine. So, for the case of this we can say Q H B, Q H A reversible and this is your Q H B so we are saying Q H B reversible is greater than equal to 0.

That is what the statement is. But being the reversible you can exchange the rows of A and B, so we can switch, because reversible engine is reversible engine. So, you can actually switch the row. It operates in the same way, switch the row of A and B basically at will. So, we can do that, if we do that, this Q H A reversible would also be greater than Q H B reversible and this implies that Q H A reversible is nothing but it is same as Q H B reversible.

So, which means clearly that for a reversible heat, essentially it does not matter whether you have a heat engine 1 or 2, both of them get the same kind of heat from the heat source, of a high temperature source, that is what it clearly points out. Now this implies, so let me just make a statement here.

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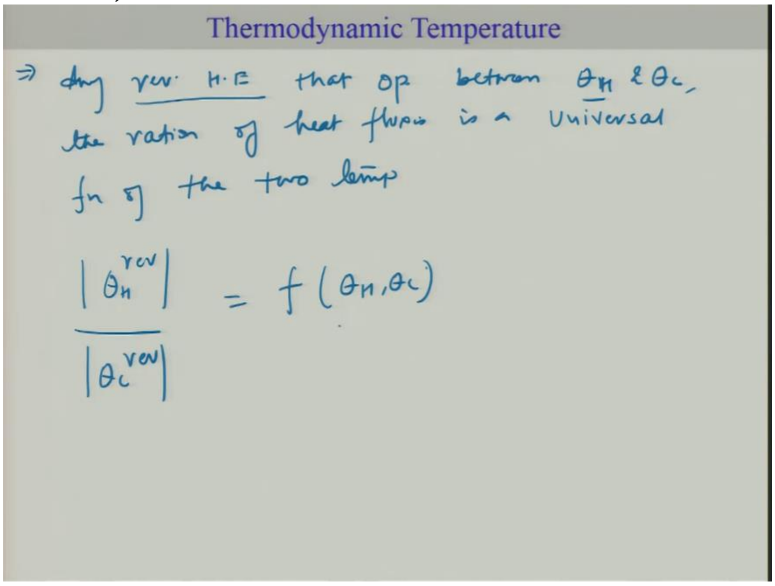

$$\frac{|\theta_H^{rev}|}{|\theta_C^{rev}|} = f(\theta_H, \theta_C)$$

So, this implies that any reversible heat engine that operates between theta H and theta C, now the ratio of heat flux is a universal function of the two temperature. So what is saying that, well it does not matter now since this has been proven, now you can clearly observe or show that essentially this any particular heat engine which operates between theta C and theta H and C, the ratio of the heat flux that means the ratio of whatever heat comes from here and here, this is nothing but the function of temperature.

Though you can show the other way round also in the same way, that for theta C and you can clearly, that depends on the temperature of the reservoir and that was the case why we are getting this so this is one statement. Now what we want to do is we want to actually determine the function.

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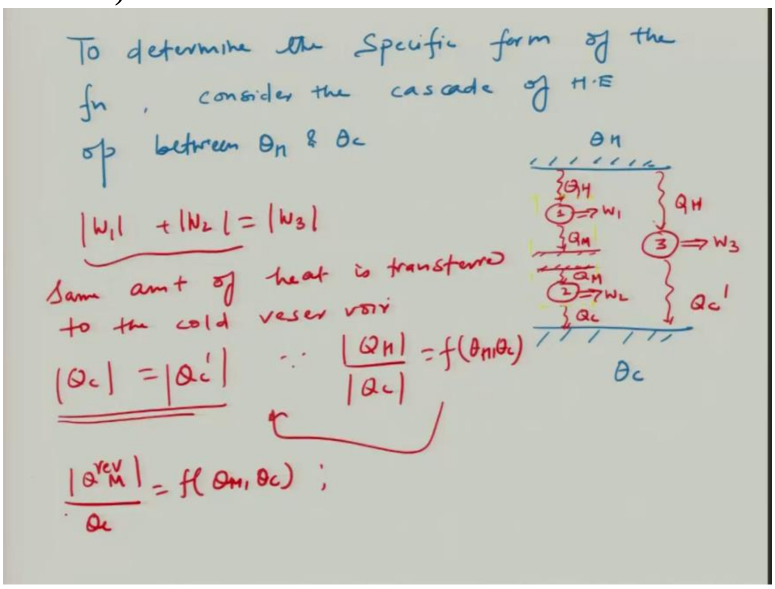

So how do you determine the function? So, to determine this function f here, specific form of the function, what we are going to do is we are going to consider a cascade, consider the cascade of the heat engine okay which operates between theta H and theta C. So, the way we can do that is the following. We can just consider this theta H, this is the thermometric temperature and this is your again theta C and now we can consider heat engine 1, this is 2 and this is your 3.

$$|W_1| + |W_2| = |W_3|$$

$$|Q_C| = |Q_C'|$$

$$\frac{|Q_H|}{|Q_C|} = f(\theta_H, \theta_C)$$

$$\frac{|Q_M^{rev}|}{\theta_C} = f(\theta_M, \theta_C)$$

So, this is 1 which takes the same heat Q H, okay. This will also take the same amount of heat flux, same Q H because it is from the same source theta H okay and it throws out certain heat first, M okay and let us say this is another thing and from here we, so here also we are going to take Q M okay and then we are going to reject Q C and this will also reject, that is to differentiate we are going to say it is Q C dash. So, these are the heat engines we have. The corresponding work are going to be W 1, W 2 and W 3, alright.

So, these are all Carnot engines, okay. These are reversible engines so W 1 plus W 2 should be equal to W 3 and since same amount of, same amount of heat should be, which essentially means that these two effectively should be same then it means that same amount of heat is transferred to the cold reservoir, because these two are same then essentially you can consider this as this kind of, so essentially Q H is, Q H is transferred, this effective combined system will have the Q H and the work output is Q O W 1 W 2 and this must be same as, as this says this must be same as W 3.

Then Q C should be equal to Q dash. So that is what we are trying to say, the same amount of heat is transferred to the cold reservoir which essentially means that Q C should be equal to Q C dash which actually should have been very obvious. okay we know that ratio of Q H and Q C should be equal to function theta H theta C. So essentially with this equality we can now plug this information here.

We can say the first one which is this, we can use that because Q C is here and Q H is here and this is the function. So essentially now we can consider this one, the second one, so if you consider second, if you consider the second engine is going to be the following. It is like Q reservoir, sorry reversible M by Q C should be equal to function of theta M theta C. So with this you can get the Q C expression from this.

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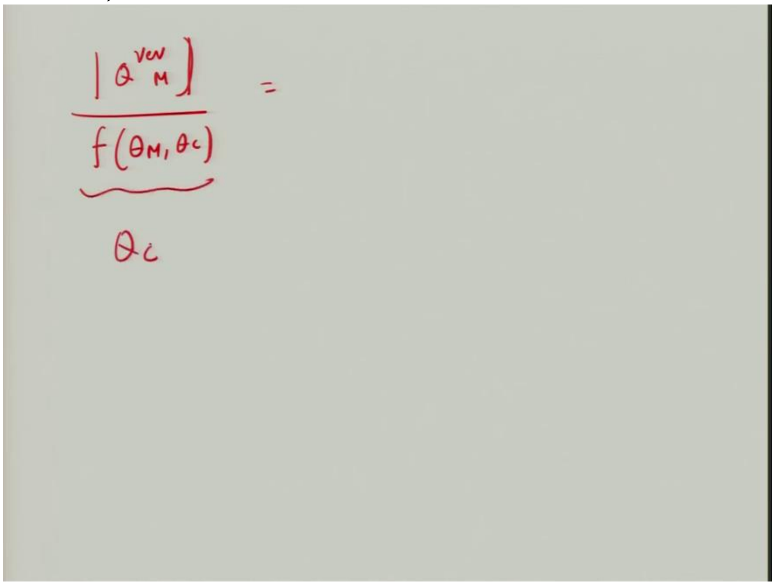

So I can write this as, so let me just again so Q C I can write as Q reversible M by N divided by f of theta M theta C. okay so that is, this is nothing but Q C, alright, That is what it was trying to look at.

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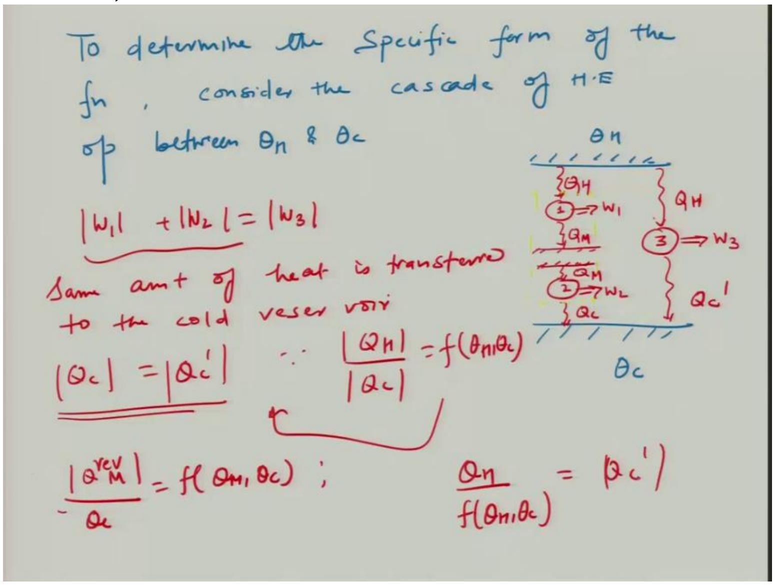

Now similarly Q C dash is nothing but, Q H divided by f of theta H theta C so that is going to be your Q C dash, this, right.

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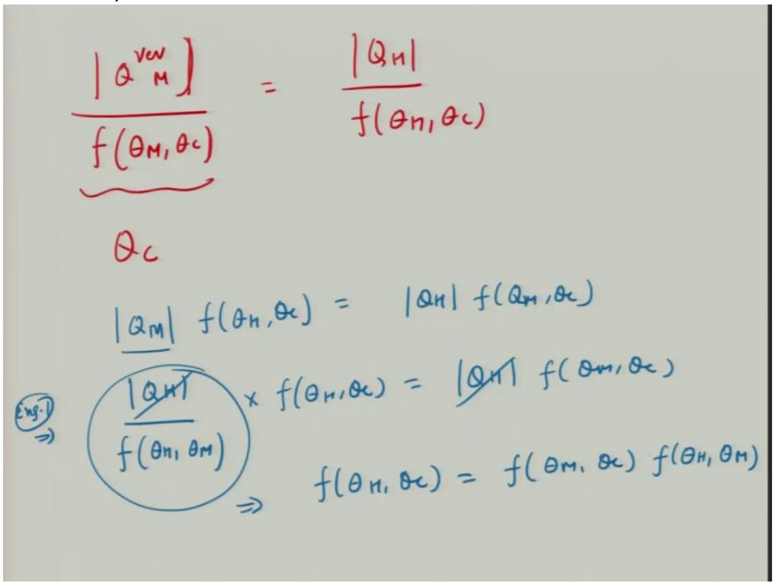

So, if you bring it here then I can write H f of theta H theta C, okay, right. So, this is what we have. Now we can rearrange quickly. So, this is going to be Q M f of theta H theta C Q H f of theta M theta C okay. Now this Q M I can write, rewrite here as Q H by f of theta H theta M. so this comes from the engine first, so this is from engine first.

$$\frac{|Q_M^{rev}|}{f(\theta_M, \theta_C)} = \frac{|Q_H|}{f(\theta_H, \theta_C)}$$

$$|Q_M|\, f(\theta_H, \theta_C) = |Q_H|\, f(\theta_M, \theta_C)$$

$$f(\theta_M, \theta_C) = f(\theta_M, \theta_C) f(\theta_H, \theta_M)$$

This implies this part and this multiplied by f of theta H theta C, Q H f of theta M theta C so this gets cancelled okay so what you have now is f of theta H theta C is nothing but the multiplication of these two function theta M theta C multiplied by f of theta H theta M.

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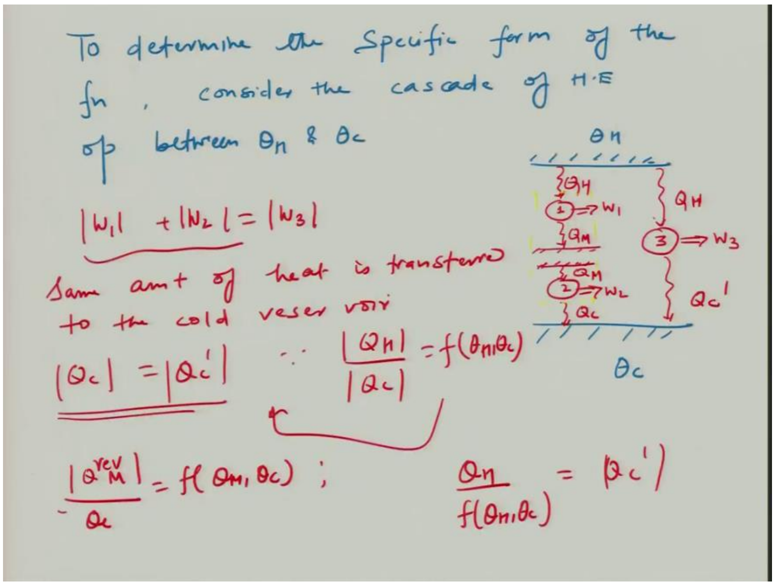

$$|W_1| + |W_2| = |W_3|$$

$$|Q_C| = |Q'_C| \ Since, \frac{|Q_H|}{|Q_C|} = f(\theta_H, \theta_C)$$

$$\frac{|Q_M^{rev}|}{Q_C} = f(\theta_M, \theta_C); \ \frac{Q_H}{f(\theta_H, \theta_C)} = |Q'_C|$$

Now this essentially means that if you have taken this, let us consider engine 3 and if you divided this into subparts which become the cascades of your engines. Essentially the function f here can be written as the multiplication of the function f corresponding to each engine,

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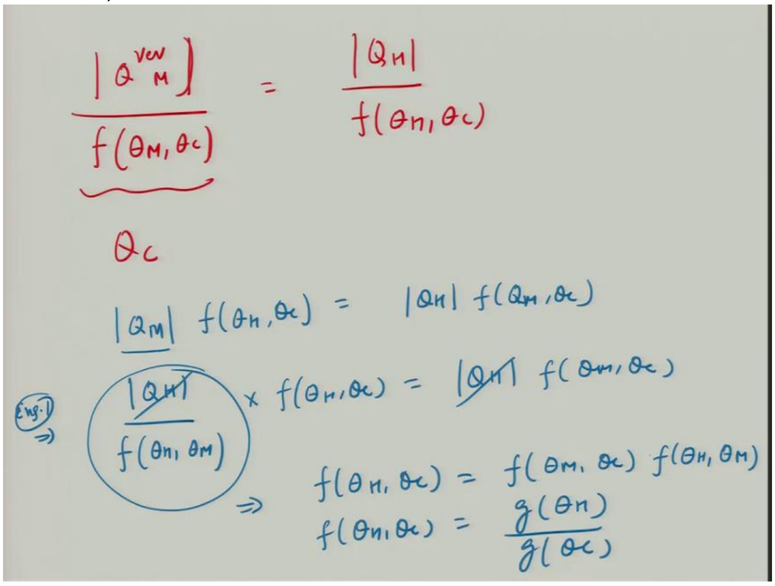

and this essentially means that you can segregate this, you can come up with the function which satisfies this. One of the functions could be that f of theta H theta C is nothing but the ratio of some kind of a function g, another you know so ratio of the function of g which just depend on theta and this will satisfy this equation. So that is something which we came up with, now this g is universal for all the Carnot engine.

$$\frac{|Q_M^{rev}|}{f(\theta_M, \theta_C)} = \frac{|Q_H|}{f(\theta_H, \theta_C)}$$

$$|Q_M|\, f(\theta_H, \theta_C) = |Q_H|\, f(\theta_M, \theta_C)$$

$$f(\theta_H, \theta_C) = f(\theta_M, \theta_C) f(\theta_H, \theta_M)$$

$$f(\theta_H, \theta_C) = \frac{g(\theta_H)}{g(\theta_C)}$$

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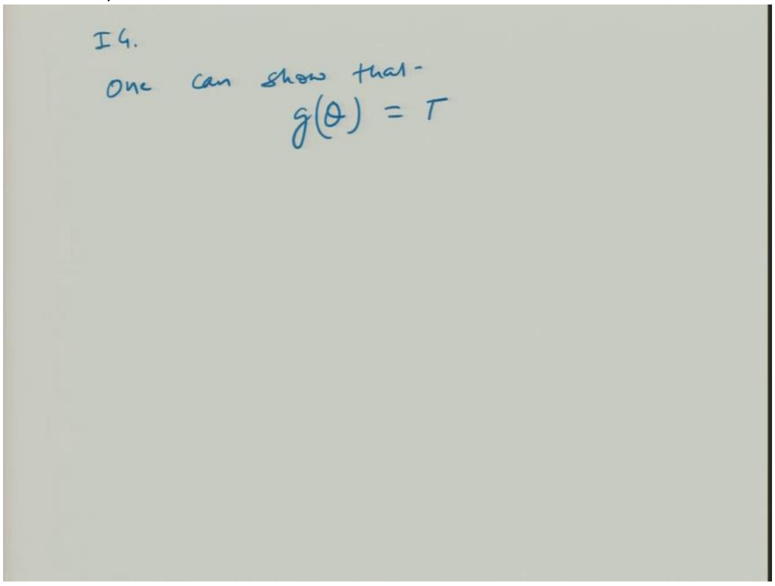

Now if you perform analysis for engine with an ideal gas, so if you use ideal gas one can show that for such an engine which is reversible engine that g of theta is nothing but T, okay so with an ideal gas part. I am using the ideal gas equation. So essentially for Carnot cycle with a working fluid of ideal gas with this analysis you can show that

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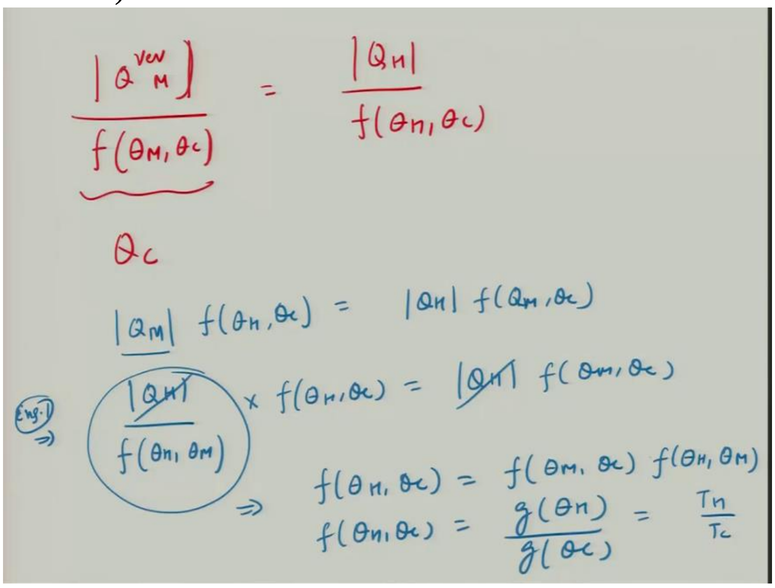

A simple ratio of these functions is nothing but, it would be simply T H by T C, that is what you are saying right.

$$\frac{|Q_M^{rev}|}{f(\theta_M, \theta_c)} = \frac{|Q_H|}{f(\theta_H, \theta_C)}$$

$$f(\theta_H, \theta_C) = \frac{g(\theta_H)}{g(\theta_C)} = \frac{T_H}{T_C}$$

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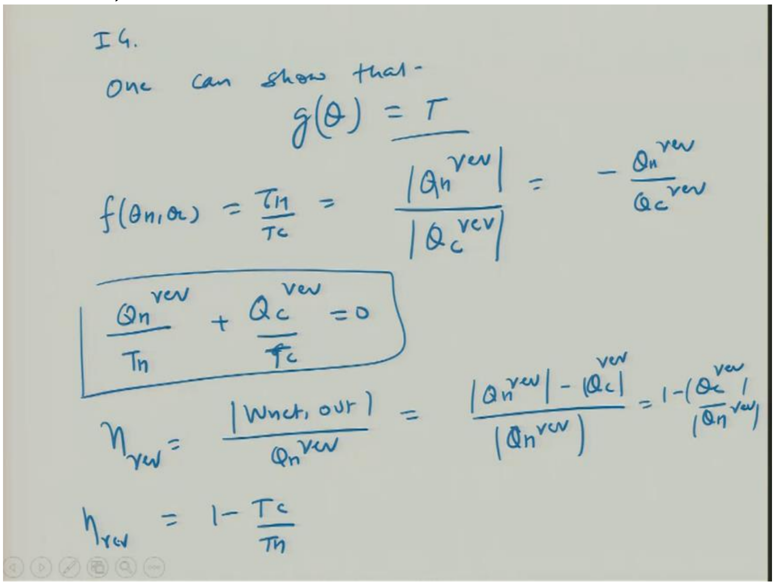

Now this brings down, brings us to the next statement that you can come up with f of theta H theta C is T H by T C, and this now brings us to a connection between the efficiency of the engine and ideal gas temperature scale. So, that is what we are saying ideal gas temperature scale can be used here. Now you bring again that same expression that f is nothing but your theta H, sorry Q H reversible and then Q C reversible.

Now if you use your, you know sign convention that this is going to be Q H reversible, minus Q H and divided by Q C. If you rearrange you are going to get Q H reversible by T H plus Q C reversible by T C. So, this is the expression we are going to get from this analysis. Now we know that for the case of reversible engine we can write this as W net out by Q H, desired output divided by desired input or required input, so this can be written as Q H reversible minus Q H minus Q C basically.

So you can write in terms of the ratios which is going to be 1 by Q C by Q H taking out this part and now I can simply replace this by your, by the temperature. So basically eta reversible is nothing but 1 minus T C by T H and this provides the upper limit for all the real engines using this analysis. So if you are using Carnot engine essentially it means the entire reversible is nothing but 1 minus T C by T H.

$$g(\theta) = T$$

$$f(\theta_H, \theta_C) = \frac{T_H}{T_C} = \frac{|Q_H^{rev}|}{|Q_C^{rev}|} = -\frac{Q_H^{rev}}{Q_C^{rev}}$$

$$\frac{Q_H^{rev}}{T_H} + \frac{Q_C^{rev}}{T_C} = 0$$

$$\eta_{rev} = \frac{|W_{net,out}|}{Q_H^{rev}} = \frac{|Q_H^{rev}| - |Q_C^{rev}|}{|Q_H^{rev}|} = 1 - \frac{|Q_C^{rev}|}{|Q_H^{rev}|}$$

$$\eta_{rev} = 1 - \frac{T_C}{T_H}$$

Similarly you can extend this exercise for coefficient of performance for the refrigerant, refrigerators so there also considering the reversible, reverse Carnot cycle you can use the ratio of the heats as the ratio of the temperatures of the reservoirs.

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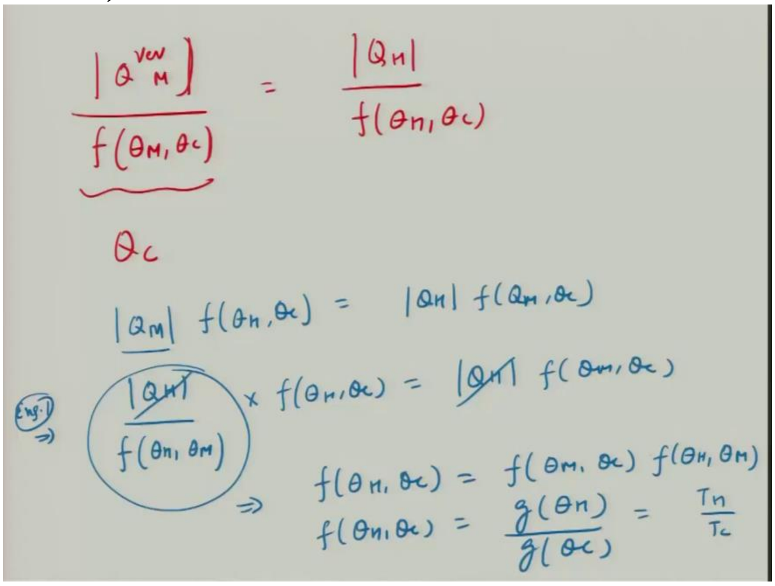

So, this was what I wanted to cover.

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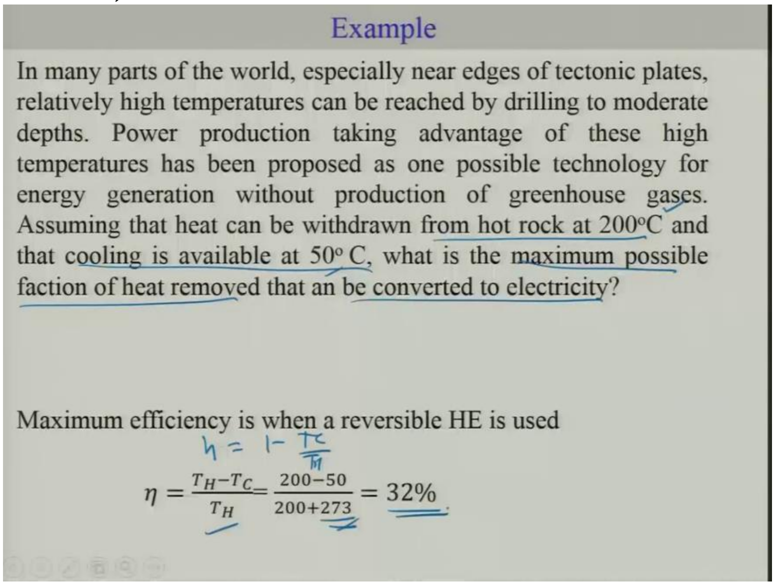

And then we can now take some examples, quick examples to express what we have learnt in the form of some analytical exercises. So, this is a question which is to make use of temperature of the tectonic plates to generate some power. So, in many parts of the world especially near edges of tectonic plates relatively high temperatures can be reached by drilling to moderate depths. Power production taking advantage of these high temperatures has been proposed as one possible technology for energy generation without production of greenhouse gases.

Assuming that the heat can be withdrawn from hot rock at 200 degree Celsius and the cooling is available at 50 degree Celsius what is the maximum possible fraction of heat removed that can be converted to electricity. So, we are being given high temperature reservoir, we are being given the low temperature reservoir and we have been asked to find out what is the basically, efficiency of any, that particular heat engine which we can come. And that maximum efficiency is possible for reversible heat engine.

$$\eta = \frac{T_H - T_C}{T_H} = \frac{200 - 50}{200 + 273} = 32\%$$

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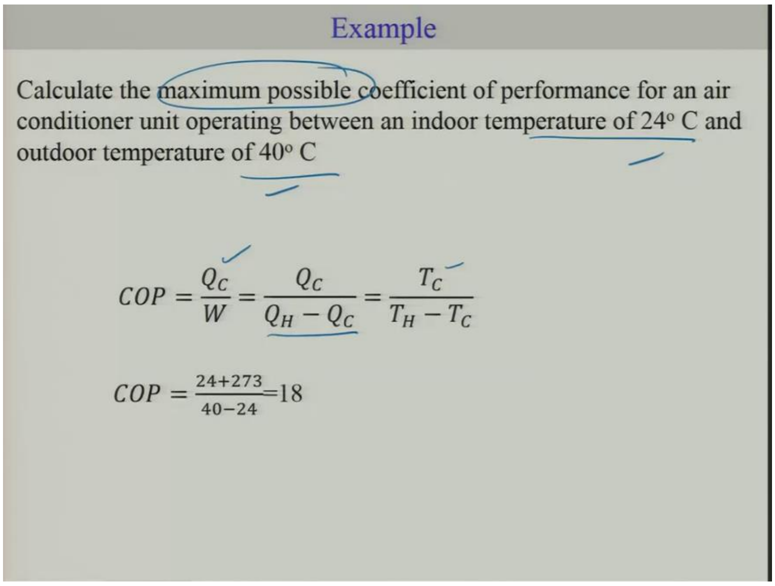

Now the second question is to calculate the maximum possible coefficient of performance of an air conditioner unit operating between an indoor temperature of 24 degree Celsius and outdoor temperature of 40 degree Celsius.

So, it is nothing but the refrigerator, again is being asked by, for maximum possible coefficient which means that we are going to consider reverse Carnot cycle, we are going to consider the same ratio of the temperatures in place of the ratios of the heats Q H and Q L.
It is the desired output by required input which is in this case of refrigerator is:

$$COP = \frac{Q_C}{W} = \frac{Q_C}{Q_H - Q_C} = \frac{T_C}{T_H - T_C}$$

$$COP = \frac{24 + 273}{40 - 24} = 18$$

So that would be the end of the class. In this we have done a rigorous exercise to define the thermometric temperature based, particularly thermodynamic temperature and what is being used is the Carnot cycle in order to define such a temperature which is of course easy for us to find out the maximum efficiency of heat engines and refrigerators and this is something which comes as a yardstick for our analysis. So that would be the end of the class and hope I will see you in the next time.