

**Chemical Engineering Thermodynamics**  
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**Lec 12**  
**First Law of thermodynamics: Examples**

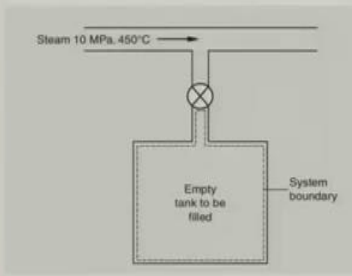
Welcome back, in today's lecture we will be trying to further work on some examples based on the first law of thermodynamics.

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**Example 1**

Steam at 10 MPa, 450°C is flowing in a pipe, as shown in Figure below. Connected to this pipe through a valve is an evacuated tank. The valve is opened and the tank fills with steam until the pressure is 10 MPa, and then the valve is closed. The process takes place adiabatically.

**(a)** Determine the final temperature of the steam in the tank.  
**(b)** Explain why the final temperature in the tank is not the same as that of the steam flowing in the pipe



Unsteady state  
Energy balance

$$dE = \delta Q + \delta W$$

$$+ \sum \left[ h_{in} + gz_{in} + \frac{v_{in}^2}{2} \right] \delta n_{in}$$

$$- \sum \left[ h_{out} + \dots \right] \delta n_{out}$$

So, this is the first example which would like to try and solve some specific questions. So, address some specific question. So, this is steam which is at 10 mega Pascal 450 degrees Celsius is flowing in the pipe. And it is connected through a valve to an empty tank. So, it is connected to the empty tank and the valve is opened and the tank fills with the steam until the pressure here finally is 10 mega Pascal.

Subsequently, once the 10 mega Pascal reached the valve is closed; but the process takes place adiabatically that means there is no heat exchange from the surrounding and what we need to do is to determine the final temperature of the steam in the tank. So, this is a first question and then explain, why the final temperature in the tank is not the same as that of the steam flowing in the pipe.

So, essentially, we need to find out the final temperature the system which we can simply do by first considering the energy balance over this particular system which is already defined by this dashed line. So, if you consider this system and then you can apply unsteady state energy balance.

$$dE = \delta Q + \delta W + \sum \left[ H_{in} + gz_{in} + \frac{v_{in}^2}{2} \right] \delta n_{in} - \sum \left[ H_{out} + gz_{out} + \frac{v_{out}^2}{2} \right] \delta n_{out}$$

Now this is a number of moles which case in similarly we can write the out part; del n out. So, in our case what we can consider is that the kinetic energy and gravitation energy these are contributions are negligible, and what we need to, then, do is to convert this E, E would be nothing but their internal energy. Other thing which we have is that the process is adiabatic that is your dQ is 0.

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Handwritten notes on a whiteboard showing the derivation of the unsteady state energy balance equation for a control volume. The equations are:

$$dU = \delta Q + \delta W + \sum H_{in} \delta n_{in} - \sum H_{out} \delta n_{out}$$

$$\frac{dU}{dt} = \dot{Q} + \dot{W} + m_{in} h_{in} - m_{out} h_{out}$$

Assumptions:

$$m_1 = 0$$

$$m_2 = \int m_{in} dt$$

$$\dot{Q} = 0$$

$$\dot{W} = 0$$

Integration:

$$\int dU = \Delta U = \int m_{in} h_{in} dt = h_{in} m_2 = m_2 u_2 = h_{in} m_2$$

$$dU = \delta Q + \delta W + \sum H_{in} \delta n_{in} - \sum H_{out} \delta n_{in}$$

$$\frac{dU}{dt} = \dot{Q} + \dot{W} + m_{in} h_{in} - m_{out} h_{out}; \quad m_{out} h_{out} = 0$$

$$m_1 = 0$$

$$m_2 = \int m_{in} dt; \quad \dot{Q} = 0; \quad \dot{W} = 0$$

$$\int dU = \Delta U = \int m_{in} h_{in} dt = h_{in} \int m_{in} dt = h_{in} m_2 = m_2 u_2$$

Depending on how you put it there, if it is a molar enthalpy, then it is going to be moles, if it is specific itself is going to be mass.

So, essentially  $U_2$  is nothing but  $H_{in}$ . Now, you can consider getting these values that means, you need to find out what is  $H_{in}$ . So, we have to resort to the tables.

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Steam table

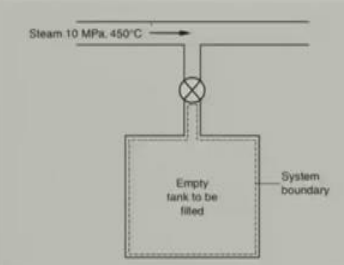
$P = 10 \text{ MPa}, 450^\circ\text{C}$   
 $h_{in} = 3241 \text{ kJ/kg}$   
 $h_{in} = u_2 = 3241 \text{ kJ/kg}$  } *flowing steam*

System:  $P = 10 \text{ MPa}$   
 $u_2 = u = 3241 \text{ kJ/kg}$   
 $= 600^\circ\text{C}$

**Example 1**

Steam at 10 MPa, 450°C is flowing in a pipe, as shown in Figure below. Connected to this pipe through a valve is an evacuated tank. The valve is opened and the tank fills with steam until the pressure is 10 MPa, and then the valve is closed. The process takes place adiabatically.

**(a)** Determine the final temperature of the steam in the tank.  
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*Unsteady state  
Energy balance*

$$dE = \delta Q + \delta W + \sum \left[ h_{in} + gz_{in} + \frac{v_{in}^2}{2} \right] \delta m_{in} - \sum \left[ h_{out} + \dots \right] \delta m_{out}$$

So essentially, we need steam table. So, if you consider  $P$  is equal to 10 mega Pascal which is a final, which is the temperature of the steam and 450 degree Celsius. Because of the steam that means is super-heated table you are going to consider, then the corresponding  $H$  in is going to be 3241 kilojoules per kg. So, that is your  $H$  in the corresponding  $U$ , of course is going to be the same.

That means the  $U_2$  of the system that means the internal energy, specific internal energy of the final state after the end of the process is going to be simply, 3241 kilojoules per kg. Now, we need to find out the temperature also. So, we are going to consider the condition of the final system which is at  $P$  is equal to pressure is equal to 10 mega Pascal. So, this is your flowing stream, right. This is your system now.

So,  $P$  is equal to 10 mega Pascal and  $U_2$  and this is  $U$ , is 3241 kilojoules per kg. So, you have to now look into these 2 terms, and then, again go back to the steam and find out what is a specific, what is the temperature for which these variables are okay. So, if you consider pressure 10 mega Pascal and then, you look at this 3241 the corresponding temperature comes out to be 600 degree Celsius.

So, what you need to look at is 10 mega Pascal in steam table and the corresponding energy is going to be given then you look at the temperature for which this corresponds to. If it is not available as far as this 3241 may not be precise, then you have to interpolate value together. So, this is something which you must have done in your first course in thermodynamics, but this is just to make sure that you revise these aspects of your understanding.

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**Example 2**

Air is contained within a piston–cylinder assembly, as shown in the Figure below. The cross-sectional area of the piston is  $0.01 \text{ m}^2$ . Initially the piston is at 1 bar and  $25^\circ\text{C}$ , 10 cm above the base of the cylinder. In this state, the spring exerts no force on the piston. The system is then reversibly heated to  $100^\circ\text{C}$ . As the spring is compressed, it exerts a force on the piston according to:

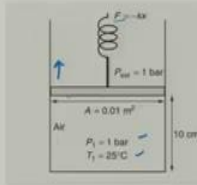
$$F = -kx$$

where  $k = 50,000 \text{ N/m}$  and  $x$  is the displacement length from its uncompressed position.

(a) Determine the work done.  
(b) Determine the heat transferred.

$$W = - \int P dv$$

$PA = kx + P_{atm}A$



So, I am going to now move to the second problem. The second problem is related to air and piston-cylinder assembly, this is the geometry of the second problem. So, what do you have is a piston with a cross sectional area  $0.01 \text{ m}^2$  and this attached to the spring with this force which is proportional to minus  $Kx$ , which it exerts on the piston. In addition to this you have the atmospheric air and hence, you have 1 bar additional pressure. So, that is additional force which will be also exerted on the piston.

The initial air is at 1 bar and 25 degrees Celsius and the volume of course is area multiplied by this height. Now, what you need to do is to find the work which it does. So, this is the initial geometry, but after this condition of the spring exerts no force on the piston, the system is subsequently reversibly heated to 100 degree Celsius. And what we need to do is to find out the work associated with this when we apply this reversible heat 200 degrees Celsius.

So, initially the temperature is 25 degree Celsius and the final temperature given 200 degree Celsius but done reversibly. So, that is a question which is given. Now, how do we start, we again start with the case of the work done; we must start with simple expression that work is nothing but:  $W = - \int P dV$ ;  $PA = kx + P_{atm}A$

So, we can write this as simply PdV and now, we can do a simple free body diagram analysis in order to find out this pressure. So, this is the force which acts as it moves upon hitting the piston will move up here. So, this is the force which acts against it and this is the force due to the here and since, this is a very slow process and hence, we can equate this. So, this force balance should make sure that this expression is valid particularly for reversible a heated system. So, you have an expression for pressure.

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The image shows a handwritten derivation on a light-colored background. The equations are as follows:

$$P = \frac{kx}{A} + P_{atm}$$

$$W = - \int \left( \frac{kx}{A} + P_{atm} \right) dV$$

$$= - \int_{V_1}^{V_2} \frac{k\Delta V}{A^2} dV - \int_{V_1}^{V_2} P_{atm} dV$$

$$dV = d(V_1 + \Delta V) = d\Delta V$$

$$W = - \int_0^{(V_2-V_1)} \frac{k\Delta V}{A^2} d\Delta V - \int_{V_1}^{V_2} P_{atm} dV$$

$$= - \frac{k}{2A^2} \Delta V^2 \Big|_0^{(V_2-V_1)} - P_{atm} (V_2 - V_1)$$

$$W = - \int \left( \frac{kx}{A} + P_{atm} \right) dV$$

$$= - \int_{V_1}^{V_2} \frac{k\Delta V}{A^2} dV - \int_{V_1}^{V_2} P_{atm} dV$$

$dV = d(V_1 + \Delta V) = d\Delta V$  (Since  $V_1$  is fixed)

$$W = - \int_0^{(V_2-V_1)} \frac{k\Delta V}{A^2} d\Delta V - \int_{V_1}^{V_2} P_{atm} dV$$

So, this is the expression.

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$$W = -\frac{k(V_2 - V_1)^2}{2A^2} - P_{atm}(V_2 - V_1)$$

Apply IG  $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} = \frac{V_2}{T_2} \left( P_{atm} + \frac{k(V_2 - V_1)}{A^2} \right)$

$T_1 = 25^\circ\text{C}$   
 $P_1 = 1 \text{ atm}$   
 $V_1 = (0.01 \times 0.1) \text{ m}^3$   
 $T_2 = 100^\circ\text{C}$

Solve  $V_2 = 0.0016 \text{ m}^3$

$\Rightarrow W = -166 \text{ J}$

$$W = -\frac{k(V_2 - V_1)^2}{2A^2} - P_{atm}(V_2 - V_1)$$

Applying Ideal Gas law,  $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} = \frac{V_2}{T_2} \left( P_{atm} + \frac{k(V_2 - V_1)}{A^2} \right)$

### Example 2

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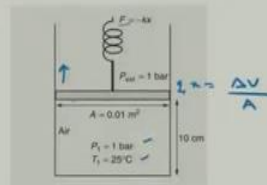
$$F = -kx$$

where  $k = 50,000 \text{ N/m}$  and  $x$  is the displacement length from its uncompressed position.

- Determine the work done.
- Determine the heat transferred.

$$W = -\int P dv$$

$$PA = kx + P_{atm}A$$



So, the final pressure can be considered as initial pressure, of course, we do have information that is the 1 bar, right, but, what about the final pressure. Final pressure would be, P atmosphere plus the additional the force due to the spring. So, this will be your final pressures now, essentially you have this information from here initially we have  $T_1$  is equal to 25 degree Celsius,  $P_1$  is 1 atmosphere; and  $V_1$  we know this is 0.01 area multiplied by height.

And  $T_2$  we know. So, with this information, we can actually solve this. So, using this exploration we can solve, we get  $V_2$  is equal to 0.00116 meter cube and now you can plug this in this expression because  $V_2$  is known, hence,  $W$  can be calculated as 166. So, the negative signs indicate that the work is done by the system on the boundary or surrounding enhanced as based on our nomenclature is indicates that it is a work which is done by the system not on the system.



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b)  $\Delta U = Q + W \Rightarrow Q = \Delta U - W$   
 $N \Delta U = N \int C_v dT = N \int (C_p - R) dT$   
 Constant A.2, Heat capacity,  $C_p$   
 $\frac{C_p}{R} = A + BT + CT^2 + DT^{-2} + ET^3$   
 $A = 3.355$   
 $B = 0.575 \times 10^{-3}$   
 $D = -0.014 \times 10^{-5}$   
 $\Delta U = 1500 \text{ J/mol}$  ;  $\Delta U = \left( \frac{P_1 V_1}{RT_1} \right) \Delta U = 630 \text{ J}$   
 $Q = 803 \text{ J}$

So, that was the first part actually the work done. The second part is to determine the heat transfer. So, we can now apply the first law of thermodynamics for this close system.

$$Q = \Delta U - W$$

$$N \Delta U = N \int C_v dT = N \int (C_p - R) dT$$

Now, this is what we know that we have already calculated it is the heat which we want.

So, Q is nothing but your del U minus W. So, we need to find out now, delta U; what about Cv, now, if, if you look at the tables in the terminal tables particularly the crowd skis table A.2 from the text book. You can find the heat capacity data, for the air. So, that will be in the mostly in the CP value.

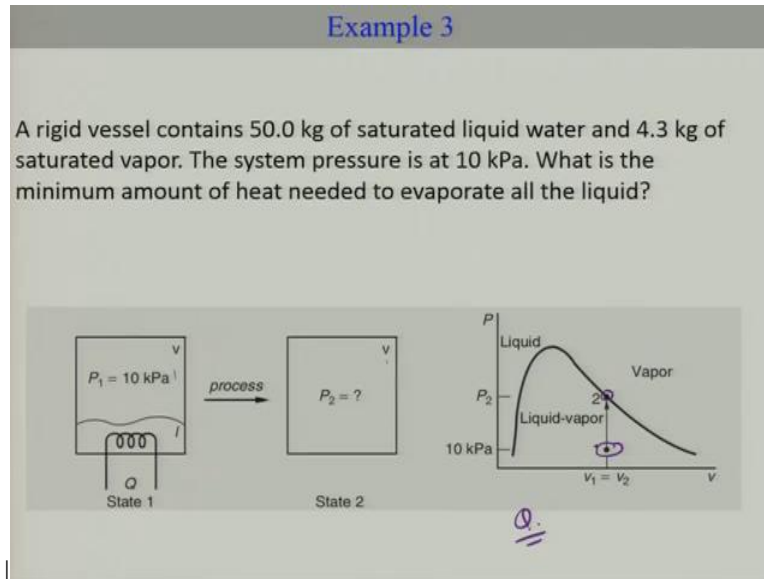
$$\frac{C_p}{R} = A + BT + CT^2 + DT^{-2} + ET^3$$

So, we know for the ideal gas, CP is nothing but CV plus R, and hence we can write this as N CP minus RdT. Now, for the case of air the C's and D's are or C's and E's are 0. So, and the values of air are 3.355 all the values of this constant ABC are 3.355, B is 0.575 into 10 to power minus 3, D

is minus 0.016 into 10 to minus 5. Now, you can plug this in, we can get delta U, this 1580 joules per mole.

If you plug in, in and integrate, you are going to get this, this is your per mole. Now, in order to get delta U, you need to find out the number of moles which we can get use the ideal gas expression. So, this comes out to be 638 joules, or you can use this here, you can obtain Q which is going to be 803 joules. So, the U is W from the previous calculation and you can obtain the value of Q. So, we are done with that.

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Now, let us move on to example 3. Now, the earlier examples we have tried are stream based, we have tried air piston, but now, here is little different here what is going to happen is that the in this case if you look at, there is a rigid vessel which contains a saturated system, water and vapor. Initial temperature or the pressure is 10 kilo Pascal and is being asked what is the minimum amount of heat needed to evaporate that means final, finally this liquid disappears and all everything is vapor.

But still we can consider to be saturates vapor and if that is the case you have a phase diagram like this or you have the PV diagram is like this where initially is somebody here at 10 kilo Pascal and basically what we are doing is, is isochoric process, where the volume is fixed and it follows this arrow line and eventually the final studies on the saturated vapor curve and what we need to find out. So, from here, to here what is the  $Q$  which is required.

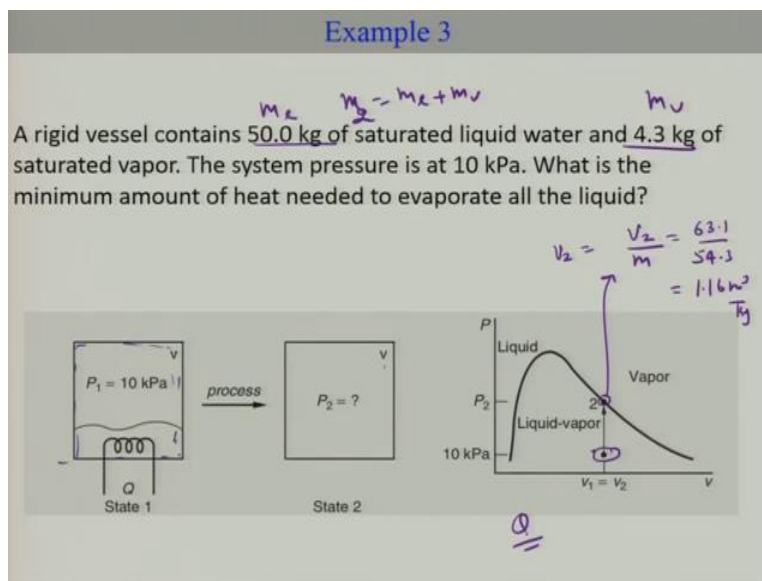
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$$\begin{aligned} \Delta K.E. &\approx 0 \\ \Delta P.B. &\approx 0 \\ \Delta U &= Q + W = 0 = U_2 - U_1 \\ U_2 &= m_2 u_2 = m_2 \checkmark u_2^v \\ U_1 &= \underbrace{m_l^l u_1^l} + \underbrace{m_v^v u_1^v} \end{aligned}$$

$m_l = 50 \text{ kg}$   
 $m_v = 4.3 \text{ kg}$   
 $m_2 = m_l + m_v$

$$V_1 = 50(0.001) + 4.3(14.67) = 63.1 \text{ m}^3 = V_2$$

specific vol. =  $v_1 = v_2 = \frac{V_1}{m}$



Alright, so, we can simply do the energy balance again. Now, with this we have  $\Delta U$  is equal to  $Q + W$ . And do we have any boundary moment if you consider this to be a system that says no. So, essentially this is nothing but. So,  $\Delta U$  can be written as simply  $Q$ . So,  $Q$  is nothing but  $U_2 - U_1$ . Now, we have to make use of the tables in this case also. So, since it is a saturated system, so, you will be having information of the specific internal energy for the liquid phase and as well as for the vapor phase.

So, if you make use of that  $U_2$  is saturated a vapor. So that is going to be simply,  $m_2 U_2$ , ok. But in the case of 1 I can write this as  $U_1 + m_2$ . So, this is nothing but a liquid here and this is  $U_1$  vapor here. So, this is the case whereas this is nothing but equivalently we are saying this is this would be  $m_2$  vapor 2. So, you just have to look into the vapor. So, similarly, you can also consider the initial volume. So, volume you can find it out okay. You also know that initial period where contains 50 kilos, you saturate liquid order 4.3.

So, you know exactly your  $m$  which is your, this is your  $m$  of liquid and this is your  $m$  of vapor and final  $m_2$  is nothing but  $m_l + m$  vapor. So,  $m_2$  we know  $m_1$  we also know, we also know. So, we can find out the specific internal energy from the table. So,  $m_l$  is 50 kg and  $V$  is 4.3 kg. So, we want to remain this constant. So is 50 times the volume of the liquid which is 0.001 plus 4.3, the one with the saturated, the specific volume of the saturated vapor is 14.67. We get 63.1 meter cube. So, that is your, of course,  $V_2$  also capital  $V_2$ .

Similarly,  $m_2$  is already known  $m_2$  is  $m_l + m_v$ . So, we know  $m_2$ . We know this, we know exactly the specific volume. So, the specific volume is  $V_1$ , this is going to be  $V_2$ . So, this is nothing but  $V_1$  by  $m$ ,  $m$  total. So, with this we can find out what is your specific volume of vapor. So, now at this condition, we need to find out the specific order to remember this that let's go back here.

So, here essentially this is your  $V_2$  to write for specific volume, this is nothing but the capital  $V_2$  divided by the total mass, because this is a condition. So, this comes out to be 63.1 which is a total volume divided by 54.3 that is 1.16 meter cube per kg. So, that is a specific volume, add the condition 2.

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### Example 3

$m_L$        $m_2 = m_L + m_V$        $m_V$

A rigid vessel contains 50.0 kg of saturated liquid water and 4.3 kg of saturated vapor. The system pressure is at 10 kPa. What is the minimum amount of heat needed to evaporate all the liquid?

State 1      process      State 2

$v_2 = \frac{V_2}{m} = \frac{63.1}{54.3} = 1.16 \frac{m^3}{kg}$

Q

$\Delta K.E \approx 0$   
 $\Delta P.B \approx 0$

$\Delta U = Q + W = 0 = U_2 - U_1$

$U_2 = m_2 u_2 = m_2 \checkmark u_2^v$

$U_1 = m_L^l u_1^l + m_V^v u_1^v$

$V_1 = 50(0.001) + 4.3(14.67) = 63.1 m^3 = V_2$

specific vol. =  $v_1 = v_2 = \frac{V_1}{m}$

$m_L = 50 kg$   
 $m_V = 4.3 kg$   
 $m = m_L + m_V$

$$\begin{aligned}
 & \text{Sp} \\
 & v_2^v = v_2 = 1.16 \\
 & \text{Look up the table \& } \\
 & \text{find } P \text{ for which sat'd specific vol.} \\
 & \quad \hookrightarrow 1.16 \\
 & \quad = 0.15 \text{ MPa} \\
 & u_2^v = 2519.6 \text{ kJ/kg} \\
 & Q = m_2 u_2^v - (m_1^l u_1^l + m_1^v u_1^v) \\
 & Q = 117 \times 10^6 \text{ J}
 \end{aligned}$$

So,  $v_2$  is nothing but vapor is nothing but  $v_2$ , this is 1.16. Now, we need to look up the table, and find  $P$  for which saturated specific volume is 1.16 and it comes out to be 0.15 mega Pascal. So, usually if you do not find precisely then you have to do interpolations. Now, corresponding internal energy of the vapor is 2519.6 kilojoules per kg and then we can find out  $Q$  is nothing but  $m_2 u_2^v$ . This is the final value minus  $m_1 u_1^l$  plus  $m_1 u_1^v$ .

So, this is something which we know from the tables, now we also know this, we also know this and hence, when you plug in all the information, I am going to get this value. So, this is a very straightforward exercise what was the key is the following that you had to find out the first total mass which you know from the information then, you want  $u_1$  you could find it out because, the information in the initial states are given to you.

So, you have to look at the saturated pressure information, from there you could get this information, the specific internal energy of the liquid and vapor. And we already know the masses hence, we know the  $u_1$ . Now, the second question was, how the important question was that we had to find out this specific internal energy for to which was the basic target in order to do that we know that volume is constant hence this information the volume, specific volume, we could find it out from this simple exercise.

The total volume divided by the total mass which will be the  $v_2$  and that will still be the vapor phase. Now, knowing that this is a vapor phase. This is a specific volume look up the table for

where this information is this. Specific value corresponding specific volume is this, and then you find out the pressure that comes out to be this, and the corresponding internal energy is this; and subsequently, you plug this information back into the equations which was the unknown at this point, but we have now this information so, you can plug in there and you obtain the final value.

So, that is the basic set of examples which I wanted to cover today. And I hope that I covered the gambit within the context of first law of. I hope that I have covered the range of the problems within the framework of the first law of thermodynamics. So, with that, I will close today's lecture. I will see you in the next time.