Chemical Engineering Thermodynamics
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Lec 11
First law of Thermodynamics for open system

Welcome back, today's lecture we have go through the First Law of Open System.

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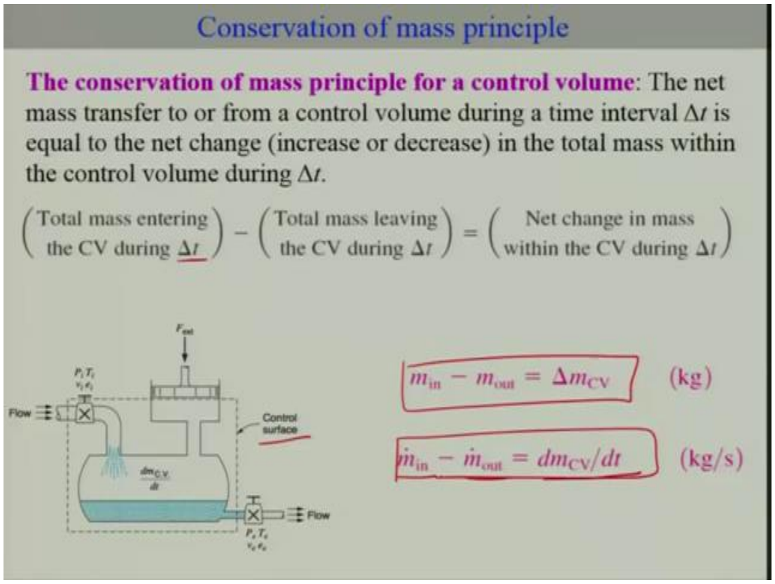

Conservation of mass principle

The conservation of mass principle for a control volume: The net mass transfer to or from a control volume during a time interval Δt is equal to the net change (increase or decrease) in the total mass within the control volume during Δt.

$$\begin{pmatrix}\text{Total mass entering} \\ \text{the CV during } \underline{\Delta t}\end{pmatrix} - \begin{pmatrix}\text{Total mass leaving} \\ \text{the CV during } \Delta t\end{pmatrix} = \begin{pmatrix}\text{Net change in mass} \\ \text{within the CV during } \Delta t\end{pmatrix}$$

$$\boxed{m_{in} - m_{out} = \Delta m_{CV}} \quad \text{(kg)}$$

$$\boxed{\dot{m}_{in} - \dot{m}_{out} = dm_{CV}/dt} \quad \text{(kg/s)}$$

Again this is something in a mode of revision so to the, to start with we will look at conservation of mass principle for a control volume. The reason why we say control volume because there is flow in or flow out in the system, so, for the case of this example, a tank is being filled continuously and taken out here with respect to this valves, one can consider this dash line as a control volume.

So, you have a flow in from this at a given, pressure, temperature, velocity and this E stands for its basically specific energy and there is a rate of mass within the control volume which keeps changing. And similarly, there is a flow out with these conditions; the exit, pressure, temperature, velocity and the effective or the specific energy here.

Now, if you are applying a mass balance on this control volume within this basically this surface, then, essentially, we can write simply total mass entering the control volume during delta t time minus total mass leaving the control volume during delta t time. So, essentially total mass entering the control volume minus total mass leaving the control volume during delta t time and that must be equal to the net change in the mass within the control volume that is nothing but this part.

So, for a given delta T it will be like this: $m_{in} - m_{out} = \Delta m_{CV}$

But, in general, we can write: $\dot{m_{in}} - \dot{m_{out}} = \frac{dm_{CV}}{dt}$

So, this is in the rate form.

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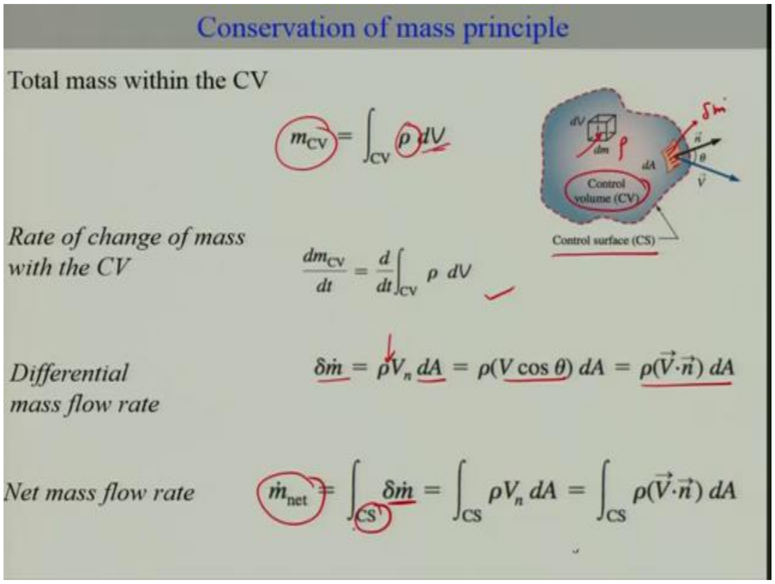

Conservation of mass principle

Total mass within the CV

$$m_{CV} = \int_{CV} \rho \, dV$$

Rate of change of mass with the CV

$$\frac{dm_{CV}}{dt} = \frac{d}{dt}\int_{CV} \rho \, dV$$

Differential mass flow rate

$$\delta\dot{m} = \rho V_n \, dA = \rho(V\cos\theta)\, dA = \rho(\vec{V}\cdot\vec{n})\, dA$$

Net mass flow rate

$$\dot{m}_{net} = \int_{CS} \delta\dot{m} = \int_{CS} \rho V_n \, dA = \int_{CS} \rho(\vec{V}\cdot\vec{n})\, dA$$

Now, one can also represent this mass of control volume in a different form. Here is, for example, in the case of this as a control volume, contain within this dashed line or curve are which is nothing but the control surface, this is your control volume, this is your control surface. One can consider the mass of the control volume nothing, but the integral of the density of the food inside multiplied by the differential change in the volume and you integrate essentially in some sense what you are saying is that you are looking at here, the density multiplied by dV and integrating over this control volume.

$$m_{CV} = \int_{CV} \rho dV$$

So that will be the mass of your control volume. The corresponding change of the mass within the control volume would be:

$$\frac{dm_{CV}}{dt} = \frac{d}{dt}\int_{CV} \rho dV$$

So, we can also write this in a differential mass flow rate, where we are interested in only in del of m dot, that will be nothing but the rho.

$$\delta \dot{m} = \rho V_n dA = \rho(V\cos\theta)dA = \rho(\overrightarrow{V}.\vec{n})dA$$

Net mass flow rate, $\dot{m_{net}} = \int_{CS} \delta \dot{m} = \int_{CS} \rho V_n dA = \int_{CS} \rho(\overrightarrow{V}.\vec{n})dA$

So, this is basically the control surface this representation in 2D as we are looking on this, but if it is 3D you are taking about all surface, which we do not see in the form of the dash line here, so, you just a integral of this d del mdot over the control surface, that is going to be net mass flow rate as referring to the what goes out of effective net mass flow rate through the control surface, ok.

So, this is nothing but only 2 specific streams, one is going out and one is coming in, then this is nothing but the difference in the two, ok; But it is just integral of over control surface. So, this now can be written as in this form rho Vn, and this can be written in this rho Vn, in the dot product form.

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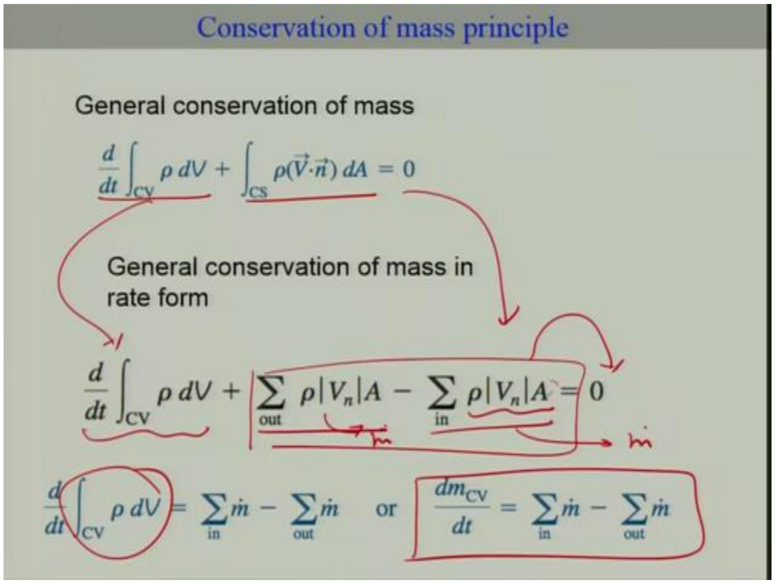

Conservation of mass principle

General conservation of mass

$$\frac{d}{dt}\int_{CV}\rho\,dV + \int_{CS}\rho(\vec{V}\cdot\vec{n})\,dA = 0$$

General conservation of mass in rate form

$$\frac{d}{dt}\int_{CV}\rho\,dV + \underbrace{\sum_{out}\rho|V_n|A}_{\dot m} - \underbrace{\sum_{in}\rho|V_n|A}_{\dot m} = 0$$

$$\frac{d}{dt}\int_{CV}\rho\,dV = \sum_{in}\dot{m} - \sum_{out}\dot{m} \quad \text{or} \quad \boxed{\frac{dm_{CV}}{dt} = \sum_{in}\dot{m} - \sum_{out}\dot{m}}$$

So, from the point of the conservation energy, what you saying is that, whatever the net flow rate which comes out or in, through the control surface that must be equal to the change in the control volume mass and that is what it is in the form of this integral. So, that is the conservation mass principle.

$$\frac{d}{dt}\int_{CV}\rho dV + \int_{CS}\rho(\overrightarrow{V}.\vec{n})dA = 0$$

One can also write this expression in the form of more expressive where we divide or segregate the inlet and outlet through the control surface in the form of here outlets or the number of streams which are getting out from the control surface and number of streams which are getting in, through the control surface so that is going to be this part and this is remain the same.

$$\frac{d}{dt}\int_{CV}\rho dV + \sum_{out}\rho|V_n|A - \sum_{in}\rho|V_n|A = 0$$

$$\frac{d}{dt}\int_{CV}\rho dV = \sum_{in}\dot{m} - \sum_{out}\dot{m} \quad or, \quad \frac{dm_{CV}}{dt} = \sum_{in}\dot{m} - \sum_{out}\dot{m}$$

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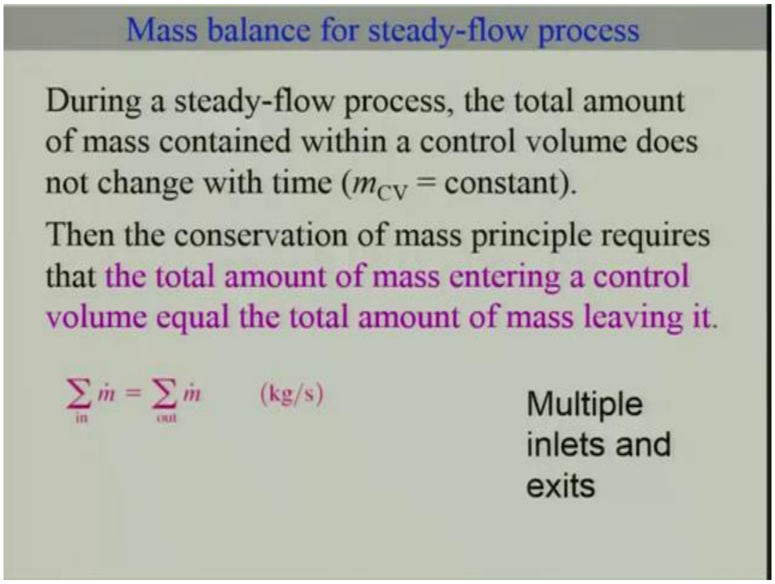

Mass balance for steady-flow process

During a steady-flow process, the total amount of mass contained within a control volume does not change with time (m_{CV} = constant).

Then the conservation of mass principle requires that the total amount of mass entering a control volume equal the total amount of mass leaving it.

$$\sum_{in} \dot{m} = \sum_{out} \dot{m} \quad (kg/s)$$

Multiple inlets and exits

So, for a steady state that means you have a steady state there is a no change in the system properties over the time, in that case the mass of the control volume should remain same at a given time, if that is a case then, essentially $\frac{dm_{CV}}{dt} = 0$, and this will lead to this expression for the case of steady state flow process where the net mass which comes in must be equal to net mass which goes out and that is the case for steady flow process.

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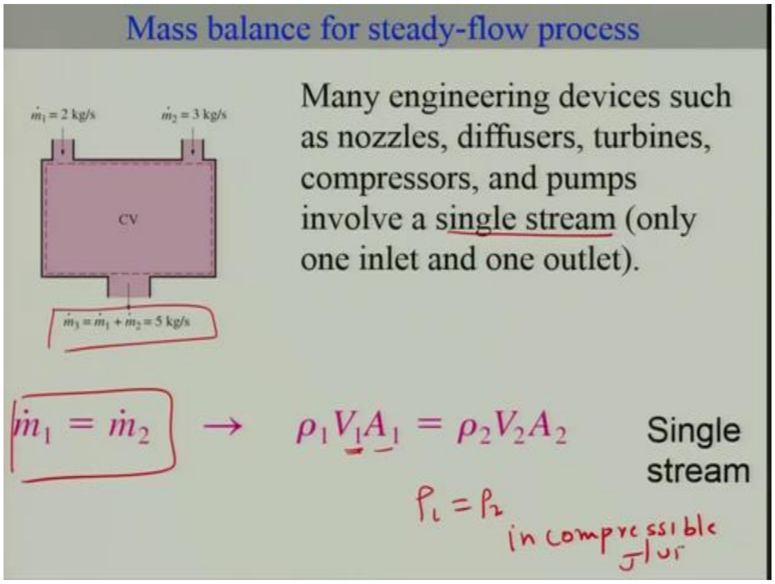

Mass balance for steady-flow process

Many engineering devices such as nozzles, diffusers, turbines, compressors, and pumps involve a single stream (only one inlet and one outlet).

$\boxed{\dot{m}_1 = \dot{m}_2}$ → $\rho_1 V_1 A_1 = \rho_2 V_2 A_2$ Single stream

$\rho_1 = \rho_2$
incompressible flow

Many engineering devices such as nozzles, diffusers, turbines, compressors and pumps involve single streams. So, we can simplify the previous expression, which is meant for multiple inlet and exits. So, if it is a single stream it is going to be only m1 m2, ok; for if it is a multiple stream this is a illustration for two inlet stream and one outlet stream.

It is clearly seen: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$. For the case of again steady state process, for the case of single stream, it is only $\dot{m}_1 = \dot{m}_2$ you can represent this in terms of density, volumetric flow and area, ok. If it is incompressible then essentially means: $\rho_1 V_1 A_1 = \rho_2 V_2 A_2$

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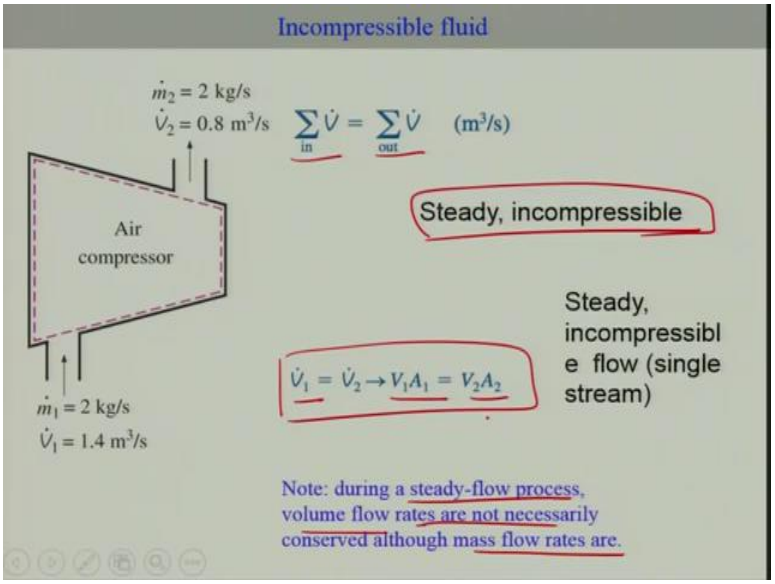

Incompressible fluid

$\dot{m}_2 = 2$ kg/s

$\dot{V}_2 = 0.8$ m^3/s

$\sum_{in} \dot{V} = \sum_{out} \dot{V}$ (m^3/s)

Air compressor

Steady, incompressible

$\dot{V}_1 = \dot{V}_2 \to V_1 A_1 = V_2 A_2$

Steady, incompressible flow (single stream)

$\dot{m}_1 = 2$ kg/s

$\dot{V}_1 = 1.4$ m^3/s

Note: during a steady-flow process, volume flow rates are not necessarily conserved although mass flow rates are.

So, in that case we can simply use summation of volumetric flow that must be same for the case of air compressor one can show this right. In this case the flow rate must be same, volumetric flow rate must be same, for the case of steady state incompressible flow. So, in this case air compressor well in this case this is compressible, so, you can apply that but for the case of air steady incompressible flow.

$$\sum_{in} \dot{V} = \sum_{out} \dot{V}$$

$$\dot{V}_1 = \dot{V}_2 \to V_1 A_1 = V_2 A_2$$

So, during the steady flow process volume flow rate are not necessarily conserved although mass flow rates are. So, this is specific this particular case where you can clearly see volume flow rate are same in specific case of incompressible conditions.

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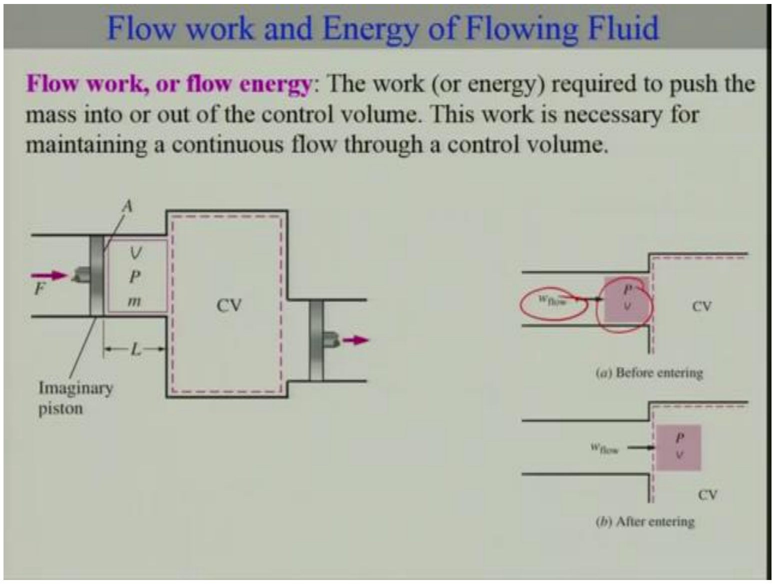

Flow work and Energy of Flowing Fluid

Flow work, or flow energy: The work (or energy) required to push the mass into or out of the control volume. This work is necessary for maintaining a continuous flow through a control volume.

Let us talk about now energy conservation, so this was all about mass conservation. So, in case of energy we need to also account for specific kind of work which is done by the fluid enters the control volume or exits the control volume and that expect is often referred as flow work or flow energy. So, in the case of simple we can use make use of simple illustration, so, you have this control volume and this is the amount of fluid which is getting in at a condition of P and V.

And so, the idea is that this work, we define a work is the work which is necessary to push this particular amount of fluid inside it, ok. So, you can represent this thing in this form that essentially you have a piston and there is a force which acts on this in order to push this fluid with a condition of V P M.

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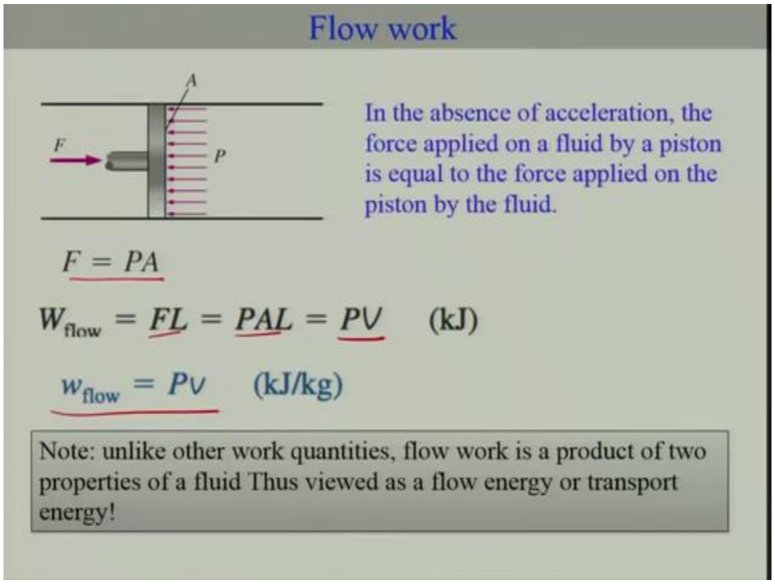

Flow work

In the absence of acceleration, the force applied on a fluid by a piston is equal to the force applied on the piston by the fluid.

$$\underline{F = PA}$$

$$W_{\text{flow}} = \underline{FL} = \underline{PAL} = \underline{PV} \quad \text{(kJ)}$$

$$\underline{w_{\text{flow}} = Pv} \quad \text{(kJ/kg)}$$

Note: unlike other work quantities, flow work is a product of two properties of a fluid Thus viewed as a flow energy or transport energy!

So, this is an imaginary thing, so how do you represent more clear more clearly so you can consider this again that you have fluid here, ok, at this condition P and this process is occurring at slowly and in that case: $F = PA$

$$W_{flow} = FL = PAL = PV$$

$$w_{flow} = Pv$$

Now, in order to push we simply use simple F into L that is the length of the distance, this piston has to travel in order to push this fluid completely in the control volume and if you make this simple expression the simplify this, then this can be shown that W flow is nothing, but PV and this is the additional energy which we provide to the control volume. Now, in order to do a complete energy balance, we have to consider this flow as a part of this fluid.

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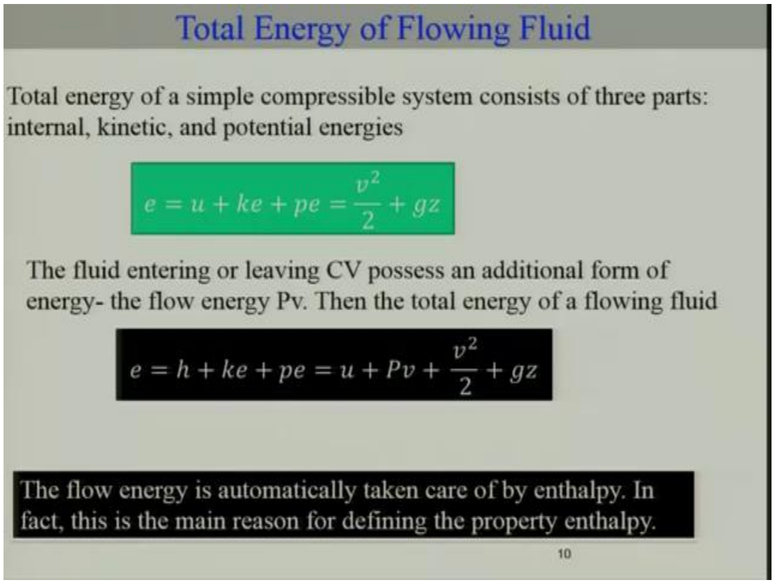

Total Energy of Flowing Fluid

Total energy of a simple compressible system consists of three parts: internal, kinetic, and potential energies

$$e = u + ke + pe = \frac{v^2}{2} + gz$$

The fluid entering or leaving CV possess an additional form of energy- the flow energy Pv. Then the total energy of a flowing fluid

$$e = h + ke + pe = u + Pv + \frac{v^2}{2} + gz$$

The flow energy is automatically taken care of by enthalpy. In fact, this is the main reason for defining the property enthalpy.

So, let us now summarize a bit of what we have learnt from the case of closed system and a flow system. So, for the case of simple compressible system, we know that energy consists of U, kinetic energy and a potential energy. So, this is a system which usually we consider in a closed system or in general where there is no flow involved, where the energy is basically nothing, but internal energy and its kinetic energy and potential energy, which is the molecular part of it. So, we can represent the kinetic energy in terms of velocity of this.

Now, for the case of control volume energy, we will be using this, but fluids which are entering or leaving the control volume contains or possess additional energy and that is the work flow or the flow energy, so we have to know account that particular energy also a part of the total energy of the following system. So, that is what we are writing here.

$$e = u + ke + pe = \frac{v^2}{2} + gz$$

Consider, so, we have now in addition to u + PV, which is nothing but the enthalpy. So, we define this as an enthalpy this is the total energy of the flow following fluid u + PV + kinetic energy + potential energy. So, you can write simply as h here. So, this precisely the reason actually to use h is, because it takes care of flow energy part, so, when if you ignore the kinetic energy and potential energy, e is nothing but simply h.

$$e = h + ke + pe = u + Pv + \frac{v^2}{2} + gz$$

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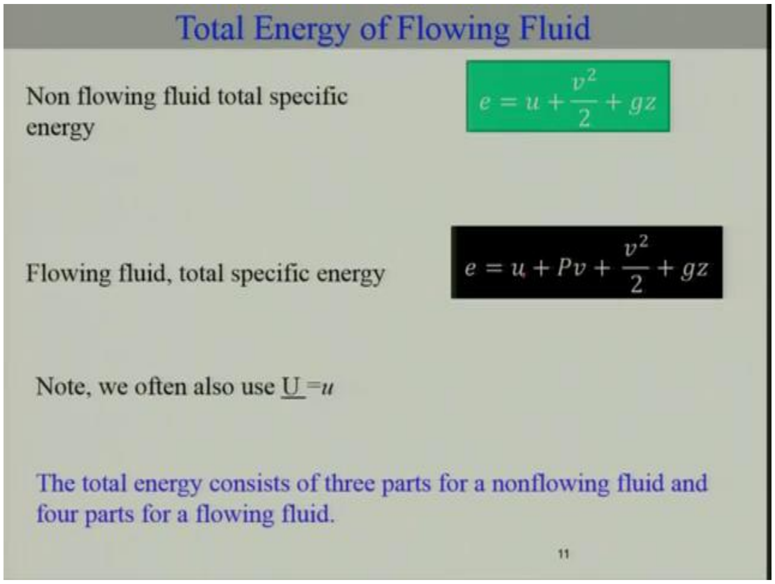

Total Energy of Flowing Fluid

Non flowing fluid total specific energy $e = u + \frac{v^2}{2} + gz$

Flowing fluid, total specific energy $e = u + Pv + \frac{v^2}{2} + gz$

Note, we often also use $\underline{U} = u$

The total energy consists of three parts for a nonflowing fluid and four parts for a flowing fluid.

So, this is what I am trying to summarize, so, remember this that we are we are considering here is nothing but mole of energy or specific energy depending on the units which you would like to use, but most of the time we will be using per mole. Sometimes, we will use per Kg as well. But again, this depends on the system which you are interested in.

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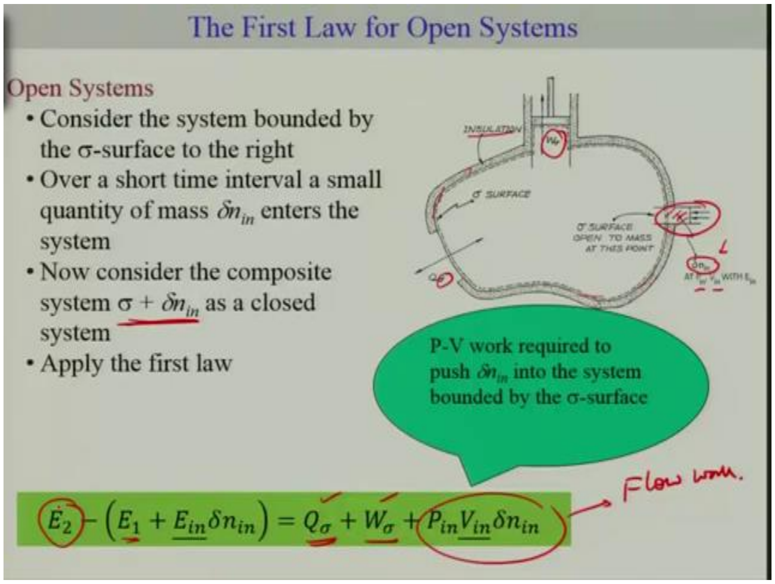

The First Law for Open Systems

Open Systems

- Consider the system bounded by the σ-surface to the right
- Over a short time interval a small quantity of mass δn_{in} enters the system
- Now consider the composite system $\sigma + \delta n_{in}$ as a closed system
- Apply the first law

P-V work required to push δn_{in} into the system bounded by the σ-surface

$$E_2 - (E_1 + \underline{E}_{in}\delta n_{in}) = Q_\sigma + W_\sigma + P_{in}\underline{V}_{in}\delta n_{in}$$

Flow work.

Alright, so now let us little bit take this idea and now apply the understanding of this energy of the flow system and concepts here, on the open system to obtain a generic expression for the first law for the open system. So, consider this particular system which is bounded by surface, now this surface which is I am showing here has certain region which is insulated, and certain which are not so there is a possibility or there is a heat transfer possible across that and this Q subscript to sigma indicates that this is occurring across this particular surface.

$$\dot{E}_2 - (E_1 + E_{in}\delta n_{in}) = Q_\sigma + W_\sigma + P_{in}V_{in}\delta n_{in}$$

Similarly, it has some piston here so it can also do some work, so the work associated with this moving boundary is also represented as W of sigma and then you have this open region where the fluids can flow back in and here this amount of fluid at given delta t is nothing but delta n_{in} at Pin Vin. And it has energy E in, so, now if you consider at a given moment for within the delta t time, that this along with this part as the complete system, then this would be a closed system.

So, that means sigma here plus this region together will be closed system and then you can apply the first law. So, this may be moving, this may be, moving a bit. But at a given time this is the only amount which gets inside, so, if you apply the First Law, what you are going to get? So, initially you have the energy of this system, this within the fluid plus this part. So, that is your, that is the E1 plus the E in. The molar energy or specific energy multiplied by the amount of mass which is associated with this.

So, remember that we are considering the sigma surface plus this as a closed system and applying it. So, this is the one way of writing this expression, but of course you can come up with the different ways, it is not the only way, to obtain the First Law for Open Systems expression. So, this is the amount of energy at the time of T and by the time this completely gets in, this is the amount, which we are saying this is E2.

But now we have to also look into the boundary condition or the boundaries here. So, you have Q which is the heat transfer plus ‘W’ which is being applied here and you have to also consider the additional work, which this particular region has done and that is given by Pin Vin delta Nin. So, this is the kind of heat, piston work around this sigma term plus this part, which is this region, ok. So, the work in the moving boundary and this work, due to the flow in, that is given by; this is flow work. So, this is the total boundary and this is the energy of this fluid; inlet minus final the E2, within this delta theta.

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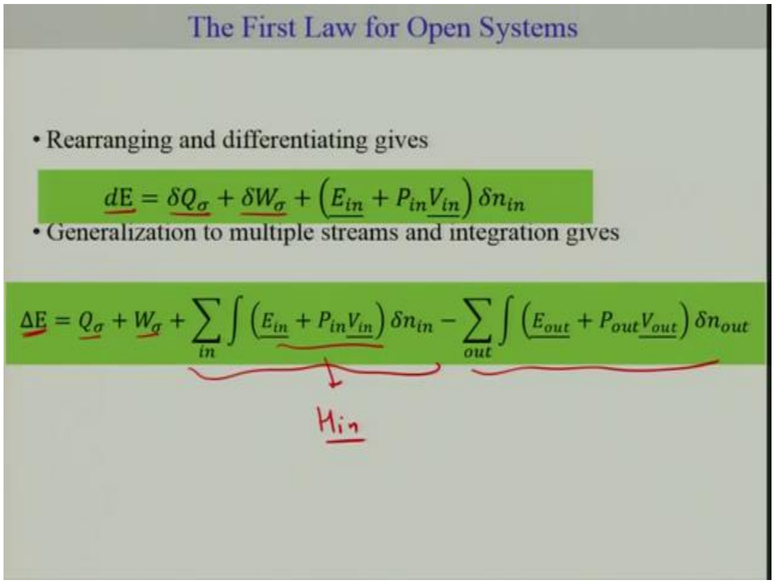

The First Law for Open Systems

- Rearranging and differentiating gives

$$\underline{dE} = \underline{\delta Q_\sigma} + \underline{\delta W_\sigma} + \left(\underline{E_{in}} + P_{in}\underline{V_{in}}\right)\delta n_{in}$$

- Generalization to multiple streams and integration gives

$$\underline{\Delta E} = \underline{Q_\sigma} + \underline{W_\sigma} + \underbrace{\sum_{in}\int \left(\underline{E_{in}} + P_{in}\underline{V_{in}}\right)\delta n_{in}}_{\underline{H_{in}}} - \underbrace{\sum_{out}\int \left(\underline{E_{out}} + P_{out}\underline{V_{out}}\right)\delta n_{out}}_{}$$

So, we can now rearrange this expression and differentiate.

$$dE = \delta Q_\sigma + \delta W_\sigma + (E_{in} + P_{in}V_{in})\delta n_{in}$$

So, if you generalize this for multiple stream and then integrate it,

$$\Delta E = Q_\sigma + W_\sigma + \sum_{in} \int (E_{in} + P_{in}V_{in})\delta n_{in} - \sum_{out} \int (E_{out} + P_{out}V_{out})\delta n_{out}$$

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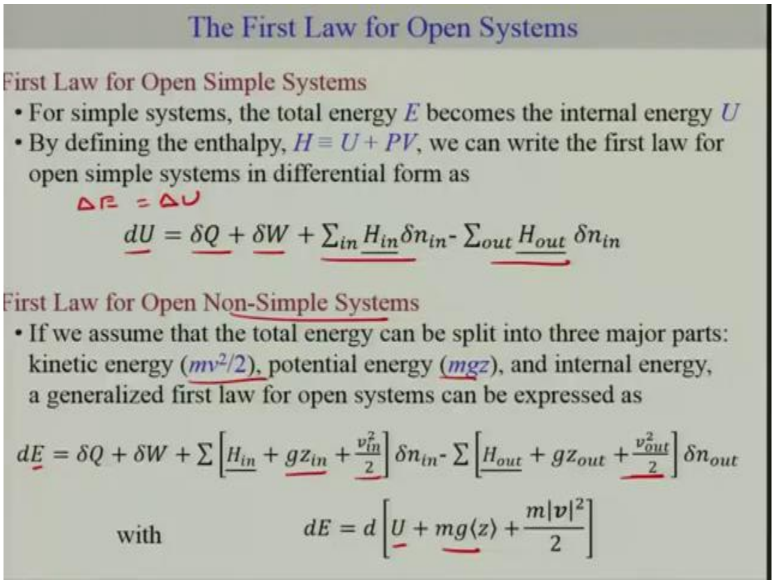

The First Law for Open Systems

First Law for Open Simple Systems

- For simple systems, the total energy E becomes the internal energy U
- By defining the enthalpy, $H \equiv U + PV$, we can write the first law for open simple systems in differential form as

$\Delta E = \Delta U$

$$\underline{dU} = \underline{\delta Q} + \underline{\delta W} + \underline{\sum_{in} H_{in}}\delta n_{in} - \underline{\sum_{out} H_{out}}\,\delta n_{in}$$

First Law for Open Non-Simple Systems

- If we assume that the total energy can be split into three major parts: kinetic energy ($mv^2/2$), potential energy (mgz), and internal energy, a generalized first law for open systems can be expressed as

$$d\underline{E} = \delta Q + \delta W + \sum\left[\underline{H_{in}} + \underline{gz_{in}} + \underline{\frac{v_{in}^2}{2}}\right]\delta n_{in} - \sum\left[\underline{H_{out}} + gz_{out} + \underline{\frac{v_{out}^2}{2}}\right]\delta n_{out}$$

with $$dE = d\left[\underline{U} + m\underline{g\langle z\rangle} + \frac{m|\boldsymbol{v}|^2}{2}\right]$$

Now, let us try to summarize again, so for a simple system, I can write:

$$\Delta E = \Delta U$$

$$dU = \delta Q + \delta W + \sum_{in} H_{in}\delta_{in} - \sum_{out} H_{out}\delta n_{in}$$

For the case of a non-simple system, we have to include the kinetic energy and potential energy.

$$dE = \delta Q + \delta W + \sum\left[H_{in} + gz_{in} + \frac{v_{in}^2}{2}\right]\delta n_{in} - \sum\left[H_{out} + gz_{out} + \frac{v_{out}^2}{2}\right]\delta n_{out}$$

With $$dE = d\left[U + mg < z > + \frac{m|v|^2}{2}\right]$$

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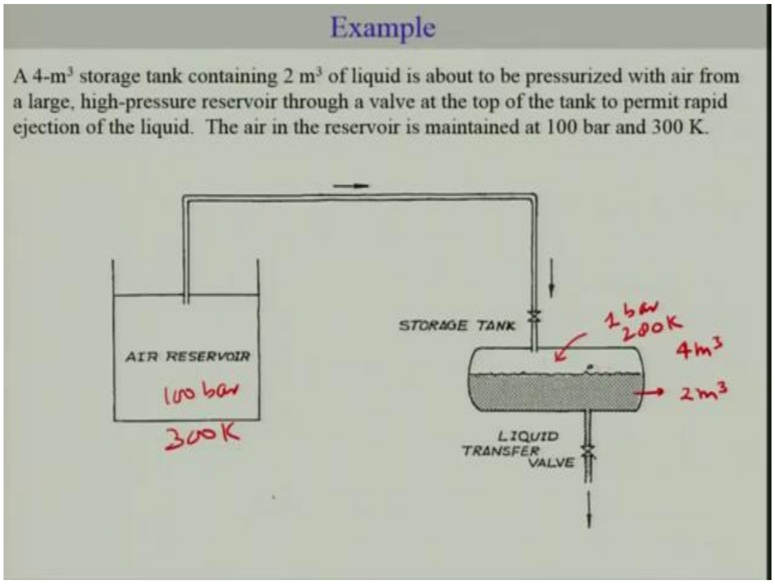

Example

A 4-m^3 storage tank containing 2 m^3 of liquid is about to be pressurized with air from a large, high-pressure reservoir through a valve at the top of the tank to permit rapid ejection of the liquid. The air in the reservoir is maintained at 100 bar and 300 K.

So, while we have just summarized and did very quick analysis of the Open System for the mass energy expression, so this is an example, it is 4 meter cube storage tank, containing 2 meter cube of liquid, so, is about to be pressurized with air, so this is basically the volume, total volume is 4 meter cube. The current liquid is 2 meter cube, and it is about to be pressurize with an air from our large high pressure reservoir, through a valve, at the top of the tank, to permit rapid ejection of the liquid. So, you pressurize it and then this is to reject liquid rapidly, the air in this reservoir is maintained at 100 bar and 300 kelvin.

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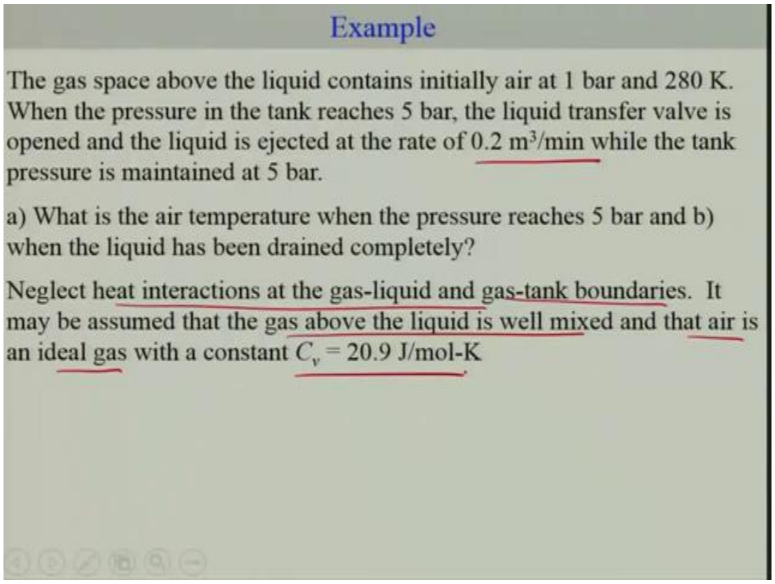
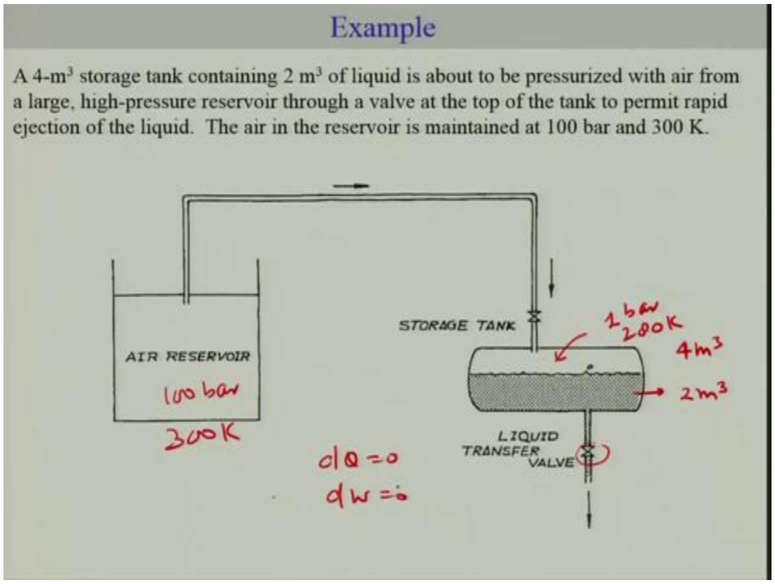


Example

The gas space above the liquid contains initially air at 1 bar and 280 K. When the pressure in the tank reaches 5 bar, the liquid transfer valve is opened and the liquid is ejected at the rate of 0.2 m^3/min while the tank pressure is maintained at 5 bar.

a) What is the air temperature when the pressure reaches 5 bar and b) when the liquid has been drained completely?

Neglect heat interactions at the gas-liquid and gas-tank boundaries. It may be assumed that the gas above the liquid is well mixed and that air is an ideal gas with a constant $C_v = 20.9$ J/mol-K



Example

A 4-m^3 storage tank containing 2 m^3 of liquid is about to be pressurized with air from a large, high-pressure reservoir through a valve at the top of the tank to permit rapid ejection of the liquid. The air in the reservoir is maintained at 100 bar and 300 K.

So, the gas space it was the liquid contains, initially air at so, as air at 1 bar and 280 kelvin, so, basically this air, which is there is 1 bar and 280 kelvin. When the pressure in the tank reaches 5 bars, the liquid transfer valve is opened and the liquid is ejected at the rate of point 2 meter cube per minute, while the tank pressure is maintained at 5 bars.

So, this is something which is very clear that, once, this storage tank is, the valve is opened, it gets pressurized until 5 bar and then we open up this valve in order to eject the liquid at the rate of 2.2 meter cube per minute. So, the question is “What is the air temperature when the pressure reaches

5 bars? That is the first part of the question and the second is “When the liquid has been drained completely what is the air temperature in this condition?

So, we have to neglect the heat interactions at the gas liquid and the gas boundaries. It is assumed that the gas above the liquid is well mixed and that the air is ideal gas with heat capacity of 20.9 Joules per Mole-Kelvin. So, this is the question, so how do you solve this problem. So, we can consider this because, we have been asked to ignore the heat interaction, ok, and there is no boundary movement or boundary work associated with the boundary, so hence we know that dQ is 0 or Q is 0 and basically dW is equal to zero, in this case.

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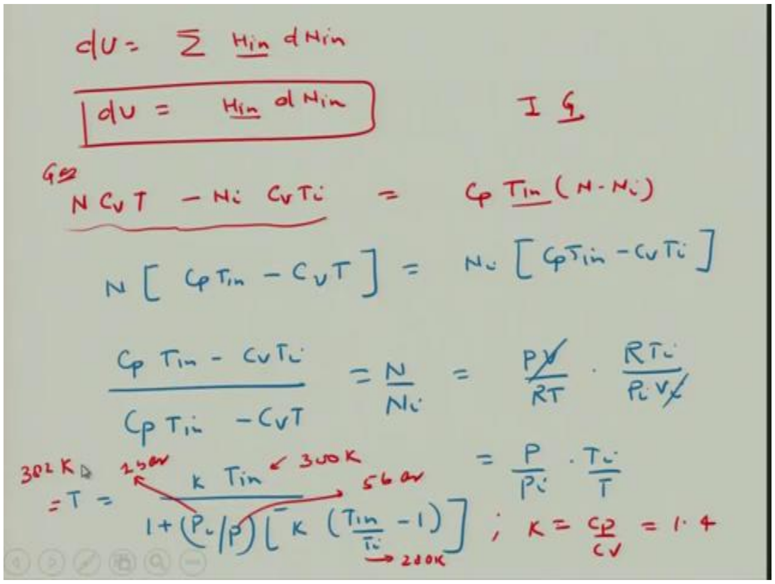


$$dU = \Sigma \underline{H_{in}}\, dN_{in}$$

$$\boxed{dU = \underline{H_{in}}\, dN_{in}}$$

I G

Gas

$$\underline{N C_v T - N_i C_v T_i} = C_p \underline{T_{in}} (N - N_i)$$

$$N [C_p T_{in} - C_v T] = N_i [C_p T_{in} - C_v T_i]$$

$$\frac{C_p T_{in} - C_v T_i}{C_p T_{in} - C_v T} = \frac{N}{N_i} = \frac{P\cancel{V}}{RT} \cdot \frac{RT_i}{P_i \cancel{V}} = \frac{P}{P_i} \cdot \frac{T_i}{T}$$

$$302\,K = T = \frac{k\, T_{in}}{1 + (P_i/P)\left[k\left(\frac{T_{in}}{T_i} - 1\right)\right]} \; ; \; k = \frac{C_p}{C_v} = 1.4$$

25 bar

300 K

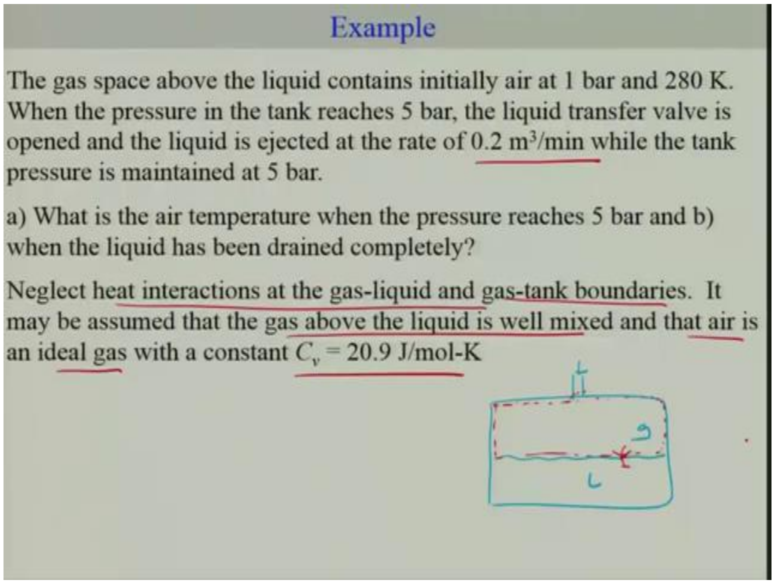
5 bar

200 K

$$dU = \sum H_{in} dN_{in}$$

$$N C_V T - N_i C_V T_i = C_P T_{in} (N - N_i)$$

$$\frac{N}{N_i} = \frac{C_p T_{in} - C_V T_i}{C_P T_{in} - C_V T} = \frac{P}{P_i} \frac{T_i}{T}$$



Example

The gas space above the liquid contains initially air at 1 bar and 280 K. When the pressure in the tank reaches 5 bar, the liquid transfer valve is opened and the liquid is ejected at the rate of 0.2 m^3/min while the tank pressure is maintained at 5 bar.

a) What is the air temperature when the pressure reaches 5 bar and b) when the liquid has been drained completely?

Neglect heat interactions at the gas-liquid and gas-tank boundaries. It may be assumed that the gas above the liquid is well mixed and that air is an ideal gas with a constant C_v = 20.9 J/mol-K

In that case, we can simply write dU of the tank, and that will be simply summation whatever comes in, because, so this is going to be ok. So, this is going to be heat enthalpy, molar heat enthalpy multiplied by the differential of whatever the moles coming in, so it could be actually mass or mole, you can consider this mass or mole, but in this case we are simply considering Hin and so here it is Hin, because there is only 1 stream, so it is going to be Hin dNin.

So, this is our basic expression here, so whatever the change in the internal energy of the tank is due to the flow in, ok. So, now with this expression we can expand or this is the ideal gas and we know when we can (ca) represent U in terms of C V, so when we integrate this, so it could be some N Cv Tf minus Ni Cv Ti is nothing but Cp Tin N minus Ni. Now, you may ask this question, “How I am writing this?”

So one other thing which I missed and which is good to tell is that, so you have this liquid, ok and then you have this gas here so this is your gas here, so what I am trying to do is, I am taking the control volume in this way. So, we are considering the inlet and then this your control volume and of course there is no heat interaction, and we are expecting that this particular kind of interface remains stable.

So, there is no exchange of the mass also, from here to here, so there is no exchange of the mass, so that does not happen, so; there is no heat interaction and there is no work and this this boundary does not move as well. So, with this we can write this expression simply for the gas and hence this N Cv Tf minus Ni Cv Ti and this is, this Hin which I can write here as (Tp T) Cp Tin.

So, what we are assuming is that the temperature of course has been also set temp, what it comes here, comes as Tin of the gas. So, what you have is Cp Tin of the particular gas and then number of moles which has been transferred, which was been nothing but N minus Ni, whatever the final number of moles minus the initial number of moles.

So, we can now simplify this expression we can write this as N Cp Tin minus Cv T, so, I think what we can do is instead of saying Tf because it is our second part, what we can do is, I can simply write T here. So, that way it is going to be simple, so this is going to be T and this is going to be Ni Cp Tin minus Cv Ti. So, with this I can get Cp Tin minus Cv Ti, Cp Tin minus Cv T, this is going to be N by Ni.

Now, considering this to be ideal gas, you can write, you can make use of ideal gas expression, this is going to be Pv by RT and this is going to be RTi by Pv Vi, ok and this I can write as P by Pi into T by Ti, because V's get cancelled, it is same. So, with this, I can rearrange this, expression and I can come up with the expression of T as K times Tin, 1 plus Pi by P, k times Tin by Ti minus 1. So, this is the final expression I can get.

If you rearrange this now with k as Cp by Cv, Cp is nothing but Cv plus I, so we will put again, the values, you are going to get 1.4, so with this your T comes out to be, so and the rest of the information we know, Tin is 300 kelvin, Ti is 280 kelvin, initial temperature, Pi is 1 bar, P is 5 bar, and 'k' you know is 1.4. So, with this if we plug in this information, the 'T' comes out to be 382 kelvin. So, that is how we can solve the first part of this problem.

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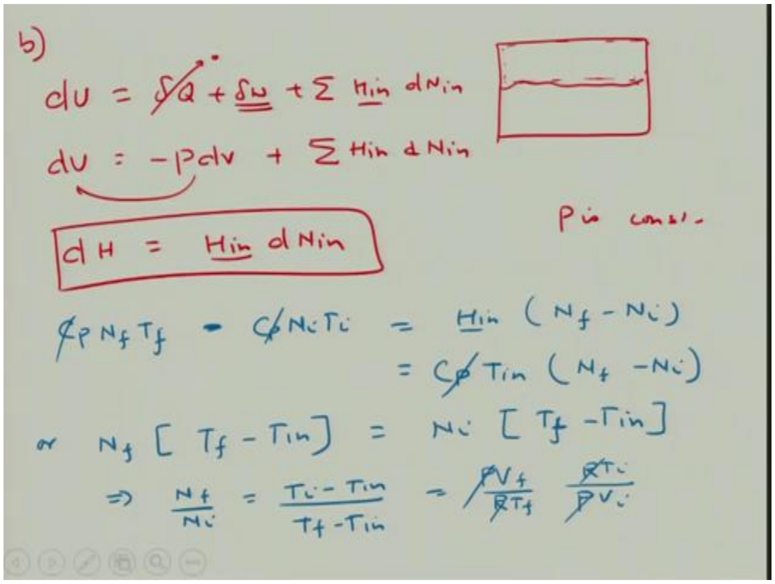

b)

$$dU = \cancel{\delta Q} + \underline{\delta W} + \Sigma\, \underline{H_{in}}\, dN_{in}$$

$$dU = -pdV + \Sigma H_{in}\, dN_{in}$$

$$\boxed{dH = \underline{H_{in}}\, dN_{in}}$$

P is const.

$$\cancel{C_p} N_f T_f - \cancel{C_p} N_i T_i = \underline{H_{in}} (N_f - N_i)$$
$$= \cancel{C_p} T_{in} (N_f - N_i)$$

$$\text{or} \quad N_f [T_f - T_{in}] = N_i [T_f - T_{in}]$$

$$\Rightarrow \frac{N_f}{N_i} = \frac{T_i - T_{in}}{T_f - T_{in}} = \frac{\cancel{P} V_f}{\cancel{R} T_f} \frac{\cancel{R} T_i}{\cancel{P} V_i}$$

Now, the second part is about, the air when the liquid has been completely drained. So this is little bit, little tricky because in this case, you consider this gas and this liquid, so this boundary, even if you consider this system, here control the control volume, this boundary is going to move and hence, there will be some work which we have to consider. So, let us consider the part b:

$$dU = \delta Q + \delta W + \sum H_{in} dN_{in}$$

$$dU = -pdV + \sum H_{in} dN_{in}$$

$$dH = H_{in} dN_{in}$$

$$C_p N_f T_f - C_p N_i T_i = H_{in}(N_f - N_i) = C_p T_{in}(N_f - N_i)$$

$$N_f[T_f - T_{in}] = N_i[T_i - T_{in}]$$

$$\frac{N_f}{N_i} = \frac{T_i - T_{in}}{T_f - T_{in}} = \frac{V_f}{T_f}\frac{T_i}{V_i}$$

Now, this part of course is 0, because there is no heat interaction, but what about this, because this is not any more 0, because this boundary is going to move, as you drain it completely, ok, so this has to be Pdv plus summation Hin dNin, ok. So, we are again considering this part here, this boundary is going to move, ok. Now, this is your dU, ok, so if you bring it here it becomes dH, ok, because pressure is constant, P is constant, it is 5 bar, so this is now with, Hin dNin, ok.

So, this is now the expression, so now the rest of the thing remains the same, ok, this is again, we have to consider molar form, so I am going to write now as quickly expression, this is going to be now Cp Nf Tf, now I can write final minus Cp Ni Ti, this is going to be Hin Nf minus Ni, ok, Hin can also be written as Cp Tin, Nf-Ni ok. Now, you can re arrange this, or I can combine this terms related to Nf that would be Nf times this, is equal to Ni Tf minus Tin, ok, Cp gets cancelled.

So with this we have, Nf by Ni is Ti minus Tin divided by Tf minus Tin and this Nf can be now written as if you make use of ideal gas, then, it is Pvf by RTf, this is going to be RTi by Pvi so this is the expression, P is going to be common, so I cancel it, so what remains is this 2 terms and now you have to rearrange this part, so this is the last few steps, so this requires a bit of rearrangement.

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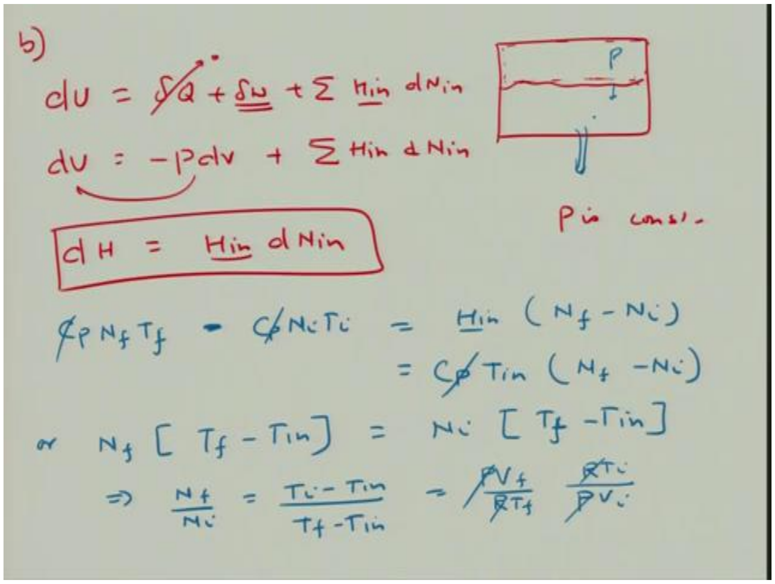

b)

$$dU = \cancel{\delta Q} + \underline{\underline{\delta W}} + \sum \underline{H_{in}}\, dN_{in}$$

$$dU = -Pdv + \sum H_{in}\, dN_{in}$$

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$$= \cancel{C_p} T_{in} (N_f - N_i)$$

or $N_f [T_f - T_{in}] = N_i [T_f - T_{in}]$

$$\Rightarrow \frac{N_f}{N_i} = \frac{T_i - T_{in}}{T_f - T_{in}} = \frac{\cancel{P} V_f}{\cancel{R} T_f} \frac{\cancel{R} T_i}{\cancel{P} V_i}$$

So basically from here you can write it like this, Vi by Vf, Tf by Ti, is equal to Tin minus Tf, Tin minus Ti, now if you rearrange this expression, the rest of basically is algebra bit. So, you can now come up with an expression as Tf as Tin, Vi by Vf, Tin by Ti minus 1 plus 1, so this is the final expression after rearranging. Now, you can plug in all the values, so if you plug in all the values, you are going to get, 336 kelvin.

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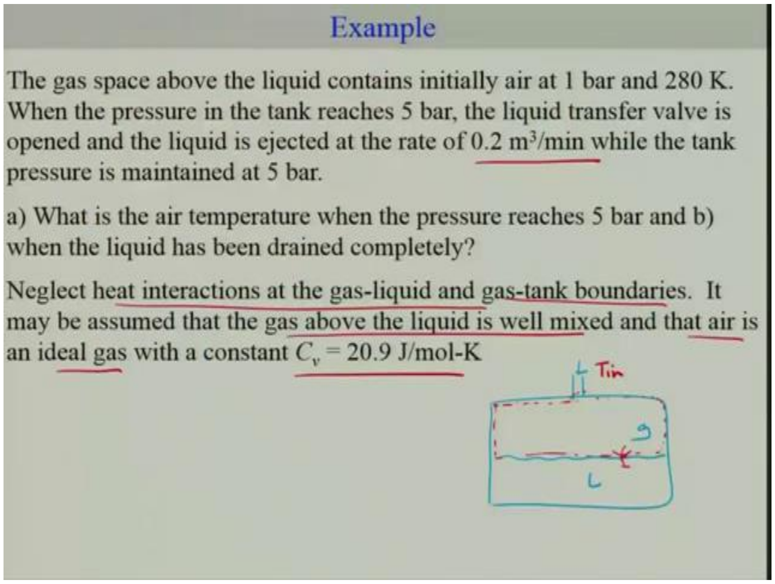

Example

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a) What is the air temperature when the pressure reaches 5 bar and b) when the liquid has been drained completely?

Neglect heat interactions at the gas-liquid and gas-tank boundaries. It may be assumed that the gas above the liquid is well mixed and that air is an ideal gas with a constant $C_v = 20.9$ J/mol-K

So, the message here is that, when the, it comes to the case of the first part, there was no boundary movement here, ok. Hence we did not considered any work associated with this because this gas liquid was frozen, or was not moving.

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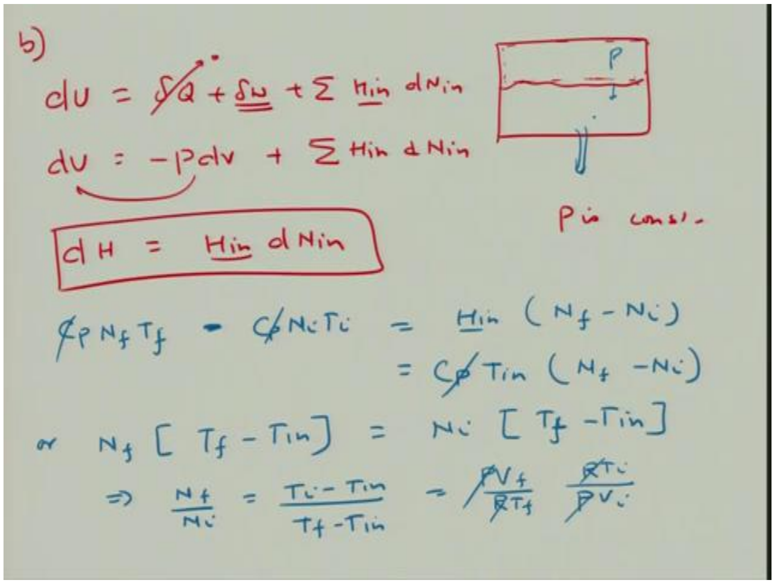

b)

$$dU = \cancel{\delta Q} + \underline{\underline{\delta W}} + \sum \underline{H_{in}}\, dN_{in}$$

$$dU = -PdV + \sum H_{in}\, dN_{in}$$

$$\boxed{dH = \underline{H_{in}}\, dN_{in}}$$

P is const.

$$\cancel{C_P} N_f T_f - \cancel{C_P} N_i T_i = \underline{H_{in}} (N_f - N_i)$$

$$= \cancel{C_P} T_{in} (N_f - N_i)$$

or $N_f [T_f - T_{in}] = N_i [T_f - T_{in}]$

$$\Rightarrow \frac{N_f}{N_i} = \frac{T_i - T_{in}}{T_f - T_{in}} \rightarrow \frac{\cancel{P} V_f}{\cancel{R} T_f} \frac{\cancel{R} T_i}{\cancel{P} V_i}$$

But in the case 2, when you deplete the liquid this will start moving, and hence you have to consider the work associated with this, with a pressure P and the changes in the volume accordingly. So, this is why we have considered this and this is nothing but, the basically the differential change in the H. So, I hope that you get the complete idea of the making use of the First Law of Open System, with this illustration, so we will be continuing some more with some more examples in the next class. So, see you next time.