Thermodynamics of Fluid Phase Equilibria

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Lecture – 06
Energy analysis of closed system

Welcome back, in this lecture we will be reviewing the Energy Analysis Of Closed System. So, as I already discussed the for the closed system you have only interaction with the surrounding is through energy ok, that means, energy can exchange with the surrounding and that would be either through work or heat of work.

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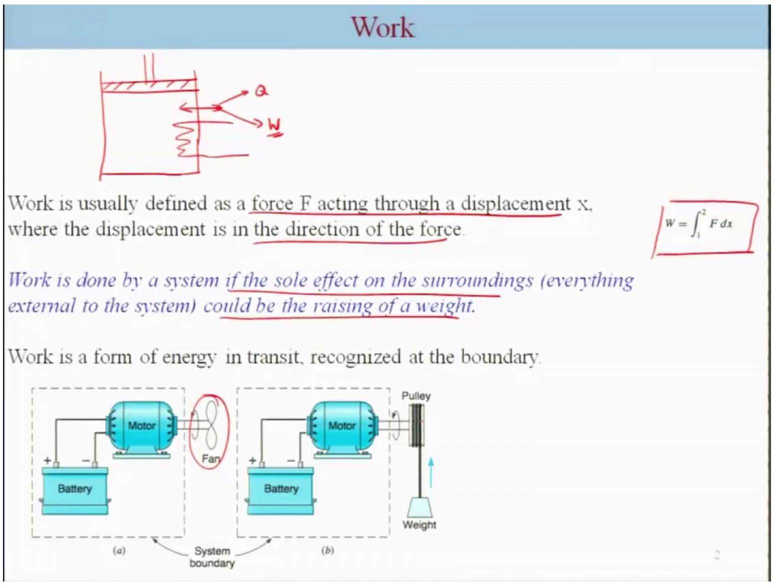

Work

Work is usually defined as a force F acting through a displacement x, where the displacement is in the direction of the force.

$$W = \int_1^2 F\,dx$$

Work is done by a system if the sole effect on the surroundings (everything external to the system) could be the raising of a weight.

Work is a form of energy in transit, recognized at the boundary.

So, let me first describe the work here. So, the work can be of different types. So, the one classical definition is defined as a force acting through a displacement where the displacement is in the direction of the force and thus we typically write the work in this form. Now for such a system of course, physically the system may not displace as such ok.

But, if this piston is movable; that means, a boundary is movable then it can displace the volume of the gas containing it, but in the other form there could be other kind of work such as electrical work which also will supply certain work to the system.

So, a generic definition is that work is done by a system if the sole effect on the surrounding could be the raising of a weight. We can take an example of a battery motor which is used to run a fan and the effect of work by the system on the surrounding that is if we consider this as a part of it, then this can be understood by the same definition by replacing the fan by a pulley weight system.

So, while the shaft rotates it is effect is basically to raise this weight through this pulley. So, this is a generic definition of the work; now as I said the work could be of different type, one is a the boundary work if the boundary work is if the boundary is moveable.

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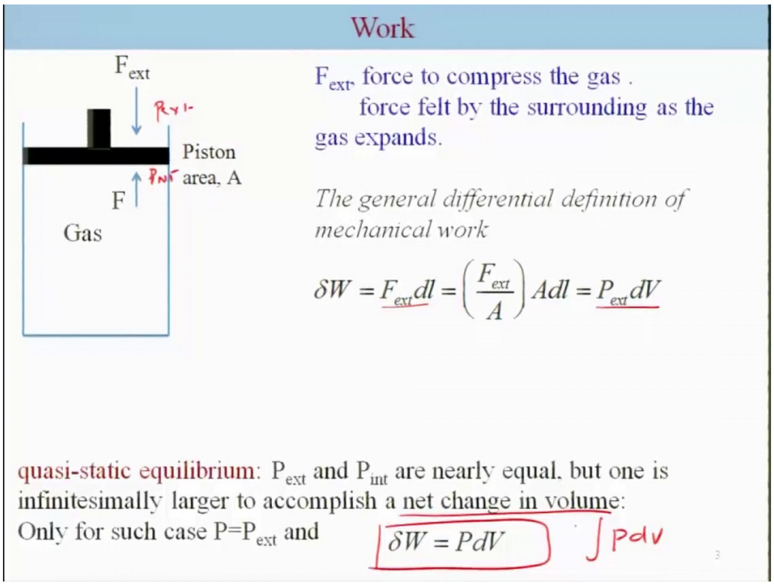

Work

F_{ext}, force to compress the gas .
force felt by the surrounding as the gas expands.

The general differential definition of mechanical work

$$\delta W = \underline{F_{ext} dl} = \left(\frac{F_{ext}}{A}\right) A dl = \underline{P_{ext} dV}$$

quasi-static equilibrium: P_{ext} and P_{int} are nearly equal, but one is infinitesimally larger to accomplish a net change in volume:
Only for such case $P=P_{ext}$ and $\boxed{\delta W = PdV}$ $\int Pdv$

Now, how do we define for such a system let us say the boundary work here now a general definition of mechanical work in this case using the piston cylinder device is simply by F external multiplied by the displacement d here, where F external is nothing, but the force to compress the gas or the force felt by the surrounding as the gas expands and you can write F external in terms of P external multiplied by dV, V where V is the volume which get displaced. For the quasi static equilibrium the P external here the P external and P internal are nearly equal, but one is in fact, a similarly larger to accomplish a net change in the volume ok.

For a very slow process you can write the differential work as simply P dV and thus you can integrate this in order to get the work you can integrate the PdV. And if you know the relation between P and V you can easily get the work out of it.

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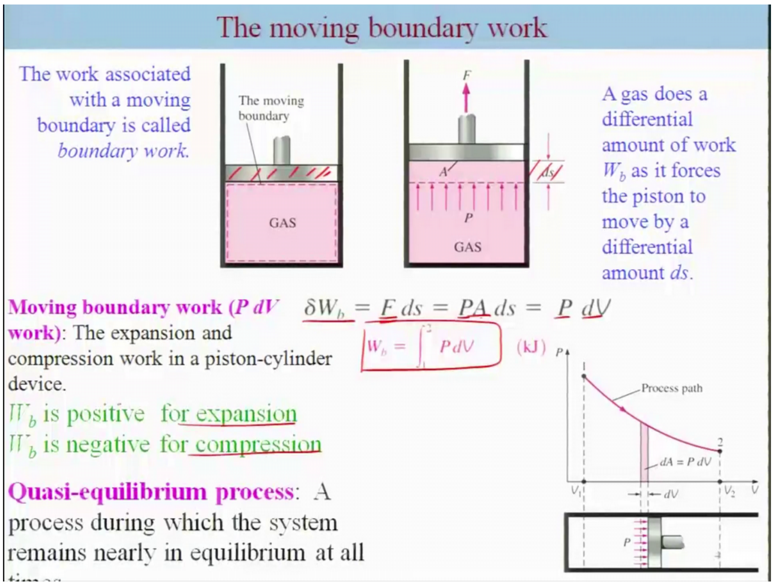

The moving boundary work

The work associated with a moving boundary is called *boundary work.*

A gas does a differential amount of work W_b as it forces the piston to move by a differential amount *ds*.

Moving boundary work (*P dV* work): The expansion and compression work in a piston-cylinder device.

$$\delta W_b = F\,ds = PA\,ds = P\,dV$$

$$\boxed{W_b = \int_1^2 P\,dV} \quad (\text{kJ})$$

W_b is positive for expansion

W_b is negative for compression

Quasi-equilibrium process: A process during which the system remains nearly in equilibrium at all

Now, as I said the boundary work is as a simplest case here, the piston can be moved and thus upon applying certain heat all sudden changes in the state conditions, so, the boundary can move and thus can undergo certain work. So, for example, for a differential change in the volume or the piston location by d s the differential amount of the work can be written as d del W b is equal to F into d s and F as I said it could be simply the pressure multiplied with the cross sectional area and this can be written as P dV ok.

So, the integral form of this would be w b is equal to integral from state 1 to state 2 and P dV. If it is a quasi equilibrium process one can draw a process da path out of it and thus you can simply integrate for such a system if you know the path, that means, you know the relation between P and V.

Now, W b for the sign based on the sign convention is positive for expansion and negative for compression ok that means, if the work is done by the system and surrounding is positive otherwise it is negative ok.

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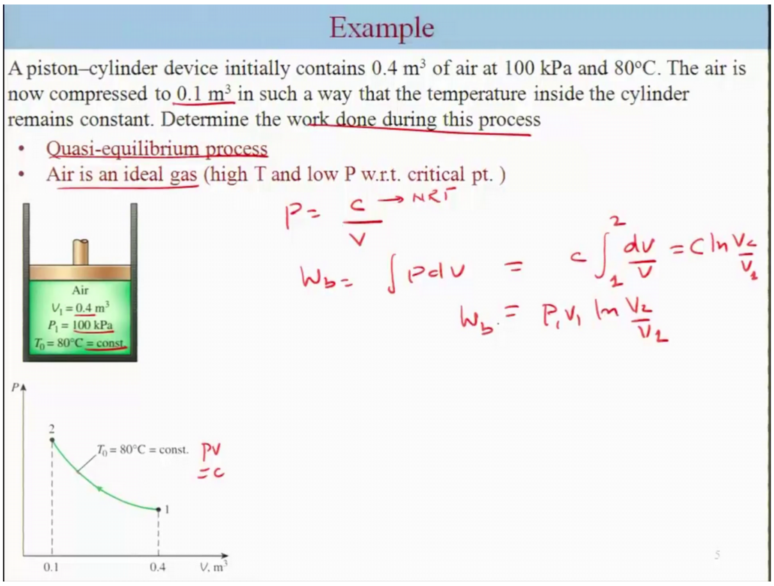

Example

A piston–cylinder device initially contains 0.4 m^3 of air at 100 kPa and 80°C. The air is now compressed to 0.1 m^3 in such a way that the temperature inside the cylinder remains constant. Determine the work done during this process

- Quasi-equilibrium process
- Air is an ideal gas (high T and low P w.r.t. critical pt.)

$P = \frac{C}{V}$ $C \rightarrow NRT$

$W_b = \int P\,dV = C\int_1^2 \frac{dV}{V} = C\ln\frac{V_2}{V_1}$

$W_b = P_1V_1 \ln\frac{V_2}{V_2}$

$PV = C$

So, let me just use this concept to do a quick example and some of you have already done this thing it is just a recap in order to speed up all the understanding for a more complicated subject or concepts in the later part of this course. So, this kind of example certainly is going to help us later on ok. So, this is a simple question the question is a piston cylinder device which has air containing at this volume 0.4 meter cube at 100 kilo Pascal and 80 degree Celsius, and it is compressed to 0.1 meter cube, in such a way that the temperature is constant,.

Now, we need to find out the work done during the process. So, what we are going to assume is that this is a quasi equili equilibrium process, further we are going to assume that the air is at the air is basically an ideal gas well because of very simple reason temperature is very high compared to it is critical point and at the word the process can be drawn on a P V diagram by this green curve from 1 to 2, such that T is equal to constant or in other word P V is equal to constant.

Because, V is equal to m N R T can be written directly like this or we can write P is equal to C some constant divided by V where C is nothing, but N R T. So, I can now write the boundary work simply as this would be your C ln V 2 by V 1 and C is nothing, but P 1 V 1 ok, ln V 2 by V 1. So, this is as very simple example from here we can get this boundary work.

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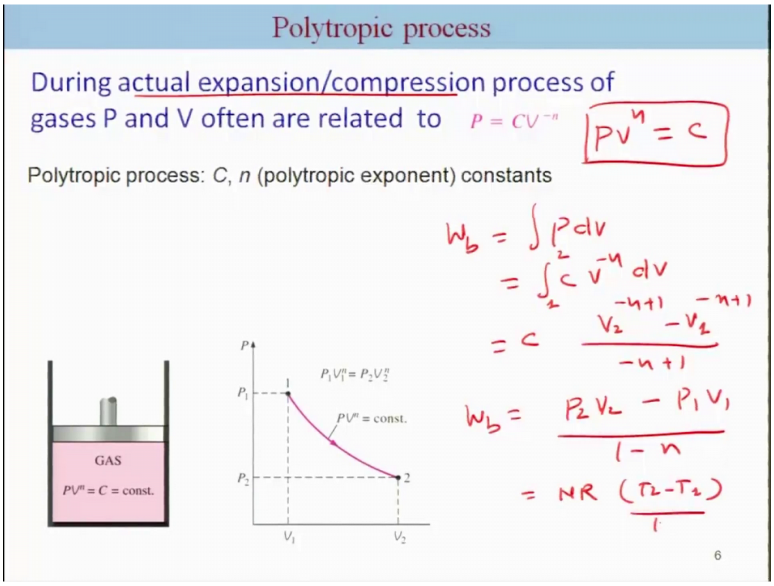

Polytropic process

During actual expansion/compression process of gases P and V often are related to $P = CV^{-n}$ $\boxed{PV^n = C}$

Polytropic process: *C*, *n* (polytropic exponent) constants

$$W_b = \int P\,dV$$

$$= \int_1^2 C\,V^{-n}\,dV$$

$$= C\,\frac{V_2^{-n+1} - V_1^{-n+1}}{-n+1}$$

$$W_b = \frac{P_2V_2 - P_1V_1}{1-n}$$

$$= NR\,\frac{(T_2 - T_1)}{1 -}$$

So, let us move on, in many processes the actual expansion and compression of the gases are related in a commonly relation which is called poly tropic process where P and V are often represented by this expression ok, P V to the power C to the power n is constant.

So, this is a your poly tropic process . So, where n can vary ok, so, from 1 to of n is a variable for n is equal to 1, of course, it is a constant temperature for particularly ideal gas. So, we can also find boundary work for such a system which is a poly tropic process. So, again for poly tropic process we can draw a diagram we can draw the path on the P V diagram. So, again we can write this boundary work in this form. Now, the P is given as a some constant C minus n, d V and you can rewrite this expression or you integrate it you should be able to get the following ok, where this is 1 to 2 and minus n plus 1.

This can be also written in this form or very simpler form P 2 V 2 because C V 2 minus n is nothing, but P 2 P 2 multiplies by the remaining term V 2 is this minus P 1 V 1 and 1 minus n. So, this is the case for a generic expression for boundary work for a poly tropic process.

Now, if you have an ideal gas you can replace P 2 V 2 by N R T and thus you can get N R number of moles on the gas constant T 2 minus T 1 1 minus n and of course, n is not equal to 1 in this case.

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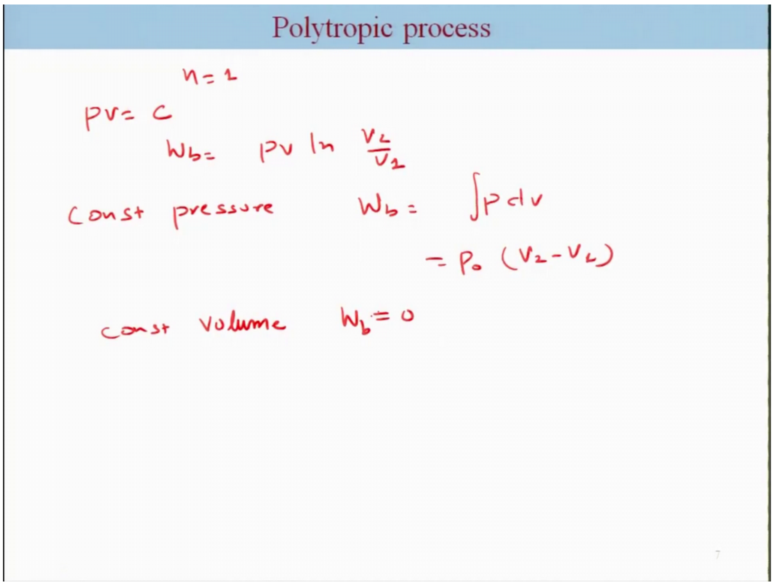

Polytropic process

$n = 1$

$PV = C$

$W_b = PV \ln \frac{V_2}{V_1}$

Const pressure $W_b = \int P\,dV$

$= P_0 (V_2 - V_1)$

const volume $W_b = 0$

You can also consider specific cases for example, if n is equal to 1 and PV is equal to C.For an ideal gas w b is going to be P V and then V 2 by V 1 which we have derived earlier also for a constant pressure W b is going to be simply P 0 which is a constant V 2 minus V 1. What about constant volume? is going to be simply 0, because the boundary is not changing at all ok. So, W b is going to be 0 for constant volume ok.

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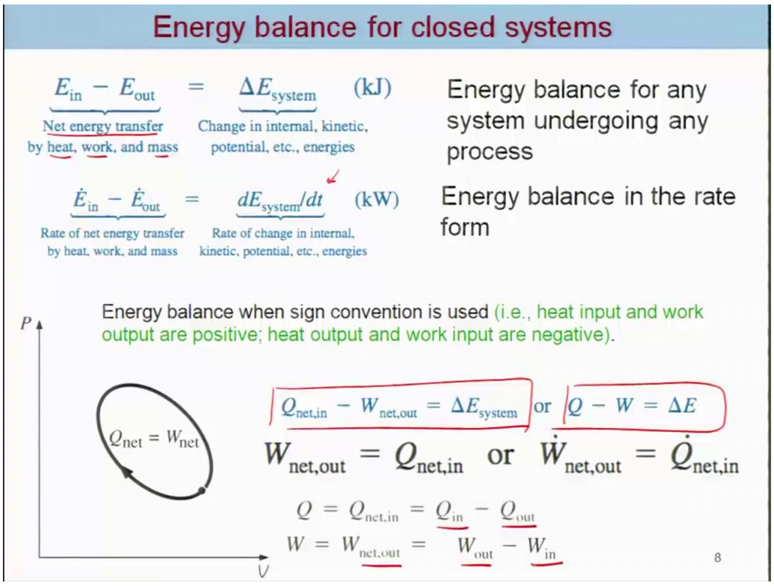

Energy balance for closed systems

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{system}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc., energies}}} \quad (\text{kJ})$$

Energy balance for any system undergoing any process

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{dE_{system}/dt}_{\substack{\text{Rate of change in internal,} \\ \text{kinetic, potential, etc., energies}}} \quad (\text{kW})$$

Energy balance in the rate form

Energy balance when sign convention is used (i.e., heat input and work output are positive; heat output and work input are negative).

$$Q_{net,in} - W_{net,out} = \Delta E_{system} \quad \text{or} \quad Q - W = \Delta E$$

$$W_{net,out} = Q_{net,in} \quad \text{or} \quad \dot{W}_{net,out} = \dot{Q}_{net,in}$$

$$Q = Q_{net,in} = Q_{in} - Q_{out}$$

$$W = W_{net,out} = W_{out} - W_{in}$$

So, having done this exercise for the boundary work, so, let us now summarize the energy balance for the closed system ok. So, as we have already discussed a generic energy balance is E in minus E out equal to delta E, E system where E in minus E out is the net energy transfer by heat work and mass.

Of course, for closed system there is no mass contribution only heat and work is going to be there whereas, for a delta E system you will have to include changes in the internal energy kinetic energy potential energy for the rate expression you include dot here which means basically is the rate of energy change with respect to time and that is also included here ok. If a sign convention is used the energy balance can be written as Q net in minus w net out equal to delta E system or in a simpler form Q minus W ok, where my definition Q is energy supplied to the system and W say as the work done by the system.

Now, for a cyclic process delta E must be 0 and in the work W net out is equal to Q net in or Q is equal to W ok, where Q net in is nothing, but Q in minus out and W net out is nothing, but W out minus W in ok. Now, this you can also derive using a generic expression here based on this without considering the sign convention.

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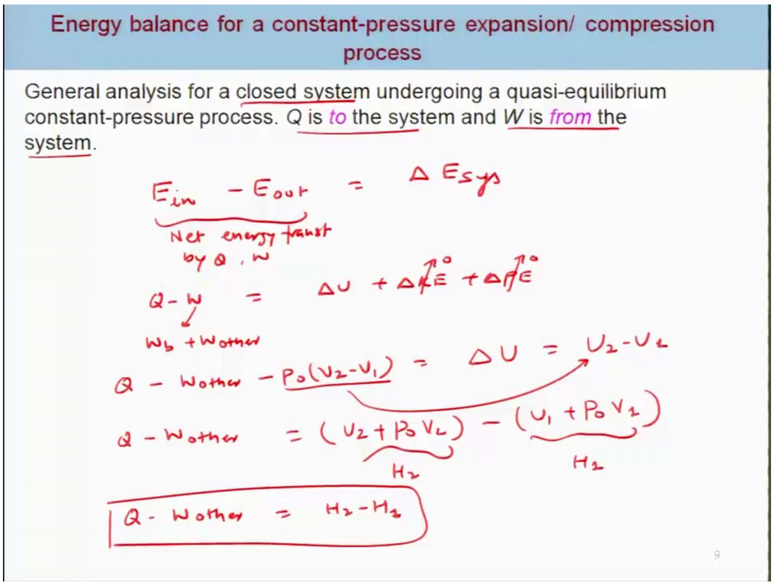

Energy balance for a constant-pressure expansion/ compression process

General analysis for a closed system undergoing a quasi-equilibrium constant-pressure process. *Q* is *to* the system and *W* is *from* the system.

$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transf by } Q,\ W} = \Delta E_{sys}$$

$$Q - \underset{W_b + W_{other}}{W} = \Delta U + \Delta \cancel{KE}^{0} + \Delta \cancel{PE}^{0}$$

$$Q - W_{other} - P_0(V_2 - V_1) = \Delta U = U_2 - U_1$$

$$Q - W_{other} = \underbrace{(U_2 + P_0 V_2)}_{H_2} - \underbrace{(U_1 + P_0 V_1)}_{H_1}$$

$$\boxed{Q - W_{other} = H_2 - H_1}$$

So, let me just now do a quick exercise on our general energy balance for a constant pressure expansion and compression process ok. So, we are going to consider is Q is applied to the system and W is done from the system. So, in this case what will be the expression for constant pressure expansion process, what will be there?

So, let me start with E in minus E out delta E system ok. Right now this is of course, for that energy transfer because of the fact that is a clau closed system. So, this will be net energy transfer by Q and W. So, Q minus W is equal to delta U plus delta K E plus delta P E and we are going to consider this to be 0. Now, this W could be W boundary and some other work. So, in that case we can write Q minus W other and W b for a constant pressure is P 0 V 2 minus V 1 and that is going to be delta u ok. So, we can take this or we can first write here is U 2 minus U 1, we can take this to this side and what remains is W other U 2 plus P 2 V 2 minus U 1 plus P 0 V 1 and this is nothing, but by definition enthalpy.

So, this will be H 2 and this will be H 1. So, for the case of a constant pressure expansion compression process for a closed system the energy valance is going to be simply H 2 minus H 1, having derived this now, we can apply it to a example.

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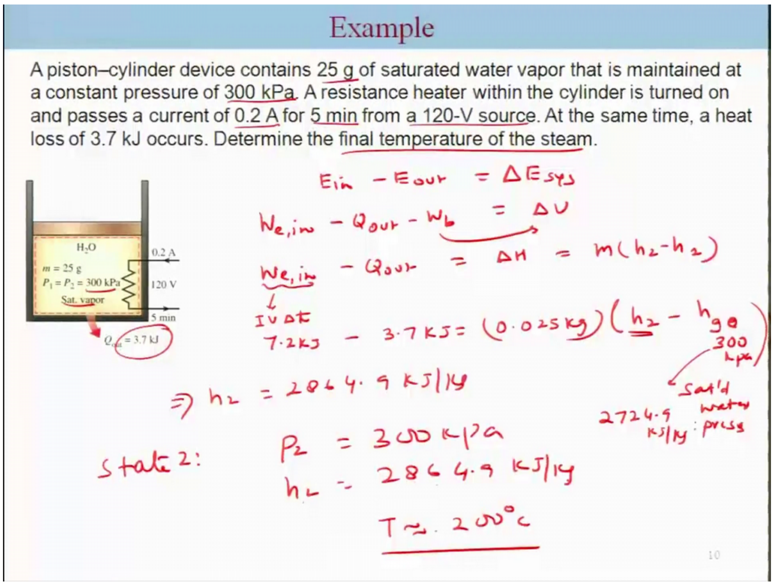

Example

A piston–cylinder device contains 25 g of saturated water vapor that is maintained at a constant pressure of 300 kPa. A resistance heater within the cylinder is turned on and passes a current of 0.2 A for 5 min from a 120-V source. At the same time, a heat loss of 3.7 kJ occurs. Determine the final temperature of the steam.

$$E_{in} - E_{out} = \Delta E_{sys}$$

$$W_{e,in} - Q_{out} - W_b = \Delta U$$

$$W_{e,in} - Q_{out} = \Delta H = m(h_2 - h_2)$$

$$\underset{\substack{\downarrow \\ IV\Delta t \\ 7.2 \text{ kJ}}}{W_{e,in}} - 3.7 \text{ kJ} = (0.025 \text{ kg})(h_2 - h_{g@300 \text{ kPa}})$$

Sat'd water press: 2724.9 kJ/kg

$$\Rightarrow h_2 = 2864.9 \text{ kJ/kg}$$

State 2: $P_2 = 300$ kPa

$h_2 = 2864.9$ kJ/kg

$T \simeq 200°C$

. So, this is our example of a piston cylinder device which contains 20 gram of saturated water vapor that is maintained at a constant pressure and that resistance heater within the cylinder is turned on and it passes a current of 0.2 ampere for 5 minutes from a 120 volt source, at the same time a heat of the heat loss of 3.7 kilojoules occurs and we to we need to find the final temperature of the system. So, we can apply that constant pressure expression. So, what we have we have again E in minus E out is delta E system.

So, ignoring the kinetic energy changes in the potential energy changes this would be delta u and if you just consider the sign which is given here which is known from the statement we can write W e in electric energy due to resistance heater minus Q out minus whatever the boundary work which is done. So, if you take out the boundary work to the other side we know this is going to be delta H or in other word m h 2 minus h 1 and we can write W e in minus Q out now what is W e in this is nothing, but your I V delta t . So, small t which is 5 minutes you have to convert it that in 2 seconds this is going to be 7.2 kilo joules, W Q out is 3.7 kilojoules and M is given to you is 0.025 kg we always prefer to use SI unit remember that and then we need to find out from here h 2 what is h 1? h 1 is a saturated vapor 300 kilo Pascal. So, this is going to be h of g at 300 kilo Pascal we look at a table of a saturated water table particularly the pressure will table.

So, from here the value comes out to be 2724.9 kilo joules per kg we plug in here and obtain the value of h 2; h 2 is 2864.9 kilo joules per kg. Now, what we have as a state 2 ? P 2 is 300 kilo Pascal and h 2 is 2864.9 kilo joules per kg,. So, now since h 2 is greater than h g, so, it is a super heated table. So, you look at the super heated table and you can find out T corresponding to which tons which is very close to the 200 degree Celsius ok.

So, super heated table 0.3 mega Pascal you look at the value you will find this h 2 will lie very close to the T you can do the interpolation or since the value is almost close I am just approximating it to 200 degree Celsius.

So, this is the example of making use of constant pressure energy balance, ok.

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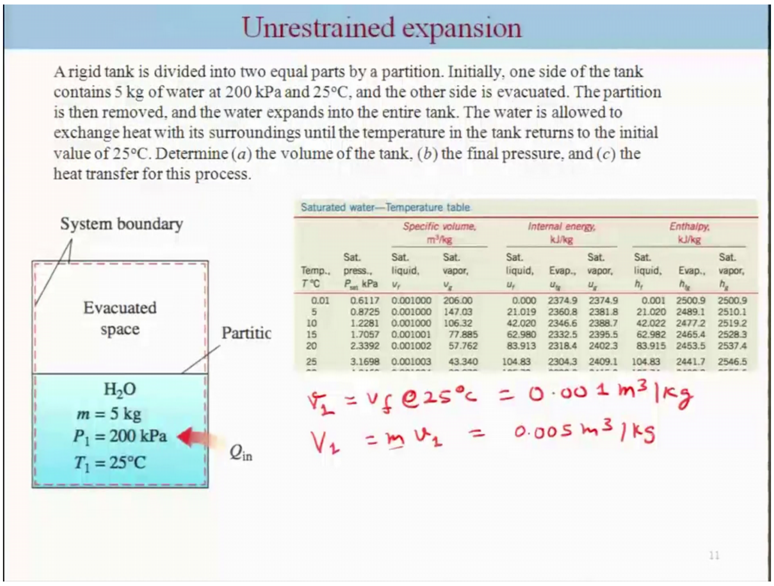

Unrestrained expansion

A rigid tank is divided into two equal parts by a partition. Initially, one side of the tank contains 5 kg of water at 200 kPa and 25°C, and the other side is evacuated. The partition is then removed, and the water expands into the entire tank. The water is allowed to exchange heat with its surroundings until the temperature in the tank returns to the initial value of 25°C. Determine (*a*) the volume of the tank, (*b*) the final pressure, and (*c*) the heat transfer for this process.

Saturated water—Temperature table

Temp., T °C	Sat. press., P_{sat} kPa	Specific volume, m³/kg Sat. liquid, v_f	Sat. vapor, v_g	Internal energy, kJ/kg Sat. liquid, u_f	Evap., u_{fg}	Sat. vapor, u_g	Enthalpy, kJ/kg Sat. liquid, h_f	Evap., h_{fg}	Sat. vapor, h_g
0.01	0.6117	0.001000	206.00	0.000	2374.9	2374.9	0.001	2500.9	2500.9
5	0.8725	0.001000	147.03	21.019	2360.8	2381.8	21.020	2489.1	2510.1
10	1.2281	0.001000	106.32	42.020	2346.6	2388.7	42.022	2477.2	2519.2
15	1.7057	0.001001	77.885	62.980	2332.5	2395.5	62.982	2465.4	2528.3
20	2.3392	0.001002	57.762	83.913	2318.4	2402.3	83.915	2453.5	2537.4
25	3.1698	0.001003	43.340	104.83	2304.3	2409.1	104.83	2441.7	2546.5

$v_1 = v_f @ 25°C = 0.001\ m^3/kg$

$V_1 = \underline{m} v_1 = 0.005\ m^3/kg$

So, we can extend this exercise for other cases such as un restrained expansion ok. So, we can extend this understanding by applying the concept of energy balance for the closed system for a case of unrestrained expansion. So, here what we have is a rigid tank which is divided into 2 equal parts and initially one side of the tank contains the water having 5 kg pressure at 200 kilo Pascal and temperature is 25 degree Celsius and other side is a vacuum, evacuate the space the partition is removed water expands in the entire tank and the water is allowed to exchange heat wi with the surrounding, until the temperature in the tank returns to 25 degree Celsius ok. What we need to find is the volume of the tank, the final pressure and the heat transfer for this process.

So, we are going to consider the system a complete tank ok. So, in that case of course, the tank volume is for is fixed, so, the boundary work is going to be 0,. So, let me just first find a couple of things we need to start with a very simple idea that what is the volume of this particular container that is we can start with the first part. So, if you look at the condition is 200 kilo Pascal 25 degree Celsius ok. So, by definition this water is in a compressed liquid and thus we can approximate the specific volume of this water as we have the specific volume of the fluid at 25 degree Celsius which is 0.001 meter cube per kg.

In the word, ok, so, we want the volume of this half volume the volume occupied initially by the water is going to be the mass multiplied by the specific volume which since this is 5 kg and you plug in this value it turns out to be 0.01 meter cube ok.

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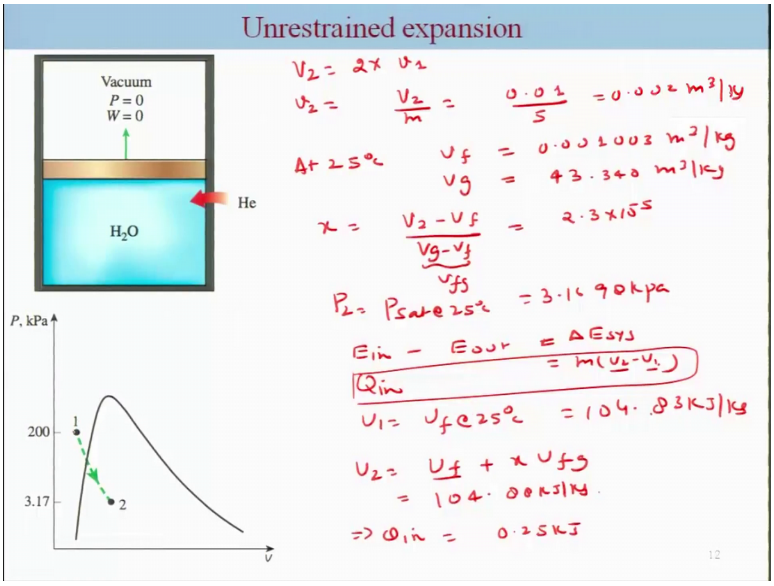

Unrestrained expansion

$V_2 = 2 \times V_1$

$v_2 = \frac{V_2}{m} = \frac{0.01}{5} = 0.002\ m^3/kg$

At 25°C $\quad v_f = 0.001003\ m^3/kg$

$v_g = 43.340\ m^3/kg$

$x = \frac{v_2 - v_f}{\underbrace{v_g - v_f}_{v_{fg}}} = 2.3 \times 10^{-5}$

$P_2 = P_{sat@25°C} = 3.1690\ kPa$

$E_{in} - E_{out} = \Delta E_{sys}$

$\boxed{Q_{in} = m(\underline{u_2} - \underline{u_1})}$

$u_1 = u_{f@25°C} = 104.83\ kJ/kg$

$u_2 = \underline{u_f} + x\, u_{fg}$

$= 104.88\ kJ/kg$

$\Rightarrow Q_{in} = 0.25\ kJ$

So, now, what we need to find out is the fa final volume. So, final volume is V 2 which is 2 times of course, which 2 times a V 1 ok. So, the what is the sa final specific volume is going to be V 2 divided by m, so, it turns out to be 0.01 divided by 5, 0.002 meter cube by kg. So, this, so, this final specific volume of state 2 is 0.02 meter cube per kg.

So, at 25 degree Celsius we can write down V f and V g and this is needed in order to find out the quality of quality of water in state 2. So, this is your 0.001003 straight from the steam table and this is 43.340 meter cube per kg ok. So, this you can see that you fa these are the values which we are using here. Now, from here we can find out quality because quality is V 2 minus V f divided by V g minus V f. So, this is nothing, but V f g. So, from here we get 2.3 into 10 to power minus 5.

Now, we write down the expression remember that also that P sate is for this P sate is P 2 is P sate at 25 degree Celsius which is 3.1698 kilo Pascal. So, we write down the expression of energy balance delta E system and this can be written as m u 2 minus u 1 and Q in is simply Q in there is no W b there is no other thing. So, it is Q in is equal to delta u which is m u 2 minus u 1. So, what would be your . So, we need to find out this two terms. So, what is a U 1? U 1 can be approximated as u f at 25 degree Celsius which is 104.83 kilo joules per kg and thus your U 2 is your U f and yeah. So, for to calculate U 2 we know already x. So, we can use U f and U f g. So, again we can go back here you we know U f and U f g we directly make use of that and x we know we get 104.88 kilo joules per kg.

We plug in this value two values here in this equation in order to get Q in ok. So, Q in is this. So, Q in comes out to be 0.25 kg 0.25 kilo joules ok. So, it is a straightforward exercise in other word what is happening is this in this process at 200 kilo Pascal at 25 degree Celsius. This was the specific volume and this was in a compressed liquid region and in this expansion process the pressure drops and it gets into that saturated region which is, 2.

So, this is how you make use of this simple energy balances in order to solve certain problems. So, this is a recap I hope that and believe that sa some of you must have gone through this kind of exercises in your earlier course of thermodynamics. But, we are trying to just recap some of the concepts.

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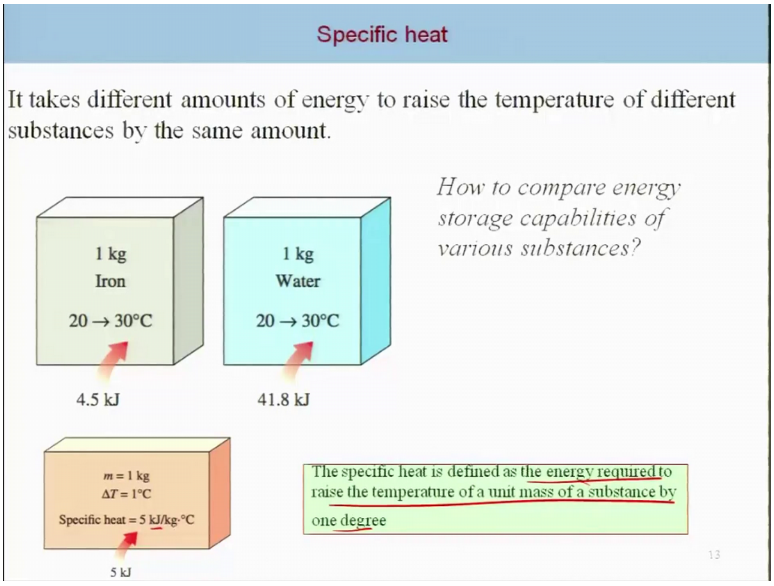

Specific heat

It takes different amounts of energy to raise the temperature of different substances by the same amount.

How to compare energy storage capabilities of various substances?

The specific heat is defined as the energy required to raise the temperature of a unit mass of a substance by one degree

So, let me just move on and I talk about specific heat here specific heat basically define in order to express different capacity of the different materials to store heat or to store energy. So, if you compare.

Let us say two materials such as iron and water. It takes different amount of energy to raise same amount of temperature ok. It takes less amount of energy for iron to change it is temperature from 20 to 30 degree Celsius for a 1 kg block of iron and if you look at the water it takes much significantly much larger amount of energy almost close to 10 times more.

So, how do you define or compare this kind of ability hold such a energy and we define in terms of a variable call specific heat and the specific heat is defined as the energy required to raise the temperature of a unit mass of a substance by 1 degree and thus it is unit is 5 it is it is unit is kilojoules kg degree Celsius or kilojoules kg Kelvin.

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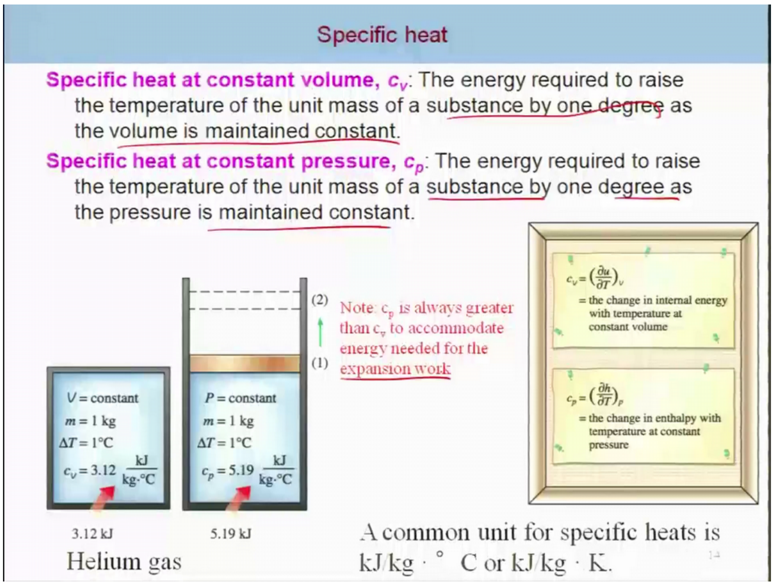

Specific heat

Specific heat at constant volume, c_v: The energy required to raise the temperature of the unit mass of a substance by one degree as the volume is maintained constant.

Specific heat at constant pressure, c_p: The energy required to raise the temperature of the unit mass of a substance by one degree as the pressure is maintained constant.

Note: c_p is always greater than c_v to accommodate energy needed for the expansion work

$c_v = \left(\frac{\partial u}{\partial T}\right)_v$ = the change in internal energy with temperature at constant volume

$c_p = \left(\frac{\partial h}{\partial T}\right)_p$ = the change in enthalpy with temperature at constant pressure

A common unit for specific heats is kJ/kg · ° C or kJ/kg · K.

Now, you can calculate specific heat E either main maintaining the volume constant or you may you can maintain the pressure constant. If it is done in a way where the volume is constant is call specific heat at constant volume which is the energy required to raise the temperature for unit mass of a system of a substance by 1 degree as the volume is maintained constant and the specific heat at constant pressure is the energy required to raise the temperature of a unit mass of a substance by 1 degree as the pressure is maintained constant.

So, in other word you can represent c v in terms of the partial derivative u with respect to T at constant volume and c p a partial derivative of enthalpy with respect to temperature pressure constant ok. Now, note that, c p is always greater than c v because this . So, in order to accommodate more energy needed to expand needed for expansion.

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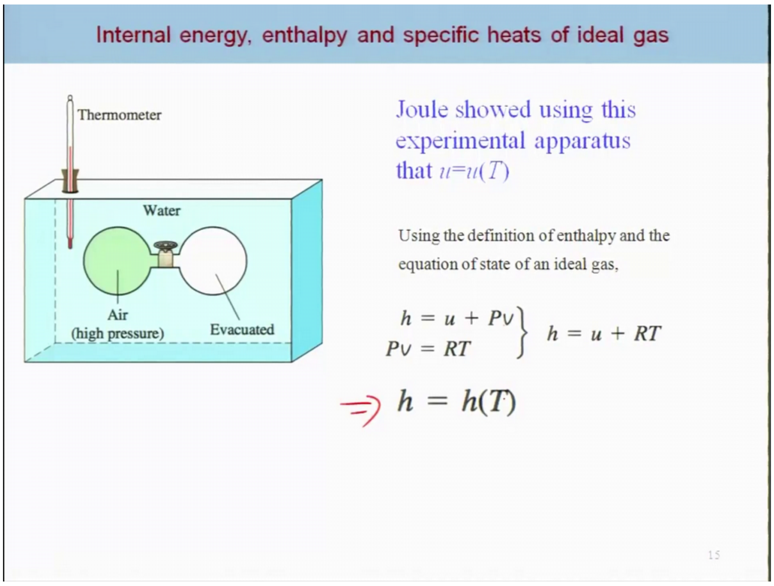

Internal energy, enthalpy and specific heats of ideal gas

Joule showed using this experimental apparatus that $u=u(T)$

Using the definition of enthalpy and the equation of state of an ideal gas,

$$\left.\begin{array}{r} h = u + Pv \\ Pv = RT \end{array}\right\} \quad h = u + RT$$

$$\Rightarrow h = h(T)$$

Now, let me just talk about these variables in terms of for particularly for ideal gas. For ideal gas joules has already shown that the internal energy is going to be independent of pressure it only depends on the temperature for by simple ex experiment.

Now, by definition enthalpy is u plus P v and since u is the only dependent on temperature for an ideal gas and P v can be written as R T. So, by definition h is equal to u plus P v and since u is independent of temperature and P v you can write it for ideal gas R T.

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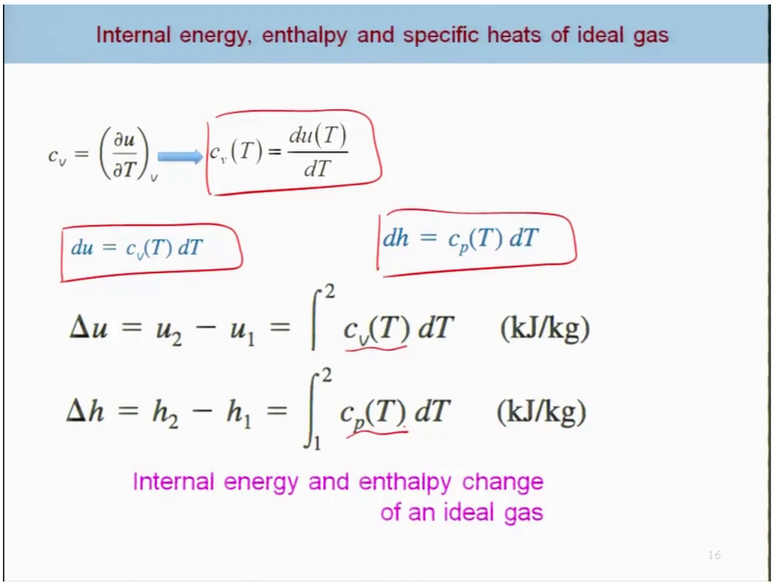

Internal energy, enthalpy and specific heats of ideal gas

$$c_v = \left(\frac{\partial u}{\partial T}\right)_v \rightarrow \boxed{c_v(T) = \frac{du(T)}{dT}}$$

$$\boxed{du = c_v(T)\, dT} \qquad \boxed{dh = c_p(T)\, dT}$$

$$\Delta u = u_2 - u_1 = \int_1^2 \underline{c_v(T)}\, dT \quad \text{(kJ/kg)}$$

$$\Delta h = h_2 - h_1 = \int_1^2 \underline{c_p(T)}\, dT \quad \text{(kJ/kg)}$$

Internal energy and enthalpy change of an ideal gas

So, as a joules has shown that find the ideal gas u is exact differentiation. So, c v now can be written as simply du by dT and simply similarly you can do the same thing for c p. In other word you can write d u as simply c v T which is dependent on temperature multiplied by d t and simply d h is equal to c p dT ok.

So, one can find out now the changes in the internal energy by just integrating this expression similarly that for enthalpy. For enthalpy will be considering c p for internal energy will be considering c v. What we need is basically now, the relation of c v as a function of temperature so that relation or that expression would be needed ok.

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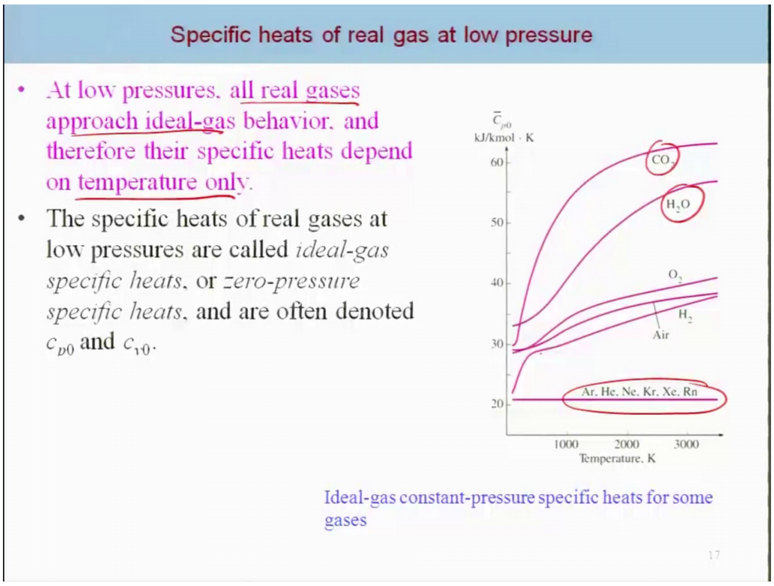

Specific heats of real gas at low pressure

- At low pressures, all real gases approach ideal-gas behavior, and therefore their specific heats depend on temperature only.
- The specific heats of real gases at low pressures are called *ideal-gas specific heats*, or *zero-pressure specific heats*, and are often denoted c_{p0} and c_{v0}.

Ideal-gas constant-pressure specific heats for some gases

Now, if you look at real gases real gases at low temp real gases as a very low pressures and very high temperature behaves like ideal gas or particularly, when you consider very low pressure. All at low pressure all the real gases approaches ideal gas and only depends on the temperature and particularly for noble gases they are only dependent on or they are almost constant. On the other hand the molecular fluids they are linearly dependent on temperature at very low pressures.

So, one can use these curves get a fit it into some kind of polynomial expression and obtain c p as a function of temperature.

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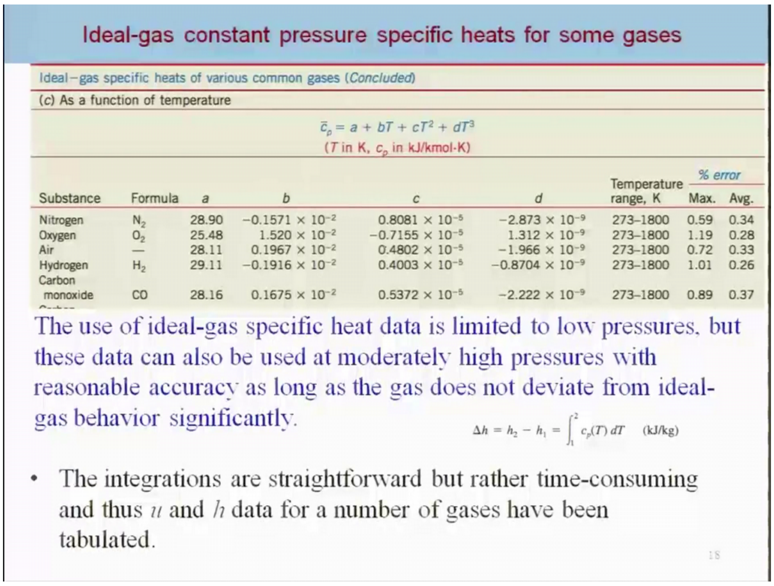

Ideal-gas constant pressure specific heats for some gases

Ideal–gas specific heats of various common gases (*Concluded*)

(*c*) As a function of temperature

$$\bar{c}_p = a + bT + cT^2 + dT^3$$

(T in K, c_p in kJ/kmol·K)

Substance	Formula	*a*	*b*	*c*	*d*	Temperature range, K	% error Max.	% error Avg.
Nitrogen	N_2	28.90	-0.1571×10^{-2}	0.8081×10^{-5}	-2.873×10^{-9}	273–1800	0.59	0.34
Oxygen	O_2	25.48	1.520×10^{-2}	-0.7155×10^{-5}	1.312×10^{-9}	273–1800	1.19	0.28
Air	—	28.11	0.1967×10^{-2}	0.4802×10^{-5}	-1.966×10^{-9}	273–1800	0.72	0.33
Hydrogen	H_2	29.11	-0.1916×10^{-2}	0.4003×10^{-5}	-0.8704×10^{-9}	273–1800	1.01	0.26
Carbon monoxide	CO	28.16	0.1675×10^{-2}	0.5372×10^{-5}	-2.222×10^{-9}	273–1800	0.89	0.37

The use of ideal-gas specific heat data is limited to low pressures, but these data can also be used at moderately high pressures with reasonable accuracy as long as the gas does not deviate from ideal-gas behavior significantly.

$$\Delta h = h_2 - h_1 = \int_1^2 c_p(T)\, dT \quad \text{(kJ/kg)}$$

- The integrations are straightforward but rather time-consuming and thus u and h data for a number of gases have been tabulated.

And, this is what often use c p, one can put it into expressions and this can be used in order to get delta h and for an ideal gas of course, we know that expression of c p minus c v is equal to r we can also get from there the expression for delta u. Now, integrations though are straightforward, but it is rather time consuming and for many gases u and h data are tabulated.

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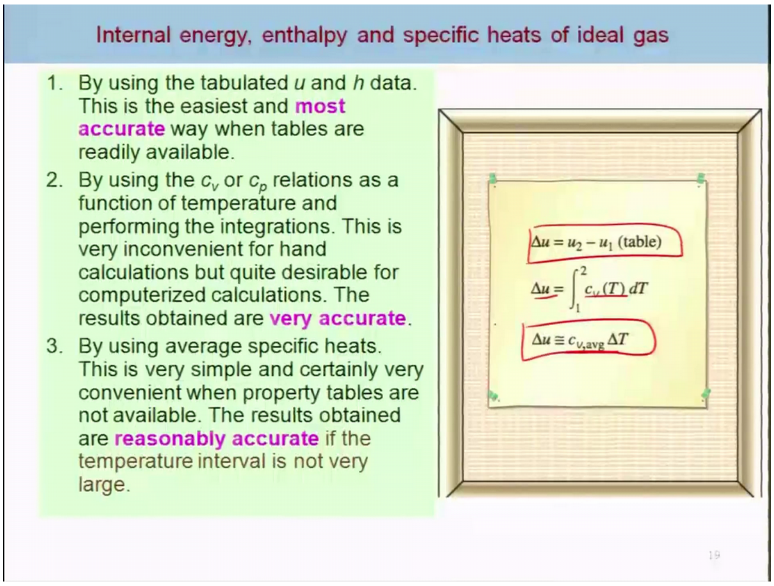

Internal energy, enthalpy and specific heats of ideal gas

1. By using the tabulated u and h data. This is the easiest and **most accurate** way when tables are readily available.
2. By using the c_v or c_p relations as a function of temperature and performing the integrations. This is very inconvenient for hand calculations but quite desirable for computerized calculations. The results obtained are **very accurate**.
3. By using average specific heats. This is very simple and certainly very convenient when property tables are not available. The results obtained are **reasonably accurate** if the temperature interval is not very large.

$$\Delta u = u_2 - u_1 \text{ (table)}$$

$$\Delta u = \int_1^2 c_v(T)\, dT$$

$$\Delta u \cong c_{v,\text{avg}}\, \Delta T$$

So, if you summarize this specific heat based on let us say calculations just we can take an example of u, internal energy you can directly take a make a use of a table in order to get the changes in the internal energy or if you have the expression of a c v as a function the temperature you can also use that or for a short temperature intervals you can just take the average c v value and obtain delta you. So, this depends on what is available and of course, the state conditions.

So, with that I will close this lecture and we have just summarized the energy balance for the closed systems. We will take up in the next lecture for the case of open system. So, I will see you in the next lecture.