

Thermodynamics of Fluid Phase Equilibria
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Lecture - 45
Models for Fugacity of Liquid Mixtures-2

Welcome back. So now, I am going to make use of Wohl's equation expression to get a Van Laar equation out of it, ok.

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Van Laar Eq
 - special case of Wohl eqn
 - molecules chemically similar but diff sizes
 E.g.: benzene (1) - 160 -> octane (2)
 $v_1^L = 89 \text{ cc/mol}$ $v_2^L = 166 \text{ cc/mol}$ at 25°C

$$g^{EX} = 2 a_{12} z_1 z_2 = 2 a_{12} \frac{x_1 v_1 \cdot x_2 v_2}{(x_1 v_1 + x_2 v_2)^2}$$

$$\frac{g^{EX}}{RT} = \frac{2 a_{12} x_1 x_2 v_1 v_2}{(x_1 v_1 + x_2 v_2)^2}$$

So, Van Laar is a special case of Wohl, so special case of Wohl it is used for mixtures having a molecules which are chemically similar, but different sizes. Some examples are benzene and iso-octane with the same example which we are we have been trying to use ok. So, if you look at let us say if it is 1 and if it is 2, it is molar volume is 89 cubic centimeter per mole whereas, here is 166 cubic centimeter per mole ok, at 25 degree Celsius, clearly suggesting that these are of different sizes.

Now, if you ignore a higher order term in Wohl expression if you consider just a first expression right and this expression if you consider you are going to get the Van Laar equation So, if you remember this z_i is equal to $x_i v_i$ divided by $x_1 v_1 + x_2 v_2$. So, let me get there and write it here. So, what I am trying to say is the following g^x by $RT x_1 v_1 + x_2 v_2$ if I consider only the first term right. So, this is nothing, but Van Laar. Now, you can plug in z here. So, this can be written as two this one and this is $x_1 v_1$

1 divided by $x_1 q_1$ plus $x_2 q_2$ this is square and this would be multiplied by $x_2 q_2$ ok.

So now, you can simplify this expression you should be getting g if I take this part the other side then I get RT and this is going to be $2 a_{12} x_1 x_2 q_1 q_2$ divided by $x_1 q_1$ plus $x_2 q_2$, ok. Now, remember that we are little bit comfortable with a 's and b 's because the reason is that that makes equation simpler and start from this one constant Margules equation.

So, we can actually write it in that form. If you do the calculations you know that from the fact that activity is related to partial Gibbs excess property and so that means, you should be able to get this expression similar to what how we got one constant Margules equation and obtain the activity coefficient ok.

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The whiteboard shows the following handwritten equations:

$$\ln \gamma_1 = \frac{A'}{\left(1 + \frac{A'}{B'} \frac{x_1}{x_2}\right)^2}; \quad \ln \gamma_2 = \frac{B'}{\left(1 + \frac{B'}{A'} \frac{x_2}{x_1}\right)^2}$$

$$A' = 2 q_1 q_{12}; \quad B' = 2 q_2 q_{12}$$

$$\frac{A'}{B'} = \frac{q_1}{q_2} = \frac{\ln \gamma_1}{\ln \gamma_2}$$

$$\frac{g^E}{RT} = \frac{A' x_1 x_2}{\left(x_1 \frac{A'}{B'} + x_2\right)} = \frac{B' x_1 x_2}{\left(x_1 + x_2 \frac{B'}{A'}\right)}$$

So, if you do that. So, I am not deriving it I get the following expression $A' + A' B' x_1$ by $x_2 + 1$ and $\ln \gamma_2$ is $B' + B' A' x_2$ by $x_1 + 1$, where A' is $2 q_1 a_{12}$. Remember that A' we referred earlier also as interaction parameter. So, indeed if you consider a_{12} is a interaction energy and q . What was q ?

Student: Effective volume.

So, it basically the q is basically the volume effective volume molecule. So, not the volume ratio as z is a volume ratio. So, A' is $2 q_1 a_{12}$, is B' is $2 q_2 a_{12}$. Now,

we need to use some kind of exponent data to obtain a dash or b dash usually a dilute solution informations are available from the experimental data. So, if you use that information from the experimental data then A dash by B dash is nothing but q_1 by q_2 ok. And what is A dash? How do you get the A dash here?

So, if you consider let us say in dilute solution x_1 goes to 0 then this will be simply $\ln \gamma_1$ infinity is nothing, but A dash. Similarly $\ln \gamma_2$ infinity, where x_2 goes to 0 will be B dash. So, A dash where B dash ratio which is q_1 by q_2 is nothing but $\ln \gamma_1$ infinity by $\ln \gamma_2$ infinity. So, this is relation you can obtain from the experiment there is a relation between these constants ok , which is related to the infinite in a dilution activity coefficient ok .

So, often we use the relation in this form, or we use in this form this is nothing but a two constant or two parameter expression or two parameter theory where I say that ok . So, this is a Van Laar expression. You can easily make use of this for benzene iso-octane as well as n-propanol and water it works very well ok . So, it captures a complexity of that nature of dissimilar things. Remember that for n propanol water it is basically hydrogen bond is also playing a role

Now, this expression I can rewrite actually, let me just further. Now, though the derivation of Van Laar suggests that it is good for solution of relatively simple system non-polar, but it is also used outside this assumption. So, it is used for polar also that is what I said n-propanol and water system ok .

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$$\frac{x_1 x_2}{g^E} = \left[x_1 \frac{A}{B} + x_2 \right] \frac{1}{A'RT} = \left[1 + \left(\frac{A}{B} - 1 \right) \right] \frac{1}{A'RT}$$

$\Rightarrow \frac{x_1 x_2}{g^E} \text{ vs } x_1 \text{ is a str. line}$
 $\Rightarrow \text{obtain } A' \& B'$

A', B' known at specific T
 customary to write at const x

$\ln \gamma_i = c + d/T$

$c=0 \Rightarrow$ solution is regular $S^E=0$
 $d=0 \Rightarrow$ solution is athermal $H^E=0$

Empirically determined

So, it is capable of reproducing data for complex mixtures. So, let me rewrite this expression rewrite this here in this form $x_1 x_2$, I now writing this one ok, $x_1 x_2$ divided by g^E this can be written as $x_1 \frac{A}{B} + x_2$, 1 by $A'RT$ or I can write this as $1 + \frac{A}{B} - 1$ by $A'RT$ ok.

So, what this suggests that $x_1 x_2$ by g^E or g^E or $g^E x$ is the same thing same nomenclature in this course versus x_1 . This versus x_1 is a straight line, and this expression this if you have the data available we can this can be used to obtain A' and B' right this is what it tells you. If you have the data of this composition and g^E is you can plug it and obtain the A' and B' out of it ok.

Now, usually A' and B' are insensitive to pressure, but not to temperature because it is it depends on the temperature but if it is insensitive to temperature then this will be a thermal solution ok. Now, practically A' and B' are often assumed to be independent temperature if the temperature range is not too great ok, this is a practical consideration.

Now, often you may have A' and B' known at specific temperature. So, and then you would like to find at different temperature ok. So, it is customary to write at constant x and then $\ln \gamma_i$ is $c + d/T$. So, you can you have the information you can get it at different temperature, but using this kind of customary expressions ok, where c and d are empirically determined.

So, usually when you see this kind of expression and if c is equal to 0 this also indicates the solution is regular. Now, this is something which I am going to spend last few lectures on it. Solution is regular which essentially means s excess entropy is 0 if d is equal to 0, that means, is independent of temperature then solution is a thermal which means H excess is ok. This is something which you can we can save by looking at it if you are aware of all the derivations, but I will just assert it now, and we will look at it in the later part of this course; that means, last few lectures ok.

So, now, that we have expressed very well the Van Laar equation the other set of equations are also available which can be derived from the again the Wohl's equation expressions.

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Molecular size equal approximation
 $q_1 = q_2 \Rightarrow x_1 q_1 + x_2 q_2 = q$
 $\Rightarrow z_i = x_i$

$$\frac{g^E}{RT} = x_1 x_2 \left[2 a_{12} + 3 a_{112} x_1 + 3 a_{122} x_2 + \dots \right]$$

$$\ln \gamma_1 = A' x_2^2 + B' x_2^3 + C' x_2^4$$

$$\ln \gamma_2 = \left(A' + \frac{3}{2} B' + 2 C' \right) x_1^2 - \left(B' + \frac{3}{2} C' \right) x_1^3 + C' x_1^4$$

Maxwell eqn
 Two constant - $c' = 0$
 Three constant - $c' \neq 0$

$$A' = \frac{q}{2} (2 a_{12} + 6 a_{112} + \dots)$$

$$B' = -\frac{q}{6} (6 a_{112} - 6 a_{122} + \dots)$$

So, if you consider molecular sizes to be equal which means q_1 is equal to q_2 this essentially means that $x_1 q_1 + x_2 q_2$ is nothing but q which essentially means that I is equal to x_i , ok. If you go back go to the definition of the z , z_i will be nothing, but x_i ok. In that case I can consider the expression of the Wohl's expression in this form ok. So, this will continue. This should have been say $x_1 x_2$ here I think yeah. So, two times right fine, x_2 and then there this a high order terms of x_1 and x_2 all right.

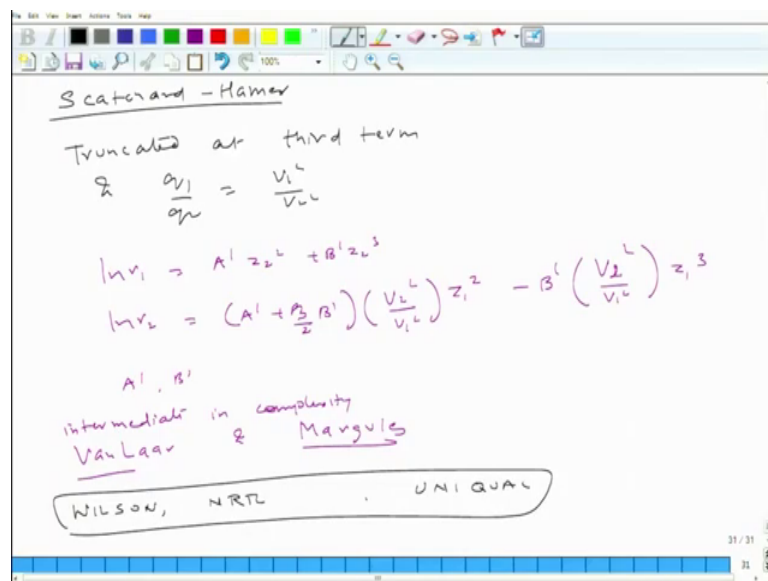
You can come up with this expression particularly containing only till x_2 or 2 till 4th order of composition this. So, again I am skipping all the derivation and you do not need to remember you know the most important thing is that how it is derived or could be

important as well as the philosophy of these expressions is actually I feel that more important rather than remembering this expressions ok.

So, this is the expression which comes out for ln for binary mixtures having the molecular sizes a constant from here we should be able to connect or get this something is you know this Margules equation of two constant and three constant for two constants c dash is equal to 0 ok, for three constants c is c is not equal to 0 ok. Where A's and B's are connected to nothing but for example, A dash is nothing, but q 2 a 12 plus 6 a 112 and so forth similarly B dash is q 6 a 112 minus 6 a 122 so forth So, once you derive you should be able to find all these things.

So, this Margules equations for two constant and three constants has a basis of molecular sizes to be equal and it is derived from the Wohl's equations ok. So, in practice you can see that you know this two constant and three constant Margules equation should not be used for the system having dissimilar sizes, but in practice they are also used ok.

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So, last equation or expression is due to Scatchard which I am going to talk and this and then I will stop and focus on some examples. So, this is a expression where the Wohl's expression is truncated at the third term, and we put the ratio of this size is as simply this as I said this one model remember that earlier I said that one model is this is nothing, but is Scatchard-Hamer equation. And here if you use this expression then the following

expression of activity coefficient comes into. So, this is again I am writing this without deriving it where A dash and B dash are functions of you know there is a's and b's.

So, I will not write it in details because I do not expect you to remember this as well, but what is important is that Scatchard-Hamer captures the complexity in between Van Laar and Margules ok, so intermediate in complexity ok. So, we may use some of this expression to do some kind of examples as a different matter, but the idea is that this is another possibility if you have intermediate in complexity now. What is the intermediate in complexity, where of course, the size differences are there that is something which is part of this the complexity.

Now, in addition to this we can also discuss various different expressions which has some sound basis Wilson NRTL which is a non-random to liquid local composition and uni quac which is universal quasi chemical theory ok. So, this is something which I will not go into details, but the one can get an expression of the activity with respect to different variables ok.

So, at this point I will stop here, and I will start I will work on some examples in the next lecture ok. So, see you in the next lecture.