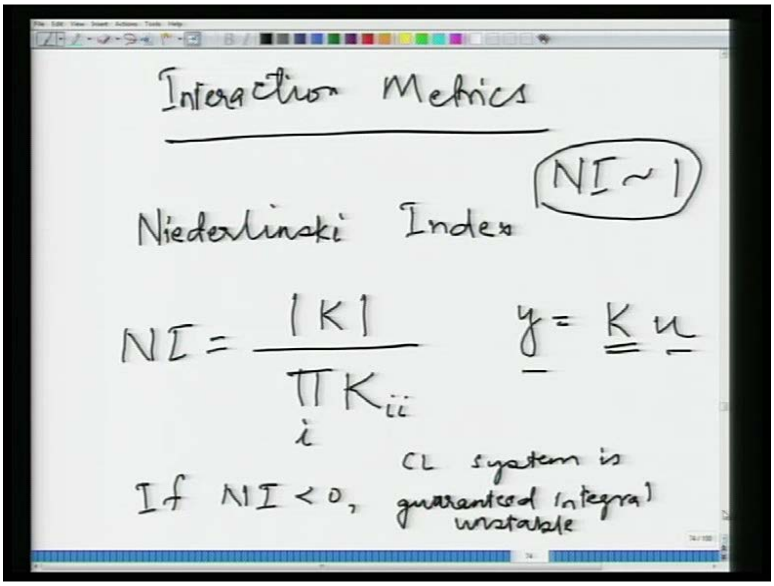
Plant wide Control of Chemical Processes
Prof. Nitin Kaistha
Department of Chemical Engineering
Indian Institute of Technology, Kanpur

Lecture - 9
RGA and Dynamic Decoupling
Steady state RGA
Dynamic decoupling
Introduction to model based control

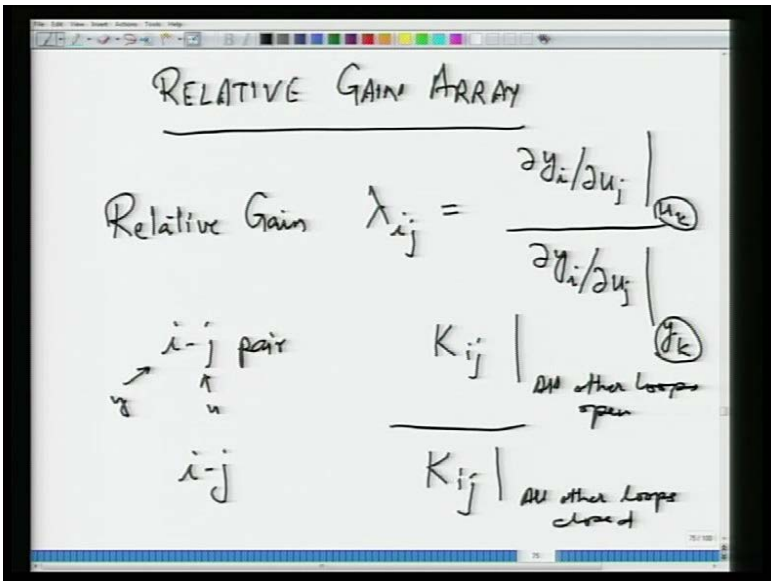
(Refer Slide Time: 00:25)



Welcome to this. We have been talking about interaction matrix, matrix that quantify the interaction in a, in a multivariable system, interaction matrix. And last time we described, what is known as the Niderlinski index? And by definition if I have a study state open loop equation, y is equal to K times u, where K is the gain, then the Niderlinski index is defined as Niderlinski index is equal to determinant of K divided by product of the diagonal terms of the gain matrix K i i. And what we said was that if the Niderlinski index is less than 0, closed loop system is guaranteed integral unstable, unstable.

So, this is actually a tool for discarding input output pairings, that would be unstable, because of unfavorable interaction, and we also said that for Niderlinski index close to 1, close to 1, this is at least for a 2 by 2 system it means that the interaction the steady state interaction is one way. So, Niderlinski index close to 1 in some sense shows, that the interaction is not as bad.

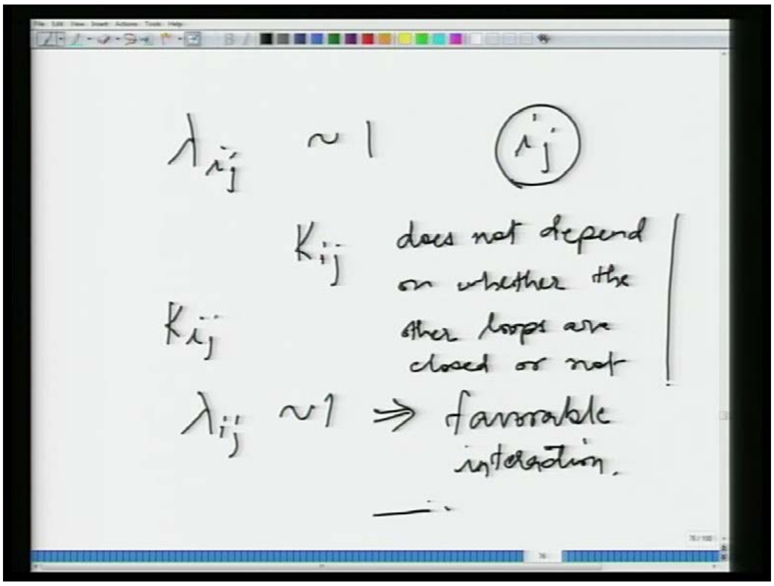
(Refer Slide Time: 02:48)



Now, traditionally we also have another interaction matrix and that is call the relative gain or rather the relative gain array, relative gain array. How do you define the relative gain? Relative gain lambda i j, where i refers to output subscript, j refer to input subscript. By definition dou y i dou u j, all other inputs are constants divided by dou y i by dou u j, all others outputs other than y i are held constants. If you to understand in a plain English, I mean what saying is if I look at the i j pair, i refers to output, j refers to input.

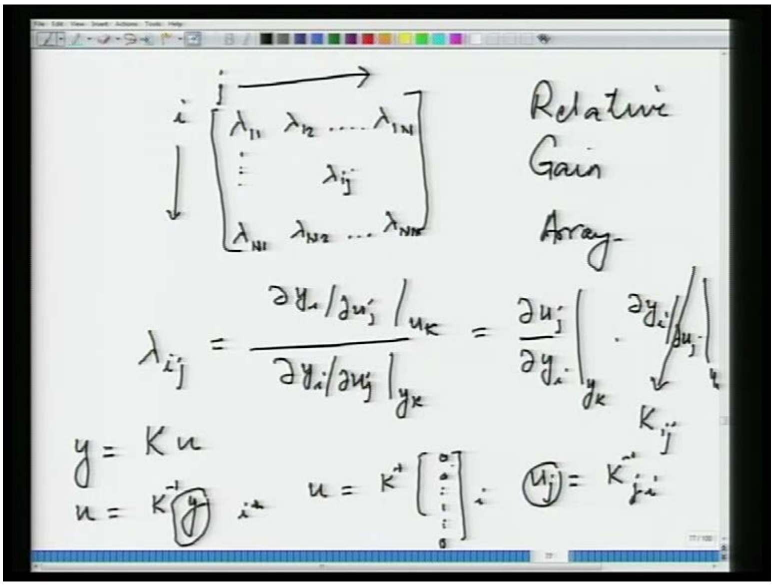
If I look at the i j paring, then what I have is the definition of relative gain is, what is the gain that this pairing c is with all other loops open, all others loops open, what that means, all other inputs are held constants, all other inputs are held constants, divided by what is the gain of this i j th pair with all other loops closed. Now, because all other loops are close, what we have is all other outputs others than y i are at their set point. So, the relative gain is the ratio of the gain, for the i j pair with all other loops open and all other loops closed, ok, so this is the English interpretation of relative gain.

(Refer Slide Time: 05:47)



Lambda i j is close to 1, what that means is whether other loops are closed or not, the i j th pair gain for lambda i j close to 1, K i j does not or depend very little, does not depend on whether the other loops are close or not, on whether the other loops are closed or not. The gain that I see K i j is in some sense independent of what is the state of the other loops, whether they are closed or not. So, relative gain close to 1 imply, let just call it favorable interaction, it does not imply in no interaction, but that the interaction is relatively mild and therefore, we call it favorable interaction.

(Refer Slide Time: 07:21)

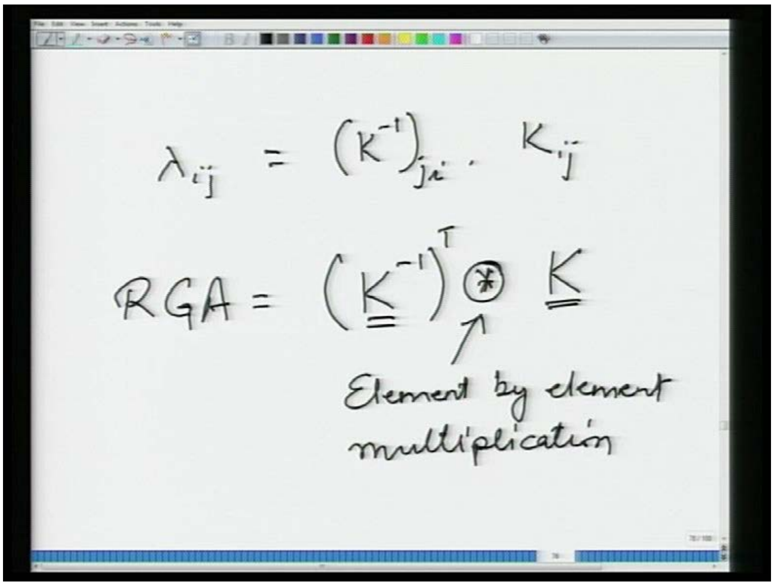


Now, if I this is my row index and let this be my column index, I can fill up lambda 1 1, lambda 1 2, lambda 1 n and so on, so forth. Somewhere there will be lambda i j and then I will have lambda n 1, lambda n 2, lambda. This is called where I have calculated the relative gain of all possible input output pairings, i pair i with j, i can take so many values, j can take so many values, for each of those when I calculate my relative gain, and put it in terms of an array, that is called the relative gain array,ok.

How do I calculate the relative gain array? well if you look at lambda i j is equal to dou y i dou u j at where all other use are held constant divided by dou y i by dou u j, where all the other outputs are held constants, which is the same as dou y i by dou u j u k times dou u j over dou y i all other outputs are held at this set point, ok.

Now, this is the open loop gain K i j, where my open loop steady state equation is y is equal to K time u, what is this guy? How do I get this guy? Well to get this guy, let me inverted and get u is equal to K inverse y, all right. Now, if all the y are held constant all the y except y i are held constant what; that means is that all the terms in this guy except the i th terms are 0, and when I do that what I will get is u will be equal to K inverse 0 0 the i th element is 1, the other elements are 0. And therefore, what I will get is the j th element, because I must look at change in the j th element u j will be equal to K inverse j comma i all right, the matrix the j i th element of the K inverse matrix will corresponds to u j.

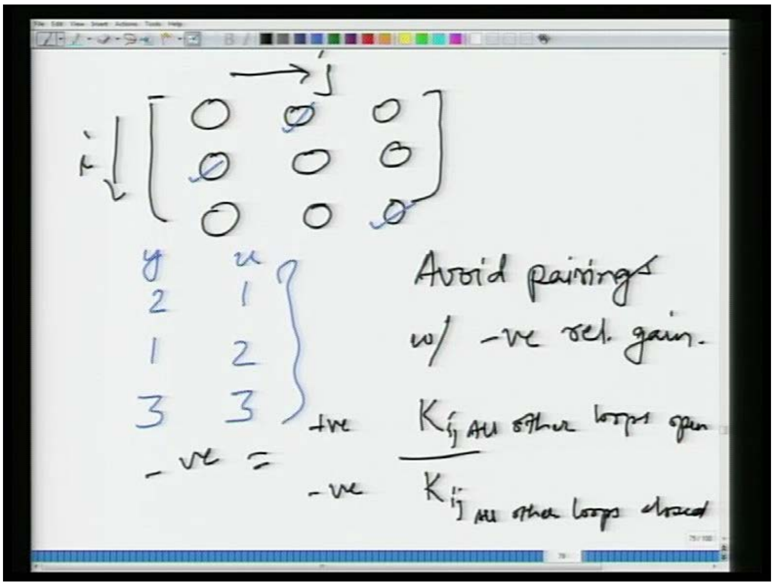
(Refer Slide Time: 11:14)



So, what I get form this is that lambda i j is equal to K inverse, this matrix its j i th elements, times the gain matrix times its i j th element, and if I have to do it for all values of i and j, what I get is my relative gain array RGA is equal to K inverse. Now, because the index is not i j its j i th therefore, I have to take a transpose had mart product, how do I defined had mart product? times K the gain matrix see.

This is element by element multiplication not matrix multiplication, but element by element multiplication. So, this is how you calculate you relative gain array, and once you calculated your relative gain array, you want to find out what is to be paired with what you look for terms that are close to 1.

(Refer Slide Time: 12:37)



And then sort of say that, the recommended paring is let say I look at the 3 by 3 relative gain array, and I find that of all the elements 1, 2, 3, 1, 2, 3, 1, 2, 3. So, this is i and this is j, what I find is that this element, this element, and this elements are close to 1, then what we say is you should be paring this is what 2 output 2 this is y with input 1, output 1 with input 2, output 3 with input 3 this is the recommended pairing, form the relative gain array, how do you calculate the relative gain array? I just told you calculate the inverse of the gain matrix take its transpose, and do element by element multiplication with the gain matrix, that is the relative gain array.

What should we avoid? Avoid pairing with negative relative gain, why is that? well a negative relative gain is saying that, all other loops open all see if I look at the plain

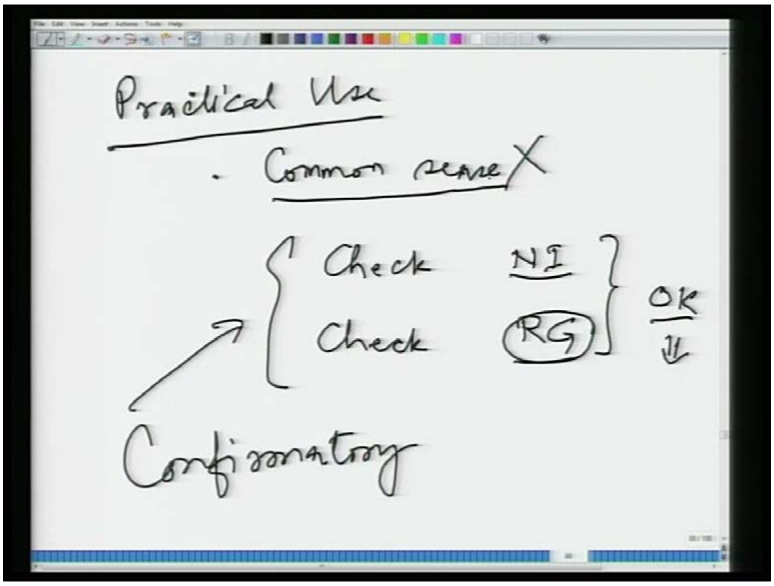
English definition of relative gain, it is the gain between the i j th paring with other loops open divided by the gain for the i j th pairing with all other loops closed. Now, if this number is negative what that means, is my controller is seeing well; what that means, let me give you an example, if you increase steam you would expect that the temperature on the tray of distillation column should increase, so gain should be positive, open loop gain you expect is to be positive. But, for some reason if the relative gain is negative, what that means, is depending on whether all loops are open or not, so if all loops are closed, if all loops are open, if you increase steam, tray temperature increases.

On the other hand, if all other loops are closed, what you get is if you are increasing steam, the tray temperature is decreasing, that is what would cause the relative gain to be… When you take the ratio, you will get a net negative number. So, that is what would cause the relative gain to be negative, and what this means is that the process gain sign changes depending on whether the other loops are on or not.

So, direct acting controller must be a reverse acting controller, and reverse acting controller must become a directing acting controller depending on the status of the other loops. And operator keep taking loops on or off you know, there will always sometime operator would like intervene switch off the loop, and take control action himself, but if the other loop gain sign has changed, that loop would go unstable, unstable in the sense instead of increasing the steam, you would keep on decreasing the steam, and the steam wall will ultimately shut down or vice versa.

So, this is something that is not desirable, must be avoided, so as a tool for rejecting IO pairings. If you look at the relative gain array, all IO pairing that have negative relative gain should not be implemented. What should be implemented? What are possible candidates for implementation are all relative, all element with all possible IO pairing with positive relative gain preferably as close as possible to 1.

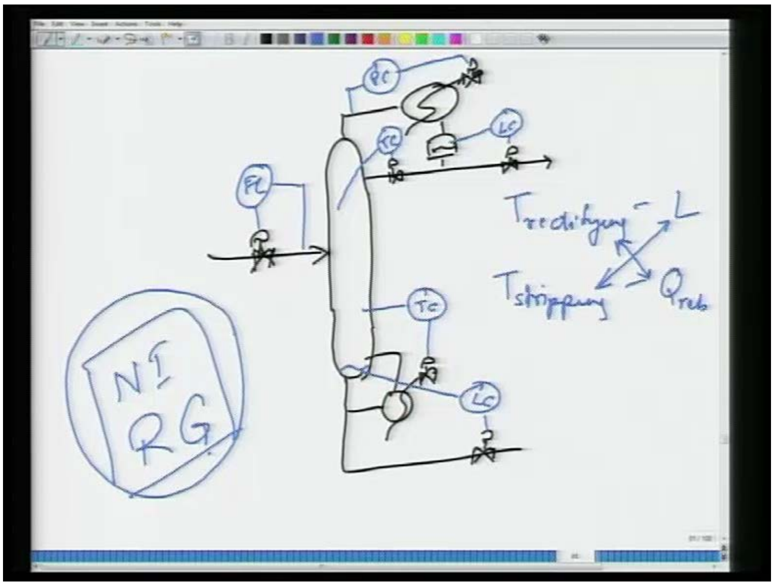
(Refer Slide Time: 16:59)



While, we have practical use in practice, when you are suppose to figure out, what should what input output pairing should be implemented? practical use would always be common sense, get the input output pairing form common sense. Once, you have that input output pairing form engineering common sense, and what constitute engineering common sense would constitute the rest of the course, but once you have the input output pairing, check Niderlinski index, check relative gain, and if these are ok, you are all right.

So, these are actually conferment in some sense conferment tools things, tools that, tools that conform your engineering judgment, that yes this is what I wanted to do, this is what is the most sensible thing to do based on engineering common sense, and look here the Niderlinski index and relative gain seems to be the suggesting the same. So, these are essentially tools to confirm conventional wisdom, ok, that is my take one it, I do not think these tools have to be used, it would be, let me put it the other way, it would be disaster to not do this, and recommend an input output pairing based purely on relative gain array. Because, that in my opinion would lead to something that should not be done, use you engineering judgment and then use these matrix Niderlinski index and relative gain to confirm that ok, what you are doing is ok. And what you would find is 9 out of 10 times the common sense approach actually works quite well, just to give you an example.

(Refer Slide Time: 19:14)



For example, if you take a distillation column here is the distillation column, a simple distillation column and do not worry, if you do not understand some of the things, that just an example. Reflex drum you put in some reflux back into the column, take out some distillate, where some reboil you are putting in steam here, that cause the vapor to boil up, and then you are taking out the bottoms.

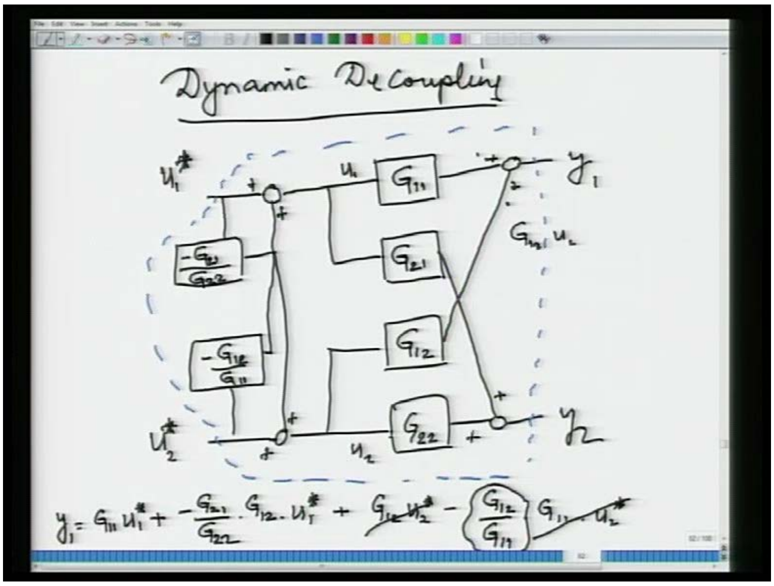
Let us say, this is under flow control, the feet is under flow control, while you got to control two labels, liquid label at the top liquid level at the bottom, let say you are doing in this way. This is probably the most common arrangement, you would also have to control the vapor pressure, the pressure of the vapor that is entrap in the column. And that the most common arrangement to do, that is this you adjust the condenser due to your cooling rate in the condenser, to control the pressure, temperature control and let say you are adjusting the reflux to maintain to a rectifying tray temperature.

Now, the choice of this two tray temperature, what would you find is if I am controlling a rectifying tray temperature using a T rectifying is control using reflux, and T striping some tray temperature in striping section is controlled using Q reboiler, the reboiler duty. If I am doing this, what you would find is typically my Niderlinski index and my relative gain would be positive, possibly close to 1, but certainly Niderlinski would not be negative.

On the other hand, if you do this kind of pairing, where you are controlling a rectifying tray temperature using reboiler duty, and striping tray temperature using reflux, and that does not make sense right. Then what you would find is that these matrix are not well behave, you may get the negative Niderlinski index or index or you may get a relative gain, that is negative, and therefore, these pairing are not recommended. And if you ask an operator for them controlling reflux, a controlling a rectifying tray temperature, the most obvious handle is reflux controlling, a striping tray temperature the most obvious handle is reboiler duty.

These matrix Niderlinski index and relative gain would conform that, indeed that is the way to do it, and they also suggest hopefully, that if you do it, the other way well that is not the way to do it, because you get a negative relative gain and or negative Niderlinski index. So, these are I would recommend these as tools to confirm your engineering wisdom, and what the effect should is to is you know, do as much as possible to gather or to become wise in the engineering common sense way.

(Refer Slide Time: 22:28)



There is other thing that needs to be discussed here, and that is called decoupling, I would call it dent dynamic decoupling, and I think this will be the last that we do, may be one more dynamic decoupling. See, if I have a multivariable 2 by 2 system, it mostly easily illustrated on a 2 by 2 system 1, 2, 3, 4 and what I have is y 1, y 2 and this is u 1, this is u 2. Notice, if I make a change here, let say this goes up as a step, if u 1 is increase as a step, then y 1 would give a response, let say it give this kind of response, y 2 would also give a response, and let just say it is a gives a response. Now, I know I am changing u 1 and therefore, I know y 2 is going to show a response. What I would like is, how should I adjust u 2? how should I adjust u 2? So, that this signal is a negative of this signal, all right then what I would get is y 2 does not show any response or a very little response right.

So, because I know I am changing u 1 therefore, I know y 2 is gone change, can I change u 2 in a such a way such that, the overall effects get cancelled and y 2 does not show any response, that is the question I am asking. And I can ask the same question, when I am making step change to u 2, how should I change u 1 such that this plus this goes to 0. So, then what I will have is I change u 1 y 1 response, y 2 shows no response, I change u 2 y 2 response, y 1 shows no response, this is what is called a decoupled system. y 1 and y 2 are not couple with u 1 and u 2, u 1 effects y 1 does not affect y 2, u 2 effects y 2 does not affect y 1, that is a decouple system.

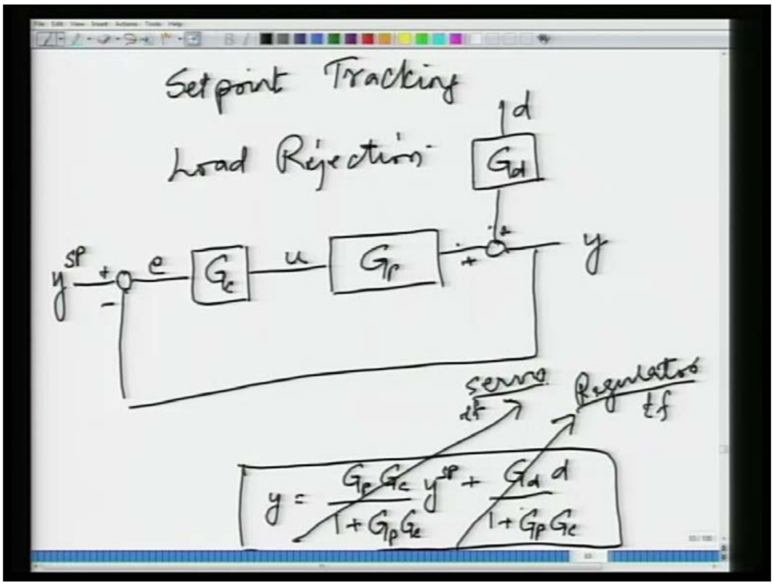
So, how do we accomplish this decoupling, I can use the philosophy of feedback control, let me call this u 1 star, and let me call this u 2 star, and you can since, I have drawn the circle, you can think I will be putting signal there, what I do is my take this guy multiplied by some transfer function and add it up here. Now, what should go in here that is the question, ok, what should go in here should be such that, this and this are negative of each other, so if u 1 is going up as a step y 2 will go to G 2 1, what I would like is that this signal should be minus G 2 1, for this signal to be minus G 2 1, this signal should be minus G 2 1 by G 2 2, while that is what goes in here, what should go in here minus G 2 1 by G 2 2.

Similarly, if I get a step here, I would like that only this changes, this change y 1 does not change, in order to do that I take this signal, pass it though a transfer function box and what should go in here, that is the question well, what should go in here should be such that a this signal is negative of this signal, if this is going up as a stop this signal will be G 1 2 time u 2. So, this signal has to be negative of that, in order of this single to be negative of that, what should this be this should be minus G 1 2 by G 1 1 well, that is will goes in here minus G 1 2 by G 1 1. Now, if you look at if you look at the input output relation in this box, if you look at input output behavior between u 1 star and y 1 and between u 2 star and y 2, if I go to the next page there will be trouble, let see if I can fit in here.

Let just see how, what is the dependence y 1 on u 1 star and u 2 star, if I if I look at, so y 1 would be equal to G 1 1 times u 1 star that is this guy, when I make a change here this decoupler will cause a change in u 2, and this change in u 2 will give me an additional signal here, what is that additional signal that would be plus minus G 2 1 by G 2 2, that is this guy times G 1 2 into u 1 star. So, that is the depends of y 1 and u 1 star, what is the dependence of y 1 on u 2 star? well if u 2 star goes up as a step, the effect on y 1 would be plus G 1 2 into u 2 star. Now, because u 2 has gone up as a step this decoupler would cause, what I am saying is this decoupler will cause u 1 to move, and what is that movement in u 1 would be minus G 1 2 over G 1 1 times G, the signal is minus G 1 2 or G 1 1 multiplied by G 1 1 times u 2 star.

Now, if this thing is physically realizable, what I mean is if this thing is physically realizable, then these two term will actually cancel out, and what I have is y 1 depends only on u 1 star, no depends on u 2 star. Similarly, what I will find is that y 2 depends only on u 2 star, no dependence on u 1 star all right. So, this is called dynamic decoupling, and I have cover it just because the hake of covering it, because it cover usually cover most of the places.

(Refer Slide Time: 31:16)



The thing that I wanted to tell you was that, in chemical process set point tracking is required, only when you are starting of the plant or shutting down the plant, that happens let us say once one two year or once in a year, set point tracking. What you are interested in is load rejection, most of the time load reduction is a kind of like autopilot, in and just to make that point let just consider this, a single input single output system G p, I have got a controller well, let see something is coming here, and what I have here is G d and this is disturbance d, and this is y, and what I have is y set point plus minus that is negative feedback, this is the error, and this is the input to the process.

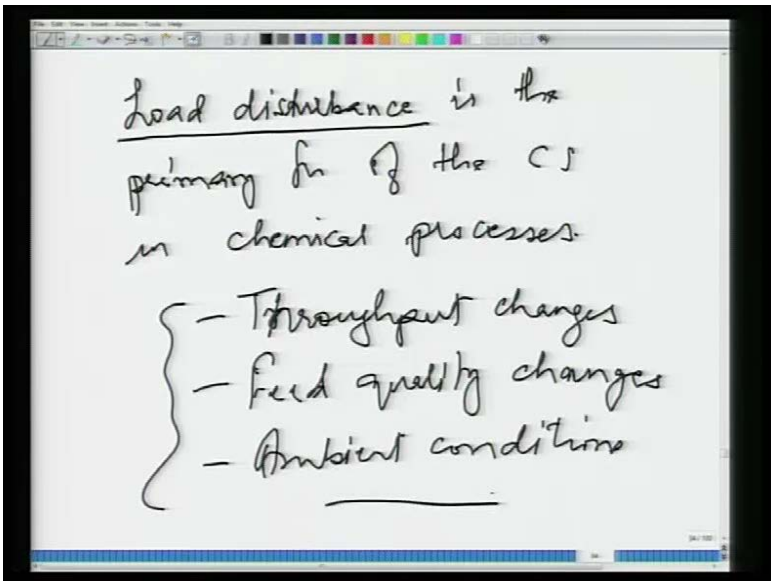
So, the disturbance d affects y through the transfer function G d, and if I look at the closed loop I mean the overall transfer function, what I will find is if d is 0, how does y set point affect, a change in y set point affect y, we saw that y, let just do this, y is equal to this signal plus, this signal that is equal to. So, this signal is G p times u or G p G c e. So, this is actually G p G c error and the error is y set point minus y and, and his signal is plus G d times d. Now, if take the y terms in the left hand side, what I get is 1 plus G p G c times y is equal to G p G c times y set point plus G d times d, and then I divide what is here what I will get is this is what I get all right.

So, how does y depend on a change in y set point or on a change in the disturbance d well, this is the equation that governs that, that shows the dependence, this is called the servo transfer function, that we have seen this is called the servo transfer function, and this is called the regulated transfer function. So, we were talking about the servo and regulated transfer function.

So, if we have a single input and single output system with a block diagram I shown here, the dependence of the output on a change in the disturbance and or change in the set point is given by this equation; and this transfer function is call the servo transfer function, and this transfer function is called the regulated transfer function. And the servo transfer function shows how the output response to change in a set point, regulated transfer function characterizes, how the output changes to change in a disturbance.

Now, notice that in both the transfer function 1 plus G p G c occurs in a denominator, and this is what actually determines the stability of the closed loop system, why am I talking about the servo and the regulated transfer function, that because when I am running a plant, I am actually more interested in ensuring that disturbances are rejected effectively. So, I am interested more in a regulator response, my interested in a servo in a servo response is not much.

(Refer Slide Time: 36:18)



Because, primary objective in chemical process is load disturbance, effective load disturbance is the primary function of the control system, of the control system in chemical processes. And what do I mean by that, what do I mean by that is what put constitute load disturbance? For example, the quality of the feed to the process changes, the quality of the feed to a distillation column changes, the quality of the feed going into processes changes, that is a disturbance into your process, my control system should be able to handle it.

Another example, I would like to increase the through put or change the through put or change the production rate in the process. So, my heating rate, cooling rate, flow rates etcetera must follow that change in through put. Must effectively execute that change in through put for me, so that is again a load disturbance more heating load less heating load. So, these are so, though put changes examples of load disturbance is through put changes or yeah through put or feed rate changes, more generally refer to as through put changes.

Another example is feed quality changes, what are other load disturbances, well ambient condition disturbance, we have talk about days are hotter than nights, winter are cooler than summer, the cooling water in rainy season is not as cold. Because, cooling tower are not as efficient, because the air is assume it and so on, so forth. So, these are all disturbances, I want to operate at access point regardless of these disturbances, and this is govern by the regulator response, ok.

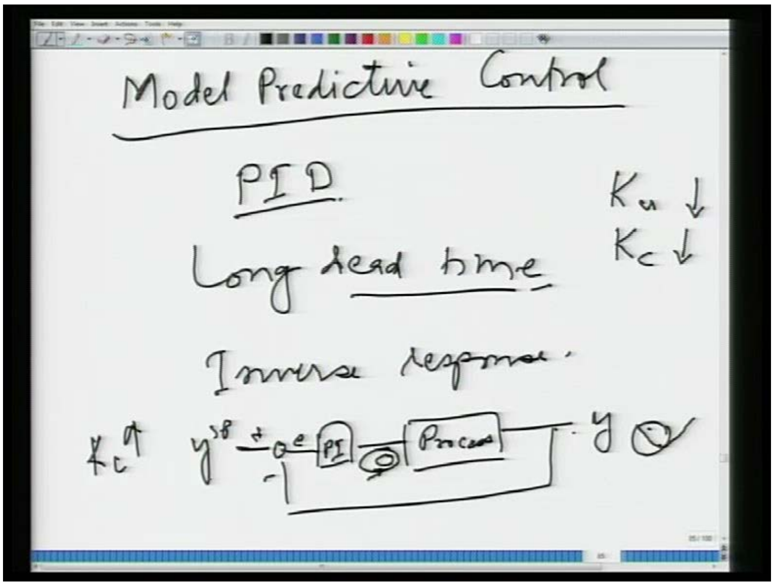
Load disturbance is the primary function set point remains the same, I would like to operate my reactor at this reaction temperature, regardless of the what is the composition of the feed, regardless of what is the flow rate that is going into the rector right. So, that is actually govern by regulator response, so the regular response is what is of interest in operating a chemical process around its design steady state, that is point number one; where is set point tracking important well aerospace system for example, their space point tracking is important, because the missile has to follow a certain trajectory, that trajectory is the set point the position set point, that your missile should track similarly, airplane, choppers, etcetera, robot, robotic arms.

So, in mechanical or an or air space system set point tracing is important, in chemical system it is load rejection, or load disturbance rejection, effective rejection or effective handling of load disturbance is what is important this point, I would like to keep in your mind. Because for example, we covered decoupling why is decoupling important? Well decoupling is important because if your objective is to track the set point, and you have a decoupled system, what happen in a decoupled system is if you change input, u only y 1 once get effected, y 2 does not get effected, if you change input u 2, y 2 gets effected, y 1 does not get affected. So, therefore the loop for y 1 and the loop for y 2 can be tuned real tight, and this loop does not disturb that loop, and that loop does not disturb this loop. So, where set point tracking is important, decoupling is useful.

Load disturbance rejection for load disturbance rejection you know, decoupling may or may not be useful, that is because the transfer function itself is different, and in a papers way back in the early 70s 1971 probably, Niderlinski actually showed in a paper, that if you implement a decoupler, your load rejection actually becomes worse, your regulator response with the vocalizes with the decoupler there, actually become worse set point tracking is better off, but there is there are cases where load rejection actually suffers with the decoupler in there.

So, implementation of a decoupler may or may not be recommended all the while. In fact, if you look at what the liben has to say he says you know, we actually do not need decoupling, but nevertheless for the sake of completion, I have covered it.

(Refer Slide Time: 41:27)



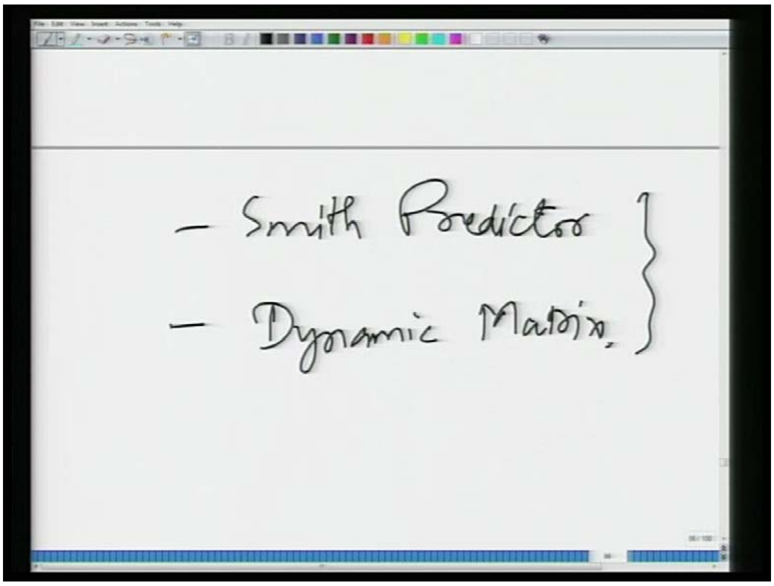
Next time what we are gone do is cover probably, the last thing that needs to be covered which is model predictive control, right now what we have seen is you know, you got controls and we have looked at the PID controller algorithm, we have look at, how to tune the tuning constant in a PID controller?

These PID controllers for example, may not work very well for system with difficult dynamics, what is an example of system with difficult dynamics? for example, system with the long dead time or system with an inverse response, which is also sometime refers to as a wrong way behavior. So, what happens if you have got a long dead time, the process with a long dead time, what happen there is I have got my, I have got my process, and I have put in my PI controller y set point y plus minus. I have put in my PI controller, I make a change here nothing happens here for a long time now, because nothing is happening to the output, the error remain unchanged the integrator in the PI controller keeps on adjusting this guy.

And what that essentially does is you cannot have your gain too large, gain cannot be increase if you tries to increase gain to get tight control of y, what you get start getting is sustain oscillation. So, for large dead time process is the ultimate gain is small, its reduced because of the large dead time and therefore, you cannot your K c your the gain that you implement in the controller also is reduced. And therefore, you know the closed loop response is not as fast an snappy, as we would like it to be should that be the case, you would like to incorporate a model that tells the controller do nothing, because the process is of that type you do something you have to wait, because nothing is gone happen to the output, because there is so many dead time in the process, how do you do that? how do you incorporate a model to accomplish that.

Similarly, in an inverse response what happen is you do something here, suppose you are increasing the steam air, you expect that temperature to go up right, but what happens is the temperature actually starts to go down, before it can come up. Now, because of this wrong way behavior again, this is worse than dead time, in the dead time nothing was happening to output here the output is moving in the wrong direction, again this would cause the ultimate gain to be reduced this inverse response behavior. And therefore, the gain that you can implement in API controller would be smaller, and with that small gain you may or may not well in the case, where you do not fast and snappy exceptable closed loop response, you have to look into model based control techniques. These model based technique is going to come, and look at next time.

(Refer Slide Time: 45:02)



And what I am going a cover next time is let see, the smith predictor, which is for Seso system, and then dynamics matrix controller. And that I believe would put in end to whatever control theory is to be told or used to be cover in the 9 or 10 lecture, that it was supposed to be covered in.

Thank you.