Plantwide Control of Chemical processes
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Lecture - 6
Systematic Tuning Using Frequency Domain Analysis

So, what we have seen is that in multi variable systems, interaction requires that the decentralized PI controllers be detuned. Usually if one of the control objectives is much more important than the other, then it is clear how the detuning must be done; you tune the one the control control loop corresponding to the more of important objective the tightest, and then take all the de tuning in the other not so important control loop. However, there are situations where both the objectives are equally important, and then we need a systematic technique to detune both the loops systematically or equally, so that the detuning gets taken about equally in both the loops. So, how do we do that? That requires some control theory, and not some actually a lot of control theory.

And what I will try to do is, go over the relevant bits, so that the systematic decentralized loop de tuning procedure can be reasonably understood. So, over the next one one and a half or two hours, what we will do is, go over the essential control theory that is necessary to understand how decentralized multi variable controllers are systematically detuned.

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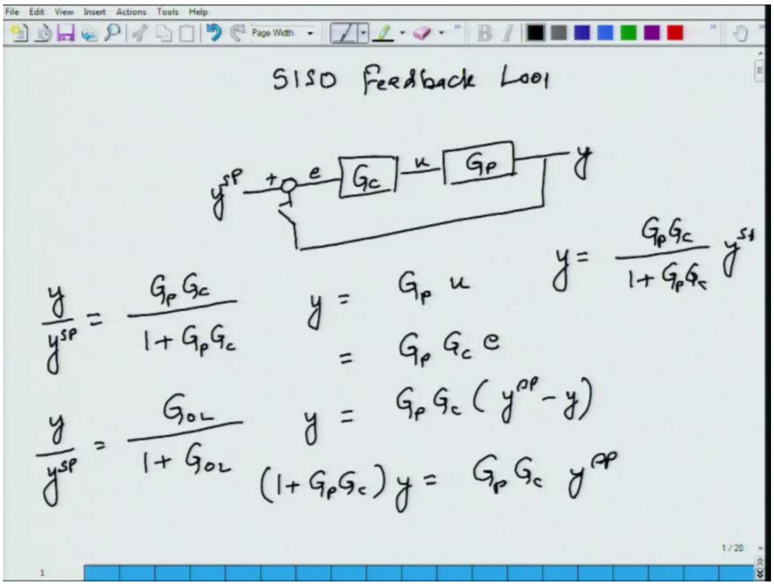

So, let us look at SISO systems; a SISO single input single output feedback loop. If you look at the single input single output feedback loop; so you got your process it has a transfer function G p, you have a controller, which has a transfer function G c, and you got the output the output is compared with its set point negative feedback, and that is it; that is your single input single output control loop. And this is called the error this is the control input and of course, that is the output.

So, if you look at the relationship mathematically, this block diagram mathematically, what you get is, y is equal to G p times u. However, u is equal to G c times e and e the error is equal to G p G c, if you can replace the error with y set point minus y. And now if you take y to the left hand side, what you will get is 1 plus G p G c of y is equal G p G c y set point. And what this gives is y is equal to G p G c divided by 1 plus G p G c y set point and therefore, what you get is that the server transfer function of the output with respect a change in the set point is actually equal to G p G c divided by 1 plus G p G c and if you look at it carefully if this loop was not on if the feedback loop was not on; that means, this loop was broken for example, let us see if this loop was off and that is a switch there which is not connected.

So, the feedback loop is off then what you see is y the change in y with respect to a change in the in y set point actually G c G p right the numerator is the forward is the transfer function in the forward path and the denominator is the 1 plus whatever is there in the whole feedback path. So, what you get essentially is. So, we call G p G c as the loop transfer function or open loop transfer function and what we.

Get is G open loop divided by 1 plus G open loop. So, this the servo transfer function of the closed loop system. So, man be we will go to the next page.

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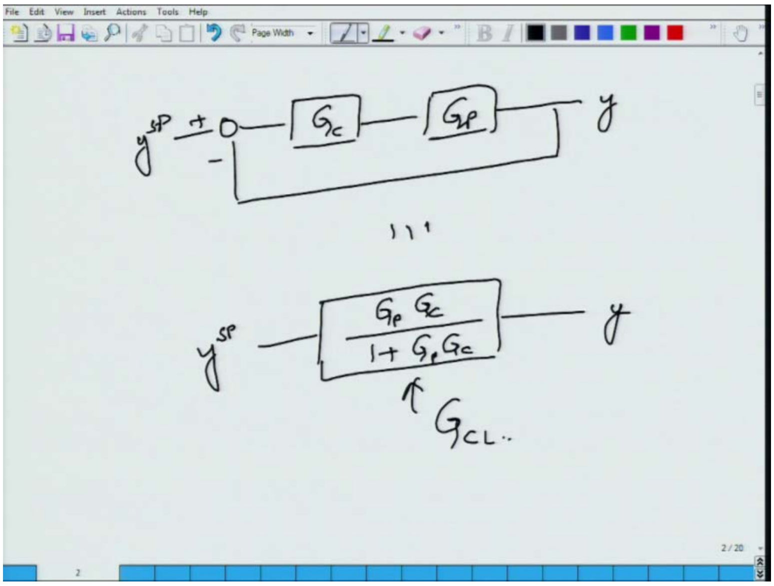

So, what we have is the simple feedback control system G c G p this thing can actually is actually equivalent to this is the y a single the above block diagram is equivalent to this simplifies to this and this I will call the closed loop transfer function.

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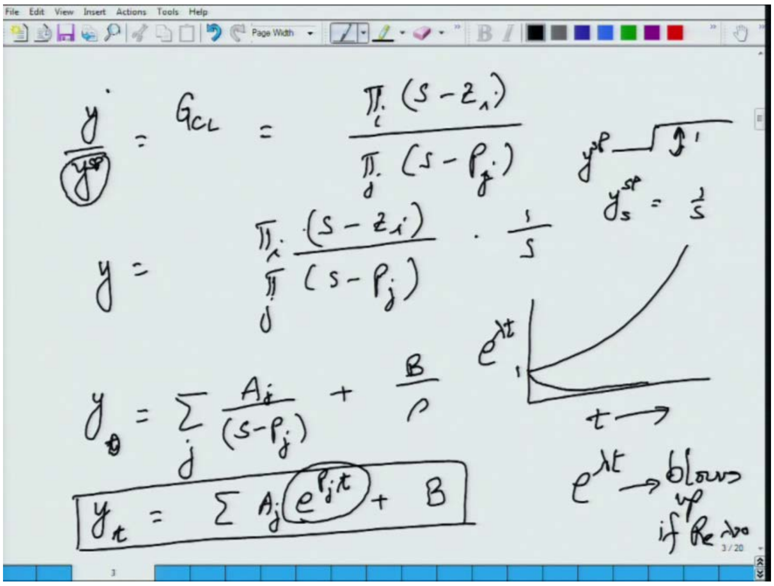

If I look at the closed loop transfer function or if I look at any transfer function if I look at any transfer function closed loop open loop whatever. So, for example, y over y set point any transfer function then it will be a bunch of poles and zeros. So, so let us say this is product of some bunch of zeros and a product of some bunch of the denominator simplifies to a bunch of poles and now let us say we say that this guy is actually a step if it is a step then what we will have is y will be equal to the Laplace transform of. So, this is product over i product over j product over i product over j this is j sorry s minus p j times whatever is the change in y set point.

So, let us say you are giving a step change in y. So, y set point is here and it changes by a unit step then what we have is the Laplace transform of y set point is will actually be equal to 1 by s and you can refer to our control book to see this and then what you will get is y actually is this guy and ultimately if you want to get the domain time domain response y t what you will have to do is take the you know separate this into partial fractions and when you do those partial fractions what you will get is y t is essentially equal to A over well a summation over j s A j over s minus p j plus may be B over s plus a constant term and this is assuming that the all the initial conditions are 0 and. So, this is y of s separated.

Into you know partial fractions using partial fractions you can get this and then what you will get is when you take the inverse Laplace transform y t will actually be equal to summation A j e to the power p j t plus B because the inverse transform inverse Laplace transform of 1 over s is one. So, therefore, you get this as the dynamic response of this closed loop system to a step change in the set point now you can see that the dynamic response is you know is coming from these terms and if you look at these exponentials exponentials if you plot in time this is time and if you plot e to the power let us say lambda t then what you will get is essentially let us see e to the power t t equal to 0 would be one and if lambda is positive things will grow up if lambda is negative things would this is.

Of course one things would decay to 0 in long time. So, if any of the poles of your transfer function is positive if any of the poles is positive what you get is you will get an e to the power that pole times t term which will blow up in time. So, what; that means, is if any of the poles of the transfer function that you are looking at has a positive is in the right half plane has a positive real part that positive real part e to the power lambda t will blow up blows up if real part of lambda is greater than 0.

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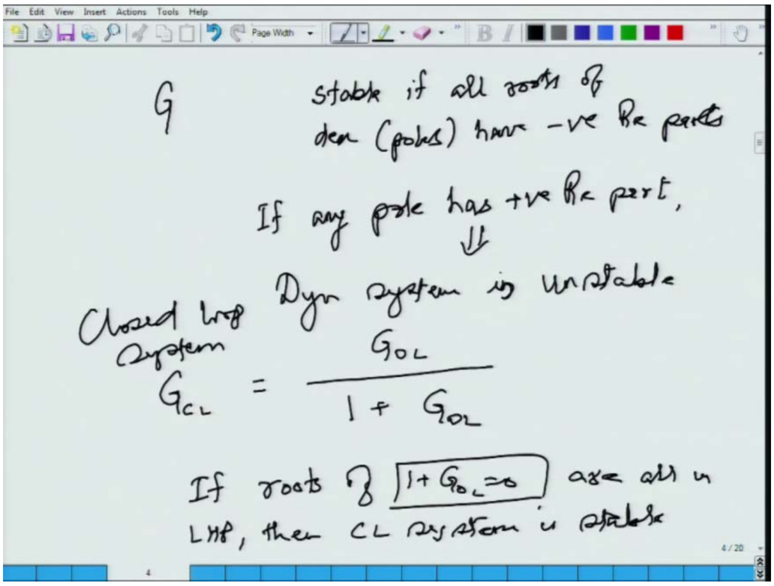

So, what this basically gives is that stability of a system if you have a system and weather transfer function G this transfer function is stable if all roots of denominator which are also referred to as poles have negative real parts what this means another way of looking an at looking at it is if any pole has.

Positive real part then that dynamic system then the system is that implies dynamic system is unstable now for closed loop systems like I showed you the closed loop system transfer function G closed loop is actually equal to the open loop or the loop transfer function divided by 1 plus G open loop.

So, closed loop closed loop system now if you look at G open fine. So, what this means is if roots of 1 plus G open loop equal to 0 if of this equation are all in left half plane; that means, they have got negative real parts then closed loop system is stable if any of the roots of this equation 1 plus G open loop equal to 0 are in the right half plane even if one of the roots is in the right half plane then that closed loop system is unstable.

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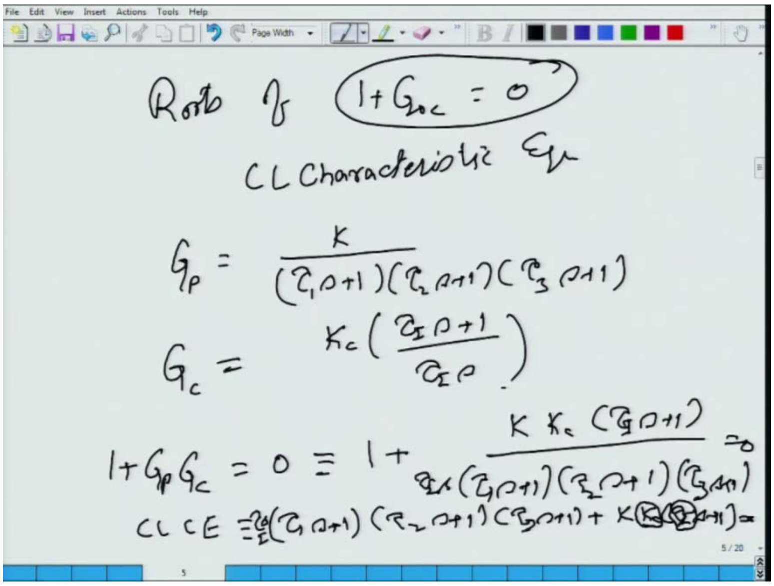

So, if you want to figure out whether a closed loop system is stable or not then what you need to do is figure out roots of 1 plus G open loop equal to 0 stability is governed by the roots of this equation this is called the closed loop characteristic equation. So, now, what you want to do is figure out whether this guy has any root in the right half plane if it has any root in the right half plane the closed loop system is gonna be unstable.

So, stability or instability burns down to figuring out whether the roots of the closed loop equation are in the right half plane or not that is the crux of the matter how do we go about figuring it out well if you have a system which has got no dead time and which is representable by you know rational transfer functions.

So, for example, G is equal to G process may be equal to I do not know a second order may be a third order system and over that tau 1 s plus 1 into tau 2 s plus 1 into that is the third order like and let us say you got a pi controller. So, the controller transfer function is gonna be K c times 1 plus 1 by tau i s. So, K c into integral time constant s plus one divided by this is a PI controller transfer function how do you get it refer to your text book.

So, then what you would say is 1 plus G p G c is equal to 0 is equivalent to 1 plus well let us see K times K c times tau i s plus 1 divided by tau 1 s plus 1 into tau 2 s plus a man let us make it a p well it does not matter tau s plus 1 into tau 3 s plus 1 and. So, what you will get is that your closed loop characteristic equation for this system is actually equivalent to tau 1 s plus 1 into tau 2 s plus 1 into tau three s plus 1 plus K times K c times tau i divided by tau i s that I have forgot.

So, there is a tau i s here tau i s plus 1 equal to 0 do not worry about the mathematics of it the only point that I want to make is that as you change K c this is the controller tuning parameter and tau i as you change K c and tau i the coefficients of the s and the constant term change if the coefficient change coefficients change then you can see that this is actually a 1 2 3 a fourth order system because the highest power of s in this equation is four.

So, it is got four roots it is got four roots and as you change the tuning parameters the position of these roots change. So, what you will find is if you have a feedback control system for this third order system let us say you have fixed tau I and you are just changing K c if you have fixed tau I and you are just changing K c what you will find is.

When K c is small all the roots of the closed loop characteristic equation are real and in the left half plane; that means, real in negative then if you crank up the K c you will start seeing that the that oscillations are coming what; that means, is and these accelerations actually die down the these oscillations decay in time what; that means, is that the roots have negative real parts; however, some of the roots are now having complex conjugate parts these complex conjugate parts imply that you get oscillations the fact that the real part is negative implies that these oscillations die down if you keep cranking up the gain further what you will find is that these that you know these oscillations die slower and slower and there comes again where the oscillations become sustained and what this is implying is that as you are.

Changing K c the closed loop characteristic equation roots are moving and they are moving from the left half plane towards the right half lane plane and when you have sustained oscillations then you got purely at least two purely imaginary roots and then when you crank up the gain further you know the real part of the root becomes positive and then you start getting these oscillations that blow up.

So, this is what we observe in practice and. So, if you have a transfer function that looks like this you can trace the roots as K c as a function of K c you can plot the locus of the roots as a function of K c and this is called the root locus from then you can figure out at what K c does the gain you know at what K c do you start getting oscillations at what K c do you have the roots moving from.

The left half plane to the right half plane and. So, on. So, forth and that gives the ultimate gain and then of course, we have seen in the past how tuning is done you crank up the gain get sustained oscillations and you say this is the limit of stability I must run my tuner my controller at a gain that is sufficiently away from the words of in stability. So, back off the gain by a factor of two etcetera etcetera etcetera all this is fine, but chemical processes are notorious in that they have got large in times.

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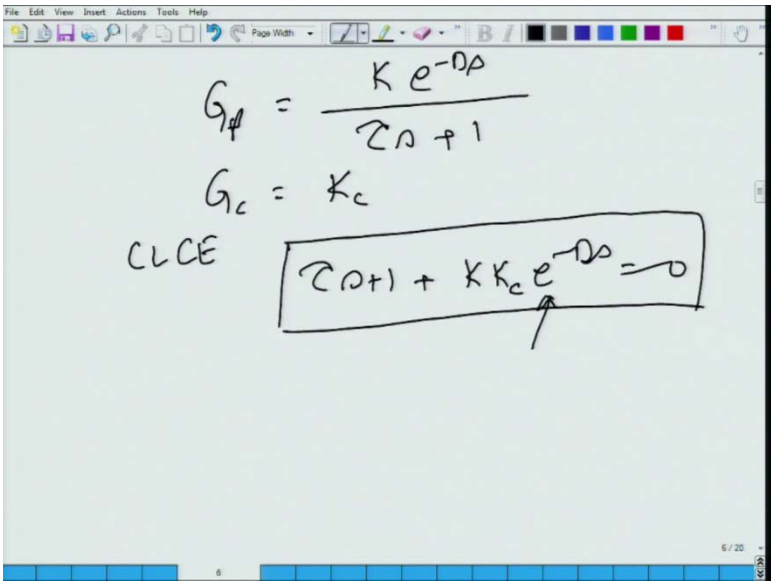

So, a very typical transfer function would be G p is equal to first order plus dead time K times e to the power minus dead time upon tau s plus one now if you look at this guy the closed loop characteristic equation for a p controller. So, and let us say G c is equal to K c; that means, it is a p controller if it is a p.

Controller the closed loop characteristic equation would be tau s plus 1 plus K times K c into e to the power minus D s is equal to 0 now you can see this is a transcendental equation because of the presence of this term. So, a you do not know how many roots there are because in a transcendental equation you cannot tell how many roots there are if it is a polynomial equation then the order of the polynomial tells you that if the order of the polynomial is four there are four roots. So, root locus tracing the locus of the roots is not an option here. So, for chemical systems root locus cannot be used, but the idea remains the same as you are changing the gain the some of the closed loop characteristic equation roots move and they tend to move towards the right half plane now for these kind of systems.

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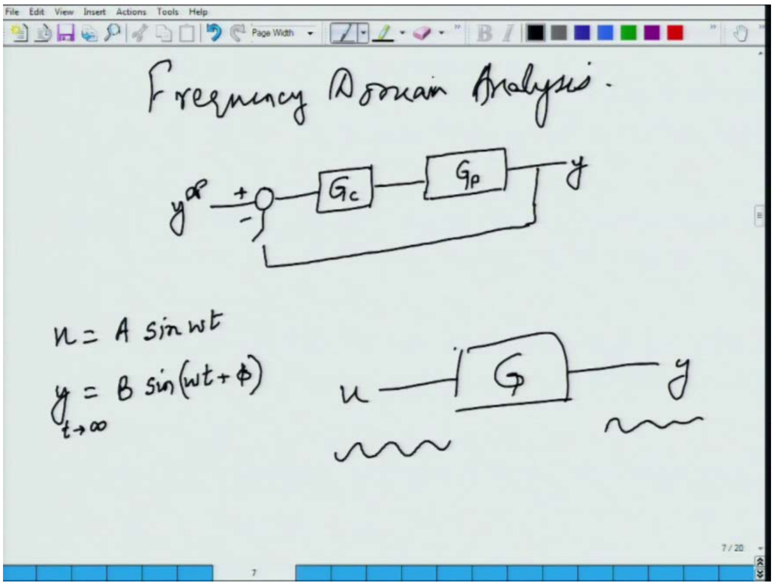

What is more relevant is frequency domain analysis and I will just go over what is frequency domain analysis very quickly frequency domain frequency domain analysis let us let us just say because we do not know of feedback systems. So, now, let us say I have I still have a process and well let us see if you have a process and even though that process may have dead time let us see let us say I have got a process with a controller and let us say I have got a set point same system now of course, I need a I need a switch here which will be very useful now there is something in linear system theory that if there is a linear dynamic system even if it has dead time and you fore set with a sinusoid; that means, I have got a linear system.

Which has got a well which has got a transfer function G and I have got an input and I have got an output if this input is a sinusoid then the output would also be a sinusoid; however, its amplitude may be more than the input sinusoid and also it will have a phase lag. So, if you force a linear system with a sinusoid after the initial transients have decayed the linear system will settle into sinusoid oscillations and the output sinusoid would be of the same frequency as the input sinusoid it is amplitude may be more or less and it will have a phase lag.

So, what we are saying is if y if u for example, you are forcing a linear system with let us say a sinusoid that is amplitude a sin omega t then after sufficient time has elapsed and you have allowed the initiate transient to span out what you will get is the output would be let us see be a different amplitude a sine wave with the phase lag phi as t tends to infinity the system will settle down into these nice into a nice.

So, I am sorry into a nice kind of system let us look at the feedback loop above and let us say the feedback is not yet closed the. So, the feedback loop is. So, the feedback loop is what shall we say and by the way this can also be represent as this guy I can put a multiplier by minus well.

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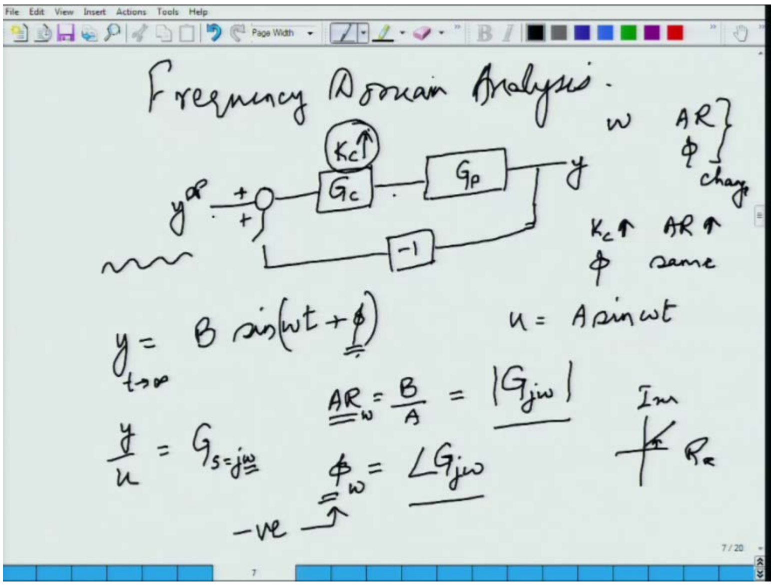

Let us see minus 1 here this is the same feedback system this is that I am multiplying by minus 1. So, let us say the input is a sine wave. So, the input is a sine wave and let us say it is amplitude is one if the input is a sine wave then this would be a sine wave if sufficient time has elapsed then this would also be a sine wave this would also be sine wave and if sufficient time has elapsed then G p is being forced by a sine wave then y would also be a sine wave and the signal here would also be a sine wave right.

So, what you get is if I am forcing my set if my set point why set point signal is moving as a sine wave I will get a sine wave here it is amplitude will be different and it’ll have a phase shift alright now let us say let us say this G c is just a gain it is a it is you know a proportional controller now if I start cranking up K c what will happen is the amplitude of the output sine wave will go up. So, this sine wave will start becoming bigger and bigger and bigger as K c is increased also the phase shift will change if you have a linear system being forced by A sine wave u is equal to K sin omega t and then the output will settle as t tends to in infinity as another sine wave which has the same frequency and a phase lag and if the relationship.

Y by u which is a transfer function is given by transfer function G then what people have shown that is the amplitude magnification which is called the amplitude ratio which is equal to B by A how much is the sine wave getting magnified by is actually given by G j omega where s in the transfer function the Laplace variable s has been replaced by G omega the magnitude of this complex number similarly the phase lag or the phase shift is given by the angle of G j omega and what I mean to say is just to just to clarify you see when you replace s in the transfer function with j omega when you put s equal to j omega for a particular value of the frequency of the sine wave for a particular value of omega what you will get is that G j omega is a complex number.

Now if you look at complex numbers complex numbers can have real parts and imaginary parts and let us say the complex number is this guy then the magnitude of that complex number is this the length of this inclined line and the angle of that complex number is this guy have measured anti clockwise positive angle is anti clockwise this everybody knows.

So, what linear system theory says is that if you are forcing a linear dynamic system with a sine wave of a certain frequency then the output sine wave after sufficient time has elapsed would be a sine wave of the same frequency, but it will magnified by an amplitude ratio and that amplitude ratio is given by the absolute value of the complex number G j omega and there will also be a phase shift phi and that phase shift will be given by the angle of the complex number G j omega alright.

So, now, why is this relevant well I will tell you why is this relevant let us say let us say you can see that the amplitude ratio and the phase shift are functions of omega right because G is a function of s if you put s equal to j omega the complex number the real part and the imaginary part of that complex number will depend on omega and as omega changes the real part and the imaginary part of that complex number changes as the real part and the imaginary part of that complex number changes well its magnitude and its angle changes if its magnitude and angle changes then; that means, the amplification of the sine wave the factor by which this sine wave is being amplified and its phase shift changed. So, the. So, the magnificent or. So, the amplitude ratio.

And the phase shift depend on the forcing frequency now because most systems are causal in the sense that if I make a change to the input the output takes time to manifest to respond to the input therefore, these for more for natural systems for most physical systems the phase lag or the phase angle is usually negative also notice that if I change this scalar factor K c then K c does not depend on s since K c does not depend on s well let us see how do I explain this K c is a pure real number if K c is a pure real number it will only effect the amplitude ratio, but not the angle well you need a complex algebra to know this, but refer to your text books. So, now, let us see what I am trying to say is let us say I start cranking up K c intuitively you can see that if the frequency.

Of the input sine wave is the same and the amplitude of the input sine wave is the same and if I am cranking the gain up if I am cranking the gain up what I will get is I will get a sin wave which is getting magnified larger because you are multiplying it by a larger number.

So, as K c is increased amplitude ratio will go up the phase angle will remain the same as omega the forcing frequency is changed amplitude ratio will change and phi will also both will also both will change now let us say I am changing the frequency and usually the phase lag is a negative angle I am changing the frequency of the forcing sine wave y set point and I get that frequency at which the phase is minus 180 degrees. So, let us see maybe I can rub all these off because this slide remains relevant rest of the stuff is just.

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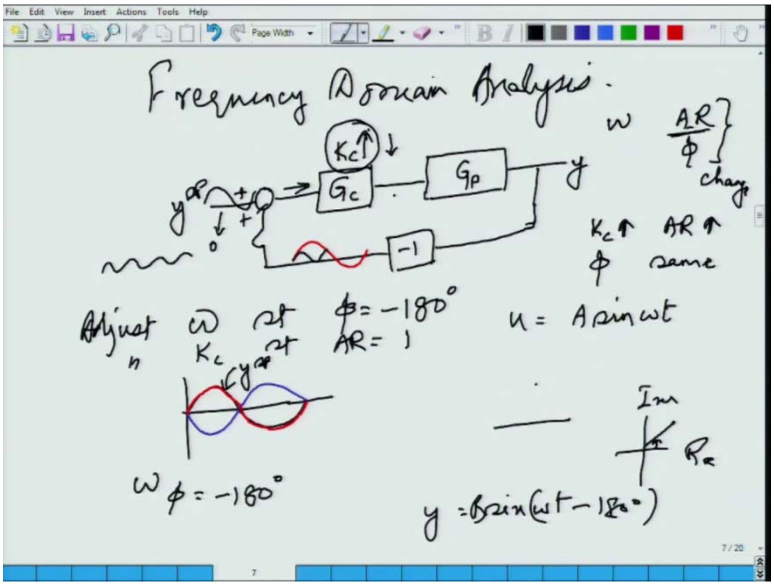

So, I adjust the forcing frequency omega such that the phase angle is minus 90 degrees or minus 180 degrees phase equal to minus 180 degrees if this is A sine. So, so I am drawing y as a sine wave let us see let this is fine. So, y is a sine wave that goes like this then if the phase lag is minus 180 degrees then well let us see. So, y would be sin omega t minus minus 180 degrees and some factor d and what; that means, is t equal to 0 sine minus 180 what is sine minus 180 sine minus 180 and we need. So, then your u is changing this is u if this is u and the output sine wave is lagged or has a minus 180 degree phase shift then what you can see is that this signal y and maybe I should draw it as a u u curve one more thing.

So, I just told you if you adjust K c you can get the amplitude ratio whatever amplitude ratio you want you can get without effecting the phase that being the case let us say I adjust the omega to get a phase shift of minus 180 degrees and I adjust K c such that amplitude ratio is equal to 1 if I have done that then the sin wave input sin wave looks like this is u meaning that this is y set point then y would look like mirror image of this guy and of course, the sine waves go on and its amplitude would be the same as the input sine wave because the amplitude ratio is 1 K c has been adjusted to give you an amplitude ratio of one yes or no forget.

So, here comes a sine wave there is the output sine wave then when you multiply this y by minus 1 you will get another sine wave and when you multiply it by one this blue sine wave will flip will flip over the time axis and then what you will get is your feedback signal you know the signal that is being added to y set point will actually look exactly like this yes or no. So, if I am at that frequency at which and at that and I adjust the K c such.

That the amplitude ratio is one and the phase shift is minus one eighty degree then this signal would be this red signal this red wave would be the same as this blue this black wave the red wave and the black wave would be the same they would exactly be the same now what do I do the feedback loop is not yet closed the feedback loop is not yet closed what I do is I shut down the sine wave the forcing sine wave.

So, this signal goes to 0 and I close this feedback loop what would happen well the forcing has gone to 0; however, there is this sine wave which keeps going back and therefore, the system will keep on oscillating on the other hand if I reduce the K c a little bit then what will happen is this sine wave would be smaller and what; that means, if there is a sine wave forcing this guy then the output sine wave is smaller and when you fit that back in smaller will become still smaller will become still smaller and basically if you stop.

Forcing the systems this oscillation will become smaller and smaller and smaller and the system will settle down. So, do you see that the frequency at which phase equal to minus 180 degrees is special and of at if at this frequency the amplitude ratio is less than 1 then when you close the feedback loop oscillations die out if the amplitude ratio is one the oscillations will remain even though you are not forcing the system similarly if the amplitude ratio is greater than 1 then those accelerations even though you are not forcing the systems will tend to blow up.

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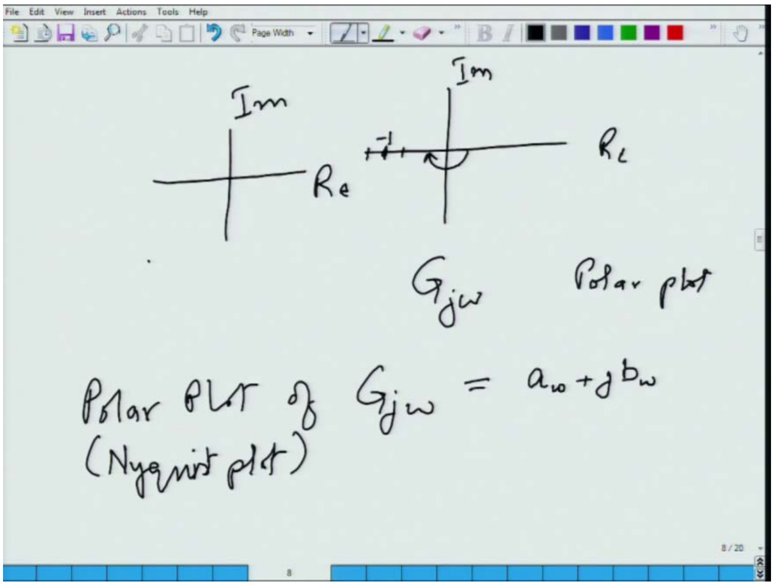

So, what we are saying is that if you have the G plane; that means, you are plotting G j omega the real part and it is imaginary part this is actually called the polar plot of G j omega and I will I will just show that to you then.

Frequency phase shift of minus 180 degrees means that the angle is that the angle is minus 180 this is the angle amplitude ratio one means we are talking about this complex number. So, this com this number minus 1 the real part is minus one and the imaginary part is 0 is actually special and what we will just discuss very soon is that if for all value if you have G j omega if when the angle is minus 180 degrees absolute value of G j omega is greater than 1; that means, G j omega is here then the system is gonna be unstable if it is here it is going to be stable if G j omega is this guy passing through minus 1 0 then it is gonna be on the on the brink of stability and instability right and let me also introduce at this point of time the polar plot of G j omega polar plot of G j omega which is nothing, but you plot the real see G j.

Omega is a complex number with the real part and the real part depends on omega plus j times B which is the imaginary part and the imaginary part also depends on omega.So, the plot of the real and imaginary parts as omega goes from 0 to infinity from low frequencies where the sine wave is really slow to very fast sine waves that is called the polar plot or Nyquist plot of a transfer function now let us say your transfer function is let us say let us say your transfer function is I do not know first order if you look at a transfer function that is first order.

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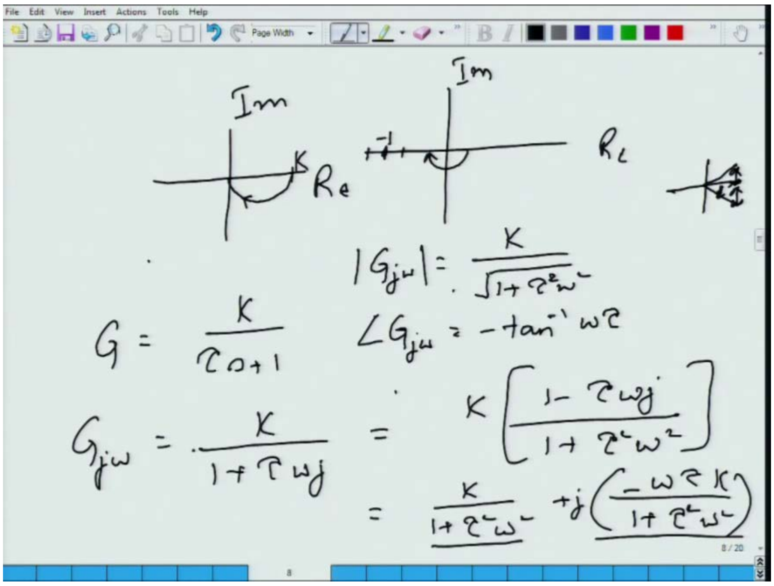

So, let us say your transfer function is G equal to I do not know K over tau s plus 1 then G j omega will be equal to 1 plus tau omega j and then if you multiply by the complex conjugate of the denominator what you will get is actually K times well 1 minus tau omega j divided by 1 plus tau square omega square and of course,.

K is multiplying everything and therefore, what you will get is the real part of this complex number is K by 1 plus tau square omega square and the imaginary part is plus j into minus omega tau divided by 1 plus tau square omega square and then you will say that the magnitude of G j omega magnitude of G j omega is gonna be oh of course, I forgot the K here magnitude is gonna be K over square root of 1 plus tau square omega square and the angle is going to be the angle of a complex number if you have a complex number what is its angle its angle is well if it is got a negative real part. So, if it is got an actually if it is got a negative imaginary part the angle is. So, this is the complex number.

That we are talking about it is got a positive real part this is positive and it is got a negative imaginary part. So, the complex number is in this fourth quadrant. So, the angle is gonna be whatever is the magnitude of the imaginary part. So, the angle is gonna be negative and the angle is negative because you are going in the clock wise direction.

So, negative tan inverse real part by imaginary part that is essentially gonna be omega tau and now you can see that as omega tends to 0 the magnitude of the complex number is K angle is 0 as omega tends to infinity magnitude tends to 0 angle tends to minus tan inverse infinity which is minus ninety degrees. So, if you look at the Nyquist plot of a first order transfer function it looks like this starts at K and as omega is increased goes this way.

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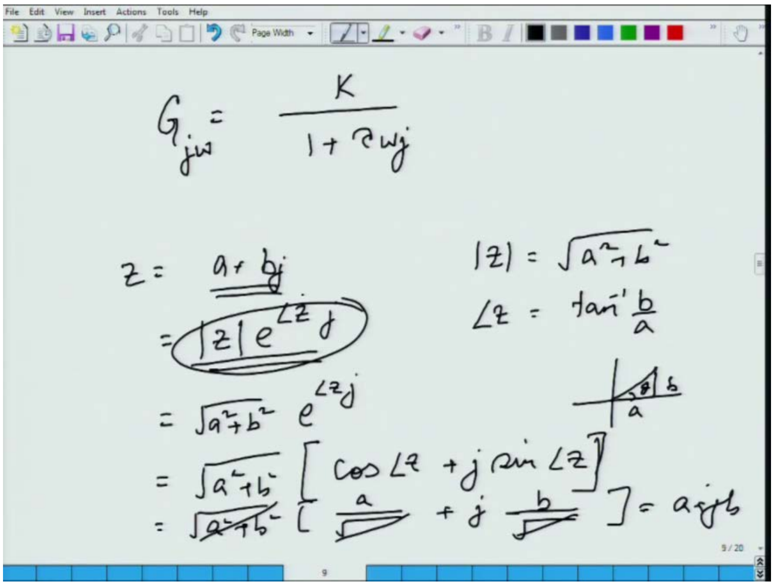

This is the Nyquist plot of a first order transfer function let us take this guy only G is equal to K over 1 plus G j omega is equal to tau omega j now if you remember any complex number for example, let us say I have I got a complex number z is equal to a plus b j this complex number can be represented as this is the Cartesian form it could also be represented as magnitude of z times e to the power of angle of z times j where magnitude of z is equal to a square plus b square and angle of z is equal to tan inverse b by a I am assuming both b by b and a are positive if they are not then you will have to think which quadrant it is and then adjust the sign; however, the equivalent well you just put.

Put this put this expressions in there and then what you will find this. So, if you have an angle this is b and this a and this is the angle then. So, what you have is this angle is phi let us let us call this angle theta or angle angle of z then this guy would be square root a square plus b square times e to the power angle.

So, e to the power angle of z j and by (()) identity you know that is equal to square root a square plus b square times cos angle of z plus j times sin angle of z and once you see what the cos of this angle is the cosine of this angle is what cos of this angle is a by square root a square plus b square. So, that is equal to square root a square plus b square times cos of the angle is a by square root a square plus b square plus j times sin of the angle is b by the same square root terms and then.

What you find is that the square root term cancels out which is equal to a plus j b. So, you see that a b j. So, you see that this is actually equal to this guy this is called the polar form representation of the same complex number a plus a plus b j it is much easier to work with these analogies.

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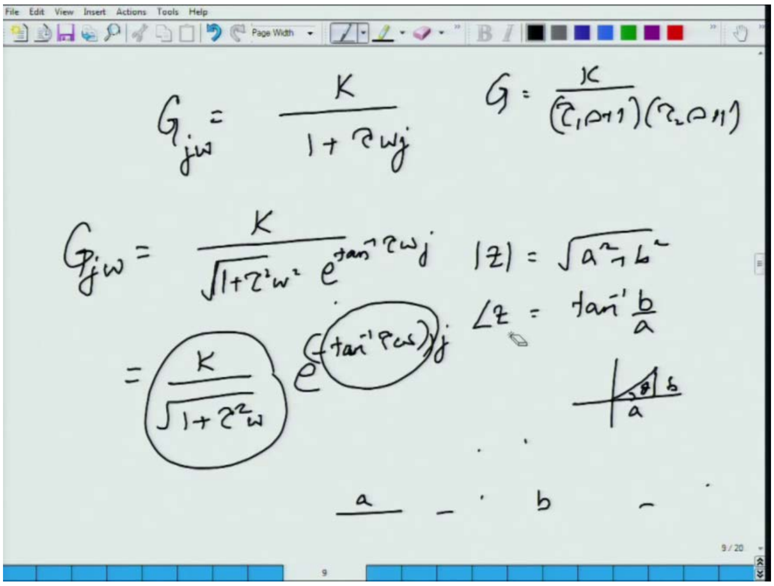

So, this is to use for this first order transfer function what do we do. So, this will be equal to G j omega will be equal to K divided by. So, K remains K and I can replace this complex number by magnitude of this guy and the magnitude of the denominator is gonna be square root 1 plus tau square omega square and the angle of this guy will be e to the power imaginary part by real part tan inverse of imaginary part by real part. So, that is gonna be tan inverse tau omega and then what will this guy turn out.

To be this guy will turn out to be K over square root 1 plus tau square omega square when I take e up e up what I will get is omega j sorry negative tan inverse tau omega times j. So, from this by inspection you can see that this guy is the magnitude and this guy is angle. So, you can see that if you work in polar form and this particularly helps us for example, if you had G is equal to 2 lags in series for example,. So, let us say it was I do not know K times tau 1 s plus 1 times tau 2 s plus then all you have to do is then G j omega for example, here let us fix this.

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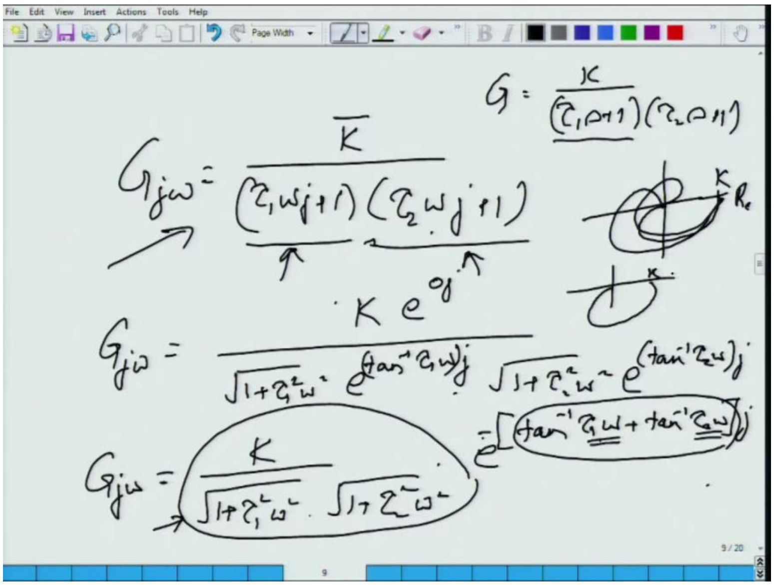

Then G j omega would be K times tau 1 omega j plus 1 tau 2 omega j plus 1 this treat this as one complex number and this as another complex number and this is another complex number which is real. So, then the total product G j omega and express each of the complex number in their respective polar forms this will be.

e to the power 0 j and the guy here will become the first complex number will become its magnitude is square root 1 plus tau 1 square omega square its angle is e to the power of tangent inverse tau 1 omega times the seconds complex number is 1 plus tau 2 square omega square and its angle missed the j this was the angle that is the j and its angle is e to the power tangent inverse tau 2 omega times j and then when you say therefore, G j omega is equal to is equal to K divided by square root of 1 plus tau 1 square omega square times square root of 1 plus tau 2 square omega square times e to the power and now you can see negative tangent inverse tau 1 omega plus tangent inverse tau 2 omega times j.

So, you can see you easily get the magnitude and you easily get the angle the same thing if you want to do with you know let us see you wanted to do the same thing in Cartesian then you would have to multiply by complex conjugate of this guy complex conjugate of this guy complex conjugate of this guy and let us say you had not just two lags may be 3 4 lags then the multiplication and all the algebra will get very messy on the other hand if you deal with the polar a polar system then what you have is that the magnitude becomes simply the multiplication of that multiplication or division of the complex number and the angle becomes the sum of the angles of the respective complex numbers if the complex number is in the numerator the angles get.

Added if the complex numbers are in the denominator the angle becomes negative that is all right. So, it is much easier to work with what shall we say to work in the polar form. So, what would be the what would be the Nyquist plot of this guy a Nyquist plot of this guy would look like this at omega equal to 0 the magnitude which is this term you put omega equal to 0 is actually K this is K angle is 0 because tan inverse 0 and then what about the angle as the angle goes omega goes very large the magnitude goes to 0 what about the angle well tan inverse omega tau 1 is omega goes to a large value goes to 90 degrees tan inverse infinity is 90 degrees this guy also goes to 90 degrees.

So, 90 plus 90 is 180 degrees and the minus sign. So, the angle at very large frequencies is minus 180 degrees and if you basically look at what the Nyquist plot look likes or what the polar plot of G j omega which is which is this guy what the polar plot of this guy looks like it actually (( )) if you had 3 lags in series what you would probably get is something that looks like I do not know the angle will become minus 270 if you had four lags in series with all real parts and if you do the algebra and I am not showing it if you had a second order denominator, but the roots were complex conjugates then what you will get is you will get something like loop roots are large complex numbers you will get something like this starting at K and. So, on. So, forth. So, plotting the Nyquist plot once you start dealing in polar co ordinates it is actually very intuitive let us look at a pure dead time.

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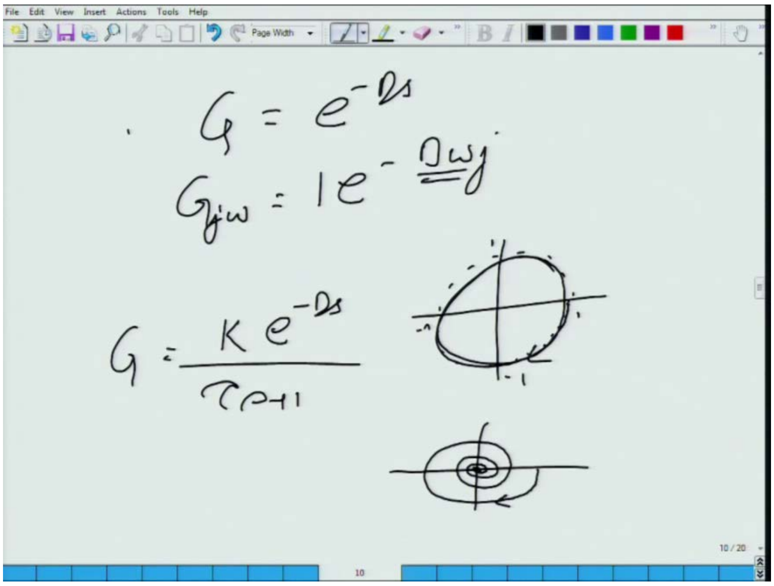

So, G is equal to e to the power minus D s and G j omega replacing s by j omega will be e to the power minus D omega j right what is the magnitude of this guy well the magnitude of this guy is one all the time what is the angle well the angle is minus D omega and as omega increases the angle becomes more and more negative what you get is magnitude is one.

So, this is the unit circle this is one this is minus 1 this is minus 1 this is plus 1. So, this is the unit circle and the angle keeps going more and more negative. So, the Nyquist plot looks like this just keep going round and round the circles it is a circle of course, it my drawing is not. So, good. So, forgive me for that now let us see you got G is equal to first order plus dead time.

What would the Nyquist plot look like well the lag will force the magnitude to 0 as omega goes large the dead time will cause the angle to keep decreasing keep becoming negative negative negative and more negative. So, what you have is you are going round and round, but the magnitude is becoming smaller and smaller ultimately at large frequency this is what the nyquist plot for a first order plus dead time would look like and of course, the direction it just follows. So, you can see that if you have the transfer function replace s by j omega the Nyquist plot is a (( )) and then what we are saying is if you look at the open loop system please remember what did we do here yes when we were looking at the amplitude ratio and the phase shift this guy was not closed this guy.

Was not closed because this guy was not closed what we are doing is we are looking at the and the amplitude ratio like I told you is well where did I tell you that let us see let us see let us see alright.

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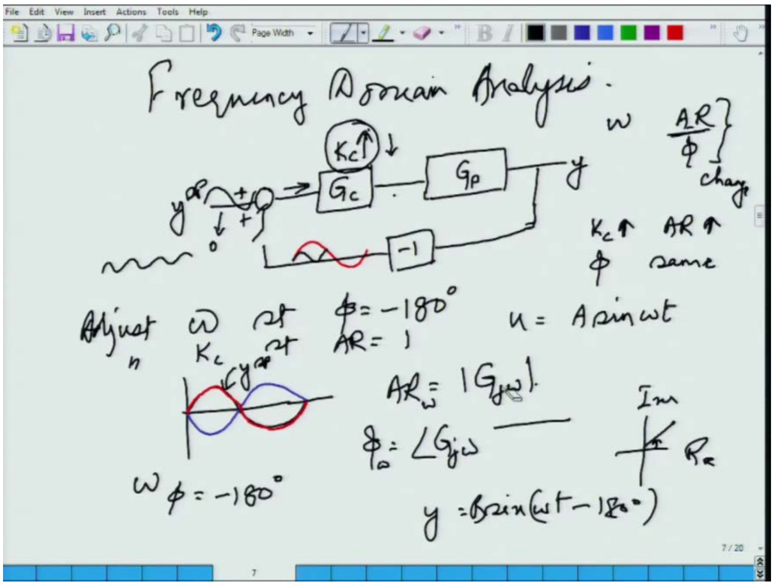

It is and the amplitude ratio is given by AR is equal to at some frequency omega is magnitude of G j omega that relates the transfer function and phi is equal to angle of G j omega at that frequency this is what I told you, but please note since the feedback loop is not closed G in this case is nothing, but G c G p G c G p and I am looking things out here. So, I am looking at this guy and. So, what was I trying to say I was trying to say you are plotting G j omega as a function of omega.

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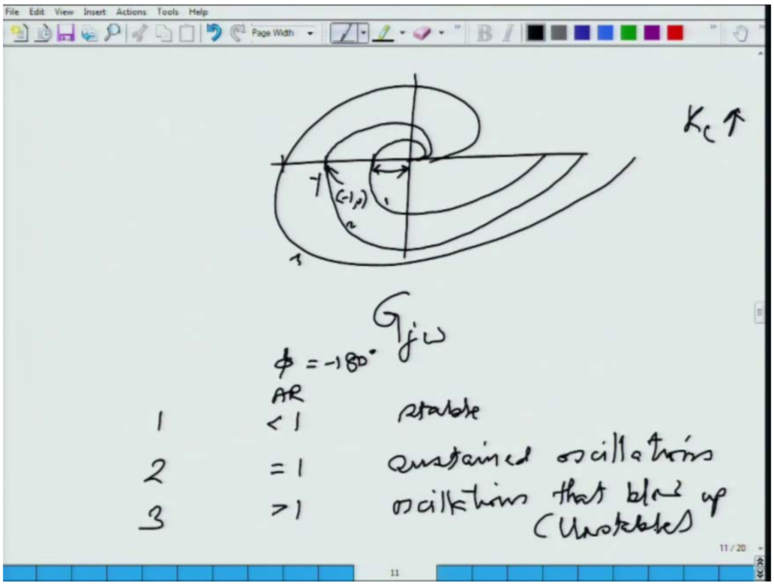

And. So, what this is saying is if you are plotting G j omega as a function of omega the amplitude ratio of the open loop. System without the feedback loop closed if that plot looks like this let us say or that plot you cranked up the gain. So, that plot looks like this or that plot looks like this you are cranking up the gain. So, so so, the Nyquist plot is moving out K c is being cranked up this guy corresponds to when the angle is minus 80 the amplitude ratio is less than less than 1 the second curve this is the first curve second curve third curve first curve when the angle is minus 180 degrees the amplitude ratio is what is the amplitude ratio for the first curve the amplitude ratio is less than 1 for the second curve when the angle is minus 180 degrees of G j omega the amplitude ratio is exactly equal to 1 and for the third curve which is this guy the out outer curve when the amplitude when the angle is.

Minus 180 degrees the amplitude ratio this point is minus 1 0 the amplitude ratio is greater than 1. So, you can clearly see that if I am plotting the Nyquist plot or the polar plot of the open loop system transfer function then if at the frequency of minus 180 degrees the amplitude ratio is less than 1 equal to 1 greater than 1. So, this guy would be stable this guy would be sustained oscillations. So, it is at the verge of instability and this guy would be oscillations that blow up. So, it is unstable oscillations that blow up. So, it is essentially unstable unstable and oscillatory.