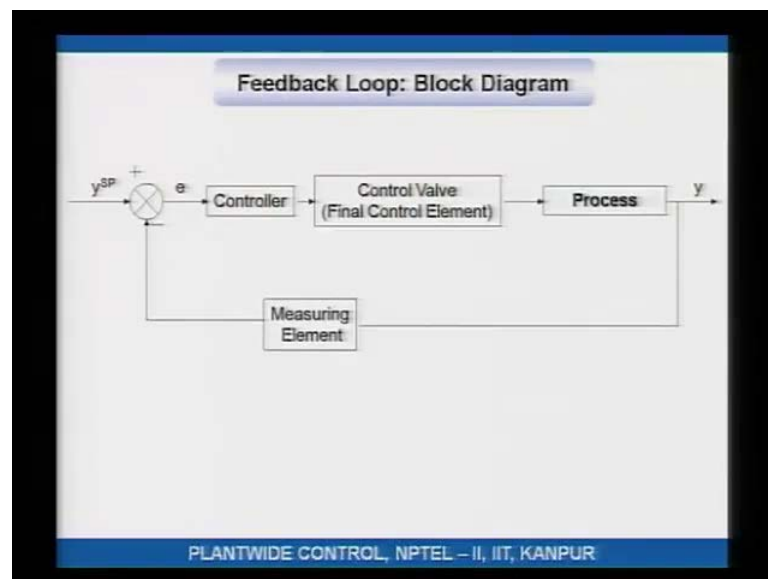


Plantwide Control of Chemical Process
Prof. Nitin Kaistha
Department of Chemical Engineering
Indian Institute of Technology, Kanpur

Lecture - 3
PID Control

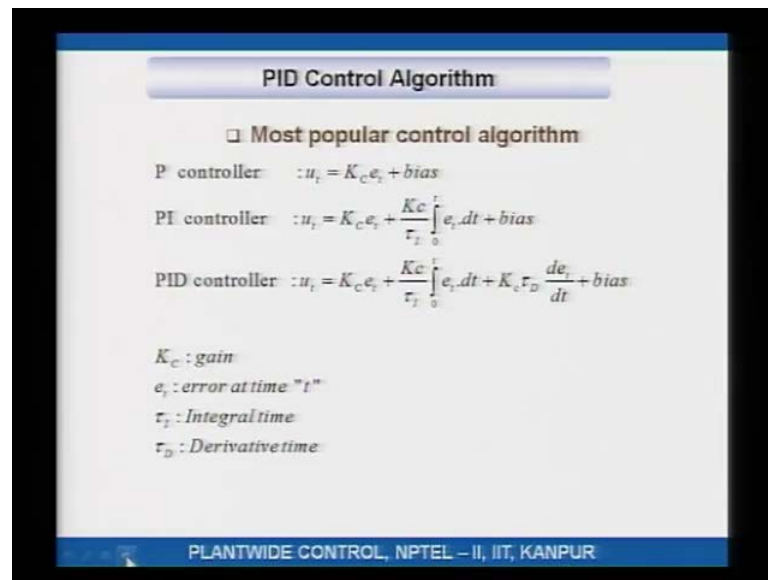
Welcome back. So, last time we looked at process dynamics as a systematic way of characterizing the response of the output to a change in a causal input. We looked at basic response types, and we also looked at combination of the basic response types to represent any type of dynamic response. We also looked at a feedback control loop and sort of defined that a control algorithm is something that takes in the deviation from the desired value in the output and converts that deviation to an equivalent control action or to an equivalent change in the input or the manipulated variable, so that the deviating variable is brought back to its target value.

(Refer Slide Time: 01:15)



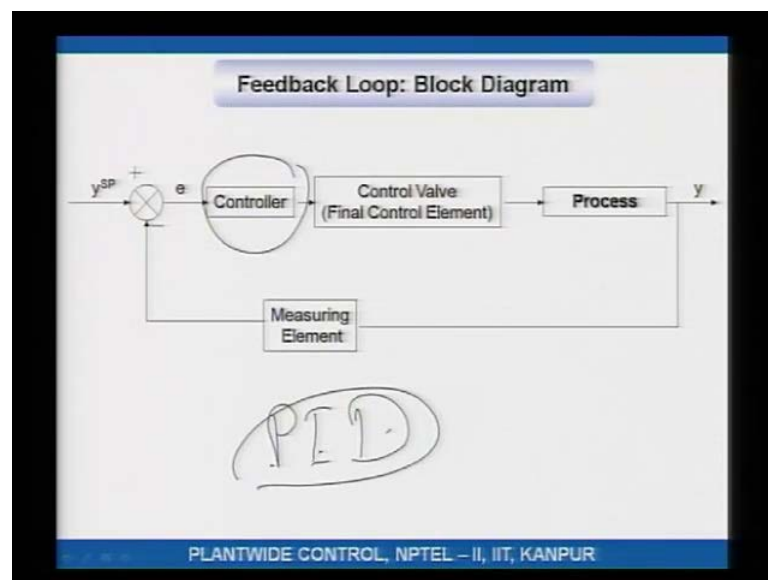
So, this is where we left ourselves last time a feedback loop with its block diagram.

(Refer Slide Time: 01:24)



Now, the most popular control algorithm which is implemented in what is here, the controller in this guide.

(Refer Slide Time: 01:34)



Which is implemented in this controller is called the PID control algorithm and the P I and D stand for proportional, integral and derivative. So, this PID control algorithm is the most commonly used control algorithm in industry and this is even 90 percent of the controllers used in industry would be of the PID type, in fact of the P I type. This is even as you know fancy model productive controllers advanced controllers have come into,

but the PID controller is still holding strong because we will go into it. Now, PID control algorithm so I think I will probably do this so you have the output from the controller which is u which is a function of time. You also have the error which is the desired value which is y set point minus where you are right now.

(Refer Slide Time: 02:14)

The image shows a whiteboard with handwritten equations for a PID controller. At the top, the error signal is defined as $e_t = y^{SP} - y_t$. Below this, the PID control law is written as $u_t = K_c \left[e_t + \frac{1}{\tau_I} \int e_t dt + \tau_D \frac{de_t}{dt} \right] + \text{bias}_p$. Arrows point from the terms in the brackets to labels: e_t is labeled 'P' (Proportional), $\frac{1}{\tau_I} \int e_t dt$ is labeled 'I' (Integral), and $\tau_D \frac{de_t}{dt}$ is labeled 'D' (Derivative). The word 'PID' is written at the bottom of the board.

So, the error in the process variable or the controlled variable, you have that signal and you would like to control this error into an equivalent motion into an equivalent controller output or controller output signal. So, the PID control algorithm is u_t is equal to a controller gain K_c times current error. So, u_t is proportional to the error K_c times e_t . So, this is called proportional action plus $\frac{1}{\tau_I}$ times integral of $e_t dt$, this is called integral action plus $\tau_D \frac{de_t}{dt}$ and we would like to close the bracket here. Again please forgive my hand writing, it is lousy and I do not see any scope of it improving.

So, this is proportional action, this is integral action and this is derivative action, hence the acronym PID controller. Now, if I just a second, there is also a bias term here plus a bias term. This bias term is there so that the output of the controller is whatever the valve position is at that point in time at steady state. This bias term is there so it is easier to understand things if you use.

(Refer Slide Time: 04:20)

The image shows a whiteboard with handwritten mathematical equations for the position and velocity forms of a PID controller. The equations are as follows:

Position form
$$u_t = K_c \left[e + \frac{1}{\tau_i} \int e dt + \tau_d \frac{de}{dt} \right] + \text{bias}$$

Velocity form
$$\frac{du_t}{dt} = K_c \left[\frac{de}{dt} + \frac{1}{\tau_i} e + \tau_d \frac{d^2e}{dt^2} \right]$$

Setting the error to zero for the position form:
$$0 = K_c \left[0 + \frac{1}{\tau_i} e_{\text{final}} + 0 \right]$$
$$0 = \frac{K_c e_{\text{final}}}{\tau_i} \Rightarrow \boxed{e_{\text{final}} = 0}$$

This is called the position form, the velocity form is if you differentiate this equation u_t is equal to K_c times e plus 1 by τ_i times integral of $e dt$ plus τ_d times de by dt plus a bias term. If you consider a system at rest, why do you have the bias I wanted to explain that. If you consider the system at rest if the system is at rest that means e is 0 everything is 0 . So, if you have so this term would at rest. The term in the bracket will go to 0 , if you do not have the bias term u_t would be 0 . What that means is the valve is either fully closed or fully opened, depending on how you have defined it. But typically at rest the value will be about 50 percent open.

So, to ensure that there is no discrepancy you will have to have this bias term, to ensure that when the system is at rest the equation holds. If you differentiate this equation, what you will get is du_t by dt is equal to bias is a constant that will go away, we will get K_c times de by dt plus 1 by τ_i differential of an integral, so that will disappear e plus τ_d times d^2e by dt^2 , second deviator of the error. This is what you will get this is. So, this is the first equation is called the position form of the PID algorithm position form and after differentiate, differentiating the equation the equation that you get after differentiating the position form what you get is called the velocity form of the PID algorithm.

Now, this velocity form is actually quite instructive, the error became non-zero because of a disturbance. Now, because the error is non-zero controller will take some action and

finally, the finally, let us say the process settles to after disturbance to some steady state. Well at that final steady state because by definition of steady state all time derivatives will go to 0. So, at that final steady state what you will have is $\frac{du}{dt}$ is 0, so that will be 0. $\frac{de}{dt}$ must also be 0, so that is 0. We do not know the error is so I will just keep it e at final steady state plus second derivative of time because by definition of steady state all time derivatives are 0 you will get this. So, what you get is 0 equal to $K_c \tau_i$ times e_{final} . For this equation to hold for 0 to remain equal to 0, for left hand side is 0, for right hand side should also be 0.

For right hand side to be 0 this implies for the equation to hold e_{final} must be equal to 0. What does this mean? This means that, if you have this error term e_{final} term came because your controller had integral action in there. So, what it means is if you have integral action in your controller then at the final steady state error that means deviation in the output will be 0. Deviation in the output from its set point will be 0. So, essentially what it means is integral action the purpose of integral action is to, ensure that there is always 0 offset at the final steady state, it removes offset. So, the purpose of integral action what this analysis is showing is that integral action actually results in 0 offset. What is the purpose of the p and d action, well we we going to we going to see that in the in the presentation.

Of course, these are the so a P controller is nothing but only the P action. P I controller has got the P action as well as the integral action. The PID controller has got everything, the P action the integral action and the derivative action. These constants, $K_c \tau_i$ and τ_d these are called actually tuning constants, $K_c \tau_i$ and τ_d these are tuning constants.

(Refer Slide Time: 09:11)

PID Control Algorithm

□ Most popular control algorithm

P controller : $u_t = K_c e_t + bias$

PI controller : $u_t = K_c e_t + \frac{K_c}{\tau_i} \int_0^t e_t dt + bias$


PID controller : $u_t = K_c e_t + \frac{K_c}{\tau_i} \int_0^t e_t dt + K_c \tau_D \frac{de_t}{dt} + bias$

K_c : gain

e_t : error at time "t"

τ_i : Integral time

τ_D : Derivative time



PLANTWIDE CONTROL, NPTEL – II, IIT, KANPUR

These are constants that are there in the hands of the designer, to get the desired control response, to get desired control performance, so tuning constants. So, these are parameters that the control designer adjusts to get the type of response or to get the type of control performance that he or she desires.

(Refer Slide Time: 09:43)

PID Control Algorithm

Position Form

Velocity form

$$u_t = K_c e_t + \frac{K_c}{\tau_i} \int_0^t e_t dt + K_c \tau_D \frac{de_t}{dt} + bias$$

$$\frac{du_t}{dt} = K_c \left(\frac{de_t}{dt} + \frac{1}{\tau_i} e_t + \tau_D \frac{d^2 e_t}{dt^2} \right)$$

Tuning Parameters		Controller Mode Function	
K_c	Controller gain	P Action	Speed of response
τ_i	Reset time	I Action	Zero offset
τ_D	Derivative time	D Action	Suppress oscillations

PLANTWIDE CONTROL, NPTEL – II, IIT, KANPUR

So, again I talked about the position form and the velocity form. The tuning parameters or the controller gain, the reset time τ_i is also known as sometimes reset time and the derivative time τ_D . Now, what is the purpose? I just explained to you that integral

action actually results in 0 offset. Well the purpose of p action or proportional action if you look at the position form or rather the velocity form, rate of change of controller output is proportional to rate of change of error. So, if the K_c is larger for a small rate of error, you will have a large rate of change of or a large rate of change of controller output.

So, because you are changing the controller output at a faster rate, hopefully your speed of response would be faster. Your deviating variable would be brought back close to its set point faster. So, the purpose of p action is speed of response, purpose of y action like I explained is 0 offset. What it essentially means is, if the error is non-zero integral action will continue to move the u the controller output will will keep on adjusting the controller output until the error is driven to 0. So, because of this integral action essentially introduces a seeking behavior, you keep on adjusting u until the error is driven to 0. So, this seeking behavior actually if you going into control theory, it actually results in some amount of destabilization. D action is actually the opposite of i action.

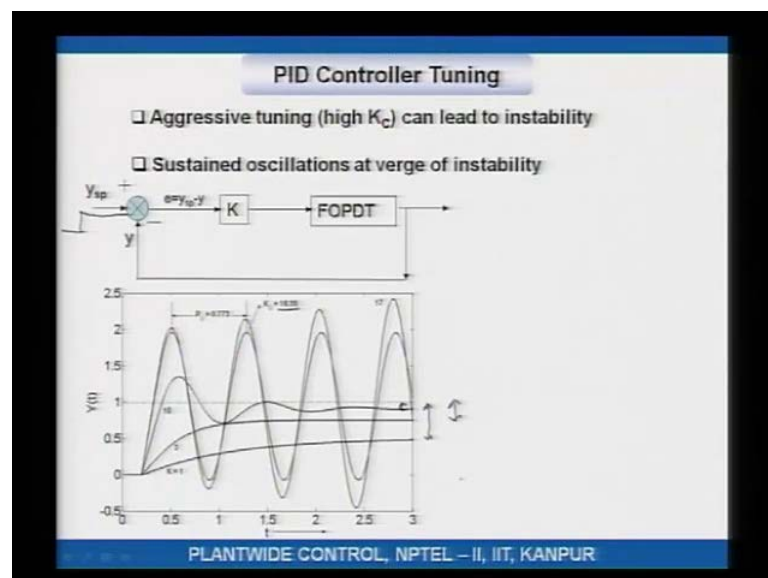
It brings in some amount of anticipation into the system and because of the anticipate rate of change, which way am I headed because of this anticipation, D action is sometimes used where you want a really fast and snappy response. The P I response is not good enough so you introduce D action so that you can use a higher gain to get a fast and snappy response that is acceptable. So, this very common example here will be controlling a reactor temperature. Tight control of the reactor temperature is very desirable because If it is highly exothermic system small deviations in reaction temperature can actually result in a thermal runaway. So, you want the temperature to be controlled really tightly. If you are using only a P I controller that is not possible, if you bring in D action then you can use larger gains in the P action, K_c larger K_c s can be used and with larger K_c s you will get a faster and snappy response.

So, D action is typically used to suppress oscillations because of the anticipatory nature and because the i action causes oscillations because you are seeking. These oscillations get suppressed because of D action and because the D action is suppressing oscillations now, you can use a higher gain K_c and this higher gain causes a faster and snappier response. Disadvantage of D action well, if you take a signal and differentiate it let us say I take a sine wave. I differentiate it I will get you know, I will get a signal which is

like this. So, the D action actually amplifies noise. If you take the derivative of a noisy signal, the differentiated signal will be much more noisier.

So, D action actually derivative action actually causes noise amplification and therefore, it is seldom used it is only used where it is absolutely essential and I just gave you an example tight reactor temperature control. Because there is noise amplification in D action typically when you are using a PID controller with the D action on, what you would have is the process signal that is coming the signal from the sensor it would typically be filtered to filter out the noise.

(Refer Slide Time: 13:50)



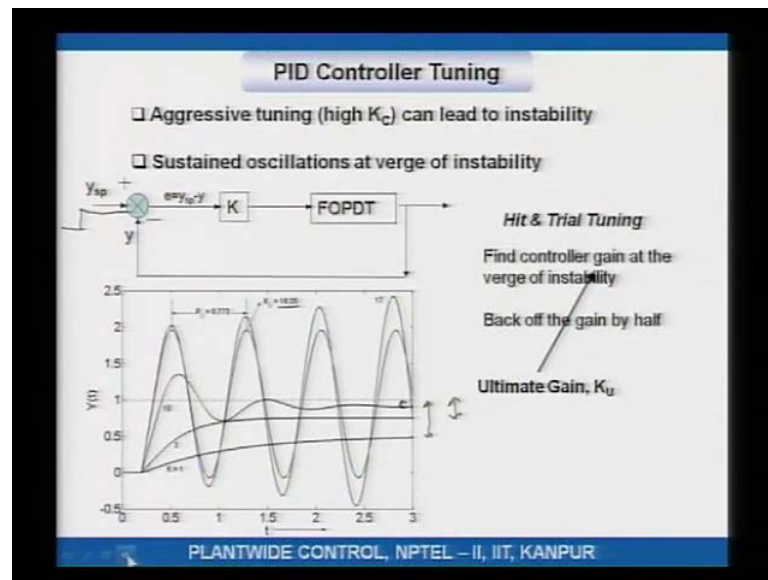
How do you tune controllers? Well two things I think it is better explain in this figure. So, let us say you got the first order plus dead time process. You are feeding back the output y getting the errors signal and let us say you are just putting it through a P only controller, which is which is which is just you know multiplying the error with a gain K and this is the output of the controller which is the input to the, to your first order plus dead time process. Now, let us say I give a step change to y set point. What I am saying is at time t equal to 0, what I am doing is y set point goes up as a step. What I have done in this figure that is there is, I have changed the gain this value of K from low to high values and seen what happens to the response as a step change in the set point is given. Alright, at K equal to 1, the response looks like this.

You want to get to 1, but because it is P only controller you go in that direction, but some offset remains. The output does not reach the set point, it goes towards the new value offset point, but it does not reach there. You increase the gain from K from 1 to 3, well output goes and the rate of change in the output is faster than with K equal to 1, but again some amount of offset remains. You increase the gain to 10, well rate of change of output is much faster and now you start getting oscillations. However, some amount of offsets still remains. If you increase the gain further to about how much is it, to 16.35 about 16, what you get is the rate of change is fast however, the oscillations now are sustained, the oscillations do not die down.

If you increase the gain further from say 16 to 17 what happens is, these oscillations actually start to blow up. So, what I am seeing here is, as I am increasing the controller gain for a step change in the set point, at low controller gains I got a stable response in the sense that my output moved from wherever it was towards the new set point. Of course, some offset remained. As I kept on increasing the gain the offset reduced as I further increase the gain, I started getting oscillations, then I reach the verge of instability and if I increase the gain further, I got an unstable response. My close loop system became unstable. So, first lesson is feedback control can actually destabilize your system, if the gain is too high you will get a unstable close loop system.

Now, if you want to choose. So, how do you choose a value of K_c ? Well rule of thumb method would be find out at what gain you start getting sustained oscillations? The implemented gain should be about half of that, so that you are always away from instability because you do not want to you do not want an unstable close loop system.

(Refer Slide Time: 17:25)



So, the hit and trial tuning method is find the controller gain at the verge of instability where you get sustained oscillations in the output. The implemented controller gain should be about half of that. So, this is the hit and trial tuning method. That value of the controller gain where you get sustained oscillations which for this example in the figure of 16.35 is refer to as the ultimate gain. The period of the oscillations is referred to as the ultimate period.

(Refer Slide Time: 17:55)

PID Controller Tuning

Empirical Tuning Procedures

Zeigler Nichols (Aggressive)

Tyres Luyben (Loose)

	P	PI	PID
Zeigler-Nichols			
K_C	$K_C/2$	$K_C/2.2$	$K_C/1.7$
τ_i	—	$P_u/1.2$	$P_u/2$
τ_d	—	—	$P_u/5$
Tyres Luyben			
K_C	—	$K_C/3.2$	$K_C/2.2$
τ_i	—	$2.2P_u$	$2.2P_u$
τ_d	—	—	$P_u/6.3$

PLANTWIDE CONTROL, NPTEL – II, IIT KANPUR

Now, based on the ultimate gain and the ultimate period of oscillations, Ziegler Nichols and Tyreus Luyben and there are various other Keihin Khone there are various other empirical tuning procedures, that use this value of the ultimate gain and the ultimate period to suggest what your control tuning should be. So, the two most popular ones are Zeigler Nichols tuning and Zeigler Nichols tuning is quite aggressive and what is very manier times used for distillation columns Tyreus Luyben tuning. Now, maybe I should explain this a little bit. Look at the table, if you are using a P only controller Zeigler Nichols tuning method is saying get your ultimate gain, where you get sustained oscillations your applied control gain should be half of that, K_u by 2. If you are using a P I controller your proportional gain should be K_u by 2.2. What that means is in a P I controller I am supposed to use a gain that is slightly lesser than a P only controller.

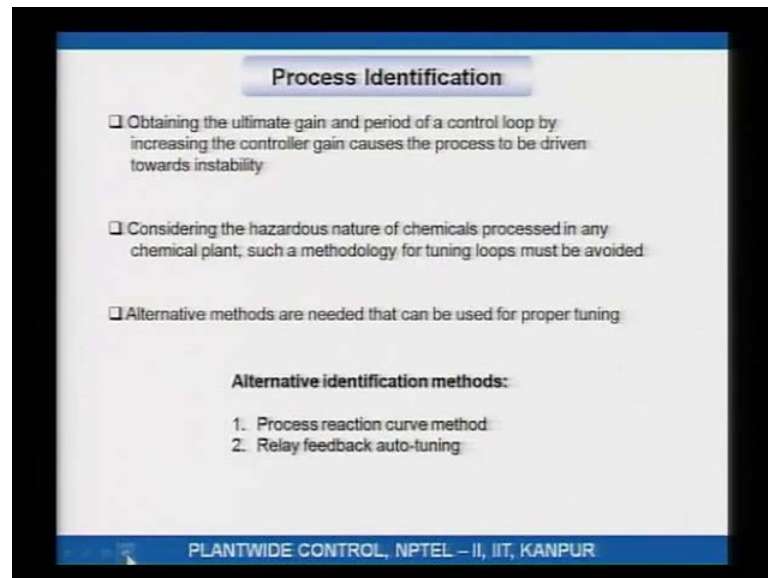
Why is the gain slightly lesser because integral action actually introduces that seeking behavior. So, what that means is because of intergral integral action your response will become oscillatory at a lower gain compared to, if you had no integral action. Now, if you introduce derivative action, notice that the controller gain is K_u by 1.7, which is higher than actually if was of pp only controller. What this is saying is, if you have introduced derivative action you can actually use a higher gain and the higher the proportional gain the faster the response the tighter the control.

So, of course, PID like I explained earlier is used and it is used when you desire a fast tight and snappy response, most common controller P I. Where do use P controllers? You use P controllers where you really do not care about offset, a very common example is level in a tank, which is just a surge drum, you really do not care whether the level is 50 percent, 55 percent or 45 percent or 60 percent or 40 percent as long as the level is between 25 and 75 percent, you are okay. So, for this type of a system where the where the level is allowed to float in a range offset is acceptable because offset is acceptable the simplest controller that will give you reasonable control performance, is a P controller simplest is best.

Therefore, level is typically controlled using P controllers. Tyreus Luyben tuning, it is more conservative you will find that the the gain implemented in Tyreus Luyben is actually significantly less than what is recommended by Zeigler Nichols and Tyreus Luyben is typically applied to distillation columns because you do not want very large and sudden changes aggressive changes in for example, the re boiler duty because that

can lead to hydraulic problems. So, Tyreus Luyben is more conservative tuning method, Zeigler Nichols is for aggressive tuning and the only point that I wanted to make here was that because of derivative action, you actually can use much higher gains and therefore, PID controllers are used where you want tight fast snappy control of the output.

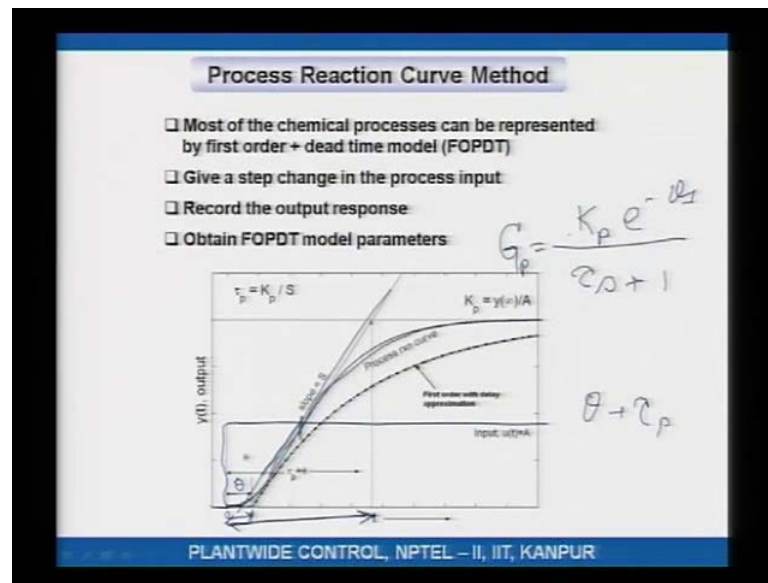
(Refer Slide Time: 21:24)



If it is an electrical system you know you have got an oscillo scope, you keep on cranking up the gain where you get sustained oscillation, you say this is my ultimate gain back of by half. In a chemical system where you are dealing with hazardous chemicals you do not have that liability of driving the process towards instability, because should something go wrong, while you got a disaster on your hands. So, because obtaining the ultimate gain and period of a control loop by increasing the control gain causes the process to be driven towards instability, you really do not have the liberty of getting the ultimate gain by the method that I just talked about cranking up the gain till you get sustained oscillations that cannot be done in a chemical process.

Therefore, alternative methods are needed which can be used for proper tuning of our PID controllers. These methods, two common ones are the process reaction curve method and the relay feedback auto tuning method and we are just going to go over these two methods in the next new slides.

(Refer Slide Time: 22:31)



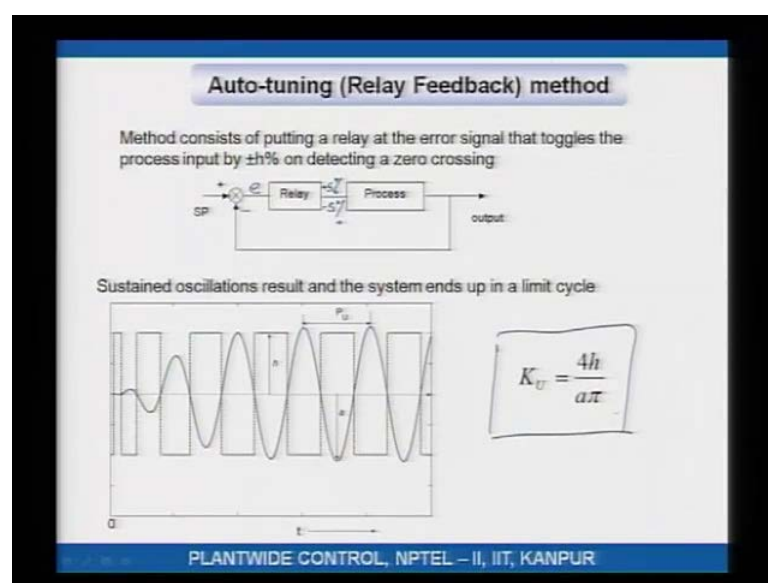
Now, the process reaction curve method essentially tries to fit a first order plus dead time model to a S shaped to a, to to an S shaped output response. Most of the chemical processes responses can be represented by a first order plus dead time model. So, what you do here is, so we have got this process reaction curve method and in the process reaction curve method what you do is, your loop is opened that means the controller is not on, you give a step change to the input and here is the step change. In response to that step change you you record the output response and this is the output response that you have this line over here.

Now, you take this output response curve find out where the, find out the inflexion point in your S shaped curve draw a tangent at the inflexion point. So, this is the inflexion point and you draw a tangent at that inflexion point this tangent in intersects the time axis here and what you say is this is my deed time theta. This tangent line is extended and wherever it wherever it reaches the final steady state value which is K_p , which is the gain of the process for a unit step. You note that time and then what you say is, whatever that time may be it is equal to theta plus tau P, so you know theta from here and then you say theta plus tau P is equal to this guide. So, then what you will say is that my first order plus dead time model is $K_p e^{-\theta s} / (\tau_p s + 1)$, this is my first order model that I have fitted.

What you will see if you do this for this example process the predicted model actually is much more sluggish than the actual process reaction curve and that is considered fine because for a more sluggish model where the response the model says that the process is responding slower, your gain that you get would be more conservative. So, you will be using lower gains, so this process reaction curve method actually gives you a very conservative estimate of your ultimate gain, ultimate period. What I am saying is you get the model based on the process reaction curve method for that model you obtain the ultimate gain or ultimate period using either a simulation or using complex algebra, which has very well covered in process control textbooks.

From that ultimate gain and ultimate period you apply Zeigler Nichols or Tyreus Luyben tuning and get your tuning parameters. The point why it is done is because the way the process reaction process reaction curve method works it gives you, tau that is larger than what the actual tau is and therefore, your tuning is more conservative. That is about it. Note that in this tuning method, the control loop has to be switched off you do a step test then you can switch on your loop back and from the step test data you get your ultimate, you fit the model get your ultimate gain, ultimate period, apply a tuning tuning procedure Zeigler Nichols, Khohin Khone, Tyreus Luyben and you get estimates for your controller tuning parameters K_c τ_i τ_d . Implement those and then you may have to fine tune them a little bit further, to get the kind of response that you want.

(Refer Slide Time: 26:35)

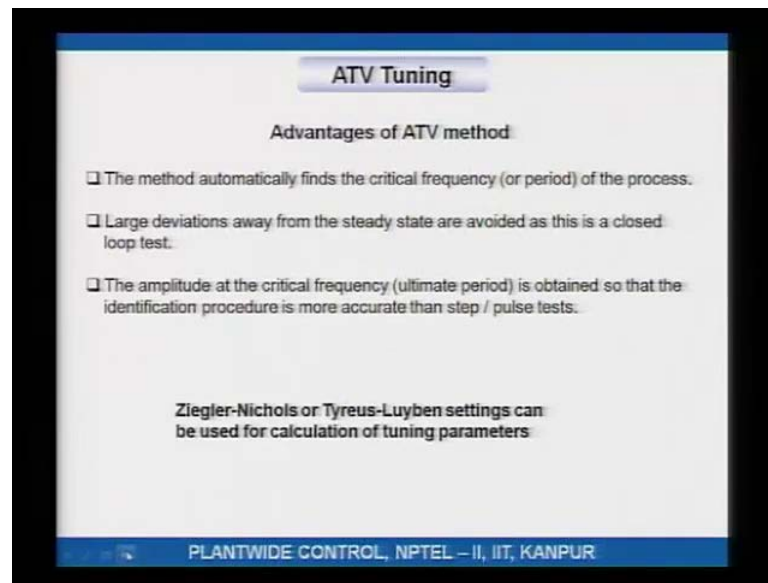


There is also what is called the auto tuning or relay feedback method. What is done here is your feedback loop is on instead of the controller what you do is put in a relay. What does the relay do? The relay takes in this error signal and if this error signal shows a 0 crossing the output of the relay relay toggles to either, let us say plus 5 percent. At the next 0 crossing of the error the output will toggle to let us say, minus 5 percent. So, plus h or minus h. I have just used as an example 5 percent. If you put in a relay in a feedback loop system, this is what happens and that is shown in the figure. What happens is the output toggles the output cross the error, crosses 0 and as the error crosses 0 your state of the relay goes from here to here.

In response to that, you know the error crosses 0 again toggles down starts to go up because the output has toggled down it starts to go back down crosses 0 again now, the output of the relay toggles back up now, the output which was going down because the input has toggled back up turns around comes back crosses 0 again. You see it results in sustained oscillations with toggling of the input to the process. The period of these sustained oscillations gives is is considered as equal to ultimate period and from the height from the amplitude of the oscillations in the output and the height of the pulse, you know plus 5 percent minus 5 percent plus 10 percent minus 10 percent, that is something that you implement.

So, from this you obtain K_u as this guy. The ultimate gain is estimated as four times the height of the pulse, the height of the relay relay toggling divided by amplitude of the oscillations in the output, divided by π . This is how you estimate K_u . Note that in this auto tuning relay feedback method, your process the feedback is on a limit cycle is introduced. Limit cycle in the meaning sustained oscillations. Note that the feedback loop is on and your process is oscillating around its steady state. So, this method is actually very popular because this essentially ensure that you do not switch off the loop, you just put in a relay from the, from this period of the sustained oscillation as well as amplitude of the amplitude of the output oscillations you can estimate K_u and then you apply an empirical tuning rule to get K_c τ_i and τ_d .

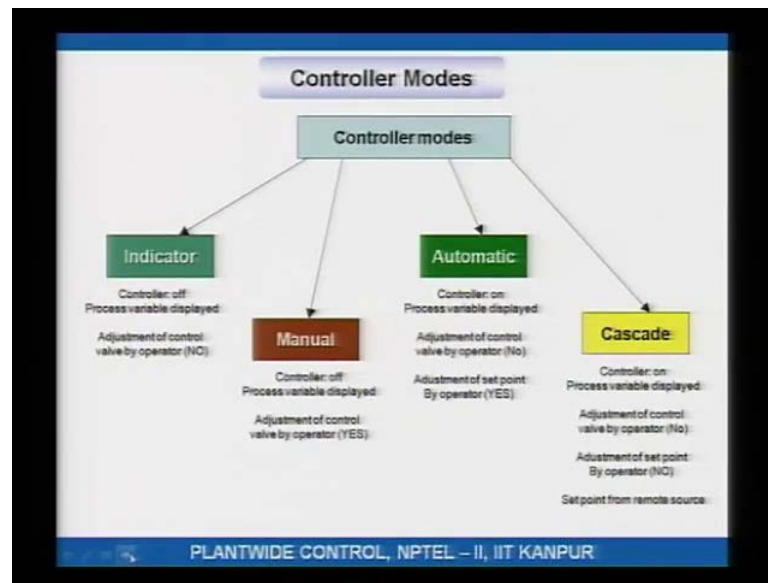
(Refer Slide Time: 29:41)



Advantages of this auto tuning method, those are few who have done some control theory you would know that there is what is known as the cross of cross over frequency, which is where the phase becomes the phase lag becomes the frequency at which the phase lag becomes 180 degrees, alright. So, this method actually results in those sustained oscillations which and the frequency of that sustained oscillations is close to the cross over frequency. Large deviations away from the steady state are avoided, that is because your process is always oscillating around its steady state. The amplitude is obtained at the critical frequency and the and that is what is important so that this identification procedure is more accurate than step or pulse test because the pulse test and step test have got more of lower frequency content.

What you are interested in, is the amplitude ratio at the critic cross over frequency. So this A T V method actually cause causes sustained oscillations near the critical frequency and the amplitude that you are getting in the output is actually closer to the amplitude ratio. Like I said before once you have an estimate of K_u and p_u you can either use Zeigler Nichols, Tyreus Luyben and there is also other one called Keihin Khone settings to calculate your tuning parameters.

(Refer Slide Time: 31:11)



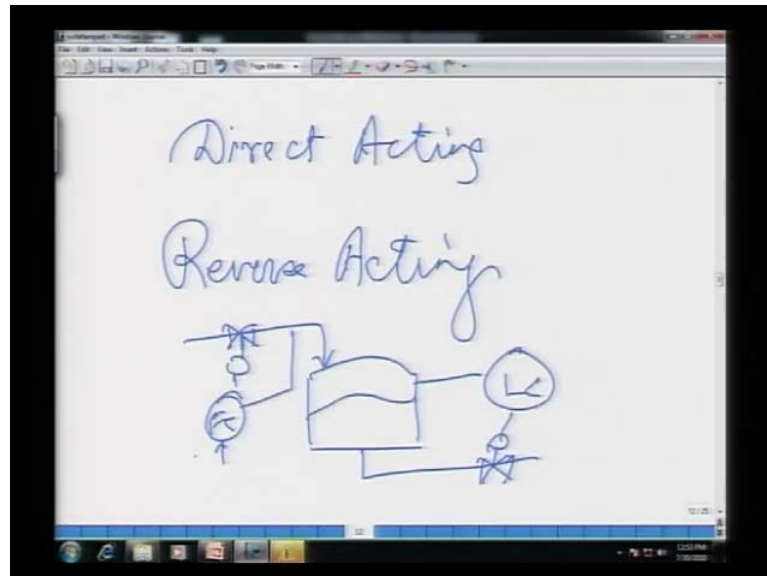
Now, we are going to talk about controller modes. Any controller it is got four modes and these four modes are indicator, manual, automatic and cascade. What happens in an indicator mode is no control action is being taken, it is the controller is simply indicating what your process variable is. You do not even, you cannot even adjust a valve position. In manual mode controller is off, value of the process variable that is to be controlled is being indicated however, since the controller is off valve is at whatever position it is. The operator can change that valve position, you can say that the valve is 50 percent open or 60 percent open or 40 percent open.

So, in manual mode the operator is specifying the valve position signal. The signal to the valve. In automatic mode the controller is switched on and now the position of the valve is governed by what should be the position of the valve is calculated by the controller because the controller is on. What does the operator do? The operator specifies what should be the set point of the output. So, in in manual mode operator specifies the valve position, in automatic mode operator specifies the set point of the output. There is also what is known as the cascade mode and what happen in the cascade mode is even this set point is coming from another controller.

So, these are the four modes of a controller. Controllers can also be reverse acting or an direct acting in what that what that means I will just explain you in a little bit. It is not

explained there. Reverse action and direct action may be that should be the last one that we do this time around

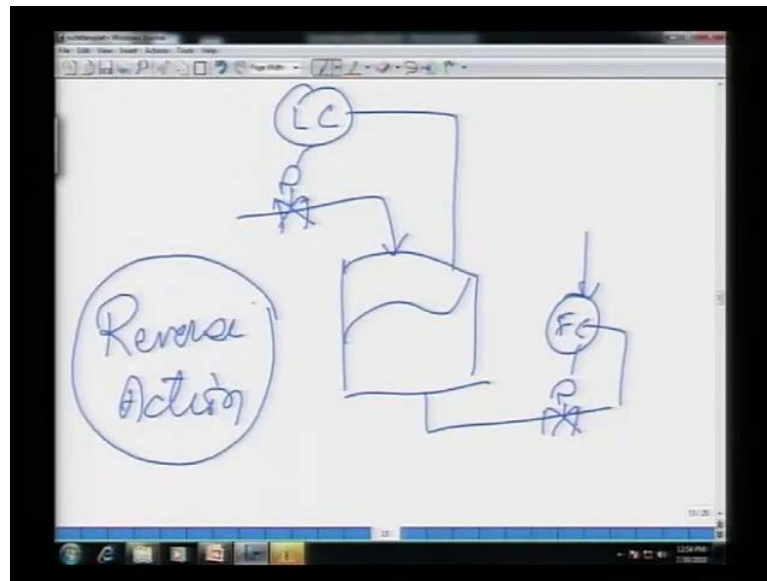
(Refer Slide Time: 33:10)



So, the controller can be direct acting or it could be reverse acting. What does it mean? Well let me explain it, is best explained you know in plain English. Direct acting is, if my output that is to be controlled is increasing, I should increase the I should open the valve, I should increase the output of the controller that is direct action. What is reverse acting? If the output is increasing beyond the set point, I should actually close the valve, I should actually decrease my valve position as just just to explain it for for example, if I have a tank and I am controlling the level of the tank this way and there is some feed that is coming and this is under flow control.

Let us say this is what I have. Let us say the flow to the tank increases, that means if the flow increases level would start to build up. If the level starts to build up, what should the controller do to the output valve? This output valve should be opened, what that means is if this if at the beginning this valve is 50 percent open the output from the level controller should increase from 50 percent in order to bring the level back, right. So, if the level is increasing the output of the output of the controller should increase, this is direct action. If you take the reverse situation and let me explain this here.

(Refer Slide Time: 34:57)



What you have here is well the flow controller is here and the level controller is now on the feed. They are controlling the level this way. Now, if the set point here is increased the flow out would increase, level would start to decrease. In order to bring this level up back up, what should I do to the inflow? Since, the level is decreasing, in order to bring the level back up I must increase the inflow. What that means is if this valve is initially 50 percent open this valve should be opened beyond 50 percent to may be 60, 65, 70 percent in order to bring the level back.

So, what we are saying here is, if the level is increasing controller output should be decreasing. Alternatively if the level is decreasing, controller output should be increasing this is called reverse action. So, we have just look that the different modes of a controller indicator, manual mode, automatic mode or cascade mode and we have also just described the action of the controller reverse or direct.