Mass transfer II

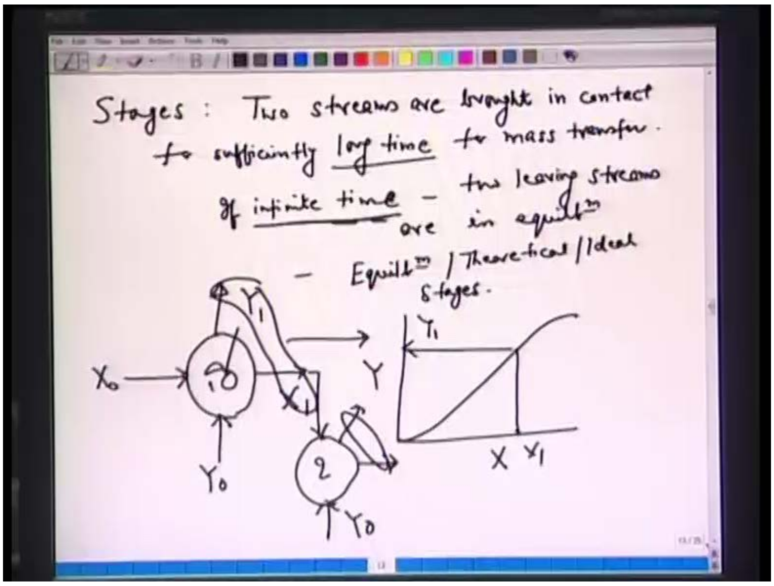
Prof. Nishith Verma
Department of Chemical Engineering
Indian Institute of Technology, Kanpur

Lecture No. # 07

In today's lecture, we will discuss mass transfer in the stagewise processes. So, previous lectures we had continuous contact process like packed column, so two streams are brought in contact whether co-currently or counter-currently. Now, we have a stages, so these are a certain devices, you can think of a mixing chamber - two streams are brought in contact, they spend some time there; then the two streams leave from the stage, they go to the second stage or second device.

We can have cascades of such stages; and similar to the previous case, we will make a material balance for a species balance on stage on every stage, and we will mark our mole fractions or mass fractions on the equilibrium curves. We will draw the operating curves, and we will see what is the driving force of mass transfer at every stages. So, with that introduction, let us start this stagewise contact process.

(Refer Slide Time: 01:21)



Stages : Two streams are brought in contact for sufficiently long time for mass transfer.

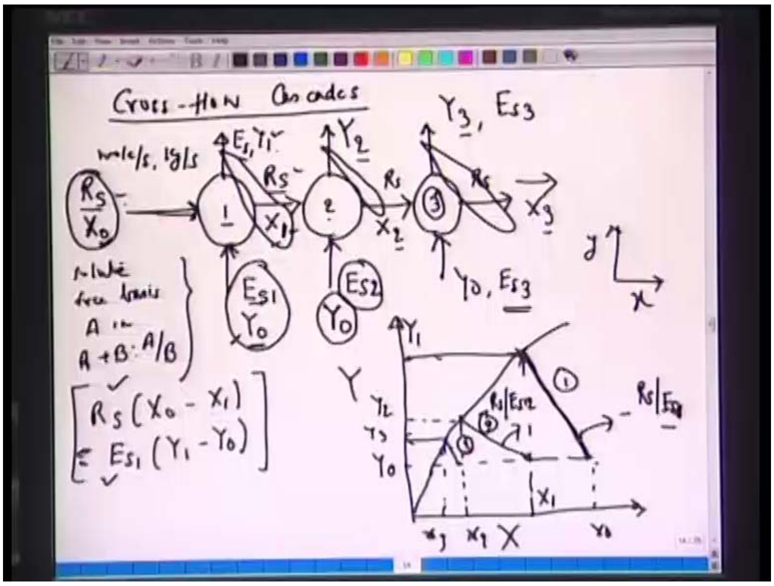
If infinite time - the leaving streams are in equilb^{m}

- Equilb^{m} / Theoretical / Ideal stages.

We have stages; two streams are brought in contact for sufficiently long time. We will see the meaning of this in a minute -what does it mean for sufficiently long time for mass transfer. So if we give theoretically if we give infinitely time if infinite time, then we can say that two leaving streams two leaving streams are in equilibrium are in equlibrium and we will call them equilibrium stage or theoretical stage or ideal stage. So all it means, two leaving streams are in equilibrium and we have given as very long time or infinitely long time. So we are talking of a device like this, one stream is brought in contact in other stream; this has a mole fractions say X, this has a mole fractions Y or X 0 Y 0, here they are informally mixed and when they leave they have the mole fractions X excuseme so we have the mole fraction X 1 and the mole fractions of the second stream as Y 1. So, if they have good mixing here and they have spend a very very long time it is possible that X 1 and Y 1 the two leaving streams they will be in equilibrium. So we can take the equilibrium curve like Y and X and we can say that X 1 and Y 1 they are on equilibrium curve; so this would be Y 1 and this would be X 1, so this is the meaning of equilibrium.

We have one stage, now we can further take this stream and can bring in contact; we can bring into it is second stage, and again we can bring in contact with another stream phase Y 0 to show that the leaving streams again here and for Y 0, if we leave it from here they would be in equilibriums. So this is the meaning of your stages in cascades or in series etcetera. So there are several combinations are possible. We can have cross co-current here, or we can have counter-current. So we will see, we will make a material balance on each of these stages. It is possible that there is some other possibility, there is another different type of stages where we have two cascades or two stages in series, then we have two stages in parallels and two streams are getting mixed etcetera. So, but for simplicity, we will take these two cases.

(Refer Slide Time: 04:40)



So, let us start with cross-flow, the most common cross-flow cascades. So, we have one stage one, we have the second stage two, we have the third stage three, say let us give one stream R with the mole fractions X 0 is brought in contact with another stream here, which is E and it has a mole fractions Y.

Now, if we recall from our previous lectures when we had discontinuous contact process - we say that it is a simpler to work on capital X and capital Y, which is solute-free basis. So We have, if we have binary components only one component diffuses from one stream to another stream, then it is convenient to work on capital X and capital Y mole fraction as well as a molar flow rate which is solute-free basis, because we realize earlier that operating lines in such cases become linear. So we can do, so our mathematical computation becomes simpler here. So we will here also will work on solute-free basis mole fractions or solute-free molar flow rates or weight mass flow rates.

So, essentially we can write this R S and E S which is mole per second or kg per second, but it is a solute-free basis. So the trans free component, if it is A in A plus B we are making a balance on A over B that is a meaning of your solid free basis. So R S X 0 it is brought in contact with E S and Y 0 and this stream leaves. Now, if it is R S this will remain same as R S molar flow rate based on A free B free basis, this will remain the A free basis will remain same as R s, composition will change from X 0 to X 1 and let say that this composition changes from Y 0 to Y 1 and we have the same flow rate E S here.

So, R S R S this will remain the same E S and E S will remain the same. Of course the total flow rates will change for both R S stream and the E S streams. Now this stream R S and X 1 and with the mole fraction X 1 again it is brought in contact with the second stream say flow rate is E S 2 here. So we can write E S 1 here and this has same mole fractions as earlier Y 0 for the simplicities.

Now, it is important to note that if we have given enough time - distance time or contact time for these two streams R S and E S in the stage one then Y 1 and X 1; they would be in equilibrium two leaving streams will be in equilibrium. Now, this is the maximum - we can have separations here, it is going from X 0 to X 1 and Y 0 to Y 1, if the two leaving streams are in equilibriums mass transfer will stop here. We cannot have separation more than X 1, to have more separations or separation higher than X 1 or higher than Y 1; we have to bring it in the second stage where again we bring this in contact with the fluids E with the with the flow rates E S 2 with the mole fractions Y 0. Now, this stream again comes in contact with the stream E the spend time here certain time here. Again, we can assume that if you have given enough time the leaving streams with the mole fractions Y 1 and the leaving streams X 2 they would be in equilibrium. So, similarly we have X 3, we have Y 0 and we have Y 3 here, excuse me this would be Y 2.

So, we are following the nomenclatures that the leaving stream have the suffix which is number three; it originates from a stage three. So we have Y 2 X 2 originating from 2; we have Y 1 E 1 originating from X - from a stage one and we have the free solvents which is Y 0 and X 0 compositions, and since we are working on solute-free basis R S R S R S will remain the same here, we have E s 3, the flow rate of the entering in the third stream and here the flow rate will remain the same as E s 3. Now, what we should do? We should mark these points different coordinates on our equilibrium curves or the plots which we had originally. So, now we are again plotting capital Y and capital X although we can also plot on a small y and a small x, but we have seen the advantages here that if we use capital Y and capital X solute-free basis from the stoichiometry, then the operating curves will become linear.

So, we have, let assume that this is our equilibrium curve Y versus X and this is the compositions of starting from X 0 and Y 0. So, let us mark this point as X 0 and let mark this point as Y 0.

If you make a material balance on stage one is a very simple R s X 0 minus X 1 will be same as excuseme will be same as E S one and Y 1 minus Y 0 which means for any stage, here stage one and two we can also we can draw this operating curve. So this is nothing but your material balance or a species balance which we have made on the transferring component say A here between the two entering streams and the leaving streams. So, let's mark this points. So, if we have X 0 Y 0 the two ways of doing this, if we know the flow rates of R S and E S 1 then we can draw the slope here; this is easier to show that let took previous cases that this will be a negative slope with R S minus R S over E S 1 - this is slope of the operating curve. And if the streams Y 1 and X 1 the two streams E and R S, they are in equilibrium then this operating curve will hit here and we have these compositions Y 1 and X 1.

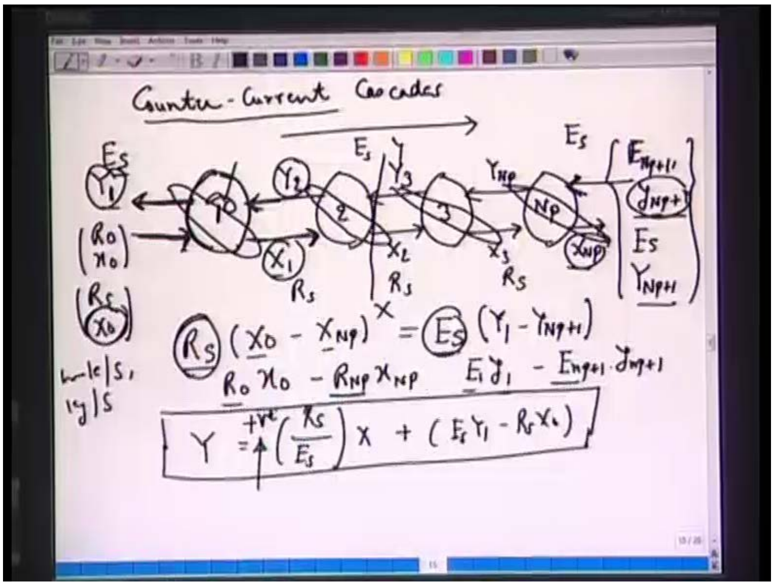
Let's see again here, see the two ways of drawing this or plotting this. If the compositions and the specification of the feet R S X 0 and the solvents E S and Y E S and Y 0 they are given we can mark these two points. If you know the flow rates we can draw the slope and we can say that if they have spent enough time or it is a ideal stages or equilibrium stages, then this line will be on the operating curve the leaving streams Y 1 and X 1 they would be in equilibrium or given Y 1 and X 1. If you mark this point one can connect these two points to have this operating curve minus R S over E S 1. So depends upon the type of problems which we have which is formulated here.

Now, when the streams enter two again it counters the same compositions Y 0, but the flow rates is E S 2; all it means you mark the same point here this is Y 1 and this is your Y 0. But now, the flow rates if you assume that E S 2 is different from E S 1 you will have a different slope here which will hit here with a different with the slope of minus R S over E S 2 . R S is the same here kg per second, mole per second of the solvent or the carrier gas is the same throughout. So we have R S however E S 1 and E S 2 are different. So we mark this take the slope and draw the way to the equilibrium all it means that all it means that the leaving streams Y 2 and X 2 they are also in equilibriums, so we have Y 2 given when equilibrium with X 2 . And similarly for the third stage it has the same composition Y 0 again with the with the flow rates is different at E S 3 we have a different slope here moving all the way till Y 3, signifying that the two leaving streams are again in equilibriums so we have Y 3 and X 3 as an marked on this equilibrium curve.

So, we have a stage one stage two stage three; we starting from X 0, we have brought down the compositions till reduce to excuseme till X 3. And the more important say there is a equilibrium curve like this and also important we note here is that this is very similar to what we had in case of continuous contact process. So, in continuous contact process also if we have two stages two streams in flowing co-currently and with give them enough resistance time then the leaving streams will also be in equilibriums, but it is a continuous contact process. So there the operating lines we can assume that it is a continuous here, since we have the stages right in one stage two streams they come in contact they spend time and the leaving streams there equilibriums , then the first from the first stage whatever the streams comes out. If you want to further purify if you want to further separate the component a we have to bring in the second stage or second tray etcetera where we bring it in contact with this second solvent Y 0 and therefore we have a stage one stage two and stage three.

If the flow rates are same in all three flow rates of the E phase E S 1 same as E S 2 same as E S 3 then the slow all the three lines will be parallel to each other. So all these three lines will be parallel that is what the simplicity etcetera, and again we want to need emphasize that we are assuming these are three one two three they are equilibrium stages; that means all the leaving streams are in equilibriums and the comparison they fall on this equilibrium curve Y X equal to f X.

(Refer Slide Time: 16:21)



Counter-Current Cascades

$R_S (X_0 - X_{NP}) = E_S (Y_1 - Y_{NP+1})$

$R_0 x_0 - R_{NP} x_{NP} \quad E_1 y_1 - E_{NP+1} \cdot y_{NP+1}$

$$\boxed{Y = +ve \left(\frac{R_S}{E_S}\right) X + (E_S Y_1 - R_S X_0)}$$

We will like to do the same things, now for counter current cascades. So, in this case again we have the stages now the stream one is stream enters from first first stage in enters the first stage while the solvent now it enters from the last stage that is a counter current flow again we can have several stages. So it is a combination of different stages cascades two streams entering counter currently one from here and second second streams solvent, we can call it in the third stage of the N p stage in arbitrarily stage, then the two streams - leaving streams again they spend they have certain resistance time and so for we can make a material balance and we can mark these points here.

So, let's start this counter current cascade. So now we have a stage one and we would like to work still one capital Y and capital X of the solute-free basis flow rates, so that our operating curves are linear. So, we have a stage one and now we bring this R 0 small x 0 or we can work on R S and X 0. And we have stages like two or we have a stages like three or we have a stages say arbitrarily N P, only difference is the same thing stream R 0 and now we have the entering stream from the other side. So, this is a phrase solvent or phrased E phase extract phase we have bringing in the opposite directions counter currently. So we follow the nomenclatures this will have a suffix E N P plus 1 and this will have a compositions say y N P plus 1. If you work on solute-free basis this has a flow rate of E S and we can say it is a y N P plus 1 like this.

So we have the streams like this streams like this, and we are following the same nomenclatures that means here we can write E S will remain the same as what enters here and we can use capital Y 1; so Y 1 means looks this subscript denotes it originates from a stage 1 this means this composition is Y 2 here this composition is Y 3 here this composition is y N P and its compositions of phrase stream entering the stream is y N P plus 1 y N P plus 1. Similar these compositions will be capital X 1 capital X 2 capital X 3 and capital X N P. And this flow rate of E S will remain the same throughout, because we are working on solute-free basis.

However E N P E N P 2 E N P 3 will vary across this stages, but we said that we are working on the solute-free basis for the simplicity. Similarly R S will remain the same throughout. Now, let's here also we can make this overall material balance but more important is to understand again the equilibriums. Now let's look at the stage one stage one these two streams are entering here with the composition X 0 and Y 2 and two streams are leaving here Y 1 and X 1.

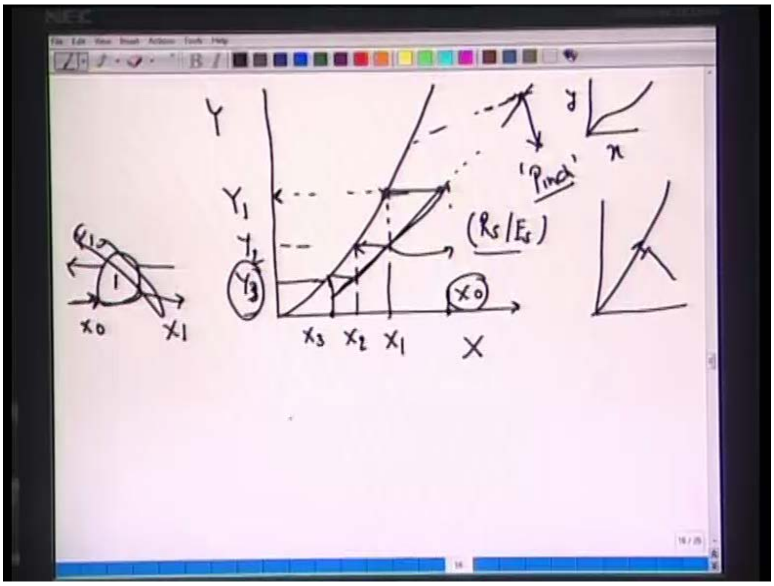
If you assume that it is an ideal stage uniform mixing and there was spent infinite time then these two streams will be in equilibriums; that means X 1 and Y 1 they can be marked on this equilibrium curve. Similarly Y 2 and X 2 will be in equilibriums Y 3 and X 3 will be in equilibriums and Y N P and X N P they will be in equilibriums. So we can mark these two points on the equilibrium curves and if you make a material balance, then all the material balance will give as given X 1, what is Y 2? Given X 0, what is Y 1 or given X N P? What is y N P plus 1?

So let's make a material balance or a species balance for the phase a we can write R S solute-free basis X 0 minus X N P. So, we have brought down the concentration mole fractions from X 0 to X N P which it same as species balance E S Y 1 leaving stream minus Y N P plus 1. So E S and R S moles per second or kg per seconds they are the same they will remain the same from the stage one to stage N P, because of solute-free basis.

One could have also written on actual flow rates total flow rates R 0 then should have been R 0 x 0 minus R N P into small x N P. So look at the difference, now R 0 is different from R N P, however you work on solute-free basis we can take this R S as a common. Similarly for this E S we could have written as E 1 Y E 1 small y 1 minus E what enters here is n P plus 1 into y n P plus 1. But if you work on solute-free basis E 1 E n P plus 1 can be replaced with E S and small mole fraction Y can be replaced with Y n X. The idea is that at any locations again if we take a stage with Y and X we can draw an operating line which would be linear Y equal to R S over E S X plus E S Y 1 minus R S X 0. So, this is our operating line in a countercurrent cascades and important again here is to notice that this is now positive - slope is positive. So what we noticed here? We have made a species balance for co-current cross current and counter current cascades very similar to the previous case where we made a species balance for co-current continuous contact process and countercurrent continuous contact process material balance overall it remains the same.

We have negative slopes in case of co-current or cross co-current and we had a positive slope in case in case of counter current flow countercurrent continuous contact process or continuous or counter current stage wise processes. However these are the discrete we have stage one stage two stage three there we had one continuous column which is a packed, although we are getting the similar mathematical expressions.

(Refer Slide Time: 23:04)



Now, let us draw or let us mark our points X 1 Y 1 X 2 Y 2 on this equilibrium curves and draw the operating curves. So we have the same capital X capital Y which will be equivalent to some small y and small x different ways of equilibrium curve, so we can draw say equilibrium curve like this. We have the streams here X 0 and we have the streams corresponding streams in Y phase or the e phase has Y 1. So we have X 0 and Y 1 this would be the stage one.

If you recall the previous, we had Y 1 and we have we have here X 0 and the streams leaving is Y 1. So you marked here Y 1 X 1 and if you look at the operating curves it is a now it is a linear and a positive slopes. So, we have a linear curve the slope of plus R S over E S and if some equilibrium curve is like this it is not it is possible that this operating curve may extend to hit this and we will call this as a pinch it is a pinch point right. In case of co-current if we recall we had the equilibrium curve like this and we had the operating curve with a negative slope which we hit hits here it reaches the equilibrium , where we have we can say that two phases are in equilibriums.

Here, we have notice the difference that we have this operating curve which has positive slope and still it is possible that in some cases this line may hit equilibrium curves depending upon what type of system we have chosen and wherever it happens we will have the pinch zero driving force and we might required infinitely not in number of stages very infinitely large number of stages. Now, let us get back to this we are marked

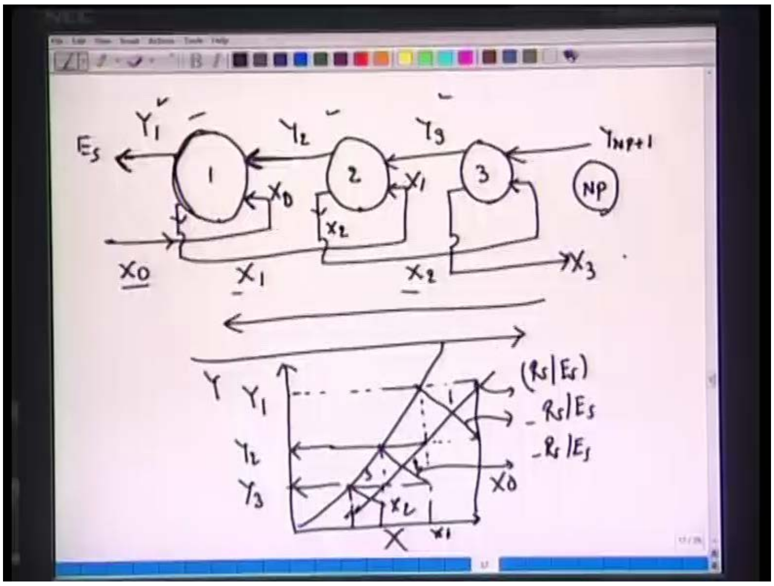
X 0 and X 1 and so this is X 1 here now we say that Y 1 is stream Y 1 is stream it is in equilibrium with the stream which leaves here Y 1 X 1. We have Y 1 here we have now X 1 here. So starting from X 0 Y 1 now we have the second stream which has compositions two streams leaving streams are in equilibrium Y 1 and X 1 we can take this X one get back to the operating curves which will give us X 2 Y 2 again Y 2 will be in equilibriums with Y 3 with X 2 here we can mark the X 2 and get back to the equilibrium curve to mark this point Y 3 and we have this X 3 here.

So all we have done we are saying that we are starting from X 0 we have brought down the mole fractions from X 0 to X 3 and for every time and we have the streams - other streams where the compositions has changed from Y 3 to Y 1. So we are the fresh solvents which was Y 3 and now the concentration has increased from Y 3 to Y 1 here.

We will also look like to look at the entire this counter-current process slightly different. If we have just one stage and we bring two stages it does not make a difference where are the inlet ports of the these two stages of these two streams.

So we have one stage, they come they can come in contact like this or they can come in contact like this or they can come in contact like this. Leaving streams if they are in equilibrium, they will be in at the equilibrium curve; all it means we can also look at every stage overall of course, is a counter-current process one stream enters here and another stream enters here. This stream leaves from here this stream leaves from here, but it is possible to draw or to make an analogy with what we had in case of cross co-current flow. So let us look at the slightly different quite interestingly this arrangement of counter-current.

(Refer Slide Time: 27:13)



We have a stage one, where now we said that earlier we had the entering streams entering like from this sideways. Now all we are saying that let us draw like this. So this stream, which has a composition of X 0, now it enters like this; and this stream which was leaving here is like Y 1. This has the flow rate of E S. Let us draw the second stage - stage two. Now this stream, which leaves from here this can go like this, and you can again enter from this stage two, this stream leaves like this with Y 2 compositions. Let us draw the third stage, so this stream, which leaves from here like this, like this, again it will enter like this and so far. The streams, which come in the top that extract phase has a composition Y 3 and so far we have these compositions say Y N P plus 1, which we drew here and this is a compositions of X N P or X. Here we have this X 3 leaving the stage number X 3 X 3 here, this will also enter some different stages here which would be marked with N P. So what will notice here that the composition is X 0, now after this composition is X 1, when in leaves here this composition X 2.

So X 0, X 1, X 2; we have Y 1, Y 2, Y 3 let us focus on this. Over all still it is a counter-current process all we have done in one stage either one and two, we have made a side wise entries. So for one stage or individual stage still one somebody can look at as if it is a stage with a co-current process stage this stream enters with the composition Y 2 leaves with Y 1 and this stream leaves with leaves with enters X 1 and leaves with enters with X 0 and leaves with X 1. Similarly we have X 1 and this is our X 2.

So, although the overall process is counter-current one stream goes like this and this stream goes like this individually we can see still one can treat as stage one two three as a co-current process, which means if you can draw the equilibrium curve of Y versus X with the equilibrium curve given as this. We have earlier we had this operating curve which was plus R S over E S as a positive for counter-current and we marked those points as X 0 then we had this Y 1 we said Y 1 is in contact in equilibrium with X 1. So we mark this point as X 1 then we had Y 2 then we had this as X 2 and then we got this as Y 3. Now this also possible to treat that since two streams are entering co-currently. We have X 0 and Y 2, so X 0 and we have this Y 2 here and since the co-current we can draw the curve like this with the slope of minus R S over E S. See we have the same stage like this now it enters here we can draw another stage one stage, stage two and we can have a stage three.

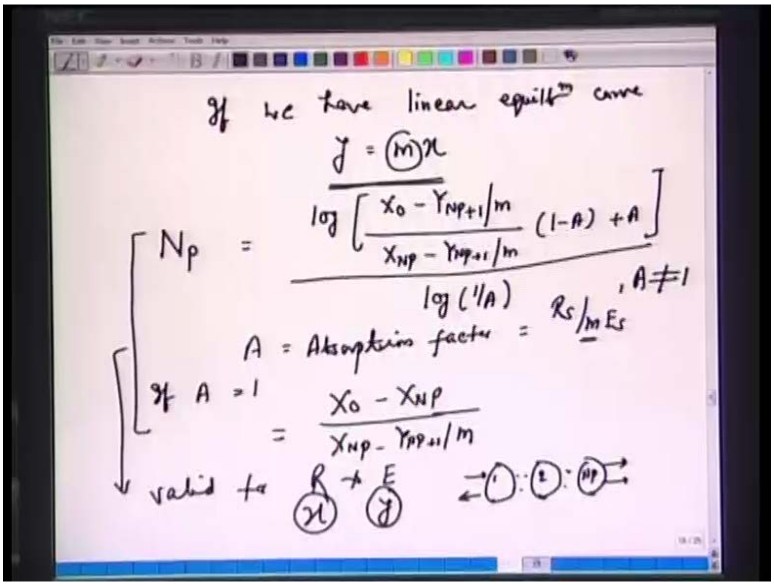
In other words, the two ways of representing this counter-current. Over all process countercurrent we can have this operating curve like this R S by E S, if you look at individually stage one two and three is possible to draw the lines like minus R over E S individually for a co-current process to achieve the same results.

So we have taken two different cast scenarios; one is a cross co-current and another is counter-current stages. We can assume that if these stages are ideal equilibriums or theoretical stages in the two leavings steams are in equilibriums. Now, we can solve this problem graphically we will take an example, where we will solve the problem graphically we will mark this points X 0 X 1 for different stages or we can also solve mathematically that means we can set up the material balance or species balance for individual stages stage one two three etcetera and we can solve the equations simultaneously both will give the same results.

In some cases it is better to work on graphically for our better understanding of the problems. In some cases when the graphical becomes quite complex it is better to solve few set of set of algebraic equations (( )) how many stages we have in… If the number of stages are very large and of course one has to write a code - programming code fortran, c etcetera and then one has to solved the compositions of the mass flow rates at every stages here.

Before we take of an example, let’s take one very special case so far we drew capital Y versus capital X and we also said that if we want to work on absolute mole fractions absolute weight basis A by A plus B or B by A plus B there is no harm except that the equilibrium curves and operating curves will not be linear and then it this brings additional computation etcetera. So, It is always wise to work on the capital Y and capital X, whether it is a contact process or it is a stagewise process. The very very special case if the equilibrium curves are is a straight line if it is a straight line then we can assume that the slope is m one can make a material balance on every stage with the slope m constant m throughout for a stage one stage two stage three, so that we can write Y equal to m x two leaving streams on equilibriums one can obtain a very analytical nice expressions. So we write down this analytical expressions you can take this example as an exercise and to see whether you get the same result or not.

(Refer Slide Time: 33:56)



If we have linear equilbm curve

$$\underline{y = \boxed{m} x}$$

$$N_p = \frac{\log\left[\frac{X_0 - Y_{NP+1}/m}{X_{NP} - Y_{NP+1}/m}(1-A) + A\right]}{\log(1/A)}, \quad A \neq 1$$

A = Absorption factor $= R_s / \underline{m} E_s$

If $A = 1$

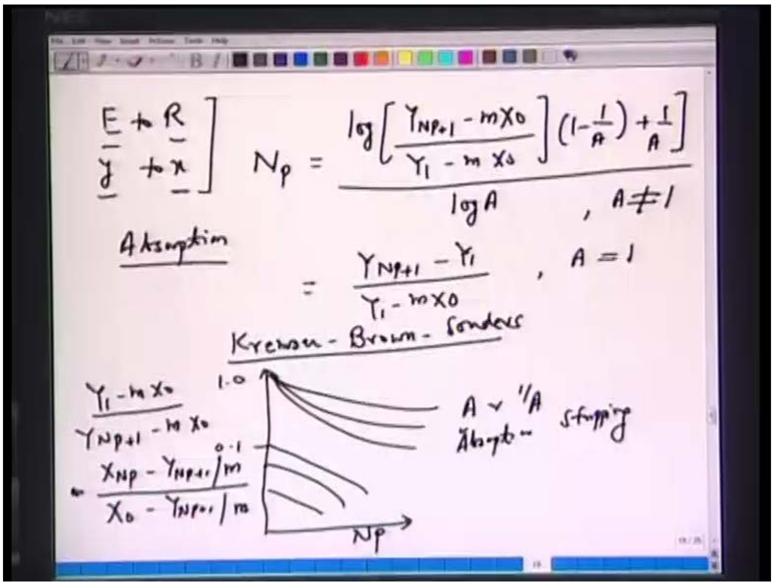
$$= \frac{X_0 - X_{NP}}{X_{NP} - Y_{NP+1}/m}$$

valid for $\underset{\boxed{x}}{R} \nrightarrow \underset{\boxed{y}}{E}$

So if we have if we have linear equilibrium curve, linear equilibrium curve like you know we can write Y equal to m x straight line, then it is possible to obtain an possible to obtain an expressions for number of a stages N P equals log as well said that you should take this example as an exercise to solve this or to obtain this expressions. So X 0 minus Y N P plus 1 over m over X N P minus Y N P plus 1 over m 1 minus A plus A divided by log 1 over A. So this A is called absorption factor and it is given as R S over m E S. So m is slope of the curve, we assuming it is linear it is a constant over the range we are considering and when we write down this expression we make note here that A does not equal to 1. And if A equals 1 then we have this different expression for N P simplified as X 0 minus X N P over X N P minus Y N P plus 1 over m and we make a note that this expression is valid or is applicable for R to E phase.

So we are discussing a transfer of components from R phase X to this Y phase and you should be able to obtain these expressions for this counter-current case which we discussed just now. So, for stage N P we should be able to obtain. Similarly if we have transfer from E to R phase we get slightly different expressions.

(Refer Slide Time: 36:28)



$$N_p = \frac{\log\left[\frac{Y_{NP+1} - mX_0}{Y_1 - mX_0}\left(1-\frac{1}{A}\right) + \frac{1}{A}\right]}{\log A}, \quad A \neq 1$$

$$= \frac{Y_{NP+1} - Y_1}{Y_1 - mX_0}, \quad A = 1$$

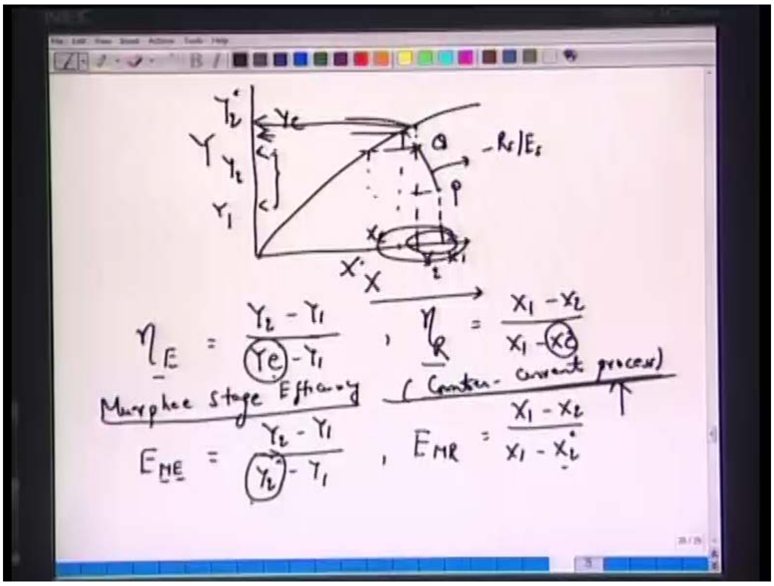
So, if E to R or y to x then we have slightly different expressions for N P log Y N P plus 1 minus m X 0 over Y 1 minus m X 0 1 minus 1 over A plus 1 over A over log A and if and here we can say that A does not equal to 1 and if A equals 1 we have simplified expressions Y N P plus 1 minus Y 1 over Y 1 minus m X 0 if A equals to 1. So 1 is like absorptions so we have absorptions from gas phase to the liquid phase and the previous example we have we have desorption or stripping. So we can may be we can go to the previous phase and we can say that the expression is holds good for a stripping or desorption so X to Y phase. And these two expressions are known as Kremser-Brown-Souders and the same equations they have also been given the text book in the graphical form. You can have N P number of stages and you will see different coordinates different numbers here, set of curves A or 1 over A stripping absorptions - case of absorption or stripping two different cases when we have mass transfer from E phase to R phase or vice versa R to E phase.

And here we have Y 1 minus m X 0 over Y N P plus 1 minus m X 0 or X N P in the second case when we have this stripping Y N P plus 1 over m over X 0 minus Y N P plus 1 over m. This was a very special case where we had we assumed that equilibrium curve is a straight line. So in that case one does not have to make a material balance for every stage ten stages twenty stages we can use this expression directly.

We have to be careful mass transfer is from which to which phase extract to (( )) E to R phase or R to E phases and we have to careful with the absorption factor A which we define as R S by m E S if E equals to 1 we notice there is similarity in the equations so we have to use the simplified equations second expressions modified equations or alternatively we can use the graph given by this Kremser-Brown-Souders; either the equations will work or the graph will work. So, what we did earlier? We assume that each stage is ideal or theoretical or equilibrium; all it means the leaving streams the coordinates of the leaving streams will be at the equilibrium and if certain non ideality or if we have not given enough resistance time or contact time to the two streams then the leaving streams will not be in equilibrium and these points will not lie on this equilibrium curve.

Otherwise mathematical treatment or the graphical treatment for the two sceneries are the same whether with equilibrium or non-equilibrium and all it means we can define efficiencies - equilibrium efficiencies They are very two common efficiencies defining cases of in case of like you know mass transfer in this stage or mass transfer is in contact process. We have modified stage efficiencies or we defined in a simple efficiency.

(Refer Slide Time: 40:54)



$$\eta_E = \frac{Y_2 - Y_1}{Y_e - Y_1}, \quad \eta_R = \frac{X_1 - X_2}{X_1 - X_e}$$

Murphee Stage Efficiency (Counter-current process)

$$E_{ME} = \frac{Y_2 - Y_1}{Y_2^* - Y_1}, \quad E_{MR} = \frac{X_1 - X_2}{X_1 - X_2^*}$$

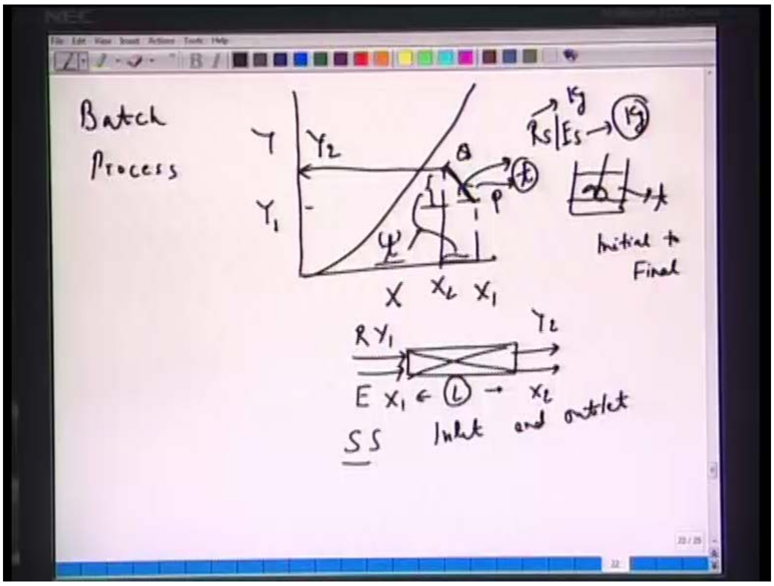
So we will have these two definitions here. So let’s redraw our equilibrium curve we had X and we had Y we have this equilibrium curve Y and X plus draw co-current so we start with P and with Q with the slope of minus R S over E S. So the leaving streams compositions here is Y 1 and this is your… So let us mark this say this is X 1 and the leaving stream here the composition is X 2 this is here Y 2 for X 1 we have this Y 1.

Now, if there an equilibrium then we can exchange this line and we will get we can mark this point as Y e equivalent to this we have this X e. For it means the maximum separations, we can have which means we can define efficiency like eta E as Y 2 minus Y 1. So this what we are achieving Y 2 minus Y 1 starting from P and ending with Q here and what maximum we can achieve is Y e minus Y 1 so all we have done. We extended this line Q to hit this equilibrium curve from where we get this Y. Similarly, we can define the efficiency based on R phase on this X basis we should be here equal to X 1 minus X 2 actual separations over what we have the maximum we can have X 1 minus X e to this. So both efficiencies are defined in case of this stagewise processes or contact process - continuous contact process. One very common efficiency is known as Murphee stage efficiencies and this efficiency this efficiency is work very nicely it works for counter-current process counter-current process and here we one defines E M E. Similarly for Murphee efficiencies based on extract phase as Y 2 minus Y 1 over Y 2 star minus Y 1.

So noting the difference now we are defining Y 2 star instead of Y e . So all it means we are saying that whatever we have the X 2 we extend this X 2 so if you recall from our previous lectures a hypothetical concentrations X 2 in equilibrium with this Y 2 star. So we are saying that bulk concentration of this phase is in equilibrium with these compositions with the second phase at Y 2 star.

Similarly we can have E M R as X 1 minus X 2, now we can have X 1 minus X 2 star. So given this Y 2 what maximum we can have in the bulk phase is X star, so of course you will notice that this definitions is applicable only for counter-current process for co-current does not make any sense, because we can never achieve more than Y e however X e etcetera. So that is why we said we noted that this Murphee stage efficiency is defined is applicable for counter-current processes.

(Refer Slide Time: 45:41)



There is one small thing here that so far we did is we have flow rates, so two streams are brought in contact either in a stage or in continuous contact process co-currently or counter-currently cross currently or counter-currently. It is also possible that we have a batch system, so if you recall from your unit kinetics reaction engineering on some in stoichiometries.

Now we are talking of two streams or two liquids say two immiscible liquids X and Y L 1 and L 2 they are brought in contact we mix them perfectly, so there is no flow rate here so far we worked on R and E or R S and E S as kg per seconds mole per second. Now we are talking of R phase or the weight of this phase is kg - phi kg; its brought in contact with 3 kg of E phase, so no kg per second. And if they are brought in contact for sometime T 1 T 2 T 3 or for a infinitely long time then we separate the two phases again R and E and take out the streams from by some decanting process by certain different unit operations.

Even in that case when there is no flow rate or we have the batch process we can still apply whatever we have done so far in all it means instead of working along the length, now we are saying that we are working along this time coordinates. So if we go back and redraw this Y versus X and we are discussing here batch process still our equilibrium curve will be the same and now we say that we start with p certain compositions of one phase as X 1 and certain compositions of another phase as Y 1 and then we spend and then we have a batch system here two phases are brought in contact they are given enough resistance time they will be separations this allow them to settle.

Now the curve will still be the same as P going to Q X and the slope of this would be now R S over E S notice the difference is E S is not kg per second it just a kg and this your kg; all it means the movement from P to Q is now on the time stayed of on the length.

So, we have said that we bring the two phase here R and E; we get them some sufficient time here, composition changes from say Y 1 to Y 2, and if we have this E phase, the composition changes from X 1 to X 2 instead of length or we are saying that we are giving them some time T. But if you make a material balance now from initial to final initial to final unlike here we have a steady state, and we say we make a material balance between inlet and outlet, but notice that we will get the same results here; now we are drawing P to Q, now the compositions here is Y 2 after certain time, composition here is X 2 after certain time. So we have achieve X 1 minus X 2 or Y 1 minus Y 2, this two difference here over a time T instead of over the length L, otherwise our mathematical treatment is same as before.

So, whether we have a batch process or we have this continuous contact process or a stagewise etcetera, material balance or the equation, mathematical form of the equations will be the similar. So it does not matter when we say that two streams are drawn in contact at a certain flow rates or they are mixed in a batch reactor or batch adsorber or batch absorber. It is just a question of working on a instead of time coordinates we are working on the length coordinates. We have the similar situations, whether we have a co-current or counter-current etcetera does not make a difference. In the next lecture, we will take an example of what we have discussed here in case of cross co-current and counter-current processes.