

Mass Transfer II
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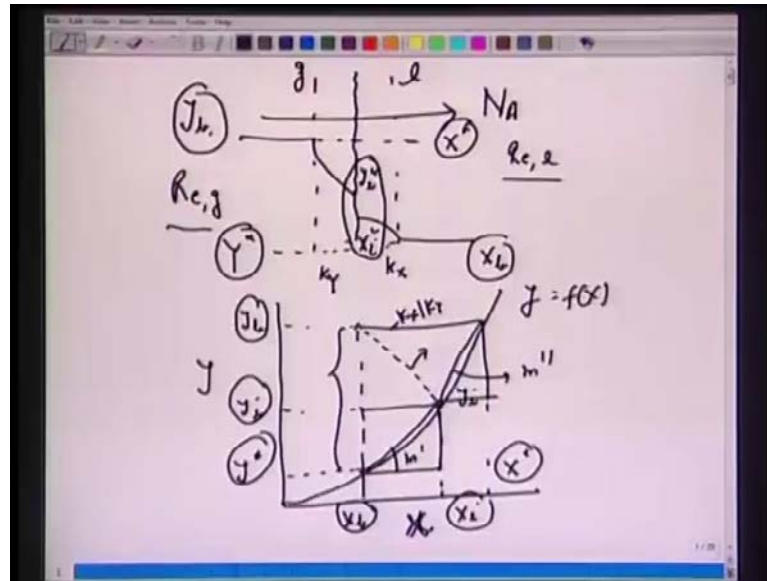
Lecture No. # 06

So, before we graphically derive the expression for overall mass transfer coefficient in terms of individual mass transfer coefficients, just try to visualize, you know, what **what** situations we have here. So, we have and say **an example** an example absorption column. So, we have an interface of gas and liquid, and say sulphur dioxide in air diffuses into water.

So, we have two individual resistances; one in the gas film and one in the liquid film. So, when we say there is a resistance in the gas film, there is a concentration drop. So, we have the bulk concentration C_B , and it drops to some interfacial concentrations, and at the interface we have the equilibrium between the gas and the liquid phase. And then, there is a concentration drop within the liquid phase before it levels to some bulk concentration in the liquid phase.

So, we have four concentrations; two at the interface in equilibrium and two bulk concentration **into** different features. We can also have a set C^* or Y^* and X^* assuming the entire bulk concentration in the liquid phase is that the interface or assuming we have the entire gas phase bulk concentrations or vapor phase concentration at the interface.

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So, with that let us try to obtain this relation between the overall mass transfer coefficient in terms of individual mass transfer coefficient. So, we have the interface, gas and the liquid, two individual **individual** film mass transfer here. So, we have the bulk concentrations, see in the gas phase mole fractions y_b . So, we can say y_b here and then it drops to the interface, say y_i , when it drops from x_i , where y_i and x_i are in equilibrium. This concentrations to this bulk phase that is x_i . Gas phase we have some Reynolds number; in the liquid phase we have some Reynolds number based on the flow rates. So, this Reynolds number decides what is the individual mass transfer coefficient in the gas phase and in the liquid phase based on Re, g and Re, l . And we have this diffusion flux, it cross this interface that N_A moles per second per meter square. We also said that for y_a, b corresponding to y_b , there will be x_b^* . We can have **X** x_b^* here, and corresponding to this x_b we can have Y star here.

So, these are the different concentration levels which we have marked here. And then corresponding to this if we derive, if we draw this line y versus x mole fractions, then we have this equilibrium curve, say solubility data, y is a function of $f(x)$. We mark on this bulk concentration. So, we have x , say let us say y_b , and we have x_b . Now, we know from slope of minus k_x and k_y , k_x to k_y we can draw this line here to obtain y_i and we can have **we have** x_i . So, we can draw this, we can complete this geometry here like this, and we have like this, if we recall this is your m dash, this triangle this slope, and this slope here is **the slope here is** m'' .

So, let us look at this, we have y_b here, y_i here, and now we have x_i and we have x_b coordinates. So, we can write **equivalent to this x** equivalent to this x_b we have, we can also have as you said earlier y^* and equivalent to this y_b we can have x^* . So, x_b , x_i , x^* and we have y_b , y_i , y^* . So, go back to the previous y_b , y_i , x_i we have x_b and equivalent to this y_b , we have x^* in equilibrium with y_b , and in equivalent to this x_b equilibrium with x_b we have this y^* . So, these points if we mark, and now we focus on this y_b minus y^* we can write as y_b minus y_i plus y_i minus y^* . So, let us go to the next.

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The image shows a whiteboard with handwritten mathematical equations. At the top, the equation is $(y_b - y^*) = (y_b - y_i) + (y_i - y^*)$. Below this, it is written as $\frac{N_A}{K_y} = \frac{N_A}{k_y} + m' \frac{N_A}{k_x}$, where $m' = \frac{y_i - y^*}{x_i - x_b}$. The term $\frac{1}{K_y}$ is boxed and labeled as the slope of the equilibrium curve. Below that, another boxed equation shows $\frac{1}{K_x} = \frac{1}{k_x} + \frac{1}{m'' k_y}$.

So, we can write here y_b minus y^* **equal** this equals y_b minus y_i - interfacial concentrations plus y_i minus y^* and this y_i minus y^* . Now, we can write it as m' in terms of now liquid phase concentration which is x_i minus x_b . Now, we have the definition for diffusion flux in terms of concentration drop and mass transfer coefficients.

So, we have now N_A over capital K_y . So, this is overall mass transfer coefficients the defined as N_A equals y_b minus y^* into K_y . So, re-call the previous lecture we had. Similarly, we can write y_b minus y_i as N_A over small k_y . So, individual film mass transfer coefficients plus m' x_i minus x_b we can write as N_A over small k_x film mass transfer coefficient in the liquid phase. So, we have we obtain the same relations which we had earlier k_y equals 1 over small k_y plus m' over k_x , where m'

dash is the slope or the of the solubility curve. So, this is the slope of the equilibrium curve or this solubility curve in this slope in present case solubility equilibrium curve.

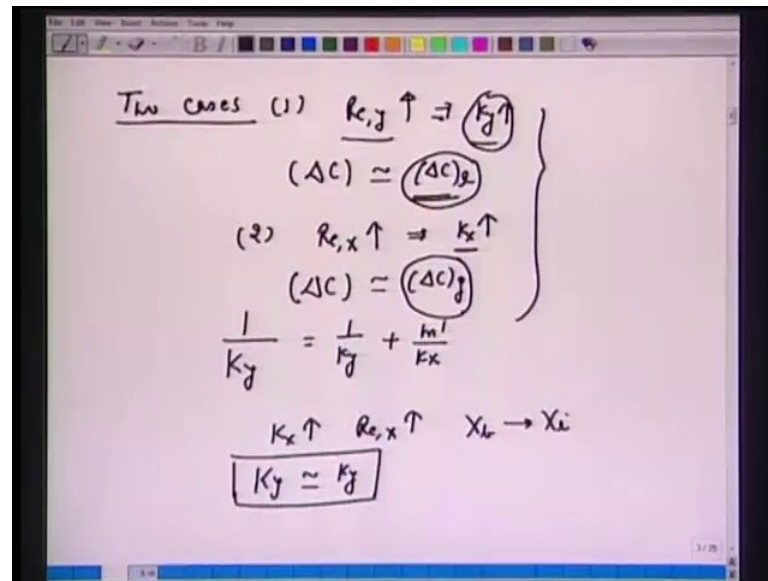
Similarly, we can write how one should obtain, we can obtain as an exercise $\frac{1}{K_x}$ as $\frac{1}{k_x} + \frac{1}{m k_y}$ double dash over k_y . So, we have obtain the similar relations which we have obtained earlier, overall mass transfer coefficients in terms of individual mass transfer coefficients k_y k_x based on the gas phase or based on this liquid phase.

Now, let us try to understand that what we have here. So, we said that we have an interface and two diffusion films right two films; one in the gas phase, one in the liquid phase. So, these are the diffusion resistances. Just like you have an electrical circuit and you apply a voltage drop across a wire and you have two resistances in series R_1 and R_2 . So, the voltage will drop from say V_a to V_i intermediate and V_i to say V_c throughout V_b . So, entire $V_a - V_b$ is distributed over two individual resistances.

Now, it is possible that we have very high Reynolds number in one phase; either the gas phase or the liquid phase. Or if we have very high Reynolds number mass transfer coefficient will be very large. So, if mass transfer coefficient is very large, then the resistance across this gas film will be very small. In other words, the drop entire drop from the bulk phase y_b or y_a we have, y_b and x_b can be assumed to be distributed only across the liquid film. Because mass transfer coefficient in the gas phase is very large. So, there is no diffusional resistance here.

Similarly, if we have very high Reynolds number in the liquid phase, you know related to the gas phase. So, we can assume that a mass transfer coefficient while get calculate that mass transfer coefficient in the liquid phase is very large. The diffusion resistance or the concentration drop from the bulk phase in the liquid to the interface is very small, and we can we can assume that entire drop or entire concentration gradient drop across the bulk liquid gas phase and the bulk liquid phase is distributed only across this liquid film and gas film.

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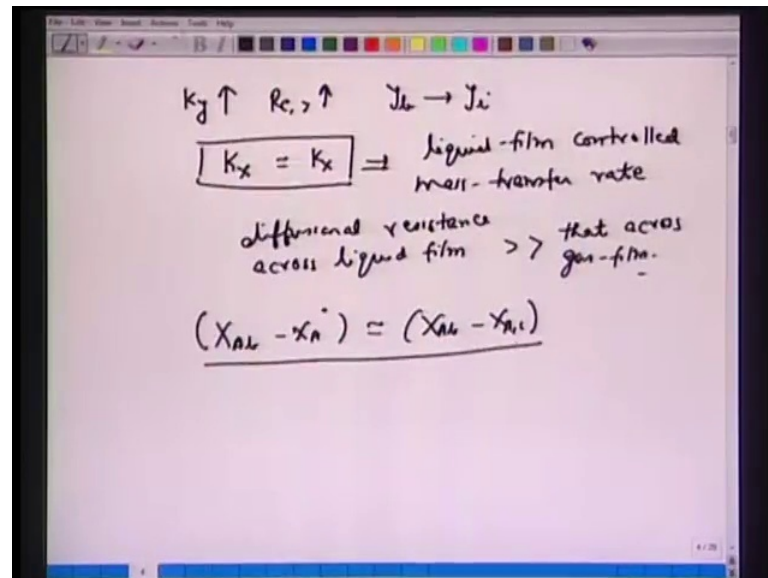


So, we can have two special cases which we summarize here that we have two stream cases for special cases. In one case, if the Reynolds number say in the y in a gas phase is very large all it means that mass transfer coefficients in this liquid in the gas phase is very large. That means entire drop concentration gradient which we have across other two films we can assume everything is concentrated in the liquid phase. Because mass transfer coefficient in the gas phase is very large, very high.

We can also have the second case when Reynolds number in the liquid **in the liquid** phase is very high. If that happens then individual film mass transfer coefficient in the liquid phase will be large, and we can assume that entire concentrations is distributed in the gas phase. So, one which is smaller here k_x is large k_y is smaller. So, we have the entire resistance across the gas phase. Unlike here, if we have the gas phase mass transfer coefficient very large that means, the entire diffusion resistance is distributed over this liquid phase. So, with that you know physical understanding, if we go back to the previous expression which we have obtained for overall mass transfer coefficients, we have $1/K_y = 1/k_{g,y} + m'/k_{l,x}$. Let us assume that k_x individual k_x is very large that means Reynolds number in the liquid phase is very large. We have the bulk phase in the liquid phase approaching to the interfacial concentration. So, there is no drop here and with that we take the limit k_x tends to infinity are very large, k_y overall mass transfer coefficients can be approximate to individual mass transfer coefficients k_y equal to $k_{y,s}$. Similarly, the second case we can have,

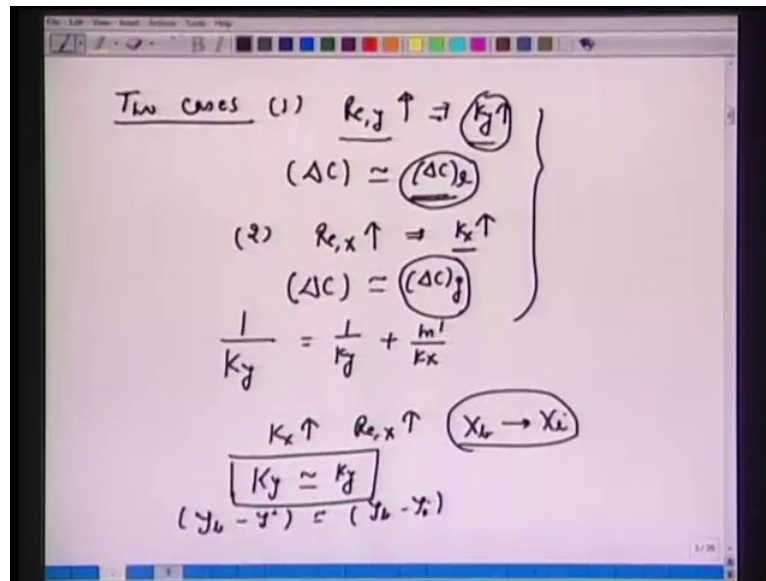
now, **second case**, if we choose here in which case **case** two if we say that a small mass transfer coefficients in the gas film will be very large, because the Reynolds number is very high.

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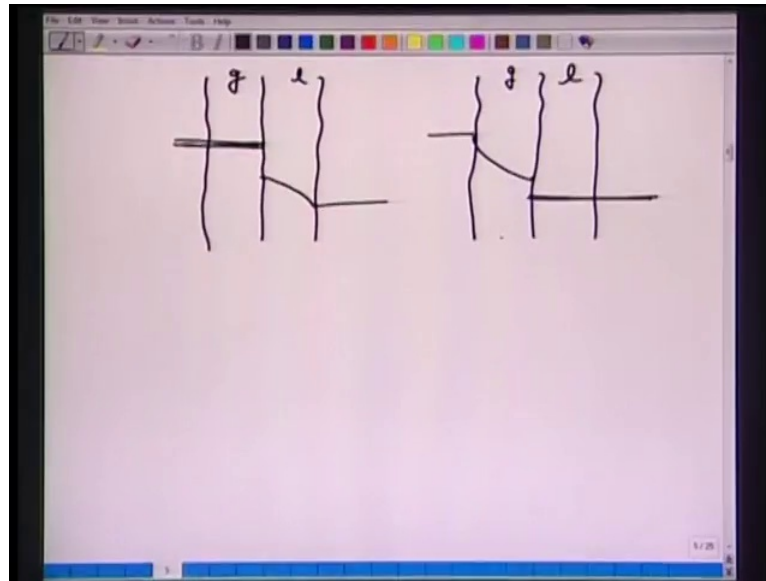
In that case bulk phase gas concentrations will approach interfacial concentrations. And we can say that capital K_x overall mass transfer coefficient in the liquid phase is approximately equal to small mass transfer coefficient in this liquid phase. Another word this one is a liquid film controlled mass transfer rate or **we see** we are saying here that diffusional resistance across liquid film is much larger than that across gas film. And in this case X_{Ab} minus X_{A^*} this will approximately equal to X_{Ab} minus X_{Ai} .

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So, we can go back and even similar situations we have in this case. The first case when we see that K_y equals k_y , we can say that y bulk phase concentrations minus y star is same as y_b minus y_i , because there is no resistance in the liquid phase. **Alright**, because mass transfer coefficient is very large, because Reynolds number is very large there. So, essentially **all these three** all these two cases or we **we** must again go back and see the physical, you know interpretations if we have a interface here and if gas phase resistance is negligible, because we have high Reynolds number. So, the bulk phase concentration will approach approximately same as interface, it will approach the interfacial concentration, and then all the drop will be in the liquid phase. If the liquid mass transfer coefficients very large, because Reynolds number is very large then we have x_b approaching to x_i and all the drop or most the drops will appear across this gas film. So, these are the two extreme cases; in reality, if both the film resistances are high then, one has to take into a count both the diffusion resistance.

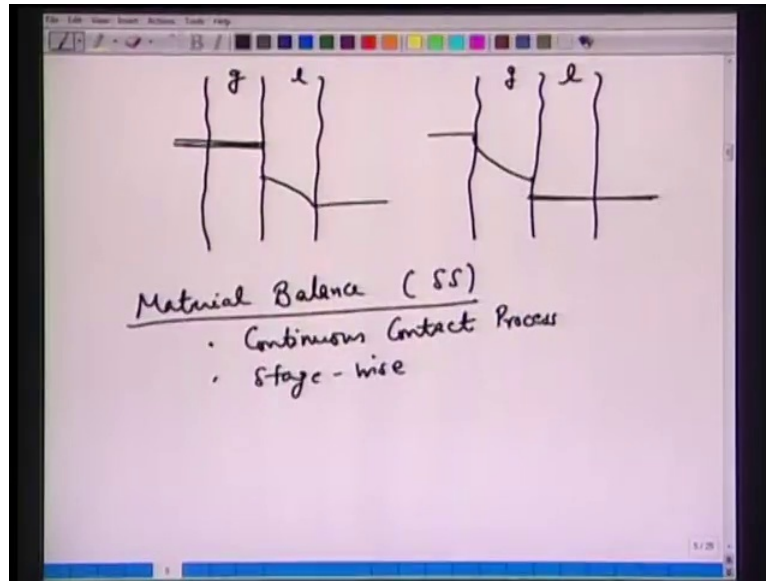
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So, in other words, graphically if you have this interface, then in one case this is your gas film, this is your liquid film, we have the situation like this and then it drops. So, all the resistances across this gas phase, here **here** it almost reaches this negligible, and then we have this drop here like this it for this liquid phase. In other case, we have all the resistances in the gas phase, then the drop will be across this gas film and here there will not be any resistance or concentration drop here in this sequence. So, there are two special cases we should keep it in mind.

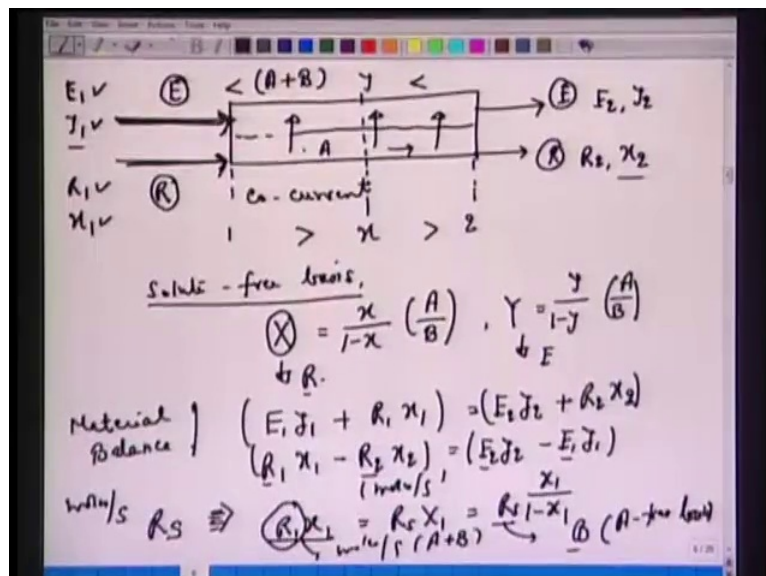
So, now what we have done lets sum up, we have equilibrium given by the thermodynamics. So, given mole fraction in x , we have y given partial pressure in liquid phase, we have gas phase concentration in the other phase. So, this is the thermodynamic. And now we can mark our operating points - mark we can our all the four concentrations with the bulk phase, in the gas phase, what is the driving force y_a minus y_i or y_a minus y^* etcetera. So, these points now we can mark based on our discussions which we had in the previous lectures. Now, we will like to draw the operating lines. Operating line is nothing but given bulk phase concentrations, what is the liquid phase concentrations, and this has to come from this material balance. Or given the liquid phase concentration anywhere in the interface or within the column how much is a gas phase concentrations. So, we have equilibrium and now we will have the operating lines.

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So, let's try **lets try** this derive this material balance in the absorption column for example. **So, we have...** So, we starting with material balance. And we assume steady state, now we have two cases; one is continuous contact process and the second is stage-wise. So, this way discussed in the previous lectures that we have two types of contact lines; one is continuous packed bed for example, in case of absorptions or we can have stage wise contact lines or cascades **in** in case of destinations. So, now we are trying to make a material balance in these two cases. So, first we start with this continuous contact process.

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So, let's see, let's say that we have this column - packed bed column in which we bring this stream we call it E symbol E. We bring in contact with another stream give a symbol of R. We can assume that for example, this is your liquid phase and this is the gas phase. So, gas phase is brought in contact with this liquid phase. And so this contact is co-current, because they are in the same directions. And we have this heated stream which is E and we have the excited stream here is R

So, let say that the flow rate here is E. So, this end is we mark it as 1, this end is marked as 2. So, we have co-current flow. Here the flow rate say molar flow rate is E_1 and the mass mole fractions of species let us take binary species A and B. So, mole fraction of A which diffuses in the two phases is y_1 . So, we have E_1 and y_1 is brought in contact with another steam R which has a flow rate of say R_1 and it as mole fractions of A given as x_1 as x_1 . So, E_1 mole per second or Kg per seconds or we have y_1 mole fractions or weight fractions, and we have R_1 mole mole per seconds or Kg per seconds, and we have subsequently mole fraction or mass fractions in this gas phase. So, they they are brought in contact let us assume that only A diffuses from R. So, throughout this contact A diffuses from B, from one stream R to this stream E and B is non diffusing components. So, at the exit, again can we have E_2 mole per second at the exit, and we have y_2 mole fractions here to symbolizes this exit end, and we have R_2 and we have x_2 mole fractions.

So, mole fractions of x now this is stream here will keep on changing along this. So, if you choose any concentration any interface here suppose we have x , x_1 will be greater than x and x will be greater than x_2 , because A diffuses from here to here. And similarly, if we have y here mole fractions in this E phase then we have R_2 y_2 greater than y and y will be greater than this y_1 alright. So, as R moves along the column both mole fractions of x and y changes. This this is stream gets enriched with A, this stream gets depleted with A.

Now, let us also define one more quantity if you re-call solute from a stoichiometry, we have solute free basis. We will see, what is the advantage of working on this solute free basis mole fractions? So, if we define X as x over $1 - x$. So, all it means the mole fractions of A A free basis. So, this is your A over B. Similarly, we can say capital Y as small y over $1 - y$. So, this is your A over B in raffinate phase in E phase, and this is

your R phase. So, we are defining capital X and capital Y, we will see, what is the advantage?

Now, let us make material balance. So, we have a steady state and we are making material balance or species balance on A specifically on A that is we get this. So, we have two streams which enter here. So, we can write $E_1 y_1$ plus $R_1 x_1$. So, this is stream brings $R_1 x_1$, this is stream brings $E_1 y_1$. This will equal to the two exit streams we have $E_2 y_2$ and we can write $R_2 x_2$. So, in equal to out; we can rearrange here $R_1 x_1$ minus $R_2 x_2$ and this will equal to $E_2 y_2$ minus $E_1 y_1$. Now, look at this term here R_1 and R_2 , E_2 and E_1 these are the moles per second at the two ends **right**. They will change because, A diffuses into B. So, this **mole flow rate** molar flow rate of A or molar flow rate of these two streams will vary along this.

However, if you have defined the quantities like capital X and capital Y equivalent to this we can also define a quantity is R_s . So, what is R_s here, it is also a moles per second of that stream on solute free basis which means we can write R_s , and we can define R_s such that we have $R_1 x_1$ equal to R_s capital X 1 which is nothing but $R_s x_1$ over $1 - x_1$. So, all we have done, we are substituting you want to substitute R_1 in terms of R_s . So, R_1 is a total molar flow rates. So, this will be moles per second or both A plus B. On the other hand, if you look at this term this **R** R_s is the molar flow rates of B 1 A. So, it is A free basis. And you will see, what is the advantage of this? So, if we define this quantity one should understand that R_s or E_s will remain the same, because it is only A diffuses. So, B does not diffuse in this directions or B does not diffuse across the interface. So, the amount of moles per second or K_g per second of B will remain same here and here. So, all we have R_s and s which will be conserved in both the phases **right**. If you work on a R actual R and E are based on mole fractions small x and small y then these two moles per second molar flow rate will change along the length.

So, it is advisable we will see the advantage in a minute to work on solute free basis that is a stoichiometry if you re-call at some level in a first or second year of your chemical in department stream.

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$R_1, R_2 \rightarrow R_s$
 $E_1, E_2 \rightarrow E_s$

$R_s (X_1 - X_2) = E_s (Y_2 - Y_1)$

$R_s (X_1 - X) = E_s (Y - Y_1)$

$Y = -\left(\frac{R_s}{E_s}\right)X + (R_s X_1 + E_s Y_1)$

Operating line

Compare this to equilibrium curve $Y^* = f(X)$

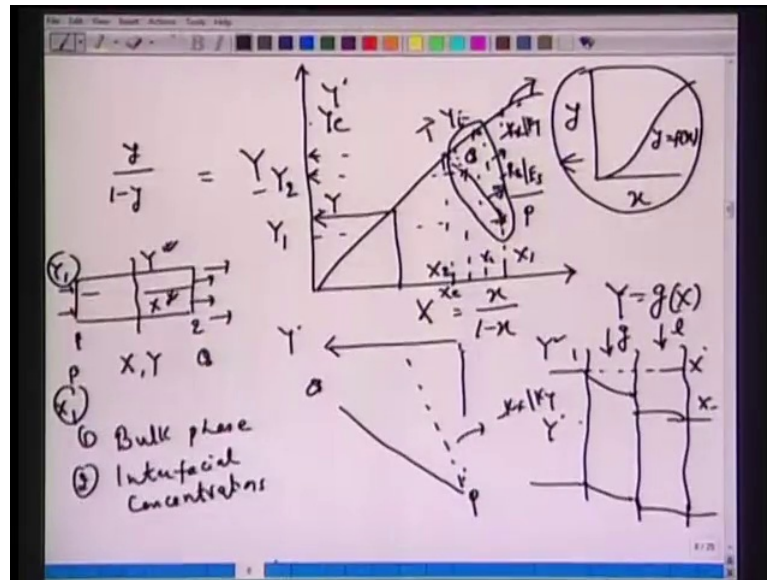
So, next if we substitute R_s , R_1 or R_2 in terms of R_s or E_1 , E_2 in terms of E_s just E_s note there is no suffix here, because R_s E_s is constant all along the length of this contact line. So, we can rearrange the same equation in terms of R capital X_1 and capital X_2 equals $E_s y_2$ minus y_1 . So, we have substituted small x_1 x_2 small y_2 y_1 in terms of their counter falls capital X and capital Y . And if this locations some intermediate in location between 1 and 2 where the mole fraction is X and Y , then we can write $R_s X_1$ minus X equals $E_s Y$ minus Y_1 . So, essentially we have made a species balance over this envelop where we have I means streams which you inter count co-currently and they exit E of x and y .

So, we can write now Y as minus R_s over E_s X plus we have $R_s X_1$ plus $E_s Y_1$. So, let us box this equation and this becomes however, as you notice from here this is the operating line **operating line**. Compare this **compare this** two equilibrium line equilibrium curve which we have Y in terms of Y^* in terms of your X . So, we have two lines here, one is the operating curves, one is this equilibrium curve. And notice this operating curve Y in terms of X is a constant is a linear, because R_s over E_s does not change. So, we have a constant slope, we can call it m x , y equals to m x plus intercept constant.

So, now we can go back and we can draw equilibrium curve and we can draw the operating curve. And again we want to re emphasize here that - one can still work on a

small y small x actual mole fractions of A in A plus B state of working on solute free basis mole fractions, and one can also work on E_1 or R_1 instead of R_s and E_s which remains constant along this. But since we have obtained in step line a linear curve in while using capital Y and capital X let us work on this solute free basis nomenclatures.

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So, we redraw our curve here, now we have capital Y and capital X . So, re-call earlier we had small y and small x . So, this is our equilibrium curve like this. So, we can always convert Y as small y over 1 minus y and capital Y X we can write as x minus 1 1 minus x over 1 minus x . So, we can always convert this our solubility curve or equilibrium curve given between y and x some function of x as capital Y as some other different function g x etcetera. So, we have say equilibrium curve like this y as x . Now, we know how to mark the points. So, we have this continuous contact line, two streams co-currently they come in and they make an exit here. We have general section here between 1 and 2 , we solve x and y . So, let us mark this point 1 as p . So, this point is p plus p has a coordinate x . This would be X_1 and this as there is a bulk phase concentration which is Y_1 . So, this is your Y_1 . So, capital Y_1 capital X_1 is marked as p .

Now, we have obtained the operating curve and you look at the operating curve which we had it was linear. So, we take the slope of minus R_s over E_s minus R_s over E_s we have this operating curve. So, what does the operating curve tells us this gives us a

relation between **between** Y and X anywhere between the location 1 and 2. So, concentration of Y in this stream both the stream they will change along this directions from 1 2. So, we have p going all the way till Q at the end T. So, here also we have two coordinates, this would be now this is your X 1 we said. So, this is now X 2 and now we have this is to start with this was Y 1 and now we have this concentration this mole fraction given as capital Y 2.

So, we have this equilibrium curve and we have this operating curve; equilibrium curve what does the equilibrium curve gives us suppose we choose any X here all it tells us how much is this equilibrium concentrations which means we if we extend this line P Q we will obtain another point here, we can give some name here say T where you can mark this point as equilibrium y e in terms of x e **alright**. So, we have 2 1 point here P Q and now we have T points here. So, let us try to understand the difference between again between the operating curve and the equilibrium curve.

So, operating curve gives us a relation between capital X mole fractions in the bulk phase in terms of the bulk phase of the other phase bulk concentration of the other phase. So, if you have x, given x, we can find out y or given y, we can find out x. What does equilibrium curve gives us? This gives us the equilibrium between the two strips. So, if we have P here and we want to find out what is the driving force, now we have to again go back and draw a slope of minus k x over minus k y. So, this operating line must not be mixed up with equilibrium line with this driving force line which is minus k x over k y. Similarly, we can draw another line. So, we have an operating curve just redraw here which is P Q and we are trying, we are drawing another line of a slope minus k x over k y. And wherever this intersects right here, this will give you interfacial concentrations y i and corresponding to this we will get x i.

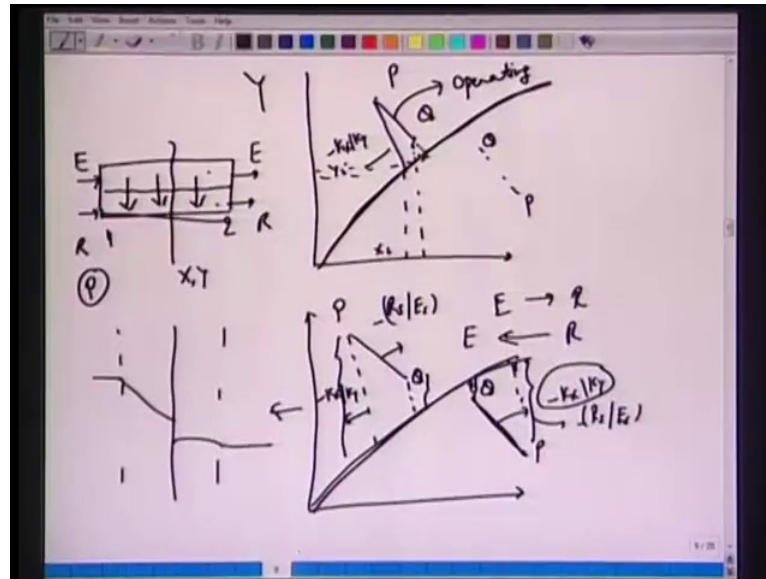
So, it is very important that we mark all these points; bulk phase concentration **bulk phase concentration**, interfacial concentrations **interfacial concentration**. So, these two phases we can mark to obtain what is the driving force for mass transfer y i minus y here. So, with these two curves - operating curves which is the linear here and the equilibrium curve y x, now **we can** we have all mole fractions of y and x between these points P and Q and this equilibrium curve.

Physically all it means that **we are** we are trying to say that we have an interface of the two phases, we have this co-current two phases a gas and the liquid gets in here, where the bulk phase it drops to some interfacial concentrations, then we have this bulk phase in the liquid phase concentration. So, we have y , we have x , we have y_i , we have x_i equivalent to this y , we can have x^* equivalent to this x , we can have y^* and these points can also be marked here if I take this point P here, and go vertically upward will it reaches the equilibrium curve that will give us y^* . So, if you take this x_1 here and go to this vertically you will get y^* .

Similarly, in some locations from 1 to 2, we have the same situations there will be interface concentrations, then there is a drop in the liquid phase, then we have this bulk phase concentrations. We like to redraw this here to in different form. So, now, we have two phases; gas and liquid, flowing counter currently, and we are saying that from the liquid phase x **there is a** or this E phase here we had E phase here. There is a diffusion of A into the other side of interface which is the gas film. We can also have different situations where from the gas phase air diffuses into this liquid. So, we have say let us assume that we have the same systems sulphur dioxide air and water. So, we have mass transfer over sulphur dioxide from air phase to the liquid phase. We can a different situation where the liquid is already saturated with sulphur dioxide, and then we bring this in contact with the gas phase. In that case there will be mass transfer from the liquid phase to the pure air. So, what we drew earlier, we have the equilibrium curve and there was a mass transfer from the x phase liquid phase to this gas phase.

Let us draw the similar situation co-current. So, two streams get in **in** one end, from one end and they exit from the other end, and now we have a transfer from the top to the bottom.

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So, let us draw this line, draw this equilibrium curve here. So, equilibrium curve will be the same Y versus X . Y versus X **excuse me** we have this equilibrium curve. Now, we are discussing we have this continuous contractive in which two streams make an end here make an entry here E and R , E and R . But now, we are saying that there is a mass transfer from E phase to this R phase.

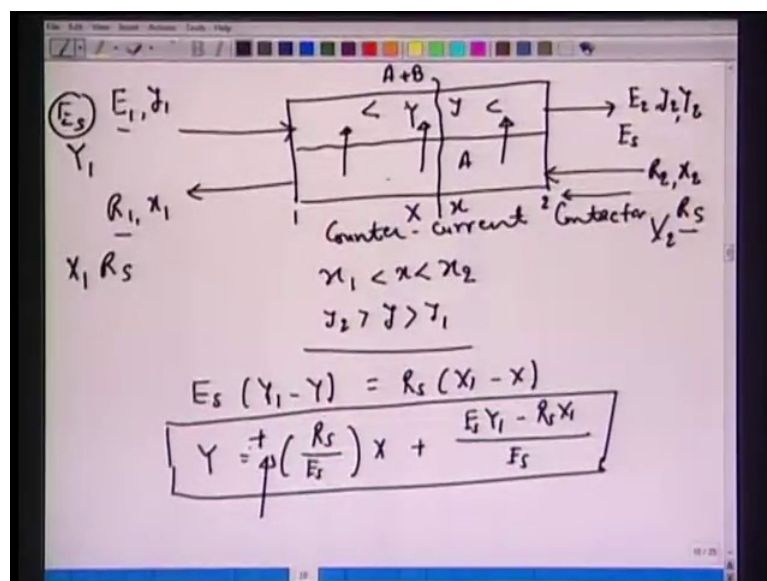
So, one can again make this material balance between 1 and 2 or between any locations between 1 and 2 x and y to obtaining we have the similar linear equation except now this point 1 if you call it P is marked here it as P and we will get a straight line - linear line which will go from P to Q as in operating curve. So, there is no difference between the two except that now we have this operating line on top of this equilibrium curve. So, here we are discussing the mass transfer from E phase to R phase. Unlike the previous case where we had this P Q here the mass transfer from this R phase to this E phase. Here also we can extend this line to hit this equilibrium which is the maximum mass transfer you can have, if you want to draw the different **different** driving force then one can draw another line with the slope of minus k_x/k_y to obtain y_i and x_i for the P . So, we will have y_i and x_i . Similarly, here also we can draw this line to obtain these two interfacial concentrations.

So, the idea is that we have one equilibrium curve and **we can have** we have the operating curves like this depending upon whether we have mass transfer from **R phase**

E phase to R or we have R phase to E phase, P Q and we have P and Q. We can also we can draw **we can draw** this slopes of constant minus k_x over k_y , here this is slope of minus R_s over R_y flow rates of the two streams, and we have minus k_x over k_y and this is the flow rate of we have R over s **excuse me**. So, this is your R . So, we have the slope of minus R_s over E_s , and we have slope of minus k_x over k_y for this for P we have minus R_s over E_s . So, all these points all these one line in here, one is the equilibrium curve and two lines here with the slope of minus k_x and k_y will give us driving force for mass transfer either here. So, we can identify these points, then we can do further calculations equivalent to this, we also have an interface and in which we have drawn a different drops or different concentration gradients.

Now, similar to this continuous co-current continuous contactor, we can also have counter-current co-current - countercurrent flow. In which case, one stream enters in from one end and the other streams E phase enters from the other end. So, both cases we have in the industrial you know, applications we have co-current and countercurrent. What is the advantages of counter-current over co-current etcetera we will see in a minute. But more important is to draw the equilibrium curve and the operating curves. In the case of co-current, we have obtained a straight line **between minus** between P and Q the two ends as slope of two flow rates R_s over E_s . So, let us see what the equation which we get here in case of counter-current.

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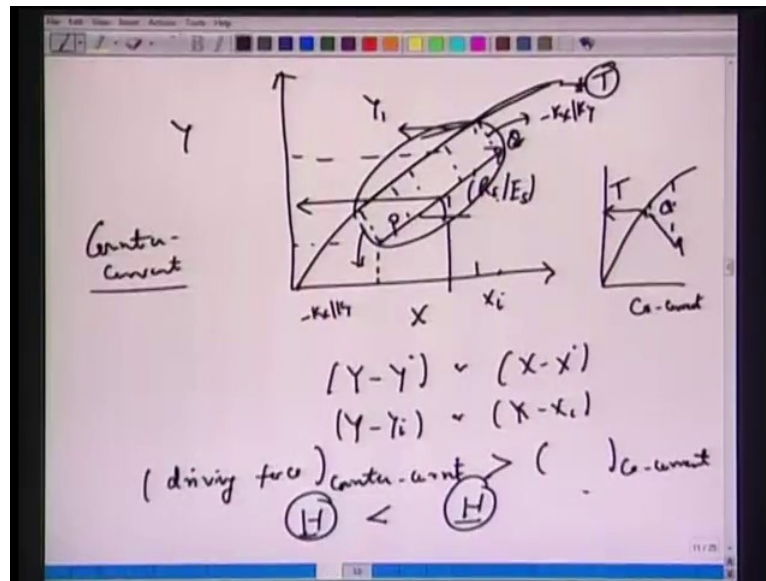
So, we have counter-current contactor. So, counter-current contactor and now let us assume that this phase E. So, we have E 1 here and the mole fraction y_1 . So, 1 and 2, and now we bring the R phase in the opposite direction from the other end. So, we have R 2 and x_2 . So, 2 mean the second end. So, this stream enters here and leaves here. So, this now becomes R 1 and x_1 and we have this stream which is now as E 2 and y_2 . So, we have essentially two phases here, and we can assume they can be a binary mixtures of A and B, and only A diffuses say from R phase to E phase. Now as x_2 as this stream enters from this direction to this here, this mole fraction will be larger. If you take any intermediate sections where the concentration mole fraction here is small x , and here it is y then we have x_2 **x 2** greater than x and greater than x_1 . And here y_2 will become greater than y_2 will become greater than y , greater than y_1 .

As we have enrichment of this stream A. So, y_2 will be greater than y , y greater than y_1 . Here we have x_2 greater than x_1 **excuse me**. So, now, if there is a mole transfer here **sorry** I think this is not correct now let us correct this. So, we have mass transfer of A from x from this R phase to E 1. So, x_2 ; so, this would be x_1 greater than x and this is greater than x_2 . So, we have mass transfer from here to here.

Now, we can also make material balance the way we did it earlier. We can write similar to this E 1 and R 1, we can define R_s and E_s . We can also define capital Y and capital X. So, if you work on these solute free base quantities. So, we have capital X 1 capital Y 1, we have capital X 2 and capital Y 2 since E_s will be constant here, we have E_s and we have R_s here. We can make the material balance the way we did earlier to obtain E_s as y_1 capital Y 1 minus y . So, we have capital X here, capital Y here equal to $R_s x_1$ minus x and now we have the operating curve as $y R_s$ over $E_s x$ plus $E_s y_1$ minus $R_s x_1$ over E_s . We can box this line this equation. Notice the difference we have now positive slope, which means if we now draw the operating curve for counter-current system starting from **P** the P and Q the two ends 1 and 2.

Now, we will get this operating curve which would be positive. So, this is different from what we had in case of co-current flow where we have the equilibrium curve which will be the same as before, and we had the operating curve which has a negative slope minus R_s over E_s .

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So, in other words, if I will draw it here, capital Y capital X we have the same equilibrium curve, now if this is point P equivalent to y_1 and x_1 then we have the slope of the curve which is positive and we have P Q. So, this is a positive slope here with the slope R_s over E_s . Here also we should be able to mark all the points here operating curves at any location between this given X we have given Y **right**. We can also draw we want to find out the driving force for mass transfer, then we can draw this line with the slope of minus k_x over k_y with the slope of minus k_x over k_y at any location between P and Q to obtain this interfacial concentrations at y_i x_i etcetera. More important here to note here is that unlike in the previous case where we had indicative slope curve between P and Q, and if we extend it to obtain equilibrium as some are location T, here we do not have that exact situation except that whenever depending upon the type of this equilibrium curve wherever we have where this line if extends and touches this line we will have 0 driving force at that locations.

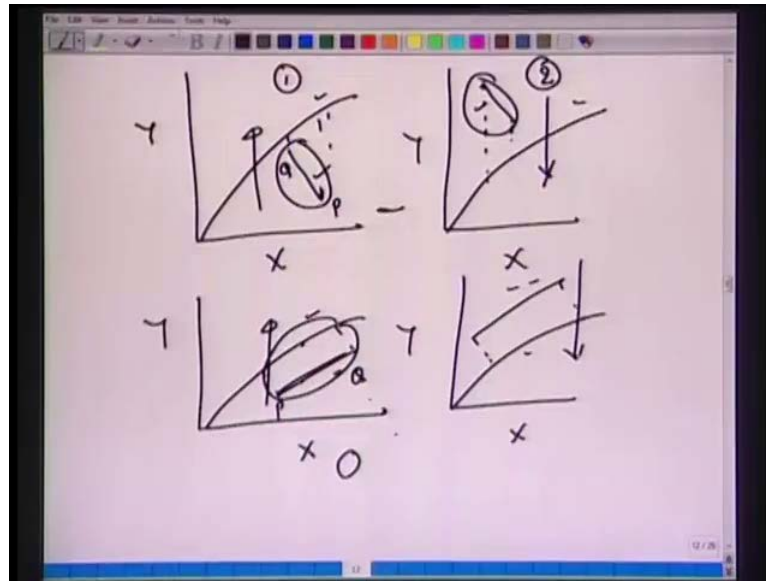
So, this point T will be equivalent to this T what we obtain in case of this co-current and the second important point to note here is that - if we draw if we find all the driving force now between P and Q at any location we draw this minus k_x minus k_y , and find this driving force of mass transfer as y minus y^* or x minus x^* or y minus y_i based on the interfacial concentrations. So, individual mass transfer coefficient for x minus x_i etcetera. It is easy to show that a driving force for mass transfer here. In case of counter-

current is much larger in general this will be greater than what we have in case of co-current.

So, very important conclusions we have of course, we marked all the points here P Q with driving force, operating curve P Q, the driving force minus $k_x x$ minus $k_y y$ in equilibrium curve more important to note here is that the driving force for mass transfer. In case of counter-current is in general is greater than driving force for co-current. And this has a huge significant implications the height of the column in this case will be smaller and the height of the column in the co-current or the size of the equipment or the co-current will be smaller than the size of this co-current.

Let us sum up what we have done so far. We have the equilibrium curve which is given by the thermodynamics, then we make material balance to obtain the operating curve. So, if it is a co-current we have negative slope, if we have counter current we have the positive slopes. And the operating curves could be below the equilibrium curve or it could be above the equilibrium curve depending upon we have mass transfer R phase to E phase or E phase to R phase. So, we should be able to draw the operating curves and equilibrium curves for all scenarios whether we have diffusion from A for one stream to this stream R to E or E to R co-current to countercurrent and we should also be able to draw the driving force for mass transfer in terms $y - y_i$ or $y - y^*$ or $x - x_i$ or $x - x^*$. So, these are the four important curves which we have learned in this course on this lecture how to draw it.

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Let us quickly draw those curves qualitatively again. So, we can draw like this Y X one then we have Y X same equilibrium curve like this. So, this is 1 this is 2. So, these are the two co-currents contactors. We can also have Y X like this, and we can have Y X with the same equilibrium curve and operating curve given like this. So, you should be able to make out that in this case in this case we have co-current, but the mass transfer is from X to Y phase where in this case this is from your Y to X phase.

So, similarly we have this certain illustrations and this we have drawn operating curve, equilibrium curve, you have drawn equilibrium curves here, operating curves **operating curves**, and we should also be able to mark all the slopes with minus k_x minus k_y to obtain the interfacial concentrations at any locations between interfacial between in the contactors the co-current or the countercurrent. In general as we said here that driving force here you can see that the difference between this and this is much larger average driving force from P Q is much larger here when in case of this co-current; which means if the driving force is large here the size of the equipment will be much smaller in case of counter current than in case of this co-current.

Now, before we take up the next lecture stage. So, we said that. So, far we had two cases one was co-current and counter-current for both for continuous contactors. So, these are like absorption columns. Similarly, we can have stages cascades just like in a distillation column. So, there also we have the same equilibrium curves or similar equilibrium

curves given by thermodynamics, and one has to make a material balance from stages between the stages; one stage, second stage, third stage these are now discrete unlike the previous case we have seen it was a continuous. So, if we have a discrete, here also we can make a material balance we can draw the operating line, operating curves and we can mark all the salient points on the curve in terms of interfacial concentrations or the bulk concentration bulk phase concentrations or equivalent y^* or x^* we discussed earlier.

So, with that **that** would be the introduction to this expression to mass transfer, before we will take up some few examples, and then we will switch or we will take up one unit operations say in term absorptions or distillation or extension etcetera. Thank you.