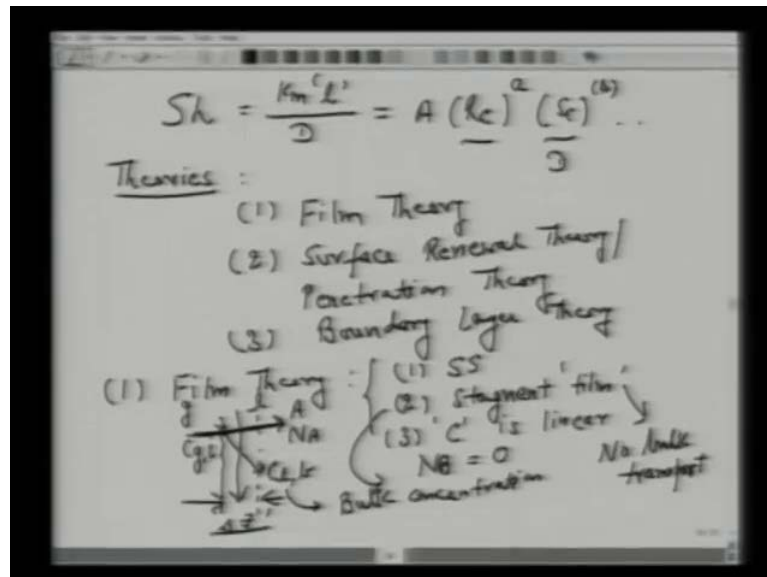


**Mass Transfer II**  
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**Lecture No. # 05**

So, essentially what we have a correlation for Sherwood number, which contains mass transfer coefficient in terms of operating conditions, like Reynolds number and Schmidt number. And, of course, this correlation as been said earlier, has to be determined experimentally.

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So, we can write Sherwood number as  $K_m$ , some characteristics length, divide by the diffusivity equals  $A$  Reynolds number to the power some coefficients small a Schmidt number the power  $b$ , may be some there are other some parameters here. So, what we have to do is to calculate Reynolds number and Schmidt number under the operating conditions. And then, the estimate, calculate, Sherwood number, and from the Sherwood number we can find mass transfer coefficients. But these are the correlations; still we also said that there are some theories, and these theories are used to find out how does in mass transfer coefficient may depend upon **same** diffusion coefficient etcetera.

So, there are three theories, let's note down these three theories: one is **one is** very popular is called Film Theory; second is Surface Renewal Theory, very similar or extended form of this is known as Penetration Theory; and third is Boundary Layer Theory. So, what we will do now, we will go through these 1, 2, 3 theories and we will stay away from, you know, regress mathematical representation or mathematical derivation, instead of that we will see what are the main postulates or the fundamental postulates of these theories.

So, the most common theory here is Film Theory. And the main postulates of this theory is that, we assume that there is a steady state one, we have stagnant film. So, we will talk of this stagnant film which is kind of hypothetical film, and we assume that concentration here in this film is linear. So, **all it** all this theory says that if we have a interface, say we have gas and liquid, then there is a hypothetical film which is stagnant. So, when we say stagnant we are saying that  $N$ , those component was diffuses in this directions  $A$  or we have the bulk flow which is 0,  $N_B$  equal to 0 here and the concentration gradient, so, say starting from  $C_g$  to this interface is linear. So, we have  $C_g$  and **sorry** this is interface  $C_g$  here, we can write  $C_{g,i}$  and we have this  $C_l$  which is bulk concentrations. So, we are introducing this terminology that there is a bulk concentration of diffusing species **alright**.

So, what this theory says let us see physically that if you have interface of gas and liquid and some species diffuses, a sulphur dioxide from air into water, then we have a hypothetical film **alright**. This film is a stagnant; there can be a flow parallel in this directions - that is possible, but the direction of this diffusion flux  $N_A$  there is no bulk transport **alright**. So, stagnant film means no bulk transport, **no bulk transport**.

So, the first main postulate here is that there is a steady state; all it means the concentration gradients starting from  $C_{g,i}$  or  $C_{g,i}$  to this  $C_l$ ,  $b$  is linear, it does not change with time and there is a hypothetical film which is a stagnant in which we can assume there is a diffusion resistance over this, there is a diffusion length  $\delta$  over which concentration drops linearly from  $C_{g,i}$  to  $C_l$ .

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The image shows handwritten notes on a whiteboard. At the top, the equation  $N_A = K_m (\Delta C)$  is written, with a definition symbol  $\therefore \text{def}$  to its right. Below this, a downward arrow points to  $(C_{g,i} - C_{l,b})$ . To the left, the equation  $(K_m = \frac{D_{AB}}{\delta})$  is written. Below this, a diagram shows a horizontal line representing a film of thickness  $\delta$ . The left side of the film is labeled  $C_{g,i}$  and the right side is labeled  $C_{l,b}$ . A double-headed arrow below the film indicates its thickness  $\delta$ . The text "diffusion resistance" is written to the right of the diagram.

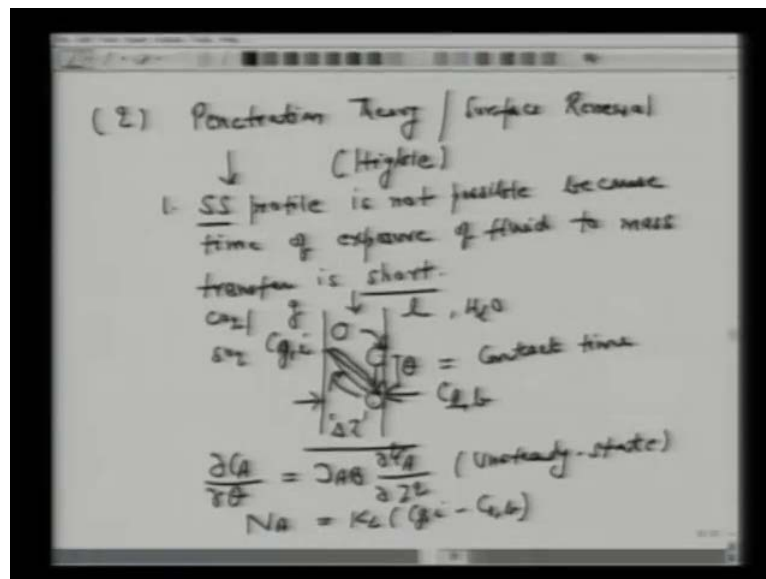
As a consequence, we can write that diffusion flux  $N_A$  still equals to mass transfer coefficients  $K_m$  into  $\Delta C$ . So, this  $\Delta C$  is nothing but  $C_{g,i}$  minus  $C_{l,b}$ . So, if you go back to the previous slide here concentration drops from  $C_{g,i}$  to  $C_{l,b}$  which means now there is no gradient **in** within the bulk of the fluid. So, whatever there is a resistance whatever is a drop in the concentrations it is within this hypothetical film.

So, we are still writing  $N_A$  equals  $K_m$  into  $\Delta C$  as a definition for mass transfer coefficients, except one can show that this  $K_m$  equals  $\frac{D_{AB}}{\delta}$ , but at least this is approximated as  $K$  - mass transfer coefficient as diffusion coefficient of  $AB$  over this film thickness or diffusion resistance.

So, we have a steady state concentration profile, one **which** does not change with time; it is a linear and there is a hypothetical film of diffusion resistance in which we can say that all concentration drops that concentration drop occurs over this film of  $\delta$  or which is equal to here use as a notation for this in this expressions we have. So, there is no concentration gradient here, and we have  $C_{g,i}$  and we have this bulk concentration which is  $C_{l,b}$ . So, this all we have this film theory. So, very, **very** common, very popular film theory and it is widely use it has only its limitations and we said earlier that there is no bulk transport in this. So, if they could be flow in this direction. That is permissible. So, we can have a flow in this direction, but **the** in the direction of diffusion flux what we writing here  $N_A$ , there is no bulk transport, there is no convection etcetera.

The second most theory is Penetration Theory or we will see that - it is a Surface Renewal Theory, which is also quite popular, and here the main fundamental postulate is that concentration profile within that film or within the system is not a steady, it is unsteady state. And the main fundamental behind is **this** that - this postulate is that or the mechanism is that the contact time between the two phases gas and liquid is very short. In other words, the contact time is so short that there is no time for a concentration to reach a steady state.

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So, if we get **get** into this physical aspect of this. So, let us start with this 2 which we can call it Penetration Theory and we will see that this Penetration Theory or a Surface Renewal Theory is quite similar there are some restriction here I will discuss this later. This was originally proposed by Higbie. So, all Penetration Theory says first steady state profile - concentration profile is not possible, because time of exposure of fluid **time of exposure of fluid** to mass transfer **to mass transfer** is short. So, look at the difference here that we do not have steady state unlike in the film theory, and why because time of contact is very short. So, if you back to the same situations what we had earlier here that we have gas and there is a liquid here.

So, still we have a diffusion resistance embedded all over this diffusion length of  $\Delta z$ , **now there is** a and we have a bulk concentrations which we can still use the same nomenclature as C l, b, and this side if we have the concentrations of C g, i is the liquid

phase and this is your gas phase. So, as per the penetration theories there are eddies. So, you have some idea of these eddies from hydrodynamics turbulent very high Reynolds number, we have these **these** eddies which move within the bulk of the fluid. So, they keep on moving within the fluids and penetration theory says that these eddies they **come in** come to the surface they spend some time here and then they go back to the bulk. So, this is the time we are talking - the contact time which is very short. And now, this time is so short that concentration profiles which develops between  $C_{j,i}$  and  $C_{a,l}$  it does not remain a steady or linear and it changes with times.

So, these are main postulate which could be very realistic. Suppose we have carbon dioxide or sulphur dioxide and we have this water here, there is a flow, we have high Reynolds number, so that there is a turbulence and we have this these eddies. So, these eddies continuously they move within the fluid as for this penetration theories the eddies come here, they spend some time, they go back to this bulk concentrations to this  $C_{g,i}$  concentration whatever we have in this gas phase.

So, without getting into too much of this mathematical derivations, one can show that one can write an expressions for unsteady state concentration profile which can be written as  $D_{AB} \frac{\partial^2 C_A}{\partial Z^2}$ . So, the main idea is that this is unsteady state. And with some initial conditions and boundary conditions one can still can come out with the expressions for. So, now if we have gas here and the situations over one can still can come out with the expressions for  $N_A$  in terms of mass transfer coefficients and  $C_{j,i}$  minus  $C_{l,b}$ .

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Handwritten notes on a whiteboard showing the derivation of the mass transfer coefficient  $K_L$ . The notes include the equation  $K_L = \sqrt{\frac{D_{AB}}{\pi \theta}}$ , the relationship  $K_L \propto D_{AB}^{1/2}$ , the Sherwood number equation  $Sh = A (Re)^a (Sc)^b$ , and the Surface Renewal Theory equation  $Sh = \frac{D_{AB}}{L} S$ . The notes also mention that  $S$  is a fractional value representing the replacement of elements.

But one can show that this  $K_L$  turns out to be that square root of  $D_{AB}$  over  $\pi \theta$ . So, what we obtain here that mass transfer coefficient is actually square root of diffusion coefficients. So, these are fundamental difference between the Penetration Theory and **and** Film Theory. In the Film Theory you saw that mass transfer coefficients is proportional to diffusivity. Here we are obtaining a relations where it says that mass transfer coefficient is a square root of diffusivity.

Now, when we talked of this correlations of Sherwood number as  $A$  Reynolds number to the power  $a$  and Schmidt number to the power  $b$  it is important to point out that is  $\nu$  over  $D_{AB}$ . So, the theory says that half of proportional to this one can make and estimate for this  $b$  here, and then can solve only for  $A$  and  $a$ . So that expression for Sherwood number is simplified in terms of  $A$  and  $a$ . So, these are the theories one makes use of you know in obtaining a correct relations or **surface** certain correlation between Sherwood number and Reynolds number and Schmidt number.

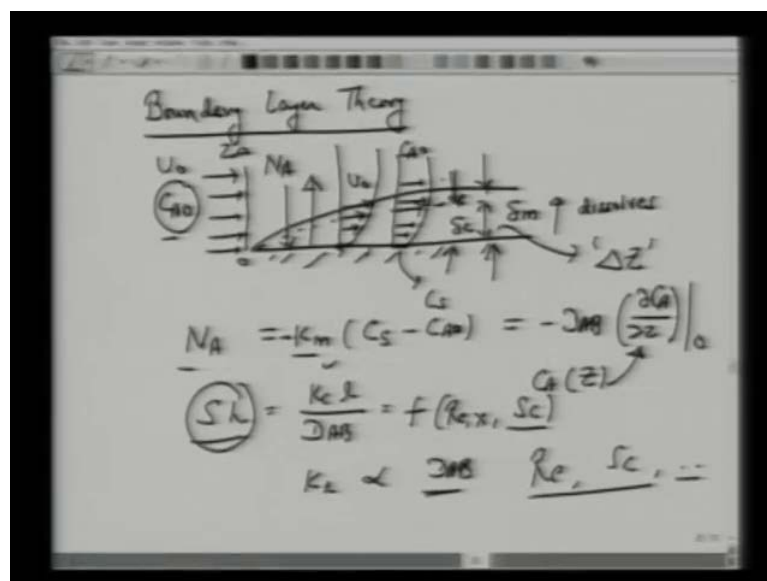
The third and extension of this we said that Penetration Theory there is another theory which is Surface Renewal Theory. Surface Renewal Theory is at bit more realistic than penetration theory although the main postulate is here is the same; **all this theory says** this theory was actually purposed by Dankwerts. And here it says that in the previous penetration theories contact time was short of course, but it was uniform constant. Here

there is a distribution of this contact time. All it means certain eddies is spend theta 1, certain eddies spend theta 2 times, certain eddies is spend **theta 3** theta 3 times.

So, but it still one can show that here also mass transfer coefficient as turns out to be a square root of diffusivity in half. Of course, may be go into further details, then there is a parameters which is S which is called fractional rate of replacement. So, there is a rate at which the surface is renewed, because of these eddies. So, one more parameter comes into this correlation, but **at the** our idea is to show that here also mass transfer coefficient is a proportional to a square root of this diffusivity.

Now, let us come to the third theory which is Boundary Layer Theory. And I am sure in the hydrodynamics and momentum balance you have heard of this over a flat surface, if its approach by a fluid, then there is a thin boundary layer in which we can say that if resistance to momentum diffusion all resistance to momentum diffusion is embedded or is confined to this within this boundary layer. Similar to momentum boundary layer we also have mass boundary layer; we also have thermal boundary layer. So, again we stay away from this mathematical **mathematical** derivation one again starts from Navier stokes equations in laminar flow. We can write down the energy balance, we can write down species balance and we can calculate the flux as long as flusy profile is known which is true in case of laminar flow.

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So, let us get back let us start this. So, Boundary Layer Theory we have a solid surface and if this approached by fluid say it constant velocity  $u = U_0$  and may be it has a constant some concentrations  $C = C_A0$ . So, we re-call **from momentum** from your boundary layer theories there is a thin boundary layer. Momentum boundary layer in which the velocity profile changes from no slip 0, velocity to this bulk velocity which is  $U_0$ .

So, all you have we are saying that the resistance to momentum diffusion is within this  $\delta_m$ , and it varies from 0 to this  $U_0$  velocity. So, similar to this boundary layer - momentum boundary layer there is also a mass boundary layer which could be thinner or could be as thick as this momentum modulate. So, without we do not get in much of this detail, so, suppose this is your mass boundary layer which is now  $\delta_c$ , now we are saying that say the velocity profile is still say lamina. So, still we have a same starting from 0 to  $U_0$  there will be concentrations gradient here also. Now that will vary **form** from some surface concentration  $C_s$  to this bulk concentration, which is  $C_A0$ .

So, again resistance to diffusion is embedded within this  $\delta_c$ . This is your not diffusion length  $\delta_z$ . And if we are ask to calculate this flux  $N_A$  either to or from this; so, we can have this plate which have some concentration may be this plate **is it is there is it** is a plate dissolves into this bulk of the fluid which has in a far stream concentration at  $C_A0$ . So, surface concentration changes with  $C_s$  **C s** and within the bulk or within this  $\delta_c$ , it changes from  $C_s$  to this  $C_s$ . One can again write down this species balance quite vigorously if you have this laminar flow if you know the velocity profile, we can do the integration to calculate this analytically diffusion flux. If you have the turbulent of course, we can make an analogy and we can come out the some correlation there also. So, if one can show here also that it can be shown that this  $N_A$  will be again equivalent to some mass transfer coefficients into this concentration drop which is from say  $C_s$  minus  $C_A0$  or we can put minus here, because  $C_A0$  is a larger than  $C_s$ .

So, if we have the diffusion flux like this  $N_A$  equal to minus  $K_m$  which can be equated to minus  $D_{AB} \frac{dC_A}{dz}$  at  $Z$  equal to 0. So, we have this  $Z$  directions and say 0. So, analytically one can solve for  $C_A$  as a function of  $z$  and then we can once we get a concentration profile, we can integrate this, we can calculate this diffusion flux equate this  $K_m$  to  $C_s$  minus  $C_A0$  to show to obtain another correlation for Sherwood number as  $K_c l D_{AB}$  and this would be some correlation as a function of Reynolds number - local Reynolds number and Schmidt number etcetera. So, idea is that all these



theories are used to get an expression for Sherwood number in terms how does mass transfer coefficient varies with this diffusion coefficients.

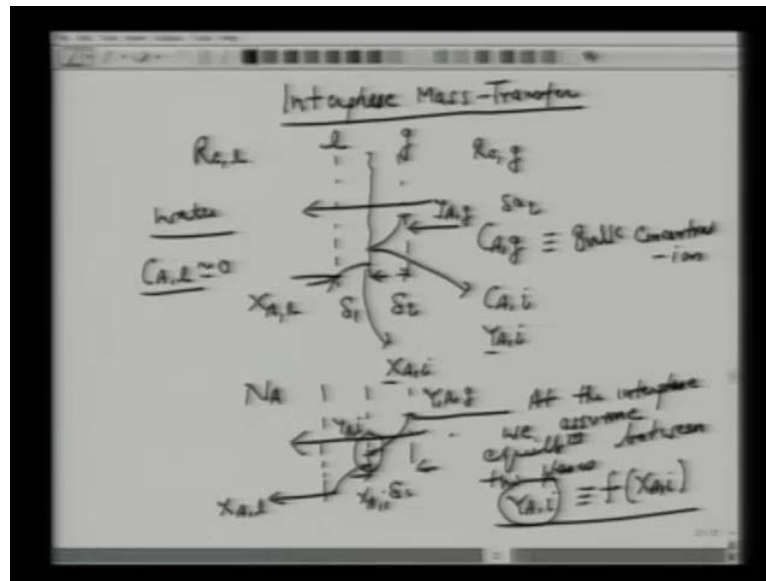
So, these are the theories more than they are some theories you know, beyond this three theories which we discussed Film theory, Penetration theory or Surface Renewal theory and Boundary Layer theory. But these are the most common theories which are going to be applied in our mass transfer to etcetera.

So, idea is that again you know whenever you are solving a real problems in mass transfer, whether it is a flow in a packed bed or a mass transfer over a flat plate or you have the interface of gas and liquid, you are supposed to go through the table of, you know, various hand book or the tray ball and get the correct correlations, calculate Reynolds number, calculate the Schmidt number or any other restriction which has given to **to** ensure that you are using correct expressions for Sherwood number to calculate this mass transfer coefficients **alright**.

So, this is the end of your introduction to your diffusion and mass transfer, we take up another topic which is also a very fundamental of your calculation or you design accepts.

Now, so far what we discussed was one film. So, we have gas and the liquid. In reality we have flow of the gas; we have the flow of the liquid. So, we can think of an interface of two phases could be gas-liquid, could be liquid-liquid, could be liquid and solid. And we can have a resistance in both the phase; essentially we are trying to combine the two resistances. And so, this topic is a new topic here, we start we call it Interphase Mass-Transfer; so, we have Interphase Mass-Transfer. So, this will be an extension of what we did in the earlier we had we talked about this hypothetical film.

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So, now if you say that we have two phases liquid and gas, both have certain Reynolds number **in a** for the liquid and we have Reynolds number for the gas. There are again we can have are two resistances - diffusion resistance; one in the liquid phase, one in the gas phase. So, we can have  $\delta_1$  and we can have  $\delta_2$ . So, two diffusion resistances, suppose this is your sulphur dioxide and this side we have water and this sulphur dioxide diffuses from across this **interphase**.

So, interphase mass-transfer again we will have a bulk concentrations. So, this concept of bulk concentration was defined earlier, we have  $C_B$  or we can say diffusion species is  $A$  in gas phase. So, this is your bulk concentration of sulphur dioxide in air. So, may be this is 1 percent or 0.1 percent. So, bulk concentrations and now we have brought in contact with water and water may be here, we have  $C_A$  in the liquid phase bulk concentration which is quite small or could be some numbers which is given. So, when we say interphase mass-transfer diffusion or we are trying to say that now the concentrations now that will drop from here from  $C_A$ ,  $g$  to the interphase concentrations which we can say it is  $C_A$ ,  $i$ .

So, concentration are drops from here  $C_A$ ,  $g$  to  $C_A$ , **c**  $i$ . Or for the gas phase we can say that is drops from  $Y_A$ ,  $g$  to  $Y_A$ ,  $i$ . So, now the concentration drops at the interphase now within the liquid side they will also be a concentration gradients. So, concentration will also drop from some interphase **interphase** concentrations, we can **denote as** denote

as  $X_A$ , i. So,  $X$  for liquid phase,  $Y$  for gas phase. And then, we have again bulk concentration at equivalent to this  $C_A$ , l as  $X_A$ , l.

So, now we are taking of four concentrations one in the bulk phase. So, that is the concept of we have concept of a resistance; all resistance is confined to this  $\Delta l$ . So, concentration drops; let us not worry about whether it is a linear or it is there is an some profile, because ultimately you are going to work on this concentration drop from here to here. Film theory of course, we will say that this concentration within the film is linear.

So, now we have four labels, one is bulk  $Y_A$ , g,  $Y_A$ , i and now we have liquid phase concentrations which is now  $X_A$ , i and then it drops to another resistance here; before we have this bulk concentration at  $X$  of  $A$  in the liquid phase. So, now we have four concentrations, and interphase mass-transfer says that at the interphase. At the interphase we assume equilibrium **equilibrium** between two phases which means  $Y_A$ , i the gas phase concentrations at the interphase is an equilibrium by some correlation by some relations with this liquid phase in the mole fraction at the interphase. So, this what we are assuming there is an equilibrium at the interphase.

However, there is a transport there is a diffusion at  $N_A$ . So, sulphur dioxide diffuses from gas side to the liquid side, because of this concentration gradient in the gas phase and within the liquid phase. And the interphase however, there is an equilibrium  $Y_A$ , i as function of some  $X_A$ , i. So, before we go for the introduce mathematics of this, let us try to understand that we have defined two concentration levels; one we have called it bulk. So, when we say bulk we are assuming that there is a inform mixings. So, that we have one concentration level either in the gas phase or in the liquid phase. Sulphur dioxide is 1 percent 10000 p p m. So, it is constant at 10000 p p m.

Now, we have water - pure water. So, as the water trickles down, of course, to start it with it is 0 concentration, but as it goes along this will get contaminated, this sulphur dioxide which diffuses from one direction to another direction. However, whatever concentration we have at any level of say this absorption column it is a constant. So, we have two bulk concentrations; one in the gas phase, one in the liquid phase. And in between we have an interphase of gas and liquid - interphase mass-transfer says that now they are two diffusion resistances; one in the gas phase, one in the liquid phase,  $\Delta l_g$   $\Delta l_l$

1, del 1 del 2; concentration since it is a diffusion resistance, all resistance will drop over this resistance.

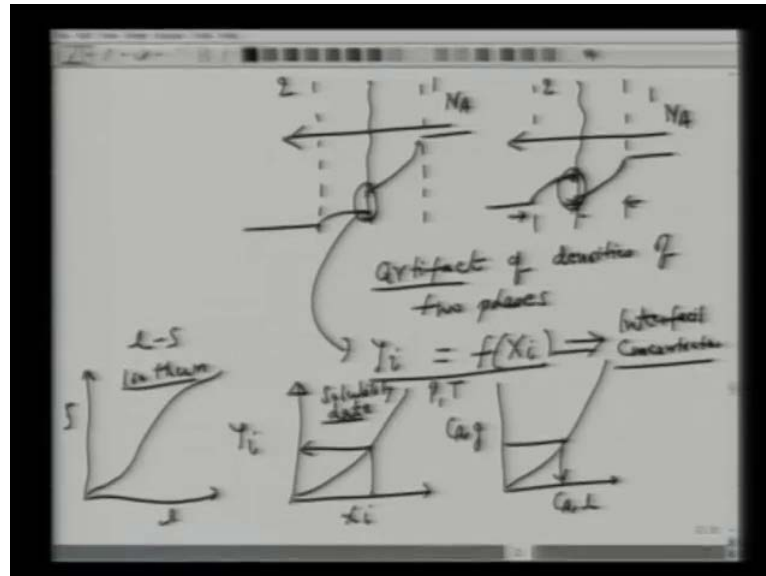
So, now concentration in the gas phase drops from  $C_{A, \text{bulk}}$  to  $C_{A, i}$  or mole fractions they convert into from concentration to mole fractions it is  $y_{A, \text{bulk}}$  to  $y_{A, i}$ ;  $i$  is the rotation for diffusion for the diffusing species which is sulphur dioxide here. So, we have  $y_{A, g}$ ,  $y_{A, i}$  and  $N$  interphase we have another concentration in the liquid phase  $X_{A, i}$  and then it drops from  $X_{A, i}$  to  $X_{A, \text{bulk}}$ . So, 1, 2 - two bulk concentration and two interface concentration while there is a diffusion, because there is a concentration gradient. So, we can apply diffusion Fick's first law; there will be diffusion flux proportional to a mass transfer coefficient here whatever we have in the gas phase multiplied by the concentration drop  $C_{A, g}$  minus  $C_{A, i}$ . And this flux will also equal to what we get in the liquid phase if there is a steady state.

So, what we get in the liquid phase is another mass transfer coefficient multiplied by the concentration drops or mole fraction drop within the liquid. So, there is a diffusion because of gradient within the two films; all resistance are diffusion or embedded within these two diffusion. What happens at the interface? Interface we have thermodynamic equilibrium. A concentration of gas phase at interface of gas and liquid is in equilibrium with this  $X_{A, i}$  concentration. So, there is some law there - equilibrium law say solubility, if we have ammonia, we have sulphur dioxide in atmosphere, we fix pressure and temperature then given pressure - partial pressure of sulphur dioxide in atmosphere; liquid, if we take water will have a maximum or a **prescribe** prescribed solubility given by thermodynamics. If we take liquid-liquid say liquid-liquid extraction, which we did earlier where we have kerosene or we had nicotine in water and we want to extract from by using this solvent kerosene. So, there also there is a there will be an interface and at which the interface concentration will be decided by the solubility of nicotine in kerosene or nicotine in water phase. So, one has to do this batch experiment equilibrium is steady to obtain this some kind of equilibrium relationship thermodynamics. If we have the solid and gas absorptions in that case also we have the some kind of equilibrium we call it adsorption isotherms.

So, first we establish our strategy is that first we establish these equilibrium concentrations. We do the equilibrium steady separately experimentally or may be is in documented in the literature that given so much of phase concentration in one phase

more fractions or **or** this concentration or the partial pressure what could be the equivalent concentration at the interface under equilibrium. Then we apply this diffusion matrix. So, now let us go back and get into this mathematics.

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So, we said that we have this interface, this draw it one more time, we have diffusion resistance, all resistance is embedded here, this could be like this, this could be like this. In some cases possible that we have concentration like this **this** and then maybe it is higher at the interface like this. So, phase 1 phase 2, this phase 1 phase 2; **these are** these two are the equilibrium this interfacial concentrations between the two phases here.

Now, whether it is larger or smaller or smaller larger it is just not a fact of density. So, let us not worry about artifact of density. Densities of two phases it is possible. So, **all it** still we have see in this case diffusion from right to the left, N A diffuses like this here also it diffuses from right to the left. So, it this does not mean that we have a diffusion from lower concentration to higher concentration. If you more important is that what happens within this film, here and here. There is a concentration drops from here to here and from here to. At the inter phase, we have equilibrium; at the inter phase we have equilibrium; still there is a diffusion, because of this gradient from here to here and from here to here. Whether it this one is higher than this, whether phase 1 has higher concentration at the lower or lower than higher or here **in the oppose** in just reverse direction - reverse way. It is all the artifact of density of phases; this could be denser phase, this could be lighter

phase, this could be lighter phase or this could be denser phase. Does not important or we are saying that inter phase concentrations  $y_i$  has there is a thermodynamics which says that the two are in equilibrium concentrations.

So, one has done this batch experiment or some we put a data where knowing  $x_i$ , we can calculate  $y_i$  or knowing  $C_{A,l}$ , we can calculate  $C_{A,g}$  in the gas phase. Or in case of say liquid-solid knowing how much is the concentrations in this liquid phase. We can calculate how much is concentrations in the solid phase. So, this is called isothermal. In case of gas-liquid, this is nothing but a solubility data. So, if we fix say pressure and temperature - total pressure and temperature and one can vary this concentration in the liquid phase calculate gas phase or given gas phase you can calculate this liquid phase concentration.

So, these are all used **to** to determine this interfacial concentration, which we say that interfacial concentrations which we say that they are the two species are at equilibrium at the interface. Although there is a concentration gradient within the bulk in this film diffusion flux. So, now if you understand this, now we can obtain in expressions assuming a steady state. So, if it is steady state whatever diffusion moles per second per meter square we have in the gas phase, we will also be same as moles per second per meter square in this liquid phase.

So, **we know** already we know the expressions for  $N_A$  in terms of mass transfer coefficients and the concentration term. So, all do we equate.

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Handwritten notes on a whiteboard showing equations for mass transfer flux and relationships between coefficients and diffusion parameters:

$$N_A \text{ (moles/l-m}^2\text{)} = K_g (Y_{A,g} - Y_{A,i}) \quad (1)$$

SS

$$= K_L (X_{A,i} - X_{A,l}) \quad (2)$$

$$(K_g, K_L) \propto \frac{D_{AB}(z+l)}{\Delta z' (\delta_g + \delta_l)} \quad (3)$$

$\propto \frac{D_{AB}^{1/2}}{\Delta z'}$

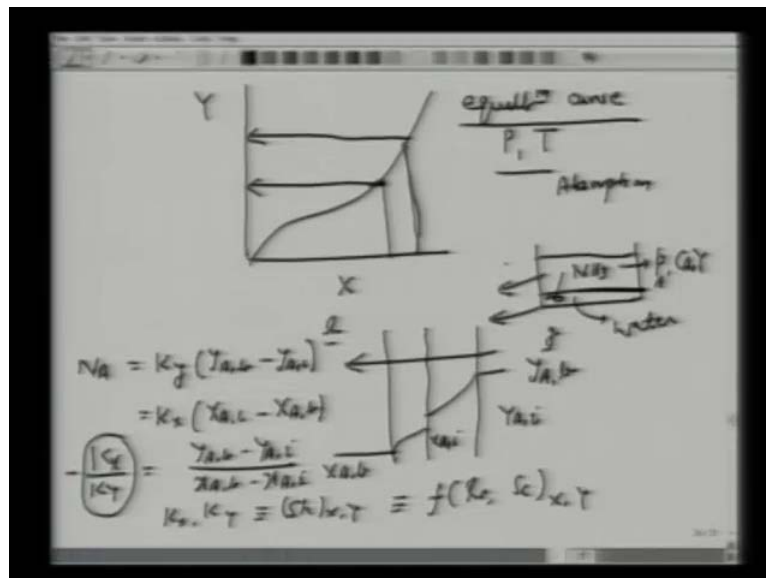
$Y_{A,i} = f(X_{A,i})$   
 Solubility data  
 Isotherm (Adsorption)

So, we can write  $N_A$  moles per second per meter square equals  $K_g$ . So, if  $K_g$  is mass transfer coefficient in the gas phase. This will equal to if over we wrote earlier we have mole fractions of A in the bulk phase; so,  $Y_{A,g}$  minus  $Y_{A,i}$ . So, this is your  $N_A$  under steady state this diffusion flux we will also be same as  $K_L$   $X_{A,i}$  minus  $X_{A,l}$  in the bulk liquid phase, here we can say if a reason those previous nomenclatures. This is nothing but  $Y_{A,g}$  and  $Y_{A,i}$  and  $X_{A,i}$  they are in equilibriums. This is given by some equilibrium data, which we have obtained from some experiment or from some reported values. This is nothing but solubility in some case if you talking of absorptions ammonia and water, is nothing but solubility data.

You are talking of adsorption of sulphur dioxide in on some solids like charcoals, then we will call it isotherms or to be more pesticides adsorption isotherms **adsorption isotherms**. So, all we are saying that under steady state, they are two diffusion resistance given by  $K_g$  and  $K_L$ . So, we re-call  $K_g$ ,  $K_L$  both will be proportional to diffusivity, diffusion coefficients either in the gas phase or in the liquid phase where in the gas phase or in the liquid phase over this film diffusion resistance **distance**  $\Delta z$  which could be  $\delta_g$  or  $\delta_l$ . So, we have **two diffusion** two mass transfer coefficients proportional to diffusion coefficients, if we applies certain theories if it is a film theory it is a proportional to  $D_{AB}$  if it is surface renewal theory and this  $D_{AB}$  to the power half over this  $\Delta z$  which is either  $\delta_g$  or  $\delta_l$

So, by this interface mass-transfer, let us summarize here all we are trying to say that interphase mass-transfer it says that interphase concentration that is an equilibrium. And under steady state two diffusion flux are the same and they are proportional to mass transfer coefficients in the gas phase multiplied by the concentration drop in the gas phase will also be equal to mass transfer coefficient the liquid phase multiplied by concentration drop in the liquid phase that is it. We have two correlations for **1** two equations, one for gas phase and one for the liquid phase. Interfacial concentration  $Y_i$  and  $X_i$  they are related and we have one more equation. So, we can say one equation 1 equation 2 and we have equation 3. These three equations will decide diffusion flux and we will take up this graphical representation of this.

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So, let us a make some graphically representation here and also do some calculations. So, first is we have a plot for  $Y$  verses  $X$ . So, this is let say it is our equilibrium curve **equilibrium curve**. And let us not worry whether it is a, we are discussing absorption or adsorptions or liquid-liquid extractions. We have in given this equilibrium curve at one **pressure** constant pressure and constant temperature.

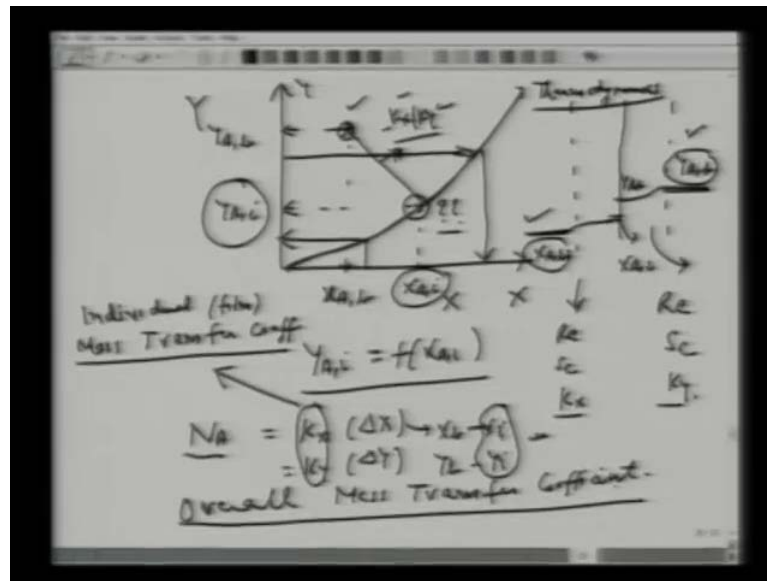
So, equilibrium curve says that given  $X$  this would be  $Y$  in one phase; suppose we have a liquid then this will be the concentration in the gas phase under equilibriums. So, all it means you want to obtain this curve for absorptions, you will be doing a very simple experiment will bring water. Water is a **absorption** absorbent and **we have** we will have



this ammonia or sulphur dioxide gas at certain partial pressure or certain concentrations or certain mole fractions. And they will be brought in contact for long time with water we can put some stir here. So, after a long time we have to measure what is a concentration here, and in gas phase and in the liquid phase or mole fraction in the gas phase or in the liquid phase and obtain this data.

Next one can again vary this partial pressure  $P_a$  and can again bring in contact with the fresh water wait for the equilibriums till we have another points. So, we have obtain all these data points for equilibrium curves. In case for the solid and liquid more solid and gas will be doing the same state. Idea is that we have obtain this equilibrium curve. And we wrote that there is an interface, two resistances here, concentration drops, like this to this. So, let us use a nomenclatures may be this is a gas size, this is the liquid size,  $A$  is a diffusion diffusing **diffusing** species,  $Y_{A,b}$  we have bulk concentrations, we have  $Y_{A,i}$ , we have  $X_{A,i}$  and we have  $X_{A,b}$ . So, two phases here recurred gas and how do we and we write the expressions for  $N_A$  once more  $N_A$  under steady state will be  $K_y$ . So,  $y$  is for gas phase will be equal to  $Y_{A,b} - Y_{A,i} = K_x (X_{A,i} - X_{A,b})$ . So, under steady state diffusion flux across this film both the film liquid and gas the other same. Which means we can write now  $K_x$  over  $K_y$  equal to  $Y_{A,b} - Y_{A,i}$  over  $X_{A,b} - X_{A,i}$  and we have  $K_x = K_y$ . So, all we know there is the ratio of mass transfer coefficient in one phase to another phase. And  $K_x$  and  $K_y$  which are obtained from Sherwood number in gas phase or in the liquid phase and this will have we **we** will shown to be this has to be shown to be a function of Reynolds number and may be the Schmidt number for the gas phase and the water phase.

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So, now let us plot can these points on X Y plot. So, we have X Y, we have equilibrium curve here between Y and X, and this is our bulk concentrations given in gas phase and water phase. So, now we can say that we have  $x$  X A bulk phase b, and here we have Y A, b the bulk phase. And let us keep this interface in mind here we have 1 and we have this 2. So, Y A, b is here and X A, b is here. All we do realizing that mass-transfer coefficient ratios is given in terms of interfacial concentrations Y A, i and X i we can take a slope of minus K x over K y, because K x over K y has been shown in terms of Y A, b X A, b, and Y A, i and X A, i. So, knowing this four concentrations we can calculate this ratios of minus K x over K y and we can plot this. Alternatively, if we know the mass transfer coefficients then we can **I** know the bulk concentrations Y A, b and X A, b. We can mark this point, we can take a slope to reach this concentration which is nothing but X A, i and Y A, i.

So, let us try to understand this again we start with equilibrium curve, one this is given from thermodynamics. So, we have already done this experiment - batch experiments and we have obtain this solubility curve or isotherms or we know the relation between Y and X at the equilibriums. What we know is possibly the bulk concentration, because we can major sulphur dioxide is 1 percent and how much is the water concentration our sulphur dioxide. So, we know this point here.

Now, if we know the operating conditions say Reynolds number and Schmidt number **Reynolds number and Schmidt number** in the gas or liquid phase we can calculate  $K_x$  and we can calculate  $K_y$ . Once we know  $K_x$  and  $K_y$  we can take the ratios and we can plot this minus  $K_x$  and  $K_y$  to raise this interface concentration which is  $Y_{A,i}$  and  $X_{A,i}$ . Or if we know this  $X_{A,i}$  and  $Y_{A,i}$  then we can join this line, and then the slope will be given by minus  $K_x$  and  $K_y$ .

So, either way it is possible. All we have done we have represented the same statements that under steady state diffusion flux in the two phases are the same and they can be represented in terms of the four concentrations - two bulk concentrations and two interface concentration; one in the one phase and the other in the second phase.

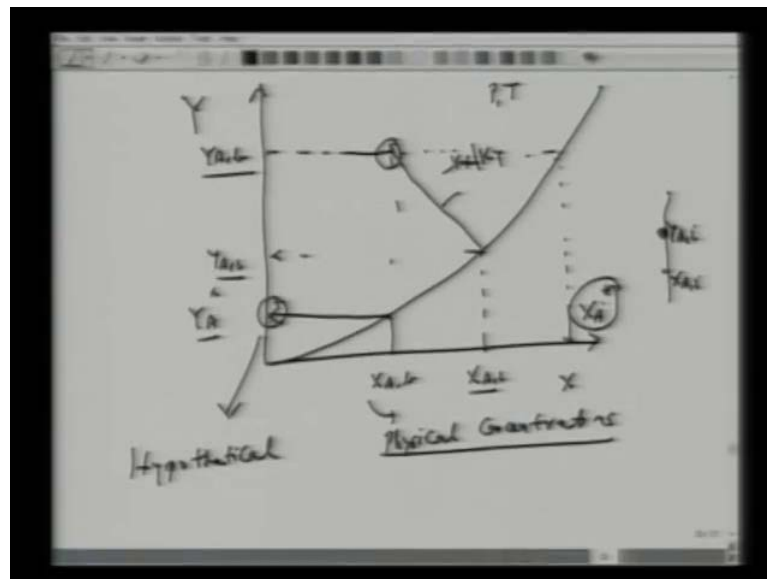
If we equate the two rates, we will get what we have shown here as a graphically; one concentration here  $Y_{A,b}$ ,  $X_{A,b}$  and  $Y_{A,i}$ ,  $X_{A,i}$ . We among the coordinates we can plot this curve. And now the problem comes here that what is the interfacial concentrations and how we can measure it. It is very difficult if you have flowing system then we have gas phase a sulphur dioxide goes from the bottom to the top and water trickles from the top to the bottom. And we have the interface **right**. If you are ask to measure the concentrations or we can do we can take the samples of gas from the bulk of the samples are liquid from the bulk, it is impossible to reach this interface concentrations, at least may be some other techniques will have to use how do we find the interfacial concentration. Because the moment we take a probe and try to reach the interface concentrations, it will spoil the concentration profile, it will spoil the hydrodynamics etcetera. So, all of you trying to say that it is a bulk concentrations which can be measured, interfacial concentration cannot be measured, one can of course, **one of course**, goes by this thermodynamics, it says that yes for given  $X_{A,i}$ ,  $Y_{A,i}$  is fixed.

If I change  $x_{A,i}$  here under equilibrium interface concentration will be so much. If I interface concentration in the gas phase is so much then in the water phase this will be so much. Or we know the relations between  $Y_{A,i}$  and  $X_{A,i}$ . Absolute values in a running in operating system it is not possible which means we have to think of some other ways how we can calculate this flux. To calculate the flux, we need  $K_x$  and concentration drop or we write in terms of mole fractions, then we have  $K_x$  into  $\Delta X$  or we have  $K_y$  into  $\Delta Y$ .  $\Delta X$  and  $X_{A,i}$  is a bulk minus  $X_{A,i}$  or we have  $Y_{A,b}$  minus  $Y_{A,i}$ . So, this are

known to us is difficult to measure. So, we will think of defining another coefficients we will call it over all mass transfer coefficients **over all mass transfer coefficient**.

So, what we have is individual mass transfer coefficients - individual or film mass transfer coefficient. There is no **harm**; one can still calculate  $K_x$  or  $K_y$  given Reynolds number and Schmidt number. But now we do not we all we know the relationship between  $Y$  and  $X_i(s)$ , But since we do not know the exact value **exact** you know, individually  $X_i(s)$  and  $Y_i(s)$  for design calculations, we need another definition here which is over all mass transfer coefficients. So, let see what is over all mass transfer coefficients.

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We have  $X$  and we have  $Y$ , we have drawn, we have the equilibrium curve. We have here  $Y_{A,b}$  - bulk concentrations in the coordinate here is  $X_{A,b}$ . If we know where two mass transfer coefficients - individual mass transfer coefficient - we draw a slope of minus  $K_x$  over  $K_y$  to obtain  $X_{A,i}$  and  $Y_{A,i}$ . This is what we did. So, now, let us define two more concentrations, for a given  $Y_{A,b}$  there is another concentration given by the equilibrium column. So, if  $Y_{A,b}$  is say 1000 ppm if  $P$  and  $T$  is fixed, we have equilibrium curve fixed. We can also get another concentrations level here which would be in equilibrium with this  $Y_{A,i}$ . So, we have  $X_{A,*}$  we will call it star. So, what is the  $X_{A,*}$ ? It is an equilibrium concentration in liquid phase given bulk concentration of  $Y_{A,b}$ .

Similarly, if we have  $X_{A,b}$  here given this  $X_{A,b}$  we can get another concentrations which we can call it  $Y_{A,star}$ . So, what is  $Y_{A,star}$ ? It is an concentration in equilibrium with  $X_{A,b}$ , but it is very important to note that in reality, we have one  $Y_{A,b}$   $X_{A,b}$ . These are the two physical or measurable concentrations.  $X_{A,i}$  and  $Y_{A,i}$  there also physical, they also exist at the interface concentration except we are trying to avoid the measurement of this concentration which is  $Y_{A,i}$  and  $X_{A,i}$ .  $X_{A,star}$  and  $Y_{A,star}$  these two are hypothetical, they do not adjust in the system.

So, why hypothetical, because  $Y_{A,star}$  you are saying that it is an equilibrium with  $X_{A,i}$ . In other words, if we have a system where interface concentration is  $X_{A,b}$  then concentration in the gas phase would be  $Y_{A,star}$ , but we do not have that. We have interfacial concentration at  $X_{A,i}$  and  $Y_{A,i}$ . So, it is hypothetical  $Y_{A,star}$  as if we have  $X_{A,b}$  in equilibrium with  $Y_{A,star}$ . Similarly, if given  $Y_{A,b}$  we have another hypothetical concentration here at  $X_{A,star}$  as if we have a gas phase at  $Y_{A,b}$  and we brought in contact with this liquid at this  $X_{A,star}$  under equilibrium these are the two concentrations.

So, all if you go back and try to re-call we have a real system or gas and the liquid; two are the bulk concentrations which are real; two are the concentration at the interface which are real; we can measure these two, we cannot measure these two. We have enough to calculate diffusion flux. Thermodynamic says that given one concentration in one phase we can calculate the other phase concentration under equilibriums. We are trying to avoid the measurements of  $X_{A,i(s)}$  and  $Y_{A,i(s)}$ . These are not the major quantities. So, we defined two more concentrations as if given gas phase concentrations. If entire concentration is brought to the entire space what would be  $X_{A,star}$  or if we have  $X_{A,b}$  here, bring the entire  $X_{A,b}$  to the interface, what would be  $Y_{A,star}$ . So,  $Y_{A,star}$  and  $X_{A,star}$  these two are the hypothetical concentrations. But none the less we will see that we will define a quantity which we said over on mass transfer coefficients which should be very handy in the design calculations, instead of working on individual mass transfer coefficients, generally we prefer to work on over on mass transfer coefficients.

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Handwritten notes on a whiteboard:

$$N_A = K_x (X_{A,i} - X_{A,b}) = K_y (Y_{A,b} - Y_{A,i}) \quad \text{--- def}$$

$$= K_x (X_{A,i} - X_{A,b}^*) = K_y (Y_{A,b} - Y_{A,i}^*)$$

Labels:  $K_{ox}$ ,  $K_{oy}$

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$$\frac{1}{K_{ox}} = \frac{1}{K_y} + \frac{m}{K_x} \quad \left[ \begin{array}{l} m = \frac{Y_{A,i} - Y_{A,i}^*}{X_{A,i} - X_{A,i}^*} \\ m = \frac{Y_{A,b} - Y_{A,b}^*}{X_{A,b} - X_{A,b}^*} \end{array} \right]$$

$$\frac{1}{K_{ox}} = \frac{1}{K_x} + \frac{1}{m K_y} \quad \left[ \begin{array}{l} m = \frac{Y_{A,i} - Y_{A,i}^*}{X_{A,i} - X_{A,i}^*} \\ m = \frac{Y_{A,b} - Y_{A,b}^*}{X_{A,b} - X_{A,b}^*} \end{array} \right]$$

Note:  $m \neq m'$

So, now let us write down the similar equations for N A we wrote N A equals X excuse me we wrote under steady state we can write N A equals K x X a, i minus X a, b will also be equal to K y as Y a, b minus Y a, i. We can also define. So, this is you can call it definitions of mass transfer coefficients in X phase or Y phase or liquid phase and the gas phase. One can also define as another over all mass transfer coefficient, which will denote by K or sometimes is denoted by K over all K ox and K oy. So, may we drop slide to bigger here this is your K x, now we can define as K x as X a star minus X a, b equal to capital K y Y a, b minus Y a star. So, look at what we have done. We have avoided using interfacial concentrations. Instead of that, you are using two quantities X a star and Y a star. What is Y a star, it is an equilibrium with Y a, b; what is Y a star it is an equilibrium with X a, b. So, these two are hypothetical, in reality bulk liquid phase and the bulk gas phase they do not see each other they are miles apart we have the interface here.

So, we are the bulk concentration of gas and the liquid here. They do not see each other, what they see at the interface what we see is the interfacial concentration X a, i and Y a, i, but we do not want to use that because that is the interfacial concentration that cannot be major; bulk concentration become major. So, to avoid that we define these two hypothetical quantities Y a star and X a star; as if we bring the entire liquid in contact with the gas we will get Y a star from thermodynamic. Or if you bring entire **y** gas phase gas **gas** in contact with the liquid we will get X a star from thermodynamics.

Now, we have all pure mathematical expressions, four equations and we will see how many unknowns **here**, we can equate this to obtain  $\frac{1}{K_{oy}}$  equals  $\frac{1}{K_y}$ . This  $K_{oy}$  is same as  $K_y$  capital  $K_y$  equals to small  $\frac{1}{K_y}$  plus we will see we will call it **m dash**  $m'$  over  $K_x$ . We will come back to this what is this  $m$  here will make use of the graphs for the interpretation of this  $y$   $m'$  this also equals to we can write  $K_{ox}$  which is nothing but capital  $K_x$  equals  $\frac{1}{k_x}$  plus  $\frac{1}{m''}$  into  $K_y$ . Where  $m'$  and  $m''$  is given as  $\frac{Y_{A,i} - Y_{A,*}}{X_{A,b} - X_{A,b}}$ . Similarly,  $m''$  equals  $\frac{Y_{a,b} - Y_{a,i}}{X_{a,b} - X_{a,b}}$ . So, this all by, you know, mathematical manipulations of this four equations, we try to eliminate, one from the other, one can show this as an expression for overall mass transfer coefficient, in terms of individual mass transfer coefficients, through these interfacial concentrations and the bulk concentrations. Of course,  $m'$  does not equal to  $m''$  or within the limit, one can show that this nothing but the slope of this equilibrium.