Mass Transfer II

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Module No. # 01

Lecture No. # 40

Today, we will like to conclude lecture on this course we started as Mass Transfer II in the first lecture all right. And if you recall now we said this course is about separation processes. So, we classified different processes as you know subgroup of unit operations all right. And then we said that for this course we had we have in the syllabus absorption, distillation, extraction, adsorption and drying.

So, these are the five different unit operations for this course all right. What we did today we will like to summarize and we will like to see, how based on our knowledge of you know different unit operations, our approach starting with thermodynamics, then the rate processes; then some certain new numeric sort of examples we can address similar other unit operations or the same unit operations with slightly more complexities all right.

So, first we said that you know we had to have a very basic knowledge of diffusion that is a very basic ingredient of mass transfer. So, in that respect we had a discussion on mass transfer coefficient, different theories you know film theory, surface renewal theory, boundary layer theories, so these are main three main theories. So, we had a discussion on how to calculate the rate. So, we developed a rate expression in terms of mass transfer coefficient and driving force for diffusion in terms of concentration gradient or in terms of partial pressure all right.

So, once we had these rate expressions, then we said that we have to start with thermodynamics or the equilibrium. So, when we have the equilibrium either in case of absorption or distillation or adsorptions then we say that rate is zero. So, we talked about the inter-phase and the inter-phase two different streams, there is no rate in other words mass transfer you know rate expressions came into del c driving force is 0.

So, then we had those thermodynamic equilibrium curve, solubility curve, established from the experimental or from certain handouts we had isotherms for adsorptions all right. For extraction we said that we need a data on immiscibility or the miscibility solubility of two different liquids.

So, once we have the thermodynamics then we said that we need to have the approach. So, the two approaches one was the rate based approach, one was the stage wise or equilibrium wise. So, in principle you know one can apply any of the two approaches to address any unit operations; but, if you have some unit operation which has trace or a stages cascades; then it make more sense when we can apply this equilibrium wises by approaches.

So, we said that two living streams they have enough resistance times they mix with each other and they live in equilibrium. And that coordinate of the two streams they fall on this equilibrium curve when we have the rate wise approach then we have need have a more rigorous approach we need to write down the differential balance species balance then we need to integrate the system and in that respect to we said that the approach of N T U or S T U make sense here that is more of an approximations but, still it one can design the column or the unit operation which it is dealing with.

So, essentially we have two approaches one is equilibrium wise approach stage wise and another is rate wise approach in which we have this N T U or S T U approach simply vice versa. Then we talked of operating lines and the equilibrium lines then the stages etcetera in that also we said that of course, we are discussing about mass transfer but, hydrodynamics also makes a plays a major role there.

So, very often when you have this countercurrent system gas liquid or even if you have the solid and the liquid or gas flows through it; then we have to worry about or we have to be concerned about pressure drop or flow digestions or loading and flooding. So, these are three major ingredients of hydrodynamic discussions and very often it happens that one has to do iterations in other words if mass transfer says that you know we have so, much of diameter of the length of this column. Then one has to go back and check from the hydrodynamics whether we have a reasonable pressure drop; because that will decide the cost of the power cost of the pump.

So, we have the economy also cost also safety issues, disposal issues and the other issues which if you have any course on design, then you will dealing with all kinds of aspect mass transfer hydrodynamics cost economy and the safety issues.

So, the way we will conclude today’s lecture is we will sum up the way we started this course and the main approach. And then we will take one very simple example where you have such certain more complex system, how we can address this based on whatever knowledge we have right. So, of course, we had only five unit operations but, you can have some new type of unit operation which is not included this course. But, still by the same approach we can reasonably we can address that type of problem. So, let us sum up. So, far what we have done in this course as far as approach is concerned.

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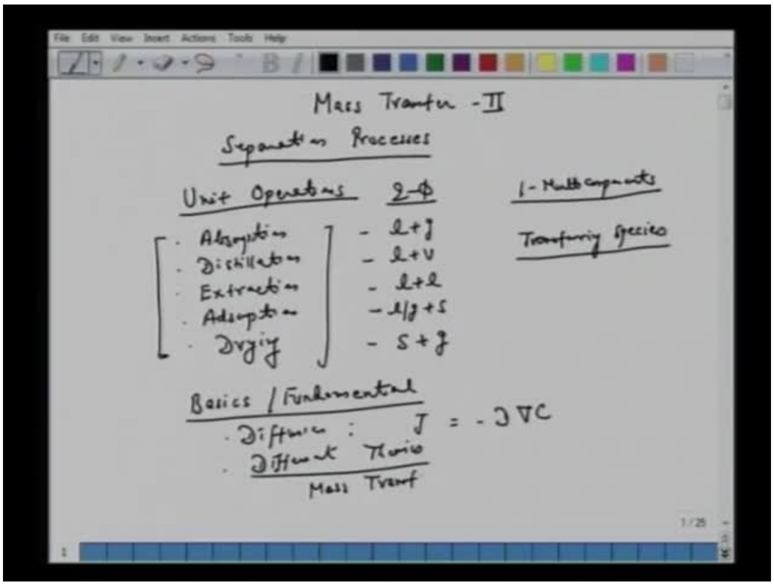

So, we started this course as mass transfer II, we said it is about separation processes and we said that we can classify different processes from the operation point of view as unit operations all right. And for this course we said we are going to have discussion on absorption then we switched to distillation plus say gas air all right.

So, we have multiphase different two at least two phase systems we have and we have generally one component or we can have multi components. So, we are talking of transferring species. So, we have transferring species which diffuses from one phase to another phase. So, when we took up this this unit operation one by one before that, we had one or two lectures on basic or fundamentals of mass transfer all right.

So, in this if you recall we started with diffusion. So, we said fixed law flux is given by minus d into concentration gradient. Then based on the diffusion we talked about different theories, we introduced a parameter very important parameter mass transfer coefficient all right.

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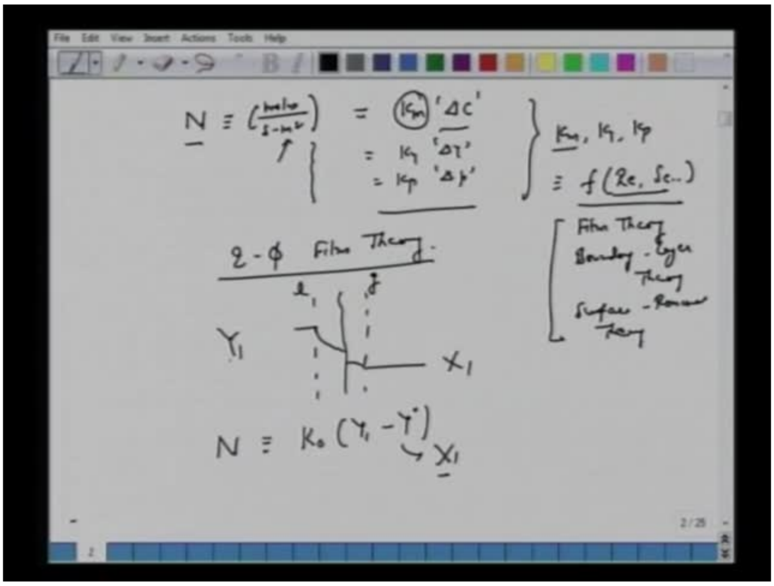

So, based on this definition of mass transfer coefficient, we say talked about the flux rate, so at what rate certain species or transferring species is getting transferred from one phase to another phase, so this meter square is the inter-phase area. And we said that by invoking this parameter mass transfer coefficient we can have driving force for mass transfer from one phase to another and we can write in several ways k m del y k p del p etcetera all right.

So, this is about the rate one has to measure in terms of mass transfer coefficients and all these mass transfer coefficients k m k y or k p they were shown to be a function of Reynolds number, Smith number etcetera. So, different theories like film theory, three most popular theory, film theory, we have boundary layer theory, we had surface renewal theory. All this theory ultimately gives us an expression for mass transfer coefficient in terms of operating parameters or the physical property of the system. Then, once we had the definition for mass transfer coefficient we talked about two phase film theory.

So, now we are talking of an inter-phase of two different phases liquid and gas; and then we said we can have two resistances where we have this concentration drop like this. So, instead of working on individual film transfer coefficients, we said that we can also write that in expressions for flux in terms of overall mass transfer coefficient; and two concentrations in the bulk phase. So, y star here could be in equilibrium with let us say with x one on the other side all right. So, essentially we have these engineering parameters to estimate this rate after this we talked about what is the step here right our approach the first step here is equilibrium one has to establish equilibrium.

One has to do the experimental one has to have do some experiment to generate this data for solubility, or for isotherms, or for or for solubility of one liquid into another liquid or one has to rely on literatures literature surveys or literature values.

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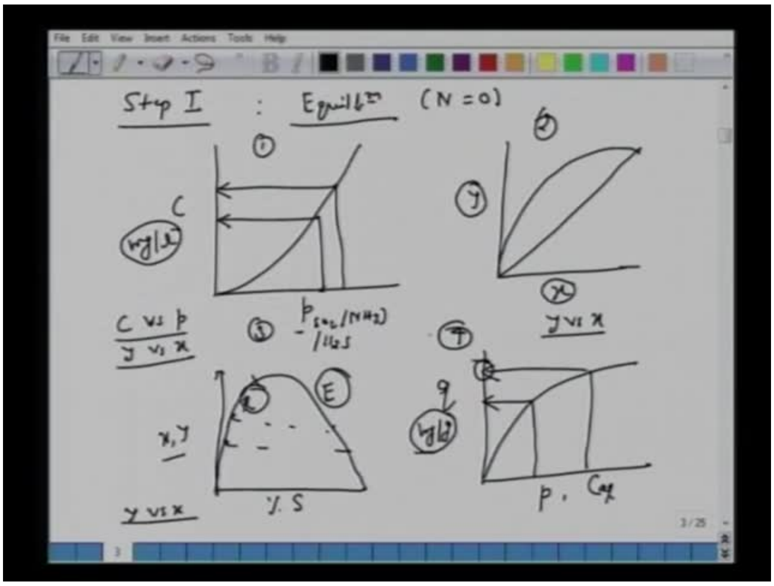

So, the step one if you want to write down a very common step for all unit operation is about equilibrium, at which rate is 0. And then if you talk of absorption we have solubility curve, so given partial pressure, what is the concentration, given partial pressure what is the concentrations. So, this is about say sulphur dioxide about ammonia all right about H 2 S in gas phase and how much is the concentration say milligram per liter in other phase. Then we came to the second unit operations which were distillations. So, here also we have different mechanism but, we have the same approach that one has to establish a relation between mole fractions in one phase to mole fraction in second phase.

So, we have this Raoult's law which gave us y versus x. Here also we can write c versus partial pressure in terms of y versus x or x versus y. So, idea is that at step one, we have to generate such kind of data from the experiment or from some literature values. We came to the third unit operation which was extractions what we have here we also have a phase diagram. So, we said some percentage amount of solvent s and here we can have x and y as in extract phase and the raffinate phase we need to know what is the equilibrium concentrations, so they connect here between this between this. So, this is also about y versus x in two phases extract and raffinate.

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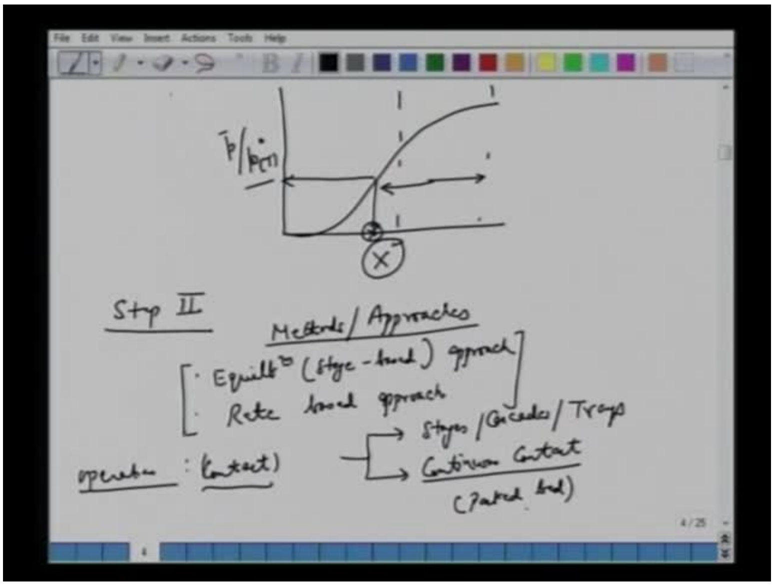

Step II

Methods/ Approaches

- Equilib^m (Stage-based) approach
- Rate based approach

operation : (contact) → Stages/ Cascades/ Trays
→ Continuous Contact (Packed bed)

Then, we addressed we talked of langmuir isotherms given partial pressure in one phase what is loading q milligram per gram. So, given partial pressure in the gas phase how much C O 2 or sulphur dioxide will be adsorbed to give this loading or even in the aqueous phase we can say that c aqueous given so much of concentrations how much is the loading of fluoride or arsenic on certain materials. So, these are the four unit operations where we talked about this equilibrium form thermodynamics.

We also had drawing the last unit operations there also we have a very similar curve like this moisture content in equilibrium as a function of saturation humidity or relative humidity relative saturation. So, partial pressure of the gas non-dimensionalized with respect to the partial pressure of liquid or water in case moisture pure moisture at certain temperature.

So, here we said we have different region for a given quality of this air moisture in certain solid cannot go below this. So, this is your x star. So, here we had this unbound moisture and the bound moisture etcetera all right.

So, our idea here is that irrespective of the unit operation, one has to start with the equilibrium, you have to establish given activity in one phase how much is the concentration in other phase or the vice versa that is equilibrium thermodynamics. Once we have established thermodynamics then comes your rate and which we have already addressed rate can be written in terms of mass transfer coefficient and the driving force. But, then we said that now about two different methods or two different approaches in one approach we have equilibrium stages and stage wise approach and the second is a rate wise approach.

So, let us see let us note down on this also that in step two now once we have thermodynamics now we are talking of two different methods; or two different approaches. So, what approaches we have? We have equilibrium approach right or we say stage based approach and the second is your rate based approach. Now, before these are the approaches we must not forget that a real system or from the operation point of view, we have different type of arrangements, so we now we are talking of contacts all right.

So, we can bring two phases in contact in stages or we called it cascades or we call it trays all right; or we can have continuous contact. So, essentially here very often we talked of packed bed for example, packed bed. Now again try to understand this and try to recall that our approach are different and the real process is different right. In principle one can apply any of the two approaches in any of those what all we have but, for our convenience if we have trays like in distillation column one we would like to apply this equilibrium based approach.

If you have a continuous packed bed like a packing or you have absorptions or you have adsorptions right one can one should apply in principle rate based approach for simplicity of course, one can always have concept of theoretical stages, theoretical plates height or distance between the two plates one can apply one can do this right. But, for our convenience if you see the real system in which we have the trays we can think of equilibrium approaches two leaving streams are in equilibrium.

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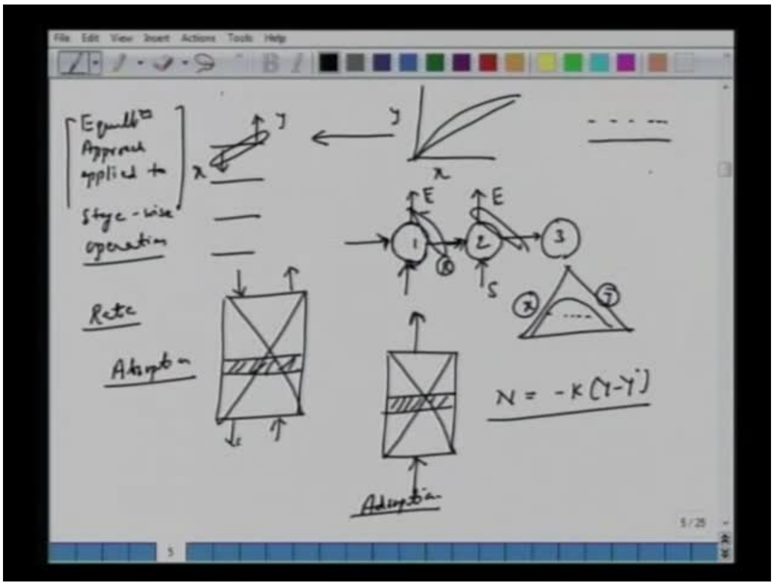

So, when we talk of say distillation we say that we have trays, so we can apply equilibrium approach the vapor which leaves and the liquid which leaves they are in equilibrium their coordinates are given in the approach one; when we had this relative volatility y versus x. So, this gives us y versus x all right, so that is the distillations when we talked of extraction then we say that well we have cascades. So, one is streams another streams give them equilibrium the region they live in equilibrium.

So, here also you bring this concern raffinate in case of extractions treat with the solvent, we have extract here two living streams are in equilibrium. And these two living streams one can mark on this equilateral phase diagram as a tie lines. So, we are here we have y and here we have x. So, we are talking of equilibrium approach or a stage wise approach approach applied to stage wise operation, like in distillation column in extraction column adsorption also we had one example where we had this type of system where the liquid was brought in contact with the solid. And then take into the second stage or more the other. When we say rate wise approach then here, we are talking of continuous packed bed column say, we can have the liquid and we can have the gas flowing in the counter current operations like in absorptions all right. Let us talk of adsorptions that also bedded. So, we have fixed packed bed and certain gas contaminated with certain species we brought in contact and it treated stream leaves now one end.

So, this is adsorption, these are all contact continuous contact wise process here we do not have stages like this, which means things like this or operations like this you are talking of rate at a finite rate from one stream to another one phase to another phase; we have finite mass transfer n equal to minus k y minus y star some kind of definition for mass transfer coefficient.

So, in principle one has to write a very rigorous species balance for this. Here we may we do not get into details of interfacial mass transfer or the time over which this stream is getting mixed with this stream and there is mass transfer. It is more like a steady state process after time t the two streams have been given enough time to mix and transfer the maximum species from one phase to another phase. Similarly, here distillations at every tray of the distillation there is pool of the liquid.

So, the when the vapor comes in and the liquid drops in there is a mass transfer there is a rate but, overall our approach is that two leaving streams where they have spent good amount of time here, so they are in equilibrium. So, we do not get into this rate based approach, here we have the approach since the process is continuous here we have the rate by use of in which case we write we wrote down generally.

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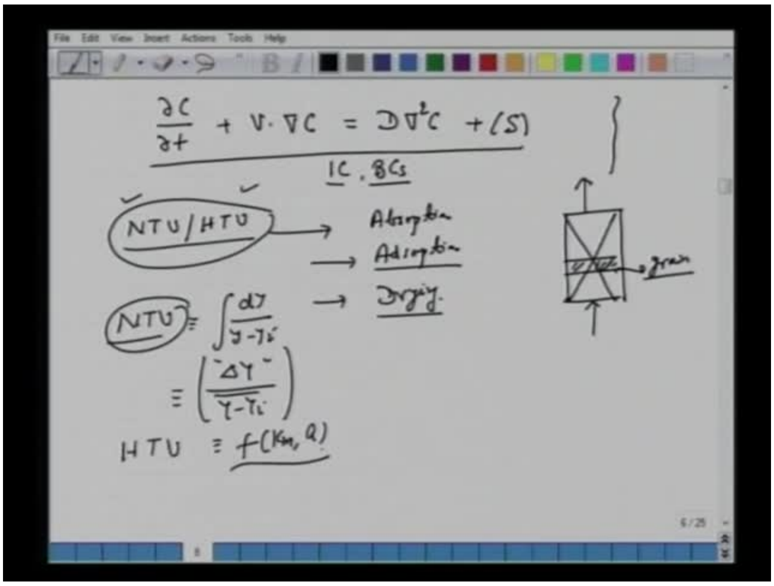

We have a very comprehensive species balance of this order a species is plot in and out by convection species is brought in and out by this dispersion or by diffusion and then we have rate at which mass is transferred from one phase to another phase. Then this has to be in principle this has to be solved by certain initial condition and certain boundary conditions all right. But, here we said that instead of solving these compressive equations these differential equations there is an approach of N T U and S T U.

So, if you recall we did mix some species balance we had simplifications, then we in invoke the concept of number of transfer unit and a height of transfer this approach can be be applied in absorption we can also apply in adsorptions. So, there also it is possible to apply this type of approach we can also apply in case of drying although we did not invoke this N T U and S T U but, a look at some of the textbook they do have this type of approach.

Drying also when you have the solids packed bed of solids say grains wheat which you want to dry off moisture for its longevity you bring in contact with hot air. So, you all very similar to what you have in case of adsorptions you have the similar situation here the moisture is driven off from the surface into this air. So, here also one can apply the concept of N T U and S T U, we said that N T U very common definition we should remember is nothing but, d y over y minus y i but, you should look at fundamentally that is nothing but, difference of whatever is the enrichment over the average driving force per mass transfer all right.

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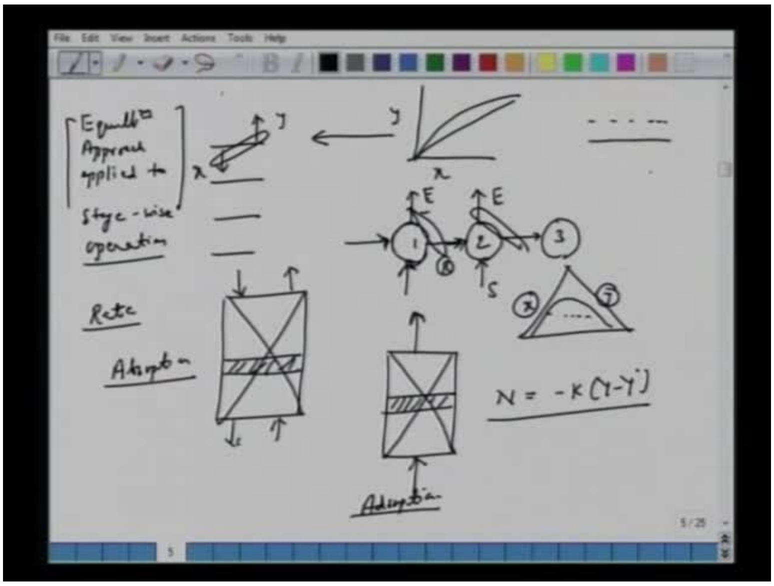

So, you bring this N T U number of transfer unit you are saying that this packed column is if they are certain physical or hypothetical they are N T us number of transfer unit. Each transfer unit has certain height which is a function of mass transfer coefficient and as some surface area per gram. Unlike you go back to the previous line where we talked of this stage wise approach there is no K all right we make overall balance between one stream and another stream.

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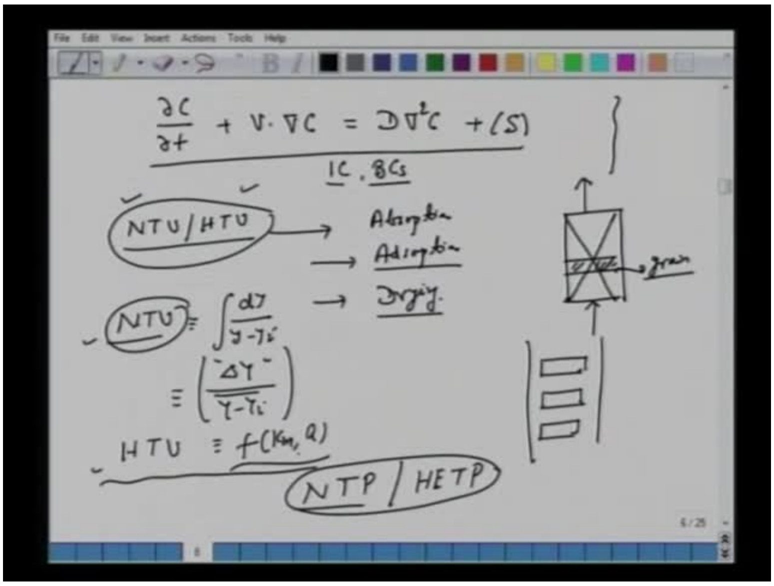

$$\frac{\partial C}{\partial t} + v\cdot\nabla C = D\nabla^2 C + (S)$$

IC, BCs

NTU/HTU → Absorption
→ Adsorption
→ Drying

$$NTU = \int \frac{dY}{Y - Y_i} = \left(\frac{\Delta Y}{Y - Y_i}\right)$$

$$HTU = f(K_a, Q)$$

NTP / HETP

So, here we have these different trays all right in case of drying this also possible that we have the trays like this. So, in principle we can also make an approach here we can also have an approach of trays wise approach. Similarly, instead of N T U and S T U if you look at some of the old textbook they also talk about N T P Number of Theoretical Plates height of a height equivalent of a theoretical plates, so it is an very old approach.

The idea of what we are trying to say here that if you have a continuous packed bed continuous contactor one has write down most fundamentally most basic a species balance then one has to integrate. But, since because after all these mathematical complexities, we have these quantities like N T Us and S T Us. One can also say that well this column is equivalent to certain trays where you can apply the concept of N T P or H T P. So, it is a different approach. So, for different unit operations absorptions we worked on N T U S T U for distillation we talked in terms of trays real trays as well as our theoretical trays. Of course, we have these equilibrium and non-equilibrium conditions etcetera. In case of adsorptions we wrote down we talked about this breakthrough curve how do we get the breakthrough curve we write down this species balance differential balance; and then we solve with a certain initial and boundary condition.

Drying we can have both trays wise we can have a continuous dryer now we give an example to remove moisture from the grain. So, there one has to write down again differential balance, one can integrate one can also invoke the conditions invoke these parameters like N T U and S T U. Extraction liquid extraction we talked about two liquids brought in contact in a stage in a mixer then we have settler. So, you said the two leaving streams on equilibrium rate is 0, maximum mass transfer we have how many stages we can also have a real industrial extraction column where the liquid two liquids they flow in the opposite direction through some contactors. There one can also bring this parameters of N T U or S T U one can work on this.

One can also take that weight other approach in terms of number of stages. So, these are the two approaches you must not mix up with it they are separate real unit operation is separate, in principle one can apply one two any of the two two different ways of contacts.

Idea here is that after deciding that what approach we have to do we need to have an equilibrium curve and we need to have an operating curve; that is what we did in case of distillations right we had number of trays you made a species balance, extractions we had a operating line and we had the tie lines.

So, whenever you have the equilibrium stage wise approach one very common approach is that to identify your two coordinates which are on equilibrium given by the tie line and the two coordinates which are on operating lines not in equilibrium. So, there is a finite mass transfer.

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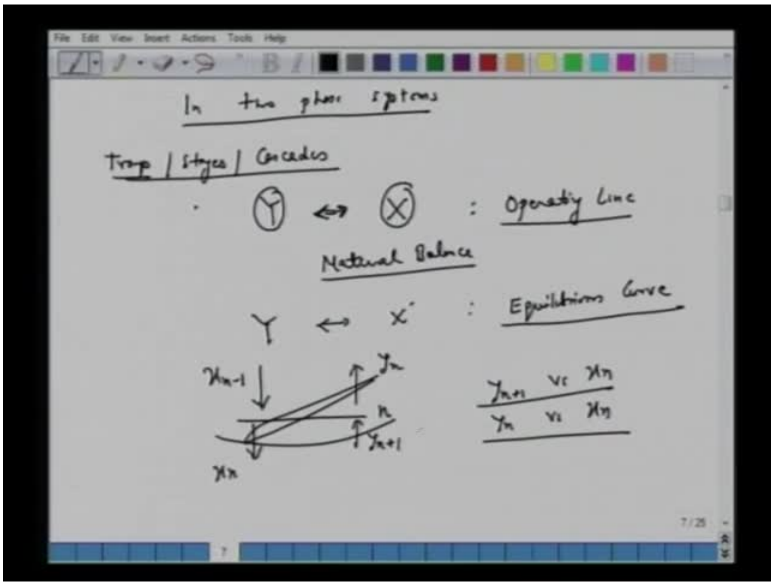

So, we will like to just sum up what we said here about this species balance. So, if you have a two phase system. So, when you have the trays wise approach or stages or you have cascades, then first is we are talking of two phases with two different concentrations mole fractions.

So, we have Y versus X given by this operating line. So, this is the most common approach we had for distillations, we had for extractions, we also did for adsorption whenever we have the stages cascades. You need to write down you need to have an operating line. So, what is the operating line? You need connections or you need some expressions which connects bulk phase concentration in one phase to another phase. Essentially you are talking of material balance or species balance between the two phases. And then you have on top of this y versus x star given by this equilibrium line all right.

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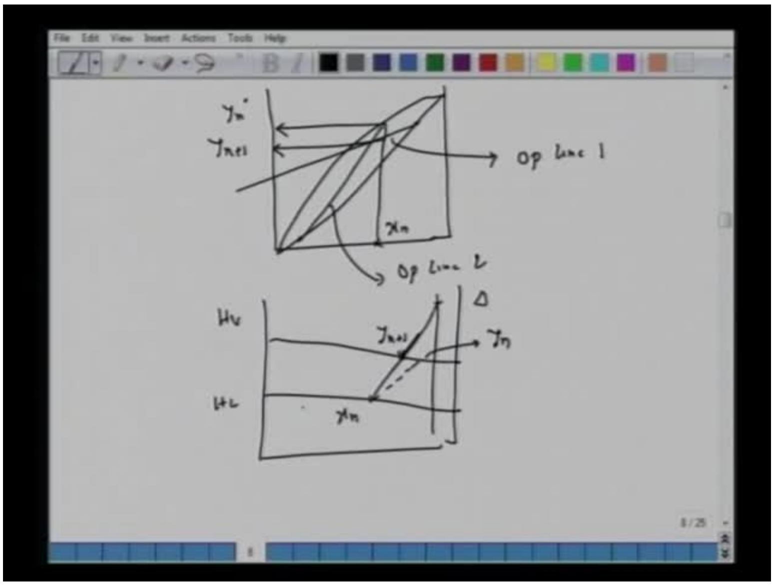

So, if you go back we talked about distillation column we take certain stage n here, we have the vapor which has a concentration y n liquid leaving that stage x n the vapor which arrives as a mole fraction y n plus 1. And the liquid which arrives this stage is x n minus one. So, we need to have a material balance, so that we have an expressions between y n plus 1 versus x n. And we have the tie line given by the equilibrium curve where we have y n versus x n. So, with this we develop these mccabe-thiele methods. So now, we are talking of thermodynamics on which we have superimposed our material balance. So, this is thermodynamics on which we have superimposed our material balance

So, we had this operating line 1 and we had operating line 2. So, what did it does for given mole fraction x if we go to the operating line if this is x n. This will give you y n plus one and for the same x n here you have y n star or vice versa we talk of this Panchon-severit methods instead of Mccabe-thiele methods we have the enthalpy curve or h v and we have h l. So, again we locate we located a differential point del. So, that we say that if we have y n plus 1 here if you connect this, this will give us x n x n on the other hand is in equilibrium with y n.

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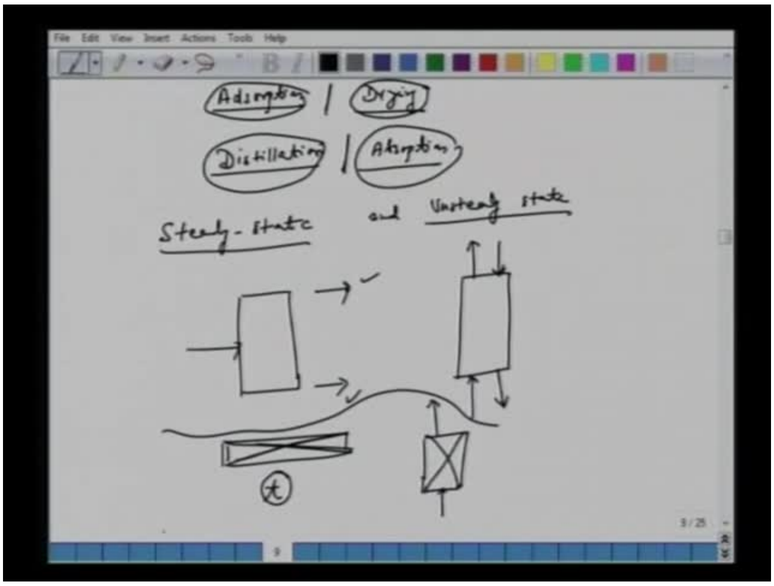

So, y n versus x n by tie line and y n plus one versus x n given by this operating line. So, what is the operating line nothing but, this mass balance or a species balance all right. So, these are all steady state process. Now let us look at certain other process like adsorption. Drying try to compare this with distillation or with let us say we have absorption. So, one thing is of course, we can always discuss in terms of distillation which is in trays adsorption we have in continuous packed bed.

Mostly adsorption you have a packed bed two streams solid phase and the gas or liquid phase drying again we have the solid phase and then we have the gas phase. So, there is a similarity and there is a contradiction between the two phases all right in two processes.

The one most another difference which you should notice is about steady state and unsteady state. So, when we say absorption or distillations we say continuously feed is fed here we are withdrawing two one bottom products and one top, one top product and one is the bottom product. When we talk of adsorption we say that gas continuously is fed in leaves from here which phase some fluent phase liquid gets in and treated liquids leave from here may be it gets regenerated and we have this continuous process. When we had these adsorptions packed bed then we talk in terms of how long will it take for the bed to be adsorbed. When you talk of drying we say that how long will it take for the solid to remove from this moisture.

So, you see the difference here that certain unit operations, we are talking in terms of steady state all right we are not interested in a startup which you will do in different course like process control all right. Here we are talking in terms of unsteady state how long will it take. So, let us again try to understand the difference between the these unit operations one is of course, we said about stage wise and contact wise that is one thing. Approach an N T U S T U or equilibrium stage that is another way of classifications.

Now, we are talking of steady state and an unsteady state, why certain processes inherently we treat them as unsteady state right. We said that adsorptions. So, we bring in some contaminated gas which we want to remove is. So, to from air we bring in contact it is zeolites, so zeolite has a finite capacity. Once it get saturated adsorbed one has to regenerated in case of absorption mind you we say that continuous where you must not forget that the liquid or the spent liquor which is also saturated has to be regenerated in the second column and has to be brought back.

So, there also we have the similar situations that certain phase is getting saturated except that the way we treat we say that is a continuous process because we have the parallel second stream. In case of adsorption what happens or drying we have trays stacks of solids which we have to dry from one moisture level to another moisture level. So, we bring in contact with certain air it gets dried stops the things bring at the phase one. We can also have continuous process for adsorption and drying what will be the constant or why we do not have so commonly?

Because now we are talking of handling of solids the solid falling from high will will break right. So, there is a breakage of the solids we do not have this kind of operational difficulty in case of liquid and vapor. So, we have a very continuous process continuously you can bring one phase, continuously you can remove the second phase.

In case of solids we cannot have we cannot think of a stages where from one stage solids will fall in the second stage and etcetera. You may see certain driers with that type of arrangement trying to make the process continuously; but, there is always very huge risk of solids getting broken into several of species and then blocking or choking occurs one has to stop it. Now you should see the different that why a process is treat as a steady state why another unsteady state because of the operation difficulties. In principle we can have adsorptions say we talked of the p s a system pressure swing adsorption.

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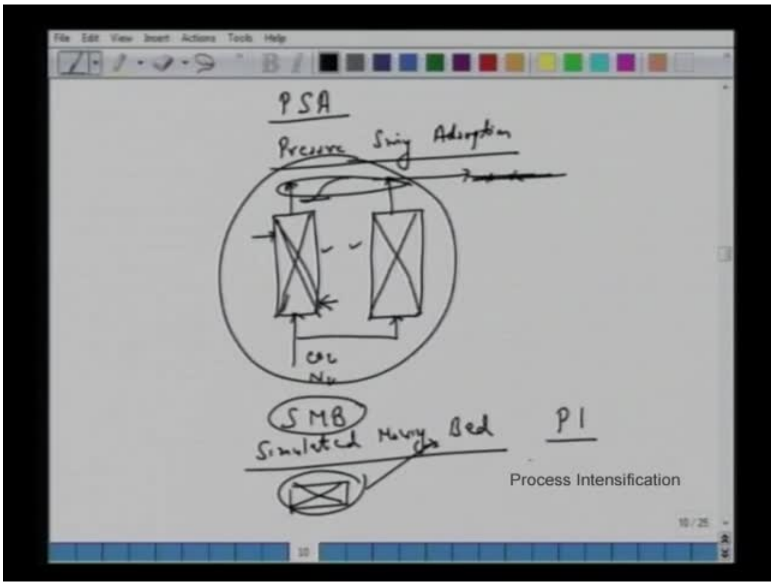

So, pressure swing adsorption in the previous class, on adsorption we had a very brief introduction this. Now if you have one bed of the solids and we bring air which we want containing say carbon dioxide in nitrogen. And treat this here of this concentration of c u two will be very small here and slowly and slowly this bed will get saturated this concentration will pick up and now we have this break through curve.

How do we make this process continuous? Well one has to bring this second adsorbent. So, that when this is adsorb then you can switch to the second bed and bring that time one can regenerate this pressure. So, here a process which is inherently unsteady state from the operation point of where seeing that this can also be treated as a continuous or a steady state process. Because here the concentration or the purity of the stream which is leaving is made deliberately deliberately constant by continuously switching from one bed to another bed. In industrially we look at you know at high graduate level you have something call SMB Simulated Moving Bed all right.

Here by sequencing by changing the sequence but, any more number of beds one can have a solid phase which is a stationary behave like a moving bed. So, we are talking into now PI Process Intensification. So, these are certain advance thing topics which you will meet meet across at the graduate levels. So, the idea here is again for the discussion that whether it is a study state or unsteady state in industries you know to begin with s plant is under start up whether it is a distillation column or adsorptions.

So, a coal plant which is now not used for long time or say the power is gone off power is off plant is shut down; when you start there is time unsteady state that is one thing. Once the process reaches a steady state then, we have a continuous purity maintain continuously concentration maintain.

Certain process like adsorptions drying where we have mostly solids. If you have the solids then also we can have continuous process like in S M B or two gets P S A system. We can have that but, its again it is a question of difficulty in the handling of the solids which has to flow by some other techniques fluidizations can make solid flow from one trays to another trays its possible.

But, we must understand that each process there is a similar each unit operation, there is a similarity there is a difference either in terms of mechanism or in terms of operations. One has to have the approach clear that first we need to have an equilibrium then we need to have operating line or a species balance then, we can address how many trays are required given the flow rates etcetera. There is another thing here about hydrodynamics.

So, in this course we had discussion on mass transfer or we deal with mass transfer aspects but, there is also very important consideration of hydrodynamics. In other words if we have a packed bed of solids and you allow gas and water to flow through this one has to be considerate about pressure drop; because that will decide the cost of the power cost of the pump; or at high flow rates or relatively high flow rates there is a possibility of solid beds getting fluidized. Once the bed gets fluidized then there is a risk of carry over house all right.

In some cases we do require fluidizations for high heat transfer coefficient or for high large mass transfer coefficients. If we have continuous packed bed say adsorption gas flowing in one direction liquid flowing from other directions, then we talked about loading and flooding it is possible that at relatively larger liquid flow rates; that liquid will prevent gas that liquid which is flowing from the top will prevent gas to flow from this.

So, there will be flooding of the column will the liquid or entire column can be loaded with the gas. So, we have that type of constant. Similarly, one has to worry about the pressure drop how much is the pressure drop in the continuous counter current process.

So, of course, in this course we did not have much discussion on hydrodynamics we just had one plot if we recall in absorptions above the Sherwood Eckert plot.

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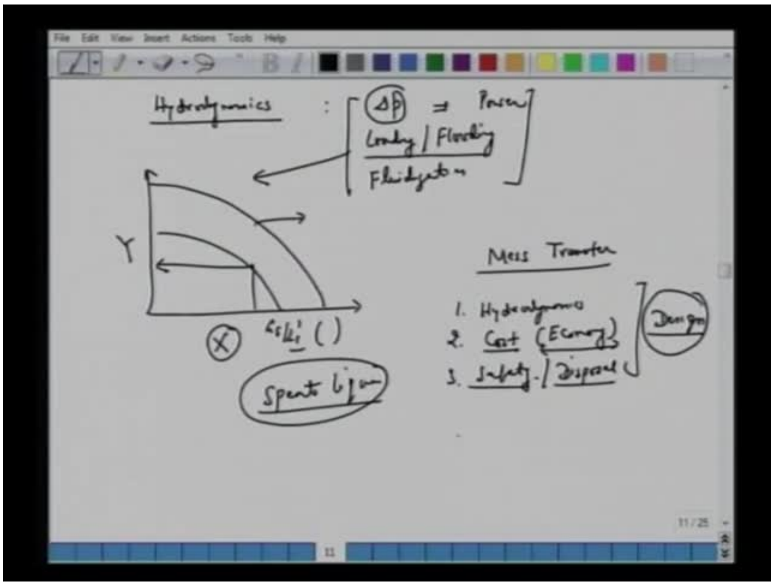

So, when we talk of this hydrodynamic hydrodynamics essentially we are concerned with the pressure drop because that decides how much is the power. One has to be considerate about loading and flooding in case of continuous counter process or fluidization.

So, we have not discuss in detail about this except, when we discuss absorption then we had this very popular graph given by Eckered or Sherwood where we talked of this loading and flooding curve or the different pressure graph y versus x certain g s over l s. If you recall and certain other quantities and we said that there is a flooding limit one has to operate at 70 percent of this gas or liquid flow rate.

So, this was the only small discussion we had hydrodynamics and you have some other codes in chemical engineering where you extensively discuss and deal with this pressure drop in multiphase system of the single phase system. So, idea is that in parallel to this mass transfer, when you design a unit operation you have to be considerate of this hydrodynamics and also cost or economy. Because in case of say absorption if we want to increase your liquid flow rates all right.

Then, the cost of the solvent itself will be large or may prevent you from going for a larger flow rate of the liquid or one has to think of this after this absorption we have this spent liquor. So, one has to regenerate all right, the cost also plays a major role in overall design design course which i am sure you will have at some level next year on wards.

One has to also worry about considerate about this safety issues or disposal all right. So now, we are talking of how are where to spend this spent liquor to we have to recover the liquid and bring it back to the column; so, there we have this cost effect or how to or where to dispose it. So, all of this is part of another course or design which will have I am sure at next third year or fourth year level.

So, idea here is what we are trying to say here that hydrodynamics and mass transfer they go hand to hand right very often you design a column then it is possible that, whatever you say for example, liquid flow rate are gas flow rate we have got. That prevents you in the you know from a smooth operations or a safe operations may be the liquid flow rate was. So, high that you have gone into the region where there is a flooding or the cost of the liquid is so, expensive you know very costly expensive liquid, so economy may not permitted.

So, if you have different some type of special affluent which you want to treat say s to s right or certain mixture of s two and n s two s you want to sweeten this natural gas. So, you have to develop some different type of solvents very special solvent which may not be like water, it is very expensive thing. So, there also one has to look at the cost analysis.

So, everything get together into in overall design of unit operation which you take you know whether it is a absorption or distillation or adsorption what we do now based on whatever discussion we had, so far, you know we take a very simple example. The idea here is to show that its slightly more complex problem which you have not addressed, so far how we can address based on the knowledge which we had. So, the most simple if you think you know you must may have liked it about mccabe-thiele methods all right.

It is about distillations you find we want to know whether want to determine how many trays we have. So, we had a feed stream, we had the condenser we had the boiler and we wrote this material balance we had this y versus x graph and we did this analysis to find out how many trays we had .

Now, let us take the same example with slightly more complex how do we make it more complex for example, let us stay that instead of one feet and two outputs top product and the bottom product we have one more side stream.

So, now we want see how we can address this you know similar this type of problem which you have not deal with so far in this lecture based on whatever we have learned. So, will have the same approach as we started today's lecture we will have the equilibrium will make a material balance; what does the objective material balance to mark certain coordinates and then how we can find the number of trays.

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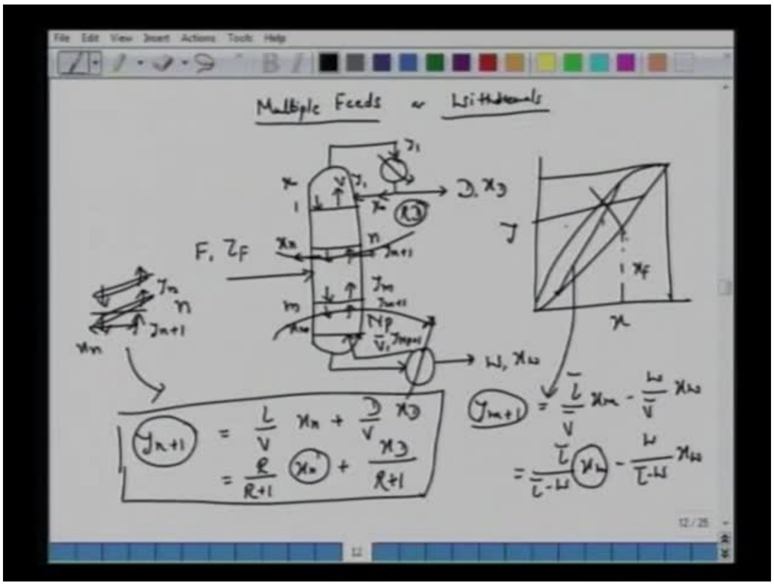

$$y_{n+1} = \frac{L}{V} x_n + \frac{D}{V} x_D = \frac{R}{R+1} x_n + \frac{x_D}{R+1}$$

$$y_{m+1} = \frac{\bar{L}}{\bar{V}} x_m - \frac{W}{\bar{V}} x_W = \frac{\bar{L}}{\bar{L}-W} x_m - \frac{W}{\bar{L}-W} x_W$$

So, again here we want to address that you know if you have meet if you come across certain more complex problem you should be in position to address those starting from this first principle. So, we can have multiple feeds or multiple with drawl. Now most of the text books or mass transfer they do address this we did not address this lecture but, we like to see how we can you know solve this type of problem.

So, let us say we have a distillation column we have a feed F mole fraction Z F top product we have condenser say let us say it is total reflux. So, we have d top product and we have this x d and this goes to the top of the first tray here. So, we had this vapor which is v y 1. So, this y one here and we have this x 0 which is a reflux here, so this reflux ratio is r. So, this stream is r into d at any stage here n we have two streams.

So, this concentration is y n plus one and this liquid which is trickling from the top is x n. Similarly, we had have this last tray n p this steam is brought here. We have a reboiler, we have the bottom product of w and x w and this vapor phase is sent back to this vapor is v bar and concentration y n p plus 1. So, if we choose any stage here m we have this vapor phase as y m all right where this is y m plus one we have this x m minus one and x m. So, to address this we had this we wrote down the operating balance equations for this and for this.

So, we took one two envelops we wrote down the first equations for the top column as y n plus 1 equals l by v x n plus d by v x d which is also we wrote as r by r plus 1 x n plus x d by r plus 1. So, what we are trying to say here that first thing to our approach was let us have first equilibrium curve. So, that is a thermodynamics we started with y versus x. Then we made this species balance, so when we say that we have written the operating balance operating equations essentially this connects your y n plus one to x n.

So, if you look at this nth tray we have y n plus 1 and we have this x n. So, two are given by this location and this stream which is coming from here is in equilibrium with the two living stream or this stream is in equilibrium with stream. So, this is your y n, y n is given related to x n by this equilibrium curve. So, we had one operating line then we had the second operating line and we had this feed line which is x n. So, its mccabe-thiele method we assume equally molar, similar to y n plus 1 we can also write y n plus 1. So, we have taken another envelop here right. So, y n plus 1 equals l bar v bar x m minus w over v bar x w or we can also write as l bar over l bar minus w x m minus w where l bar minus w into x w x w.

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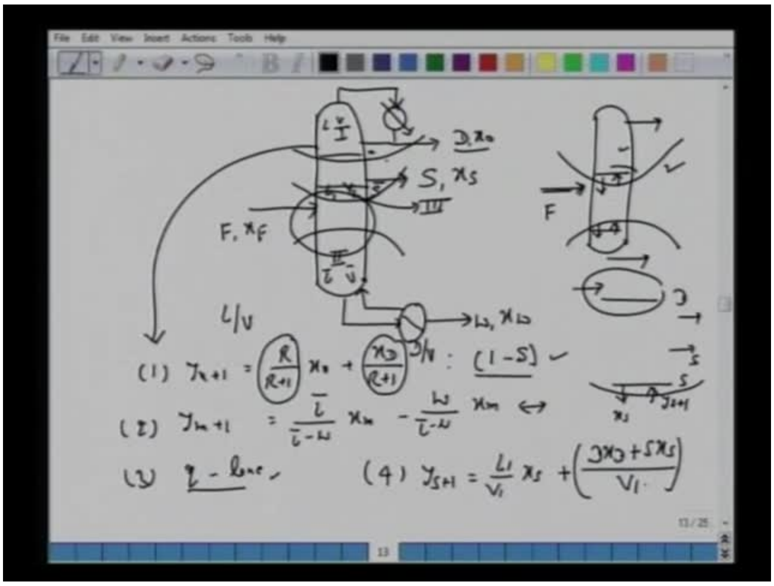

$$(1)\ y_{n+1} = \frac{R}{R+1} x_n + \frac{x_D}{R+1}$$

D/W : (1 - S)

$$(2)\ y_{m+1} = \frac{\bar{L}}{\bar{L}-W} x_m - \frac{W}{\bar{L}-W} x_m$$

(3) q - line

$$(4)\ y_{s+1} = \frac{L_1}{V_1} x_s + \left(\frac{Dx_D + Sx_s}{V_1}\right)$$

So, again this operating line two which is given by this it connects y m plus 1 to x m, so then we had these number of trays. Now, what we are saying here now we have a distillation column similar to the previous one we have f we are withdrawing this product d x zero we have reboiler here. So, this reflux vapor and we have this bottom product w and x w f and x f. Now, we are saying that lets assume there is one more side stream we are withdrawing one more product at certain trace s with the flow rate s and x s.

Now, the question is that how many trays we require? For the idea of this taking this example is to show that we should be able to address you know similar even if more complex problem than what we did, so far in this course by the same approach what we have learnt thermodynamics equilibrium and the operating lines. It should connect the two bulk phase concentrations the liquid and the vapor, so let us see how we address this problem.

Now, we start, so when you have a problem like this you must realize that one has to divide this distillation column into more than two regions. In the traditional distillation column or the one which we discussed in this course we had one product here and another product here and there was one feed.

So, we took one envelop like this and we made this material balance between two streams. We take another envelop here region two where we had this balance between this phase and this phase. So, we got the two operating lines and one we had this q line given by when you made a balance between the trays on which the feed enters.

So, with this three, we had mccabe-thiele method you could find out number of trays. Now, what we do here you must realize that if you take this envelop like this we have the similarly, exactly same situations like this. If you take envelop like this you have the same situations like this all it means now we need one two and we need to take one more balance like this another region like this. So, we have for this region, we have first operating line which will be same as before y n plus 1 equal to r over r plus 1 x n plus x d over r plus 1.

So, this operating line holds good between tray 1 and the trays above s because this same as above you know this s here. So, the situation is same as this second operating line we have for the bottom stripping sections, which is y m plus one equal to l bar over l bar minus w x m minus w over l bar minus w and x m. Third you have this q line which will be also be similar to what we had here intersection of two operating lines gives q line or we know the q line and know one of our this operating line we can find the second one.

So, for this column as well we have these same q line similar equations. Now we need one more equations for this, so we need operating line four now without making material balance by analogy you should be able to write down these equations for this operating line between you know the bottom plate. And this top and the top top product here just underneath this s plate the way we did it here for this nth plate all right you should be able to write down by analogy you can always go back and make you know starting from the first principle that this s th plate. So, we have y s plus one and we have x s.

So, take this envelop now we have two products d and s. So, what do we get here or let us write y s plus 1 now we have r over r plus 1 what was r over r plus 1 this was l by v. So, here we have l and v here we have l bar and v bar. So, flow rates are constant in mccabe-thiele equimolar flow rates in here and the flow rate will change why because we have this one more withdrawal. So, we have another region here where be the flow rate is l 1 and v 1 that means now, we can write by just by this analogy that this number is y s plus 1 equal to l 1 over v 1 x s plus; we have here instead of x t over r plus 1 we had d by v all right.

So, instead of here we had this d by v, so what we should get here by the material balance d x d plus s x s over v one you start from the first principle you should be able to convince yourself that you are getting the same equations here.

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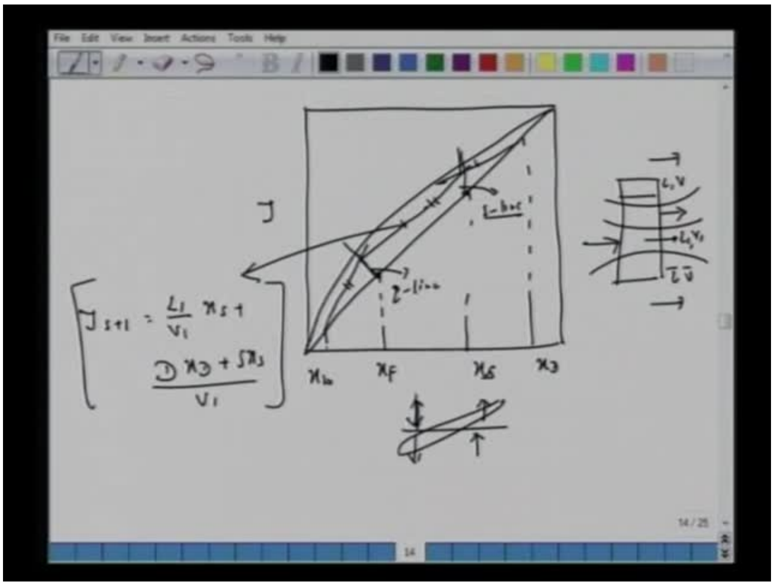

So, what happens to now mccabe-thiele plot, now you have two side streams again you start from y versus x all right you have x d top product you have feed x f you have the bottom product x w and now also you have x s all right.

So, now you have four streams two side streams x s and x d and we have now one feed entry and one we top product x w here. So, what you will expect first operating line which we wrote it will go from here to here. Second operating line for the bottom product which we wrote like this will go from here to here. So, there is a feed here feed will give this feed line q line all right. Beyond this and this now we have different operating line this operating line what we wrote was y s plus 1 equal to l 1 v 1 x s plus d x d plus s x s over v 1, so it has a different slope and different intersects. So, here now again when you have this x s you have the second very equivalent to this q line all right.

So, you must understand that even if we have one feed we have two product now we bring one more product output here our approach has not changed one has to write down one material balance for this, one for this, one for this. So, that we have we at any traces we have two streams, two incoming stream and the two leaving streams are in equilibrium; whether you are in this region or in this region.

Of course, the flow rate here was l v here we have l bar v bar and here we have l one v one. So, we had one feed line one line for x s that we are withdrawing it right if you intersect these two lines you will get another line here if you intersect this line and this line you will get a q line. So, if you know the q line this line you can draw this line if you know this line and q line you can draw the other line.

So, our approach our understanding of this mccabe-thiele method the same this is q line you can call it this as a s line. So, in this example again we are trying to reemphasize that you know in a course or in the in the syllabus. If we have slightly more complex problem given the lecture notes textbook you know one can also approach the similar problems by starting from the first principle.

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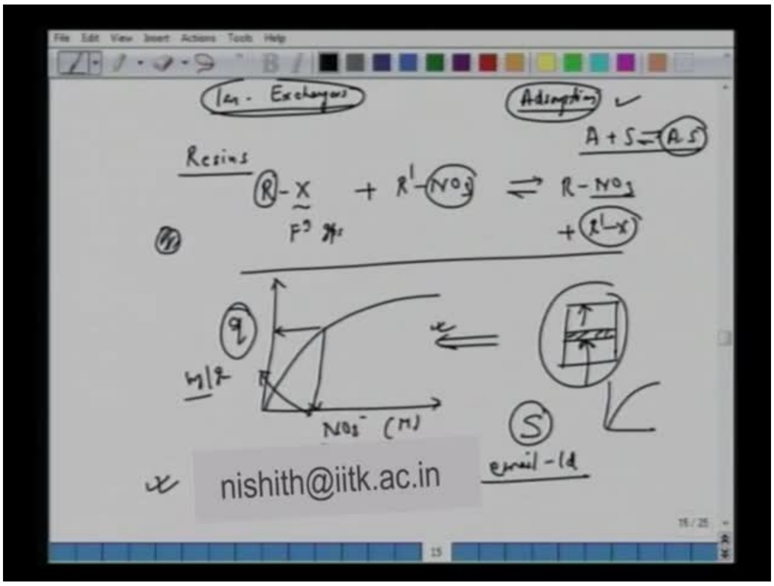

We like to conclude today's lecture with one more example if you recall we said ion exchangers we talked about this ion exchangers we did not have mathematical discussion on this we said that ion exchangers is very similar two adsorption all right. What are the ion exchangers in water purification use solid resins. So, you have certain molecules a substrate on which you had some functional groups. So, these are functional group certain polymeric materials you want to remove nitrate say calcium nitrate sodium nitrate you want to nitrate from waste water you bring in contact with this all you do you exchange ions you know three goes with R and R dash goes with x.

So, essentially you have replaced this nitrate which may be it was more harmful than the one which you have brought in the waste water. Now this reaction here each resin is a solid material it has pores inside when you bring in this contact with this they will in equilibrium here. So, what is the approach of handling ion exchangers like adsorptions one has to develop this equilibrium curve. Given this nitric acid concentrations nitrate ions concentrations in water what is the loading here q.

So, very similar to the isotherm of adsorption you have also this equilibrium curve where you write down this capacity milligram, per gram for this nitrate ions adsorbed or separated by this ratio’s. So, just the names are different fundamental reactions may be different for adsorption we say that A brings in contact with S, it is A dot S some physisorption chemisorption some bond formation has taken place here more like it is a chemical reactions.

But, still you have the similar approach here in case of adsorption we took a species balance differential balance at a distance of x we wrought down wrote down the species in species out by convections. We write it down the similar balance for this ion exchanger its only the rate equilibrium which would be different for this from this at equilibrium here q versus nitric acid will follow one isotherms for adsorption this is a different isotherms.

So, although ion exchangers in this course we did not deal we did not discuss but, it is you should see that it follows the same approach adsorptions. Your basic thing is that your isotherm is different you can go through the textbook again by Treybal or by Dutta all right you will see the same approach. So, again we like to conclude today's lecture that we discussed five unit operations which are very common in chemical engineering absorption, extraction, adsorption, drawing and distillations last but, not the least most popular one.

But, other unit operations like leaching solid and liquid ion exchangers right or we talked about certain more complex problem in the same unit operations instead of having two products we have now three products instead of having one feed we can have two feeds we will have the same approach one should must develop this you know very basic fundamentals of you know mass transfer starting from equilibrium rate process N T U. S T U you can address wide range of complex problems or different unit operations.

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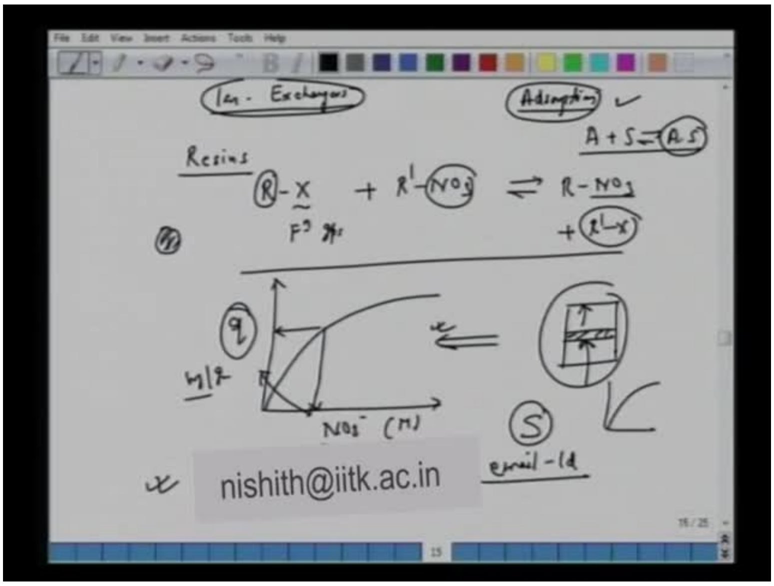

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So, I hope you have enjoyed this course and i can always be contacted by this or at this eemail address nishith at the rate of i i t k dot a c dot in. So, if you have any doubts any question any concern you can contact me at this email address all right. So, with this we have completed NPTEL lecture on mass transfer two.

Thank you.