Mass Transfer II
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Lecture No. # 04

So, now let us make material balance over time on del t.

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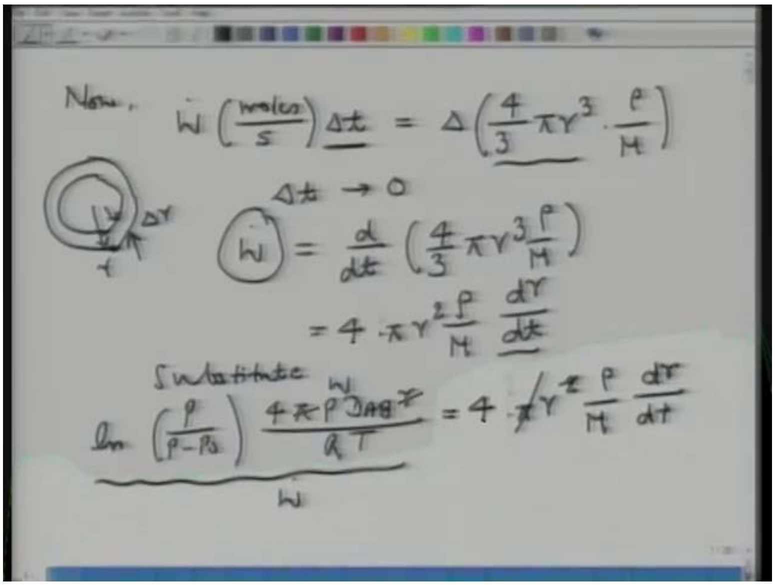

Now,

$$\dot{W}\left(\frac{\text{moles}}{s}\right)\underline{\Delta t} = \Delta\left(\underbrace{\frac{4}{3}\pi r^3}\cdot\frac{\rho}{M}\right)$$

$$\Delta t \to 0$$

$$\boxed{\dot{W}} = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\frac{\rho}{M}\right)$$

$$= 4\pi r^2\frac{\rho}{M}\underline{\frac{dr}{dt}}$$

Substitute

$$\underbrace{\ln\left(\frac{P}{P-P_S}\right)\frac{\overset{\dot{W}}{4\pi P D_{AB} r}}{RT}}_{\dot{W}} = 4\cancel{\pi}r^{\cancel{2}}\frac{\rho}{M}\frac{dr}{dt}$$

So, now we have W dot, which is moles per second, at what rate naphthalene evaporates over del t, and this we equate with rate of change in the mass. So, the volume is 4 by 3 pi r cube. So, this would be the volume multiplied by the density divide by the molecule.

So, essentially we are saying that over this del t time, this naphthalene ball evaporates, and the size decreases by del r. So, we have del r and over this, over this time, the mass decreases or the radius decreases from r to r minus del r. So, which means if we take a limit of del t tends to 0, we can write del W dot as d by dt of 4 by 3 pi r cube cube rho over M, which can be written as 4 by 3 pi r square rho over m dr by dt.

So, we have rate of change of radius of this naphthalene ball equated with respect to with rate of change of this W dot, at what rate the naphthalene diffuses over the length r to infinity. And already we have obtained this expression for W dot diffusion flux moles per second or kg per seconds in our previous lecture, previous slide here. So, if we just substitute W dot, we have ln P over P minus P 0 4 pi P, P is a total pressure, D AB binary diffusivity - diffusion coefficient of naphthalene in air - over R T equals 4 by 3 pi r square rho over M dr by dt. So, we can simplify r with this 4, 4 pi, pi will get cancelled with this, and now we can integrate. So, entire expressions which is x actually W dot, we are assuming that it remains constant over the integral.

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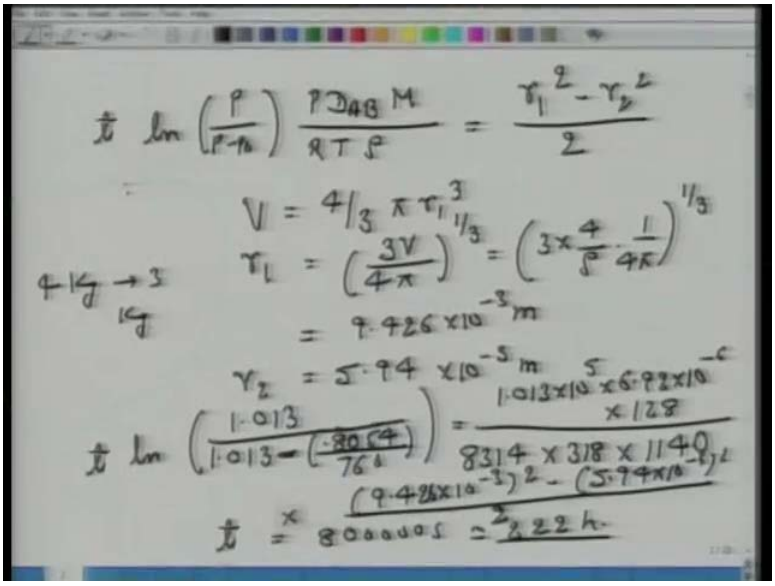

$$t \ln\left(\frac{P}{P-P_0}\right)\frac{P D_{AB} M}{R T \rho} = \frac{r_1^2 - r_2^2}{2}$$

4 kg → 3 kg

$$V = 4/3\,\pi r_1^3$$

$$r_1 = \left(\frac{3V}{4\pi}\right)^{1/3} = \left(3\times\frac{4}{\rho}\cdot\frac{1}{4\pi}\right)^{1/3}$$

$$= 9.426\times10^{-3}\,m$$

$$r_2 = 5.94\times10^{-3}\,m$$

$$t \ln\left(\frac{1.013}{1.013-\left(\frac{.8054}{760}\right)}\right) = \frac{1.013\times10^{5}\times6.92\times10^{-6}\times128}{8314\times318\times1140}$$

$$\frac{(9.426\times10^{-3})^2-(5.94\times10^{-3})^2}{2}$$

$$t \approx 8000.00\,s \simeq 2.22\,h$$

So, dr by dt if we integrate from r 1 to r 2, we will have total time for decrease in the radius from r 1 to r 2 as t ln P over P minus P 0 P D AB M over R T rho equals r 1 is square minus r 2 is square by 2. So, this is on integrations. Now, all we have to do is, substitute the correct values of r 1 and r 2 and find out the total time, which is the question asked.

So, the volume V is 4 by 3 pi r 1 cube equal to 0; so, r 1 is 3V over 4 pi 1 over 3, and the volume we can write in terms of 4 by rho into 1 over 4 pi; so, rho is a density here, and if we recall this t equal to 0, to start with the weight is 4 kg, of the naphthalene's weight.

So, we have r 1 here, which if we substitute, we should get 9. 426 10 to power minus 3 meter; similarly at r 2, when the mass becomes 3 kg here, if we just do normal substitutions, we can get r 2 as 5 . 94 10 to power minus 3 meter. And then, that if we substitute in our expression, which we have obtain on integrations we have, t ln, t is a total pressure 1.013 over 1.013 minus the vapour pressure or sublimation pressure is 0.8054 over 760 equals 1.013 10 to power 5 total pressure diffusion coefficient is given 6.9210 power minus 6, molecular weight is 128, r is 8314, temperature 38 Kelvin, density is 1140; this equals r 1 square minus r 2 square and r 1 is 9.426 10 to power minus 3 square minus 5.9410 to power minus 3 square over 2.

And then, if we calculate t will equal approximately 8000 seconds, which is nothing but 222 hours. So, it takes a long time for the mass of naphthalene ball to get down from 4 kg to 3 kg. So, the example, which we have chosen here is nothing but diffusion in a spherical coordinates, and you should be able to compare this example to the previous example which we had, where we had this Cartesian coordinate system.

So, two examples are similar: one is in Cartesian coordinate, and one is in spherical coordinates; the difference is that under study state, in case of coordinates - Cartesian coordinates kg per second or kg per second per meter square of diffusion plane remain constant. In case of spherical coordinate, this only the kg per second, which is conserved, kg per second per meter square or moles per second per meter square, it changes with the radius.

So, these are the three different examples we have taken, for very basic examples to calculate diffusion flux any over a diffusion length del Z given operating conditions like pressure, temperature and the concentration gradient.

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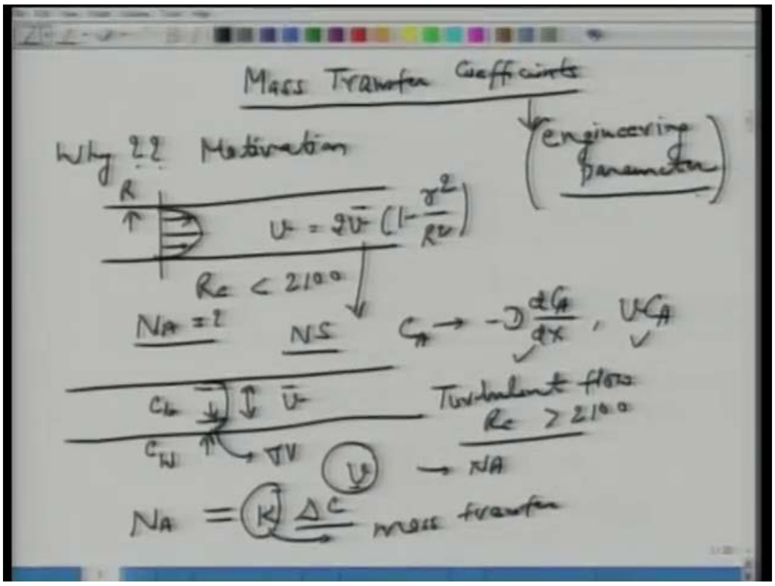

Now, what we do next? We take up a new topic - Mass Transfer Coefficients; we may be familiar with this Mass transfer coefficients; now, before we get into this, we must ask this question - what is this coefficient? Is it a fundamental variable? Is it a physical variable defined here or it something engineering parameters?

The answer is that mass transfer coefficient is not a fundamental variable, fundamental quantity, it is a engineering parameters; it has been introduce or it has been define to make our computation simpler. Now, if we can go back to all those three examples, we said that we had one example, where we had counter diffusion or where we had a stagnant, you know, medium B etcetera; we could calculate, how much the diffusion flux over this diffusion length set, the problem comes when the flow becomes complex.

So, it is a hydrodynamics; we said in the beginning class, you know, you should have some knowledge of hydrodynamics. So, it is a hydrodynamics, the fluid flow and if the fluid flow is complex, it may be difficult, by difficult to solve for the velocity profile, and then predict is diffusion flux; in those cases and mass transfer coefficients become an engineering parameters, which is we have to estimate from some given operating conditions then, we can we can apply this to calculate this diffusion flux.

So, let us take few examples, where if we are trying to say that, we should make use of mass transfer coefficients or we may not be able to make use of that general expression, which we obtain for diffusion flux, in terms of diffusion length etcetera.

So for example, so the question we are trying to answer here is that, why there is a necessity for defined for defining this mass transfer coefficient? or what is this? Motivation; so, if we recall, if we have a tube, and if the Reynolds number is less than 2100, we have well defined velocity profile, which is parabolics; V equal to we can write 2 V bar, V bar is (( )) velocity 1 minus r square over r square.

So, this would be the radius of (( )); now, if we have the velocity profile like this, in a tubular, and if we are asked to calculate diffusion flux N A, it is not be a difficult thing, in the sense that we can substitute this velocity profile in our Navier Stokes equations, one can solve for the concentration profile and then, we can calculate diffusion flux etcetera. We know the bulk transport, it would be V into C. So, once we know the diffusion flux, once we know the bulk transport, one can predict the diffusion flux.

So, the problem is not here; this is a straightforward case, where we have a tubular flow and we have well defined velocity profile, things cloud be more complicated, if the

Reynolds number goes very high; say turbulent, if we have turbulent, then we call from hydrodynamics, you will have a tubular flow and the velocity profile will start becoming, will become flatter.

So, here we have velocity profile, where in the core, the velocity is almost really constant at some V velocity or V bar, but near the wall, we have a gradient - a velocity gradient; now as it is, in the case of turbulence or turbulent flow, and the Reynolds number is much larger or larger than 2100, there is un predictions, there is a un certainty in predicting this velocity; as a consequence, we cannot apply exactly the Navier stokes equations, and we can solve for this diffusion flux; in those cases, we will like to say that yes, if we know the concentration gradient or we know the concentration difference from C wall to say C bulk; one can one should be able to write that diffusion flux equals some constant K into this concentration difference (( )). So, this is what we are trying to say the here that this K is nothing but mass transfer coefficients.

In principle if hydrodynamic is well defined, if we have the velocity profile like in laminar flow or viscous flow, there is no necessity for mass transfer coefficients or defining this mass transfer coefficients; it is very important that we understand the motivation, mass transfer coefficient is an engineering parameter. It is an engineering parameter, it is not a fundamental variable and this type of parameter is required, when we have uncertainty in hydrodynamics, in the prediction of velocity profiles, there this mass transfer coefficients become become quite (( )).

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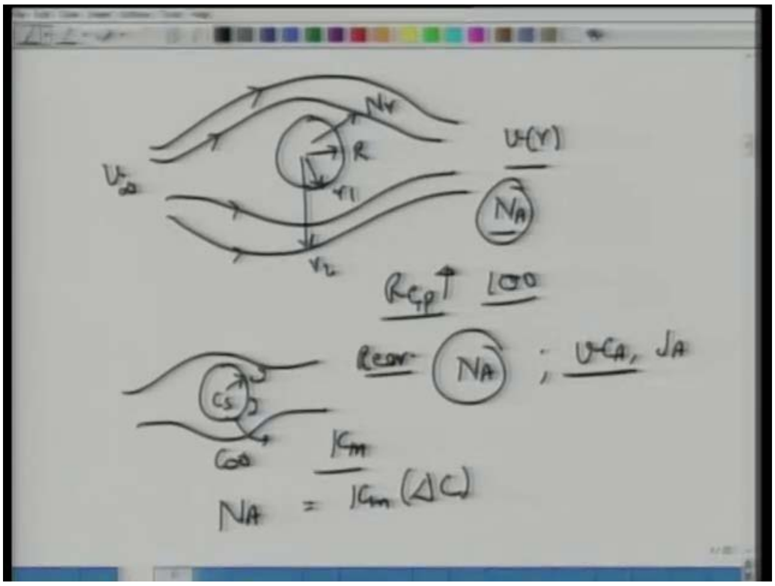

We will take, for example, once again suppose, we have this sphere and this naphthalene ball, it evaporates similar to the previous case, where we have this velocity profile like this; so the fluid approaches from infinity to this V infinity, we have this radius of this ball, which is R and we have been asked the same questions - what is the diffusion flux here? N r; now, if the velocity profile is known to us; V r we can one can predict this diffusion flux N A over a length say between r r 1 or between r 2.

If the velocity profile is well known defined, one can calculate this N A starting from navier stokes equation, spherical coordinates etcetera; the problems comes, when we have very high reynolds number; and also very large say, but say 100 Reynolds number based on the particle size if it is very large say 100, then there is uncertainty in the velocity profile; and if you recall from hydrodynamics, you will have cases of boundary layer separation etcetera or the eddies, which is form at the of this rear end.

In which case, it is a to predict diffusion flux, in terms of your bulk transport VCA and diffusion flux, it becomes quite, it involves lot of error; in the in these cases is possible to define a mass transfer coefficient quantity and to say that, we have the flux now, which we can write as diffusion fluxed N A as mass transfer coefficient into some concentration difference, where del C is concentrations difference between C S at the surface and this C infinity.

So, we are trying to make a case here, what is the motivation of mass transfer coefficients ? Why we have to define this quantity and where we can get away without defining this mass transfer coefficients, we saw two examples, where we have very straightforward case, where we have a tube and certain pool of the liquid evaporates or we have a naphthalene ball, which evaporates; in those cases we said that, let us assume that N B is 0, you know bulk transport is 0, and then we can calculate this diffusion flux.

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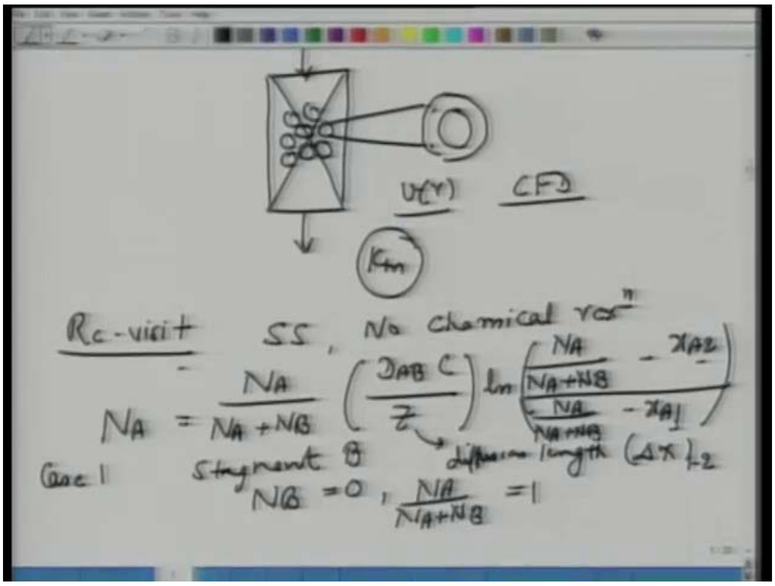

Re-visit SS, No chemical rxn

$$N_A = \frac{N_A}{N_A+N_B}\left(\frac{D_{AB}\,C}{z}\right)\ln\left(\frac{\frac{N_A}{N_A+N_B}-x_{A2}}{\frac{N_A}{N_A+N_B}-x_{A1}}\right)$$

$z \rightarrow$ diffusion length $(\Delta x)_{1-2}$

Case 1 Stagnant B

$N_B = 0$, $\frac{N_A}{N_A+N_B} = 1$

So, if suppose we have a packed bed, you know certain more complex situations, like we have packed bed, in which different spherical particles or stack stack like this, and the fluid goes like this.

So, if we look at one sphere or one spherical particles, there is a quite complex velocity profile surrounded around this, to predict velocity profile exactly, one requires you know, things likes software engineering to like CFD - Computational Fluid Dynamics - then maybe you to some extent, we can predict the exact nature of this process; otherwise it is a very difficult to predict in the fluid flow inside the pack.

In such cases, again the concept of mass transfer coefficients is quite important here. So, we will like to go back to again the general expression, which we obtain for diffusion flux and see, how the same expression can also be rearranged or can also be modified to introduce these engineering parameters, K mass transfer coefficients.

So, let us re-visit this expressions for, now we assume a study state, no chemical reactions. So, we have N A equals N A over N A plus N B we wrote D AB C over Z ln NA over N A plus N B minus x A2 we can write in terms of mole fractions at plane 2 over N A N A plus N B minus x A1.

So, this Z is nothing but diffusion flux - diffusion length - and x A1 and x A2 is your mole fractions across the plane 1 and 2; now, if you take the again the simpler case of case 1, where we say that B is a stagnant, stagnant B, we can write N B equal to 0 with N B equal to 0, N A over N A plus B is 1.

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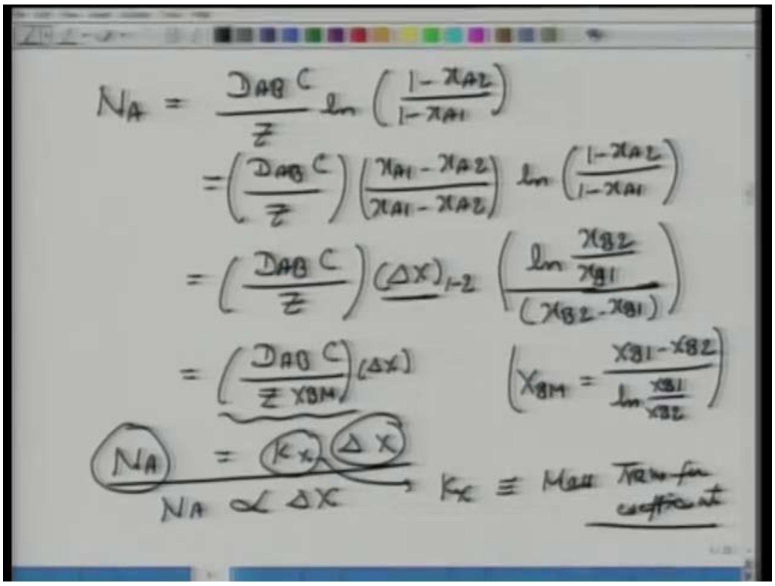

$$N_A = \frac{D_{AB}C}{Z}\ln\left(\frac{1-x_{A2}}{1-x_{A1}}\right)$$

$$= \left(\frac{D_{AB}C}{Z}\right)\left(\frac{x_{A1}-x_{A2}}{x_{A1}-x_{A2}}\right)\ln\left(\frac{1-x_{A2}}{1-x_{A1}}\right)$$

$$= \left(\frac{D_{AB}C}{Z}\right)\underline{(\Delta x)}_{1-2}\left(\frac{\ln\frac{x_{B2}}{x_{B1}}}{(x_{B2}-x_{B1})}\right)$$

$$= \underbrace{\left(\frac{D_{AB}C}{Z\,x_{BM}}\right)}(\Delta x) \qquad \left(x_{BM} = \frac{x_{B1}-x_{B2}}{\ln\frac{x_{B1}}{x_{B2}}}\right)$$

$$\underline{\boxed{N_A} = \boxed{k_x}\boxed{\Delta x}} \rightarrow k_x \equiv \text{Mass } \underline{\text{Transfer coefficient}}$$

$$N_A \propto \Delta x$$

And then, N A equals or simplified to D AB C Z ln 1 minus x A2 over 1 minus x A1.We can also write like D AB C Z we can divide and multiply by x A1 minus x A2 over x A1 minus x A2 ln 1 minus x A2 over 1 minus x A1 all we are trying to do the similar expressions we have we have rearrange written like this, and then further we can write D AB C over Z del X.

So, now we are introducing this mole fraction difference between plane 1 and 2, which is nothing but x A1 and x A2 and here what we have, we have ln x B2 mole fraction of B at plane 2 1 minus x A2 and B is a stagnant here, x A1 and x A1 minus x A x A 2 can also be written sorry this should be x B1.

So, we have x B1 and then, x A1 minus x A2 can also be written as x B2 minus x B1. So, subtract 1 minus this, which we now we have an expression for mole flux, in terms of D AB C Z into ln if you recall, if you try to recognize this term here, we have x B2 minus x B1, a quantity like this, we call it x BM. So, logarithmic average which is nothing but x B1 minus x B2 over ln x B1 x B2.

We just like, we have an arithmetic average or geometrical average or harmonic average, we have been able to obtain an expressions for diffusion flux of A, when B is a stagnant in terms of known quantities here, and concentration difference we have x BM.

Now, here comes the concept of this mass transfer coefficients, the entire quantity on the left in the parenthesis, we can call it as K x into del X. So, what we have obtained here? We are trying to say that diffusion flux, it is a proportional to del X and the proportional constant, which we got as K x or we are writing as K x, we are calling it as a Mass Transfer Coefficient.

So, again let us go back, what we said in the beginning? What is the motivation of introducing this mass transfer coefficients? What was the objective - we said that in those cases, where the hydrodynamics are complex, it is difficult to solve the velocity profile or there is uncertainty in solving the velocity profile of fluid flow; just like a flow in a packed bed very complex situation there or we have high Reynolds number, when we have a turbulent flow not viscous, not laminar then, there is a uncertainty in predicting the velocity profile.

In those cases, it is not possible to solve analytically, an expression for diffusion flux which we have obtain so far, we took two examples of case 1 and case 2. So, all we are trying to say here that, we are trying to introduce the concept of mass transfer coefficient, is the weight is defined, diffusion flux N A equals to quantity K X, which we are calling at a mass transfer coefficients into del X; del X is nothing but your concentration difference, is your mole fraction across the plane 1 and 2.

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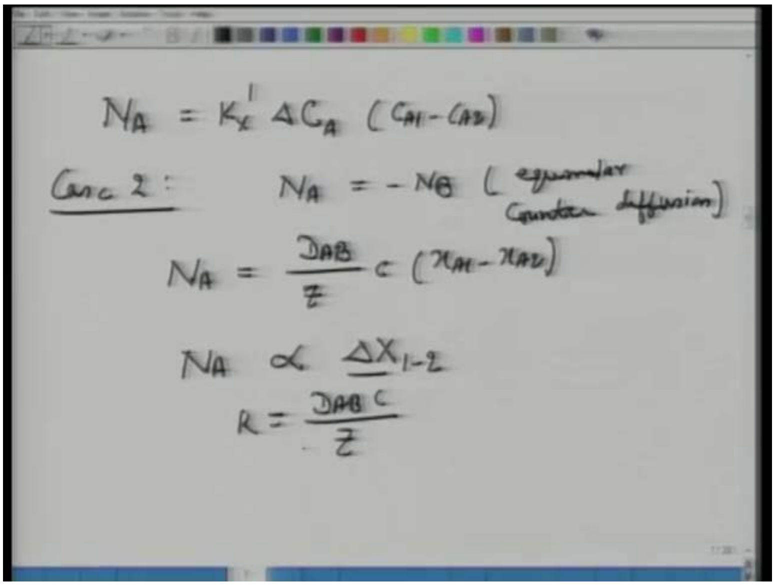

$N_A = K_c' \Delta C_A \; (C_{A1} - C_{A2})$

Case 2: $N_A = -N_B$ (equimolar counter diffusion)

$N_A = \frac{D_{AB}}{Z} c \, (x_{A1} - x_{A2})$

$N_A \propto \Delta x_{1-2}$

$K = \frac{D_{AB} c}{Z}$

we can also write N A equals K X some different prime, we can give into del C, where C is now the concentration difference C A1 minus C A2.

So, there are different ways of writing this diffusion flux; in case of the case 1, when the when the other medium, other component B was stationaries, we can write N A like K X or some proportional to concentration difference or mole fractions; let us take case 2, when we said that we have equimolar diffusion. Let us see, if we can write in a similar expression or we can develop similar expressions for diffusion flux. So, case two we studied in the last class earlier classes, we had N A equals minus N A which is equimolar counter diffusion.

So, when we have N A equals N B, then N A equals we can go back to the same expressions, which we have started earlier; we can substitute N A equal to minus N B to write N A D AB Z C x A1 minus x A2, which is even simpler than the previous case, and again we have the same situations we are obtaining expressions for N A, we are seeing that is proportional to mole fractions across the plane 1 and 2, and here the proportional constant K equals D AB C over Z.

So two cases which we studied here, very simple very fundamental; they form the basis for all our calculations or all the examples, which we had in the previous classes.

Now, we are obtained to say that we have same expressions, we have arrange in a different way, and we are getting the similar expressions that diffusion flux is a proportional to concentration across plane 1 and 2 or mole fractions across 1 and 2 or if we have gas instead of liquid, we can say that diffusion flux is a proportional to difference between the partial pressure at plane 1 and plane 2.

In these cases, of course, there is no necessity, the example which we took to define mass transfer coefficients, but as we said earlier that if we have very complex hydrodynamic situations where the velocity profile cannot be solved with accuracies mass transfer coefficient becomes an engineering tool, we can define N A diffusion flux equals a quantity K M mass transfer coefficient multiplied by concentration difference - the potential difference between the mole fraction, difference between the concentration difference between the partial pressure of 1 and 2, and all complexities of hydrodynamics will be now embedded in mass transfer coefficients.

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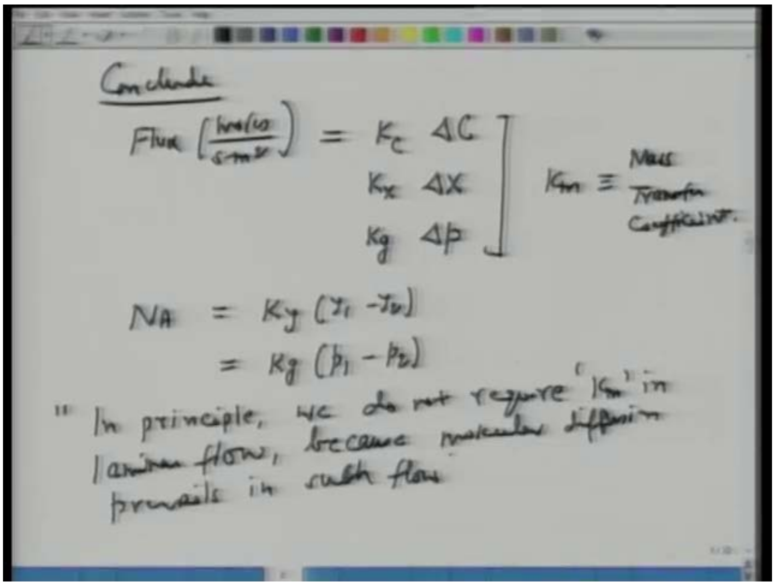

Conclude

$$\text{Flux}\left(\frac{\text{kmoles}}{\text{s m}^2}\right) = \left.\begin{matrix} K_c\ \Delta C \\ K_x\ \Delta X \\ K_g\ \Delta p \end{matrix}\right] \quad K_m \equiv \text{Mass Transfer Coefficient.}$$

$$N_A = K_y(y_1 - y_2)$$
$$= K_g(p_1 - p_2)$$

" In principle, we do not require 'K_m' in laminar flow, because molecular diffusion prevails in such flow

So that means, we can conclude that flux moles per second per meter square can be written as K M into del C or can be written as del X or it can be written as K into del P; of course, the values of K will be different, different in all cases.

Generally we write K x, we write k g, if it is a gas here and instead of if we using concentrations, we can write as k c, all of this is a mass transfer coefficients k m mass transfer coefficients.

So all it means, if we have gas system, we can also write N A as k y into y 1 minus y 2; here y 1, y 2 are the mole fractions or we can write as k g P 1 minus P 2 where P 1 and P 2, these are the partial pressure of 1 at the plane 1 and 2.

Let us write down here, the In principle, in principle we do not require K for mass transfer coefficient k m in laminar flow, why because laminar flow is well defined is a flow, we have seen in case of tube, we have analytical Navier Stokes solutions V equal to V max 1 minus r square over r square.

So those cases, where the velocity profiles are well known; there is no need to to define K m to calculate a molar molar diffusion flux, but we do not require K M in laminar flow, because molecular diffusion prevails prevails in such flows.

In fact, this should like to recall, the hydrodynamics - the concept of friction factor and track coefficients, if you are asked to calculate pressure drop in a tube, you can use fanning equations, equations which contains friction factor, we can also use Eigen Poisson's equations, if the Reynolds number is less than 2100.

But if the Reynolds number is larger, much larger than 2100, then is use friction factor fanning equations, which contains friction factor, then we rely on experimental data or Moody's plot, which is obtain from some experimental data to predict this to calculate or estimate this friction factor.

So, the idea is that use of fanning equations, we can use fanning equations very well in case of laminar as well in case of turbulent; in case of laminar flow, if we use friction factor is 16 over Reynolds number, which can be obtain if we are equate this fanning equation with Eigen Poisson's equations or we did Eigen Poisson's equation, we had the velocity profile, we said that the pressure drop equate with the shear stress multiplied by the surface area etcetera.

One can calculate and expression for pressure, then you define friction factor in terms of pressure drop and the kinetic energy per unit volume etcetera; show that friction factor 16 by reynolds number, but if the flow is turbulent then, we have to rely on certain experimental data, because in turbulent there is uncertainty uncertainty in predicting velocity profile.

Similarly, C D track coefficients, we said C D equals to D 4 by Reynolds number, if Reynolds number is less than 1 or if we have a viscous form, but how do we get C D equal to 24 by Reynolds number, we start from the velocity profile, we take a spherical geometries, solve Navier Stokes equation, the spherical coordinates, we obtain the velocity profile we solved the for the gradient then, we calculate shear stress and then, we obtain the force 6 by mu v r which we call it stokes law.

If we take this force and defined track coefficients then one can show that C D equal to 24 by Reynolds number is nothing but a calculated, its analytical expression for that coefficients, what happens? If the Reynolds number is very large in that case C D 1 has to rely on certain plots or experimental data.

So, the motivation of defining mass transfer coefficient here is also the same, if the flow is well described like in laminar flow in viscous flow, there is no need to define mass transfer coefficient; we can use expressions for N A in terms of diffusion length Z and we can do the calculations, but if there is a complexities like in a packed bed tower or turbulent flow in a tube, otherwise or the flow past is simple sphere, but at high Reynolds number.

So, wherever there is a complex situations, velocity profile is not accurately known; mass transfer coefficient quantity is defined to predict this diffusion flux in terms of mass transfer coefficient and concentration difference, but the partial pressure across that diffusion flame and diffusion planes.

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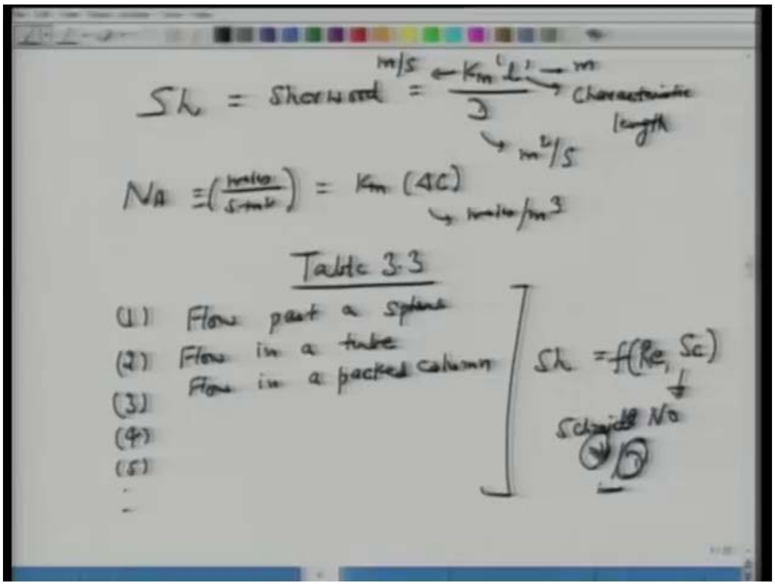

$$Sh = \text{Sherwood} = \frac{\overset{m/s}{K_m}\,\overset{m}{L}}{\underset{m^2/s}{D}}$$

Characteristic length

$$N_A = \left(\frac{\text{moles}}{s\,m^2}\right) = K_m \underset{\text{moles}/m^3}{(\Delta C)}$$

Table 3.3

(1) Flow past a sphere
(2) Flow in a tube
(3) Flow in a packed column
(4)
(5)

$$Sh = f(Re, Sc)$$

Schmidt No

How do we get mass transfer coefficients? One has to rely extensively on experimental data and in those cases, we will see that we defined a quantity like Sherwood number, which is k m l over diffusion coefficients this l is characteristics length which means different geometry, different situations l will be different, d is diffusion coefficients, which is meter square per second, l is meter and k m is meter per second mass transfer coefficients.

If we define as N A moles per second per meter square equals k m into del C, concentration different moles per cubic meter. So, when literature you will see that, there are several cases, which people have a studied and there they have estimated mass transfer coefficient in term in terms of certain one-dimensionless groups like Reynolds number, Schmidt number to from, which we obtain this mass transfer coefficient game and then, we can predict diffusion coefficients.

So, you are required to visit a table in (( )) which is table 3.1, you will see that table given in (( )) 3.3 (( )), where we have several such cases 1, 2 listed; we will just note down few of them 1, 1 is very common, if you have flow past is sphere.

So, if you have a flow past sphere or you have flow in in a tube or flow in a packed column. So, there are several such examples given in this table, where Sherwood number in all these cases, Sherwood number has been given in terms of mostly, in terms of Reynolds number and Schmidt. So, S C is Schmidt number, which is nu over d. So, kinematic viscosity nu beta square per second and we have this binary mass transfer coefficients beta square per second dimensionless numbers. So, you are required to visit this table 3.3, and see how we can defined this mass transfer coefficients ; the idea is that all these correlations, which have been developed in terms of Reynolds number and Schmidt number.

They are extensively dependent on experimental data. So, when you take up a problem or in example, you must do a literature survey you must go to visit such table like 3.3 (( )) or various chemical engineering handbook to use an appropriate correlations, from which you can calculate this mass transfer coefficients.

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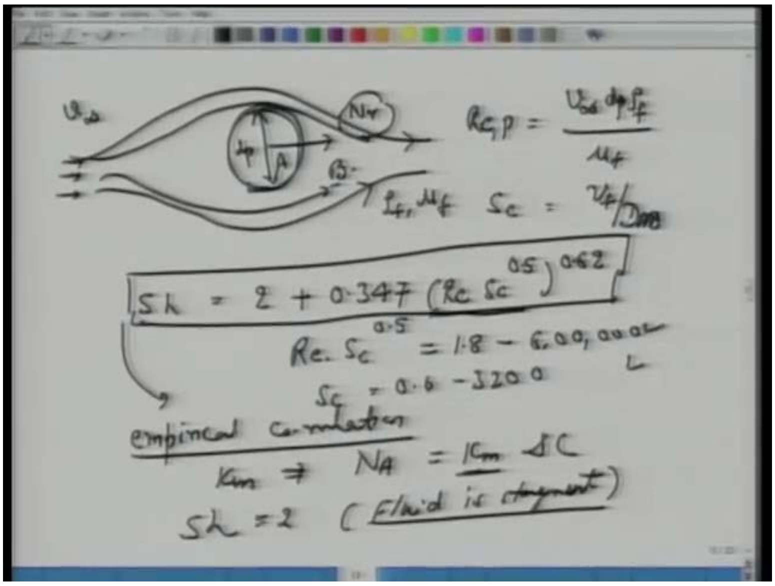

Then you can predict there is you know diffusion flux we just for an example, suppose we have what is given you have singular sphere and we have a flow like this right. So, it is a similar example that we have a sugar ball or salt or naphthalene ball and we are we have been asked to predict this mass diffusion flux N.

So, if we have some fluid flow here, there is a Reynolds number based on the particle size, which we can define asV infinity flow velocity at a very far a stream is v infinity, d p diametric of the particle, rho of the fluid and mu a rho f and mu f these are the physical properties of the fluid. We can define Reynolds number; similarly, we have Schmidt number nu f over D AB binary diffusion coefficients of A in B. So, in these cases correlation is given as Sherwood number equals 2 plus 0.347 Reynolds number, Schmidt number, it is 0.0 .5, 0.62 more important to note here is that the author has given the range of Reynolds number into Schmidt number to the power 0.5 this quantity. This expression holds good with the values is between 1.8 to 600000, and Schmidt number between 0.6 to 3200.

Now, this is an empirical correlations empirical correlations all it means, this has been obtained based on certain experimental conditions, where we meet these two restrictions and then, once we know Sherwood number, we can calculate K m, once we have K m, we can predict diffusion flux N A as K m into del C. The idea here is that Reynolds number and a Schmidt number certain conditions is not possible to defined this velocity profile or to calculate the velocity profile and to obtain the expressions for N A, if fluid is a stagnant you can make out the this term will drop and then, we have Sherwood number equal to 2.

Ah Sherwood number equal to 2 when the fluid is stagnant, one can obtain analytically it is an exercise does not it is a size, you can try Sherwood number to obtain Sherwood number equal to 2, if the fluid is stagnant. So, we are trying to give the same message again and again at definition for mass transfer coefficient, its purely arbitrarily it is an engineering parameter it is not a fundamental variable it has been defined to make our calculations simpler in those cases where there is a complexities of defining or solving this velocity profile.

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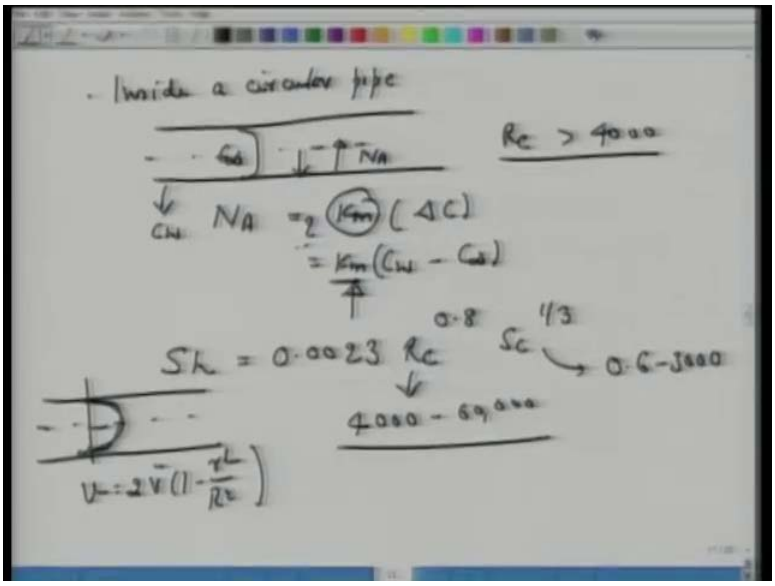

- Inside a circular pipe

$Re > 4000$

$N_A = K_m (\Delta C)$

$= K_m (C_w - C_b)$

$Sh = 0.0023 \, Re^{0.8} \, Sc^{1/3}$

$Re \rightarrow 4000 - 60,000$

$Sc \rightarrow 0.6 - 3000$

$v = 2\bar{v}\left(1 - \frac{r^2}{R^2}\right)$

So, similarly we can take one more example like, we have flow inside a circular pipe. So, if the flow is inside this pipe, the Reynolds number is very large or say Reynolds number is 4000 and greater than the velocity profile will not be parabolic, and we have some flat velocity profile; in this case if you are ask to calculate, diffusion flux from the wall let us say, we have certain species was diffuses from the wall or to the wall; in this case N A can also be written as mass transfer coefficient into concentration difference which means; we can write k m as C wall minus c (( )).

So, we have C W wall and we have very far from the wall C. So, this is the advantage of defining this mass transfer coefficients, complexities of this complexities of the velocity profile, Reynolds number is embedded now in mass transfer coefficients. And how do we get came? So, there is a correlations given in terms of we get Sherwood number, which would be 0.0023 Reynolds number, the power 0.8 Schmidt number the power 1 by 3 and then the author has given a range that this equation is valid between 4000 and 60000, look at the range here is the turbulent flow, it is not like the previous case or the simpler case.

When we have the flow is laminar, and we have the velocity parabolic velocity profile like V equals 2 V bar 1 minus r square over R square. So, in case of turbulent flow, where there is uncertainty uncertainty predict in this velocity profile, you can come out with this correlation author has come out with this correlation based on the experimental data with the range with the restriction the Reynolds number is between 4000 and 60000 Schmidt number is between 0.6 to 3000

So, once again we have given this example that would why where there is a necessity of defining this mass transfer coefficients; similarly, we can take one more example, which is more engineering flow in a packed bed. So, if you recall in the first first two lectures, we talked of this packed columns, we have the absorption column and we have sulphur dioxide which air and we want to remove by absorptions. So, generally in these cases, you have a column pack with certain packings to improve contact between the gas and the liquid.

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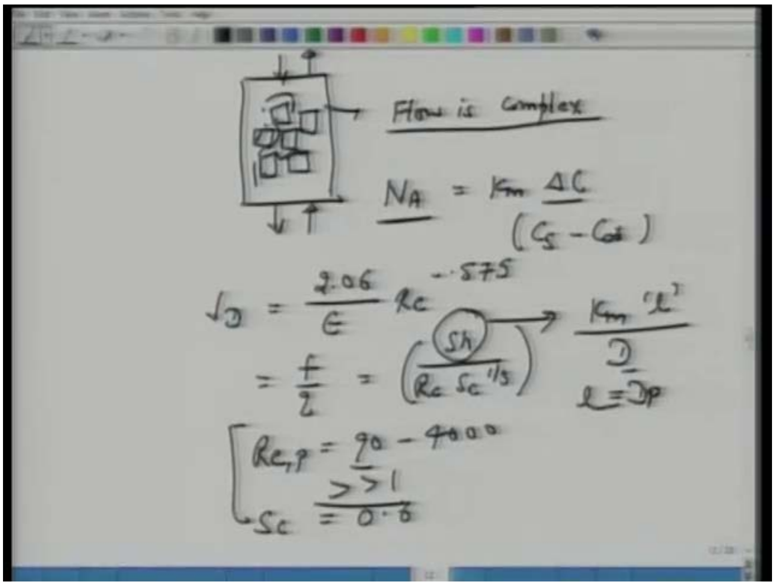

So, if you have the packing like this, where we have certain packings, start now could be spherical could be cylindrical etcetera. So, quite complex velocity profile, the liquid goes like this and gas flows through this, cross this between the pores - micro pores of cylinders and this air this gap here, the flow is quite complex, in which case again the necessity of mass transfer coefficient becomes indispensible.

And this case, also we can write mass diffusion flux N A as K m into del C concentration difference between the surface and bulk. So, infinity away from this packing etcetera. So, in this case also Sherwood number, now in this case diffusion flux or sorry I am (( )) mass transfer coefficient is estimated from factor called J D, which is related to 2.06 over epsilon Reynolds number minus 5.75 equal to friction factor by 2, but this J D is also equals Sherwood number over Reynolds number into Schmidt number 1 over 3.

Again, we have the similar story here to calculate mass transfer coefficients, we required Sherwood number, which is K m characteristics length over diffusivity; in this case l would be equal to the diameter of this particle of the packing etcetera or equivalent diameter of this packing. So, from here, we can calculate the mass transfer coefficients and can predict the diffusion flux; in this case also there is a restriction the Reynolds number must be between 90 to 4000.

So, in this Reynolds number is we can defined with respect to the diameter of the particles and whether Reynolds the Reynolds number is 90 to 4000; all it means, must larger than 1 and we have a turbulent flowing.

So, the velocity profile is quite complex and they hence, there is a necessity for defining mass transfer coefficient Schmidt number is given as 0 .6.

So, essentially, again we have taken three examples to show that mass transfer coefficient is an engineering parameters, it has to be estimated, it has to be correlated from the correlations available in the literature, if the situation which you are trying to model or which you want to study for that situation, if the mass transfer coefficient is not given and one has to obtain experimentally.

In those situations where the Reynolds number is very small flow is well defined viscous laminar, one can you know invoke this Navier Stokes equation calculate velocity profile or the velocity superimpose on species balance and can solved for the concentration gradients, once we know the concentration, we can solve for diffusion flux.

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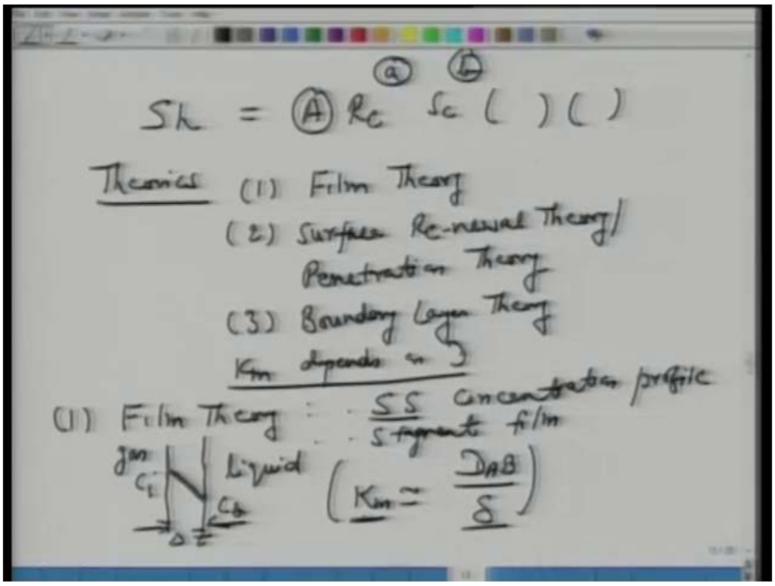

$$Sh = \textcircled{A}\, Re^{\textcircled{a}}\, Sc^{\textcircled{b}} (\ )(\ )$$

Theories (1) Film Theory
(2) Surface Re-newal Theory/
Penetration Theory
(3) Boundary Layer Theory
K_m depends on δ

(1) Film Theory : SS Concentration profile
: Stagnant film

$$\left(K_m = \frac{D_{AB}}{\delta}\right)$$

So, what we do here now we take up three well known theories. So, we are writing here that all this Sherwood number in generals can be written as a function of Reynolds number, hence with in general of course, there could be more complexities here.

But the question is that in all these cases A a and b this this numbers have to be determined experimentally, there has to be some experiment in this to a calculate this this different coefficients here there are well theory, well known theories like film theory surface renewal theory and boundary layer theories these three theories are quite popular to which tells us that this mass transfer coefficient; in some cases will be proportional to diffusion coefficient, in some cases in this will be a proportional to square root of diffusion coefficient etcetera. So, these theories are also widely used to predict or to obtain such correlations between Sherwood number, Reynolds number and Schmidt number.

So, we will take up these theories 1 by 1, we will not get into mathematical details, but just to get an idea that how does this Sherwood number is dependent on this diffusion coefficients. So, there are well three theories there are several theories the 1 the 3, which are very common one is film theory the second is surface renewal (( )) or (( )) we will see there is some difference between surface renewal theory and penetration theory.

And third is boundary layer theory, you may have done some extent boundary layer theory in your previous class etcetera, on fluid mechanics or hydro dynamics, we will just go through very briefly through this theories to see that, how does mass transfer coefficients is depends on what is K m depends on diffusion coefficients.

So, the first very popular theory is film theory and this film theory is applicable, in general if you assume that, we have a steady state concentration profile and we have we have stagnant film. So, essentially what this theory says that, we have say two streams of suppose, we have gas here and we have liquid across this we assume that the film here the concentration profile, in this liquid phase is linear that is one it is a steady state profile does not change with time.

So, we have say C g here and we have concentration gradient just at the interface and we can use C i. So, we have C i here and we have this C bulk or concentration at if of very far distance we have this diffusion length del Z. So, the fluid fluid is a stagnant film is a stagnant here, and we have a steady state concentration profile; in this case, one can show that this mass the film theory predicts that mass transfer coefficient is approximately equal to D AB over del.

Now, again we are not going into details, but theory suggets suggests that mass transfer coefficient is proportional to diffusion coefficients and one over del. So, which means larger the thickness film thickness smaller is the mass transfer coefficients

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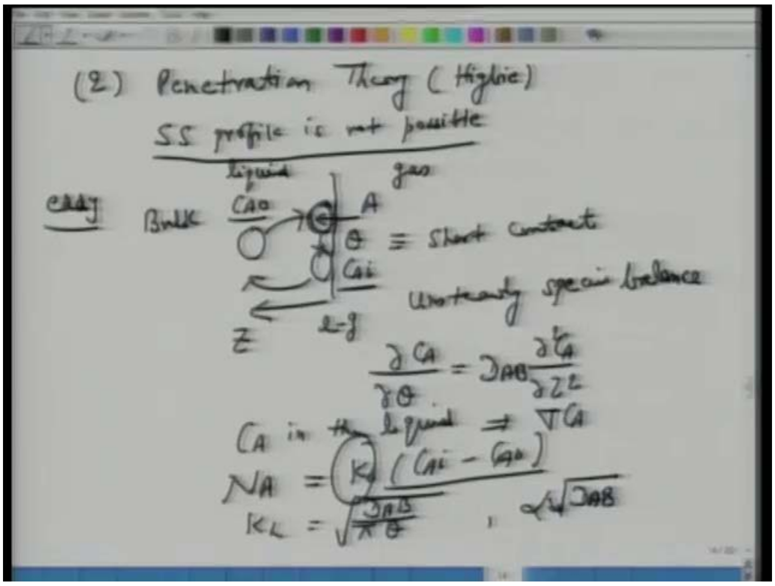

(2) Penetration Theory (Higbie)

S.S profile is not possible

θ ≡ Short contact

unsteady species balance

$$\frac{\partial C_A}{\partial \theta} = D_{AB}\frac{\partial^2 C_A}{\partial z^2}$$

C_A in the liquid ⇉ ∇C_A

$$N_A = \boxed{K_L}(C_{Ai} - C_{A0})$$

$$K_L = \sqrt{\frac{D_{AB}}{\pi\theta}}, \quad \propto \sqrt{D_{AB}}$$

Similarly, we have penetration theory, this was originally proposed by higbie and this penetration theory it says that steady state profile is not possible profile is not possible. So, this is in contrary to what we have in case the diffusion flux.

So, let us see what try to understand what is the penetration theory says, which is a different from the film theory here also we have suppose, we have this gas here and we have the liquid here. So, as what we have in case of penetration theories, there is something called eddies; if you recall from here hydro dynamics at high reynolds number, we have this hypothetical pockets of liquid, which moves from the bulk to the surface.

This eddy is stays here for sometime, in which this gas molecule the diffuses with insulate. So, there is certain contact time of this eddies at this surface. So, this is your liquid gas surface at which, there is a contact time of theta when the eddies is stays, before it gets back to the bulk.

So, in other words is a very short contact short contact between the two phases and we have a mass transfer from the gas to the liquid, in which case, it is not possible for this steady state profile to establish or or profile to reach a steady state, in that case penetration theory becomes quite popular and more realistic, it has been shown that in such cases and a steady state species balance can be written del C over del theta, if A is component, which is diffusing from gas to liquid phase equals D AB del square C A over del Z square.

So, z would be this direction, one can start from here. So, t equal to 0, may be the concentration here was C A 0 and at certain time this interface is exposed to C i then we can write this expression I can solve to obtain concentration profile of C A in this liquid, and once we have concentration file in the liquid, we can calculate the concentration gradient and one can calculate this diffusion flux.

In this case, it has been shown that diffusion flux N A will also be equal to k L C A i minus C A 0 this what we will expect proportional to mass transfer coefficients and concentration difference between the interface and the bulk concentrations except this K L, now equals to square root of D AB over pi theta.

So, this is a difference between the film theory here now k L is proportional to the square root of D AB, unlike the film theory is predicts mass transfer coefficients is linearly proportional to diffusion coefficients. So, one we had film theory, when we assume that the profile is a steady state film is a stagnant and it is linear concentration gradient is linear between the two planes, unlike in case of penetration theory, which says that short there is a very short contact between the gas and liquid; in other words there is no time for the concentration profile to reach a steady state. In that case, we write another steady state balance, one can solve for concentration gradient and can calculate the diffusion flux show that mass transfer coefficient is a square root proportional to square root diffusion coefficients.

Similarly, we have one more theory, which is a boundary layer theories you may done in case of hydrodynamics.

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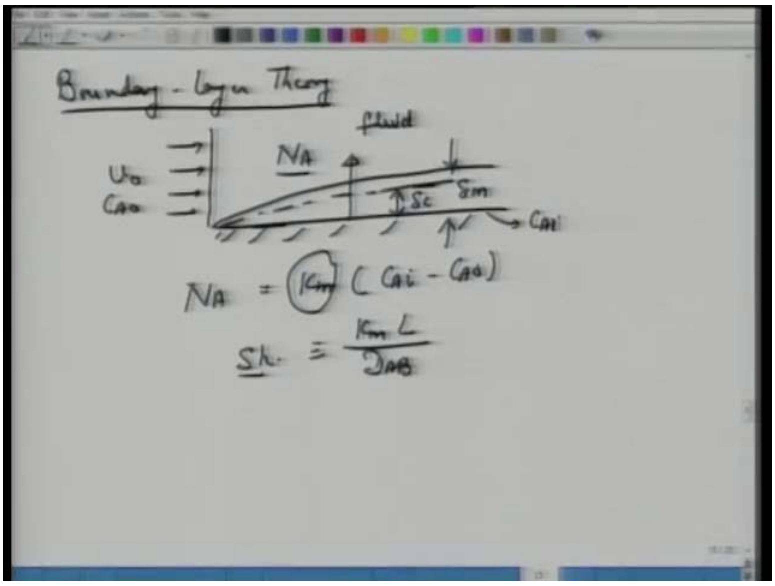

Boundary - Layer Theory

$$N_A = \boxed{K_m} \left(C_{Ai} - C_{A0} \right)$$

$$Sh = \frac{K_m L}{D_{AB}}$$

So, we will like to conclude with the third theory, which is also popular boundary layer theory; in this case also we have what we have is a flat plate and it is approach approach by velocity fluid with U 0 and C A 0 concentrations, the surface concentration is C Ai, you can think of certain solutes is dissolved in this liquid or this fluid.

Then, there is a boundary layer developed by this del M, we can have similar to this boundary momentum, boundary layer there is a concentration boundary layer del C, and if you are ask to calculate the diffusion flux N A, we can also write here N A equals mass transfer coefficients into C Ai minus C Ao.

And in this case, it has been shown this one can also calculate this mass transfer coefficients in terms of Sherwood number etcetera which is k m into the length of the plate L over diffusion coefficients D AB. So, these are the three theories, which are quite popular to estimate mass transfer coefficients; of course, we have to rely on the extensive experimental data to predict this mass transfer coefficients.