Mass Transfer II
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Lecture No. # 39

So, in the last lecture, we had a discussion on continuous counter-current dryer and the way we try to model it; we divided this dryer into three regions and we said that the division of such regions is quite arbitrary; arbitrary in the sense that in some cases, it is possible that you have a very small length for heating of the solid surface. So, that can be made negligible. In some dryers, it is possible that after the moisture content, has may reduce to certain level, then we do not have the further requirement of heating of the solid or adjustment of the solid temperature, because may be there is a risk the that solid made may decompose.

So, in that case, we said that in general, we have from the modeling point of view, three regions. In one region when the solid enters, it is an unbound moisturizer. So, there we have cost and drawing rate, then moisture content decreases to critical X c; then beyond that we had this bound moisturizer. So, there the moisture content decreases from X c to whatever is a desirable concentration.

Now, there also we made some simplifications. We assume that from mathematical point of view, we assume that there is a linear decrease all the way till whatever is a requirement for, in our example, even if we have hit or we have gone, pass this capillary effects easier.

Then, we can have some region, where there is a post heating; so, these are the three regions. We had this specific focus in the last lecture, then we derive certain expressions. How long will it take or what is this theta resistance time? We have to give to this solid in contact with this hot air to bring a concentration from decrease from one level to another level.

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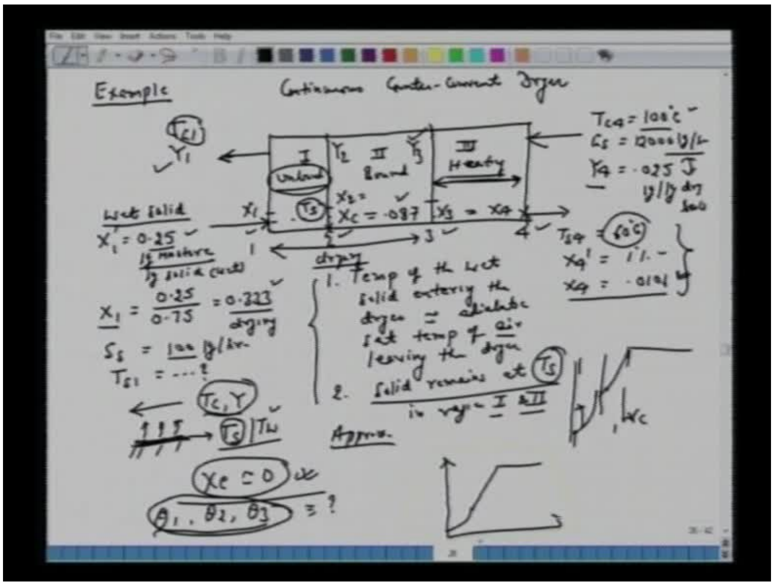

So, in today's lecture, we take an example of this continuous counter-current dryer. So, this is an example on continuous counter-current dryer. So, the problem statement reads like this, let us draw this schematic first and let us divide into three regions. So, we have region 1, 2, 3; let us mark it 1 on this interface, here 2, 3, 4. So, now, it reads that solids enter from one end, it is a wet solid; so, it has mixture content 25 percent or 0.25. Now, when we say 0.25, this is kg of moisture per kg of solid wet basis.

So, for our case, we will write it as X prime for the wet basis; so that we can calculate X or X 1 for this entrance here as 0.25 equal to 0.75 equal to 0.333. So, the wet solid enters with the moisture content dry basis at 0.333. The flow rate of this solid is given as 100 kg per hour. Now, the temperature is not given here; so, we will come back to this T 1. If you call it T s1 is not given to dry this solid, we have hot air entering from the other end. So, the temperature here is T G4 4 G for the gas, and 4 for the exit end. Temperature here is say 100 degree centigrade and this gas flow rate or air flow rate G s is 12000 kg per hour and the humidity or the moisture content in the air. So that is Y 4 is 0.025 kg per kg dry basis. So, this flow rate is also on the dry basis and it is says that this solid has to be dried to the temperature T s4 60 degrees centigrade and the moisture content X 4 prime wet basis is 1 percent. So, quickly we can calculate here X 4 as 0.0101; so the solid has to be tried from 0.333 to 0.0101 flow rate of the solid is given, flow rate of air is given, moisture content of Y 4 is given here and the temperature has given.

So, this region is so the gas which comes out has temperature T G1, which is not known to us; it has humidity Y 1, which is also not known to us and inter-phase here all thus for the gas are the for the solid. We have to calculate whatever be the temperature or this moisture content in the air phase or in the solid phase.

So, now, when we said, we have divided into three regions: first region is unbound, second is bound, the third is heating or post trying.

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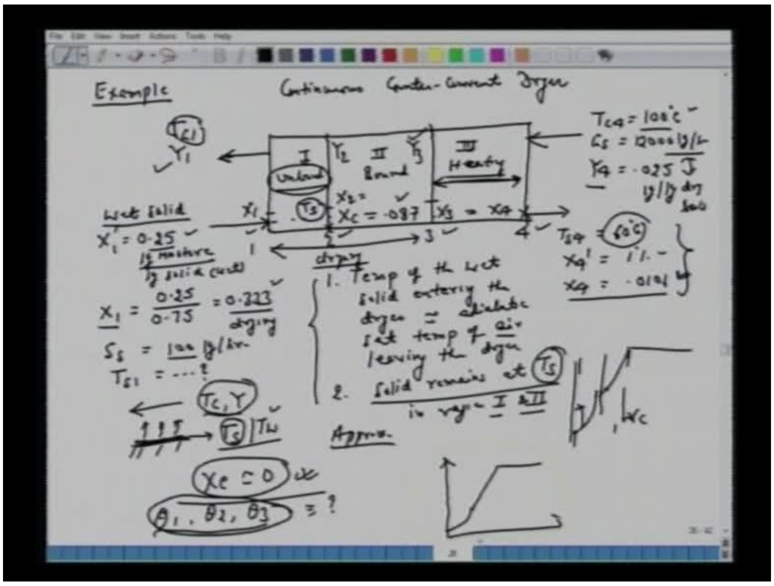

So, now, the problem also read like this, which we should note down- temperature of the wet solid entering the dryer is approximately equal to adiabatic saturation temperature of the air leaving the dryer.

So, we should pay attention here, that the solid temperature of this wet is same as not T G1, same as the adiabatic saturation temperature of air leaving this dryer that is 1, we will come back to this and see what is it mean. Number 2, you also assume, that solid remains at this adiabatic saturation temperature T s throughout in this region 1 and 2.

So, before you know anything else, let us discuss what it is mean. If you recall in our previous discussion, we say that when we have a solid bring in contact with some hot air, which has temperature T c and humidity Y, thus film on this. So, if the solid is moist covered with some thin film; we are talking on this unbound region. The moisture will evaporate, water will evaporate and this solid temperature will decrease till it reaches an equilibrium T s.

So, under steady state, we have this surface temperate as adiabatic saturation temperature or this could be wet bulb temperature, we talked about this; if we neglect the radiation effect or conduction effect, then the surface temperature will be wet bulb temperature. Otherwise, it reaches the adiabatic saturation temperature corresponding to this air quality. So that is very important that we understand here and then, once it reaches T s; so, we can assume the throughout the solid has surface temperature is T s.

So, that is why it says solid remains at T s not only in this region 1, in the region 2 also. Now, this is first assumptions that solid remains at T s in the region 1, where we have this unbound moisture is are quite acceptable. However, when we have the region 2, where now, the drawing rate has decreased and may be it goes, pass this, after going pass this, X c it reaches the region, where we have the capillary effects. So, now, we are talking about moisture, which is being driven from inside their; it is a bit rigorous assumption to assume that this solid are surface temperature will remain at T s.

So, none the less in this, there is an approximation that throughout in this drawing region. So, region 1 and 2, it is a region where we have this dryer, whether bound moisture or unbound moisture; solid remains at this T s temperature.

So and this T s is same as adiabatic saturation temperature of this air, which leaves. So, that is a very rigorous two assumptions, we have made here for our calculations, but you will notice that the first one for the region 1, it is a quite very good assumptions. For the reason 2, bound moisture’s may be if we reaches this capital regions, then it is possible that (( )) there will may be some error there.

None the less now, after it has reached this first unbound region moisture, content is now X c and it says problem X c is given to us value is given as 0.087.

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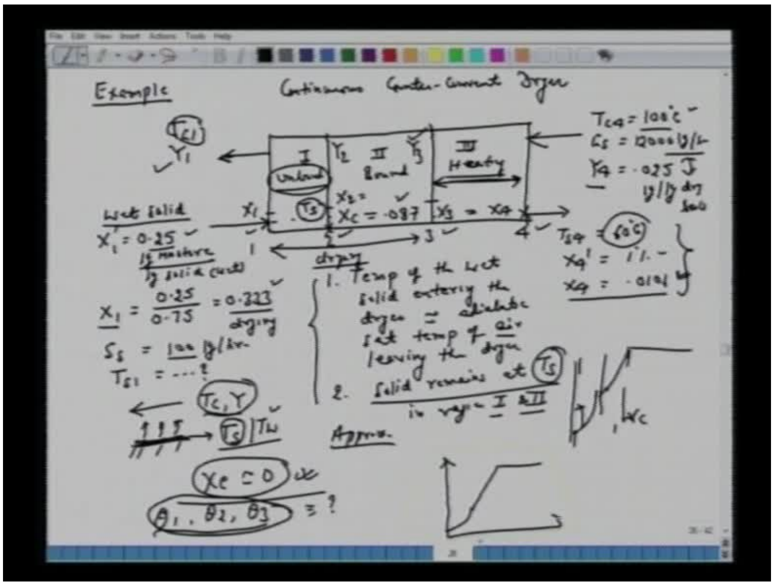

So, again notice here if you recall, we said that in this drawing, this critical moisture content or equilibrium moisture content, they have to come from equilibrium, this characteristics curve. In other words, one has to do this experiment; you rotate the same solids and simulate in the lab under more or less with the same Reynolds number of the gas fluids, the same temperature or the solid surfaces to which it exposed in the laboratory.

So, one has to establish this drawing curve, which we have been using it quite rigorously; first lecture on this topic. So, in this problem X is given as 0.087 kg of moisture per kg of dry solid and it says that X e equilibrium is approximately 0.

Now, this is also very good assumptions may be, the moisture content in the solid and equilibrium is very very small, actually it cannot be 0 and none the less for our case, we assume it is 0.

Now, all physical properties or some thermodynamics properties are given to us, but before that let us mark it here. Let us try to understand that now we have, X c is known to us 0.087. So, moisture content will decrease from here to here 1 to 2. So, X 1 0.25 now X 2 is same as X c 0.087. Now it will further decrease at X 3, but we do not know what is X 3 but notice that this is a heating region that means solid temperature only changes, gas temperature only changes, moisture content will remain the same .So, we know X 4 that means X 3 equal to X 4 equal to 0.0101.

So, X 1 is known to us, X 2 is known to us, X 3 is known to us, X 4 is known to us. So we know all the moisture content, surface temperature solid T s is not known here. here or here but we know the exit temperature, which is 60 degrees centigrade that is a quality of power product.

Let us look, at the gas phase - air humidity is known, and air temperature is known. So nothing happens here in the sense that there is no moisture, evaporations from the solid, just a heating that means Y 3 humidity of the air at this inter-phase 3 and 4 will be same as 0.03. So, we know Y 3, Y 2 is not known to us nor Y 1 is known to us. So the question here is find out total theta, theta 1 region 1, theta 2 region 2, theta 3 region 3, how much contact time, how much resistance time we have to give, to bring this solid content from here to here with the specification whatever is given to us here.

So the problem like this if we recall for (( )) the previous lectures discussion, when we had this analytical solutions, analytical expressions. One has to identify what are the humilities and the solid moisture content.

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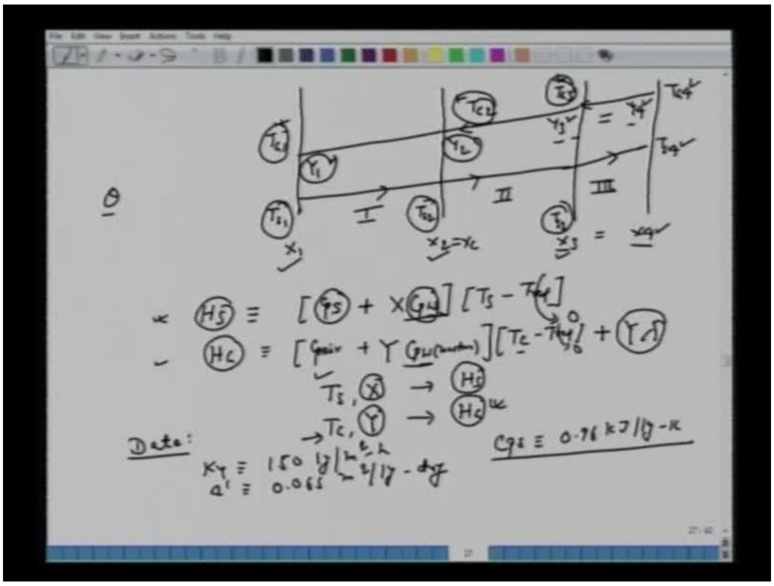

So, before that lets draw another schematics, for our calculation purpose that now we have three regions solid temperature it increases like this, you will expect. So, we have T s1 enters, T s2, T s3 then it further increases to this T s4 moisture content X 1, X 2, X 3 ,X 4.

This could that we draw some kind of schematics here; air temperature will decrease; so you have T G4 decreases T G3. T G3 here decreases to T G2 and then decreases to T G1, which is the exit temperature. Humidity corresponding, humidity is Y 1, Y 2, Y 3, Y 4. So, before doing any calculations we may be we should draw this and see what is given to us, what is not given to us. X 1 is given to us. Let us put a tick here, X 2 is given to us, X 3 is given to us, X 2 is nothing but X c critical moisture content and X 4 was given to us. But we said that X 3 equal to X 4 because this region 4. So, all the four moisture contents are known to us.

T s4,this exit temperature of the solid, which is also given to us 60 degree centigrade; so we put a tick here. T G4 is given to us 100 degree centigrade, Y 4 is given to us, Y 3 is same as Y 4 because there is no evaporation here. There i s no drying here in this reason 3 and 4 so Y 3 is also known to us.

Now, what are things not given to us - T G1, Y 1, T G2, Y 2. So, these are you have to calculate T G3 and here Y 3. So Y 3 is given to us. So, what is the approach? So first thing is that before we make any calculation for theta, we should try to make overall enthalpy balance. Overall moisture content balance or over the certain segments; so that we try to evaluate as much as quantity given to us here.

So, in this case, again now, when we have want to calculate the enthalpy, so let say how do we calculate solid enthalpy at any locations between 1 and 2, H s will be equal to say c ps. So, this will be the specific heat of the solid, say if you choose 1 kg of or 1 unit weight of the solid, this will contain X amount of moisture; so we will have C pw, so 1 kg contains X moisture content. T s solid temperature at any location here minus some T reference. Generally, you can choose T reference as 0, there is no harm in this as long as we are consistent here.

Then comes, H G air content, what is the specific enthalpy kilo joule per kg of air here. So, similar to this c ps, now you have c p air plus; instead of X you have Y. So that is a humidity c pw but this is now moisture. Specific heat of vapor or in the solid in the which is the liquid will be different from what we have in this moisture, we call it moisture on the vapor phase; so that is one place all the both are H 2 O water one is a condense phase liquid and one is water vapor here into T G minus reference temperature, which we have 0 plus there should be one more quantity here that is your Y into lambda.

So, from this reference temperature, we have raise the temperature to this T G and then we have evaporated this moisture here or water here, by this latent heat of vaporizations. So, these are the two general expressions one for solid, one for air, you should notice that that means, if you knows T s and X we can calculate H s or given H s, if you know X we can calculate T s.

Similarly, for air if we know T g ,if we know Y, we can calculate H G or if you know H G enthalpy at any locations. We know the humidity, we can calculate this air temperature. So, with this over view then we should make some, we should make our general calculations here what is given, what is not given to us.

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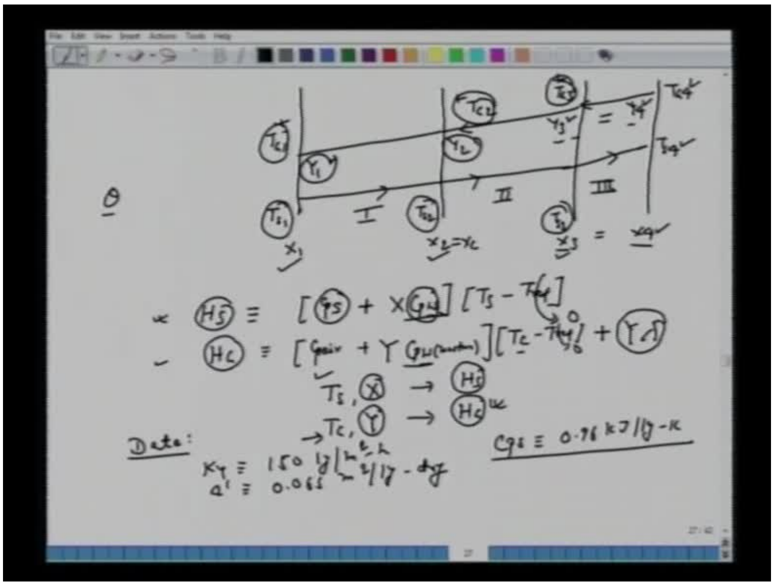

So, if you recall in the earlier class also, we had started with region 3, all then we had region 1 and then we went to region 2 to calculate all this theta. Here let us make note of certain other properties which are given to us. So what is data? some data thermo dynamical thermo physical data are given to us; so K y mass transfer coefficients 150 kg meter square per hour a prime, we said that it a specific effective area of drying. So for the solid this is 0.065 meter square per kg dry basis c ps. So a specific heat of this solid alone no water this is given as 0.96 kilo joule per kg per Kelvin.

So, these are the physical thermodynamic or physical properties given to here. Now, we can calculate all these quantities, which we are supposed to calculate here T G1, T G2, Y 1, Y 2, T G3, Y 3 is known to us because Y 4 ,T s1, X 1, X 2, X 3, T s1 is also not given to us T s2, T s3. So, these are the circles, which we have to calculate, the one we have ticked here X 1, X 2, X 3, Y 3, Y 4, T G4, T s4 they are known or we have at least make may this small calculation that X 3 equal to X 4 and Y 3 equal to Y 4.

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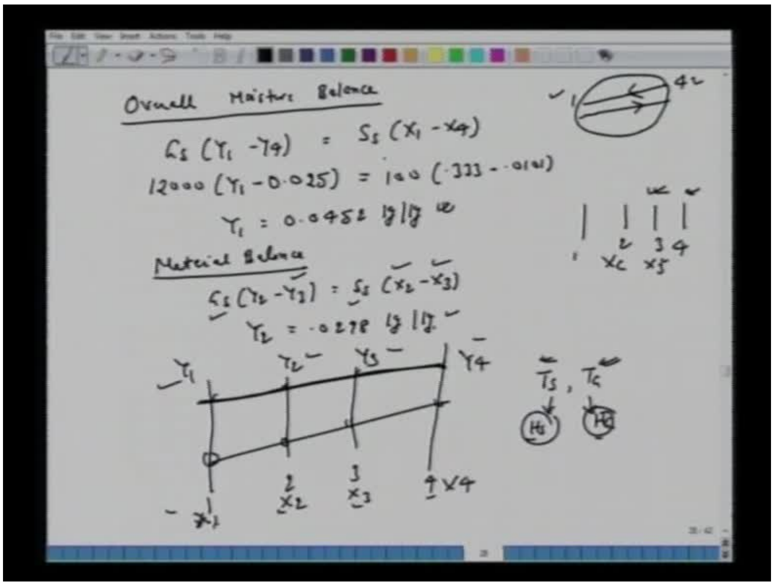

First thing, we would like to do is that is a good practice, we should make overall moisture balance. Let see, what we can calculate here in the gas phase as well as in the solid phase, so let us make it overall moisture balance would be G s Y 1 minus Y 4. So, inlet minus outlet, equal to S s X 1 minus X 4; so, it is a continuous counter-current 1 4 air goes like this solid goes like this. So, whatever is lost in the solid is gain by this air here.

Now, look at this we know G s let us put the number 12000 Y 1 is not known to us so this is so we have 1 here and 4 here at the end so its overall moisture balance so Y 1 is not given to us so Y 1 minus Y 4 which is 0.025 so that is here S s is 100 X 1 is here moisture content we calculated 0.333 minus X 4 we have to bring down to this level of 0.0101

If you do this Y 1 equal to 0.0452 kg per kg or dry basis, so we have calculated Y 1 now; so you can go back to the previous slide and you should put a tick here. So, Y 1 we have calculated it has the first thing we have calculated from overall balance. So, this is your Y 1 after that what we should do. Here again, you should inspect that what are the things we can calculate by another balance over the regions. So, let see if you can make another material balance between 2 and 3.

So, this is 1, this is 2, this is 3. Lets note down here G s Y 2 minus Y 3 equal to S s X 2 minus X 3. So, note it this that Y 3 is known. So, let us make it 4 here Y 3 is known to us because Y 4 same as Y 3 no moisture evaporation; so Y 3 is given to us G s S s is known to us, X 2 is known to us, that X 2 equal to X c and X 3 equal to X 4. So, this also known to us, so from this we can get Y 2 0.0298 kg per kg here. So, go back to this now same schematics which we drew here we had 4 regions 1, 2, 3, 4.

All X 1, X 2, X 3, X 4; Y 4, Y 3, Y 2, and Y 1 now we have calculated all four of these quantities, that means, now if you know the temperature at any locations, if you know the temperature of solid or if you the temperature of gas, then we can calculate enthalpy H s or we can calculate enthalpy in the air phase H e or if you know H s and H e and since, we know all the four; here the humidity and the moisture content, at the four locations we can calculate temperature of solid temperature of this gas.

So, this what we are supposed to do here before making other calculations, you draw the nice schematics and put as many as quantities, which are given to us, Then more important here is that definition for your enthalpy for the solid and enthalpy for this air, if you know the moisture contents, either in the solid phase or the humidity moisture content in the air phase then, knowing the temperature we can calculate the enthalpy or if you know the enthalpy and if you know the moisture content in any other two phases we can calculate the temperature.

So, then we can go back and directly substitute in the expressions for your theta resistance time or the time required for heating or decreasing, this moisture content is from one level to another level.

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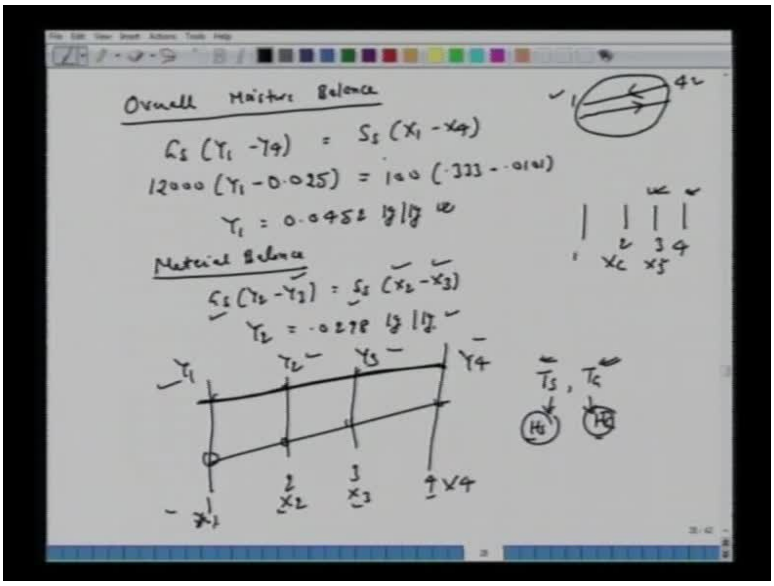

So, here all we have done. So, we just made overall material balance, we made certain approximations there X 3 equal to X 4 no heating etcetera; so there we had, we calculated Y 1 humidity, we should not give to us and then from Y 1 we calculated Y 2. So, we made overall moisture balance and then between region 2 and 3, we made another moisture balance.

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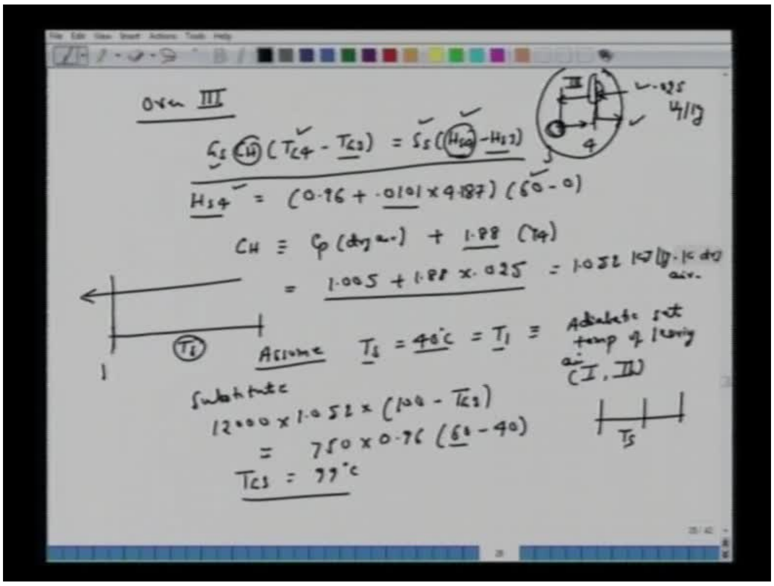

Now, let us make some other calculations here, it is now let us start with the region over 3. So that is what we did, when we developed our expressions for theta content, now we are talking of the last 3 and 4 region. Again recall our discussion, here we have the solid leaving like this and air coming like this, for this when we wrote down this balance for energy balance here or the heat load, we said that we have to make certain approximations certain iterations. So, we have to assume certain surface temperature.

So, if you write down this energy balance again G s, let say C H T G4 minus T G3, now between there is this reason 3, whatever is lost by this solid energy is given by this air. So, we have S s H s4 minus H s3.

So, T G4 let us write here, so that is given to us; that is known to us but no other quantities. So of course the flow rates are known to us S s G s S s4 here we have to see that how we can calculate other quantities like H s4 and H s3. So, we follow the same approach what we had before, how we can get H s4?

So, let us write down the expression for H s4. H s4 would be c ps same expression, which is 0.96 plus moisture content 0.0101 multiplied by specific heat of water; so 4.187 into this temperature, so 60 minus 0 that is the reference temperature.

So, now this enthalpy for this at the number 4, here here we know the temperature 60 and we know this moisture content 0.0101. So, which we means, we can get H s4. so we can calculate this quantity H s4 here.

Now, let us see what is a C H? So, from here C H of course we require to know what is a C H here. So C H would be again, this is specific heat of your… this gas which enters. So, the C H, we can use as C p of a dry air plus, we have C pw of moisture content of moisture 1.88 and then you have into Y 4 and but Y 4 is same as Y 3 here. Because there is no heating here. So, Y 4, Y 3 they are known we can write C H as 1.005 plus 1.88 into 0.025. So, that is your 0.025 humidity is given to us kg per kg. So, if we calculate this, you have the number 1.052 kilo joule per kg of dry air.

So, after this now, one has to make if you recall our discussions, we do not know other temperatures here, we do not know surface temperature here and we said that the solid remains at this are temperature, which is adiabatic saturation temperature of the air which leads at the exit 1. So, we have to assume this number, this is approach we had earlier also. One has to assume the surface temperature. Let us assume this T s is 40 degree centigrade.

So, what is this T s? T s is nothing but T 1 solid enters at adiabatic saturation temperature of leaving air and it remains at that temperature. So, T 1 is the solid temperature just same as T s and which is same as adiabatic saturation temperature of leaving air and this will remain in the region 1 and the region 2.

So, now we assume this and we substitute, so if you substitute - let say G s 12000 C H, we will calculated 1.052, just now T G4 is 100 degree centigrade temperature here of air, 100 minus T G3 is not known to us, T G3 S s solid flow rate, you know 750 into now H s4 minus H s3. So, which is nothing but solid specific heat, which is 0.96 H s3 here into 60 minus 40, this is what we are doing here 60 minus 40.

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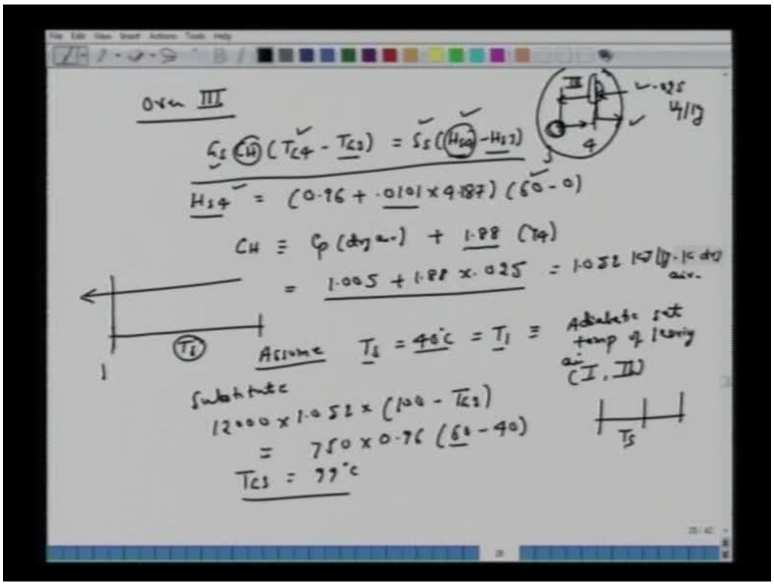

So, 60 the solid temperature, that is, the leaving temperature of 60 and we have assume 40 degree centigrade and solid remains at this temperature throughout on the region right here. This temperature is constant at T s; so, minus 40. So, from this, one can get T G3; if you calculate T G3 is 99 degree centigrade. So that is the way we read earlier; we have making a balance over this region 3 after, we have made over all balance moisture content. So, now, we get T G3. So, this is the way we have to move and to next region and see we can how we can calculate here.

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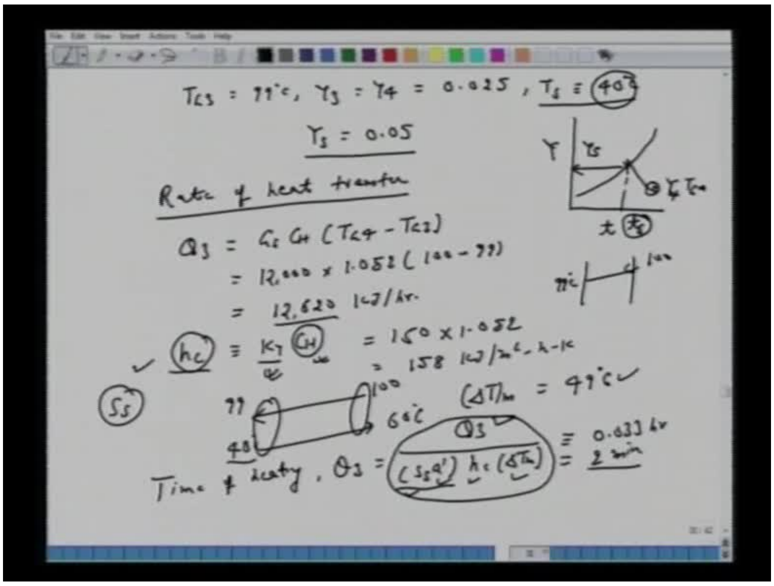

Now, if you know T G3, which is - we calculated as 99 degree centigrade Y 3 equal to Y 4 3 and 4 is in 0.025, T s we have calculated or we have assume as 40 degree centigrade.

Now, if you recall we have to check whether this T s 40 degree centigrade, what we have assumed is not correct or not. So, for that also if you recall our previous discussions, if you have a psychometric chart and we know the quality of air, which is entering here; so, we know this humidity, we know this air temperature T G4, which was 100 degree centigrade, we know Y 4; you will be adiabatic saturation line, which will hit this Y versus t degree centigrade and this would be your t s. So, from this t s, if you read, if you know this temperature, if you go back and see you will get back this 40 degree centigrade. So, that means our assumption given to us was correct here. And once, you know this T s, we have this Y s equal to 0.05. So, we can read from this Y s.

Now, let us look at rate of heat transfer. So, we are still working on this region 3 rate of heat transfer. So, this region 3 heat load Q 3 is G s C H T G4 minus T G3, which is 12000 into 1.052 into 100 minus 99.

So, that is - the temperature drops from 100 degree centigrade air to the region here 99 degrees centigrade. So, if you calculate this, we have 12620 kilo joule per hour; h c heat transfer coefficients is nothing but K y into C H, we have to calculate all this quantity, because we are going to use expressions for the heating in this region.

So, in that we required h c and if you recall the definition for C H humidity heat of this, air is ratios of heat transfer coefficient and mass transfer coefficient. In the example, mass transfer coefficient was given to us and humidity, we had calculated from this 150 into 1.052 is 158 kilo joule per meter square per hour per Kelvin.

So, we know this air temperature decreases from 100 to 99 and the solid temperature has increased from 40, which was adiabatic saturation temperature to 60 degree centigrade. So, we can find out del T 1 del T 2 L m T d I leave this an exercise very simple delta T m, will be calculated as 49 degree centigrade.

You go back to the expression time of heating for this region, if you know heat transfer coefficient, we know the flow rate S s, we know this driving force for heat transfer, time of heating, theta 3 if you call it is very analogues to mass transfer Q 3 divided by effective weight amount of the solid in this region into h c del T m. Q 3 is we have calculated just now, S s is given to us a dash is given to us h c we know delta t we have calculated here if we do this theta we will get approximately 0.033 hour. So, if you this theta 3 this will be 0.033 hour or this will be equal to approximately 2 minute we should spend some time here, how we are getting this expressions.

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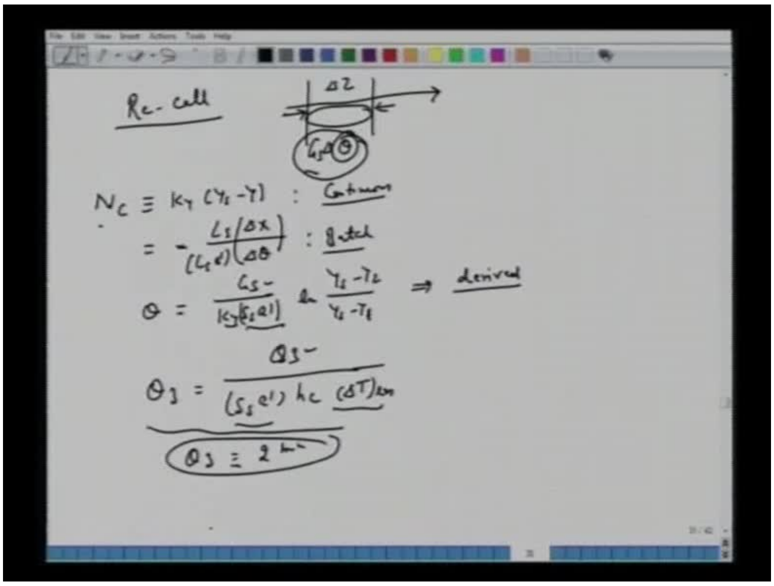

So, if you go back and recall mass transfer, we talked over this region of very small region del z. So, we are giving some theta distance time; so this G s spend say del theta. So, we have so much of kg of solid over this time del theta; so, it is like a batch of course we have the continuous process, solid continues to flows like this, but we are saying that over del T certain batch of solid is getting heated. So, what we wrote in case of mass transfer coefficient N c as K y Y s minus Y and we equated this to L s del X del theta with L s a prime.

So, what we have this a batch, we trying to make an analogy with the batch of operation, with which continues; so, you should go back to the previous lecture and see how we have obtained the expressions for heat transfer coefficient.

So, similarly, based on this what we did? We integrated or we wrote like, theta. How long will take as G s K y S s a prime l n Y s minus Y 2 over Y s minus Y 1? So, this was derived or mass transfer aspect here, making the same analogy that certain batch of the solid is getting heated in this region. All we have this time of heating theta 3 as Q 3; so how much is the heating load divided by S s a prime? So, there is analogy S s a prime K y; we have h c G s, we have this Q 3 and we have this del T l m logarithmic driving force here or temperature.

So, we have the same analogy, what we had did here, in the case of heat transfer to mass transfer to obtain theta 3 as 2 minute very approximately 2 minutes on this. So, now, what we should do here? Again, since, we know this theta 3; now, we should move to the region 2 or region 1 to see what calculation we can.