Mass Transfer II

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So, in the previous lecture we discussed characteristics curve or drying and we also took and took up an example there. So, essentially we had several regions of drying the most the fundamental was the first region where we have constant thin film of the liquid. So, essentially we had evaporation rate same as the drying rate. So, that is why it is the most fundamental. In the sense that whether we have a drying from a solid surface which is covered with a thin film of the liquid or we have just a pool of the liquid. As long as the temperature is same, it will exert the same vapor pressure. And we will have the same evaporation rate same as the drying rate.

Then of course on the drying rate start decreasing, because of dry patches which are start appearing on the solid surface. So, although the intrinsic rate is again the same as you know the evaporation rate. But since we are reporting the drying rate based on the total surface area there is an artifact of that and the drying rate decreases. So, in the last lecture we also took up an example where we calculated the total time of drying. Including the constant rate which is like unbound moisture and then we had this decreasing rate which is the bound moisture. Till we hit the critical moisture content and then we go till we read reach the equilibrium curve moisture concentrations.

In today’s lecture what we do, we study the heat transfer accepts. It since it is a drying one has to supply heat very substantial amount of heat for the moisture to evaporate or to pull the moisture from the within the solid surface or from the pores of the solid surface. And if you recall, we also said that when we go into that bound moisture region where the concentrations or the drying rate is starts decreasing non-linearly. Then, we have this capillary effects, the moisture has to diffuse by the notation diffusion they have come to the top of the surface. So, the things are quite complicated.

So, what we did today we discuss the heat transfer accepts in conjunction with this constant rate period. Where we have the constant rate evaporations or things are well understood well define less of mathematical steps. So, again we take this example for a batch system. We have a solid surface and air will flow pass the solid surface, essentially we have a cross circulations. So, the batch one crosses circulation seconds two and third we have this evaporation rate constant so, unbound moisture. With this three we will address how the heat transfer has the effect on the drawing rate. And again when we say the heat transfer, we should think of we have a some trays on which we kept the solid surface and then it comes in contact with the hot air.

Now, we can also have the radiation effect which means there could be a solid wall a radiating surface at a very high temperature. So, from which this moist solid which we want to dry can receive the heat. So, if that happens then one has also taken into a count for this ray heat of radiations in addition to that we can also have convective effect. So, all this three on conduction convections and radiations have to be taken into count to calculate the heat received by the solid surface. And then we can calculate how much your drying rate is. So, essentially it is a combined effect of heat transfer and the mass transfer.

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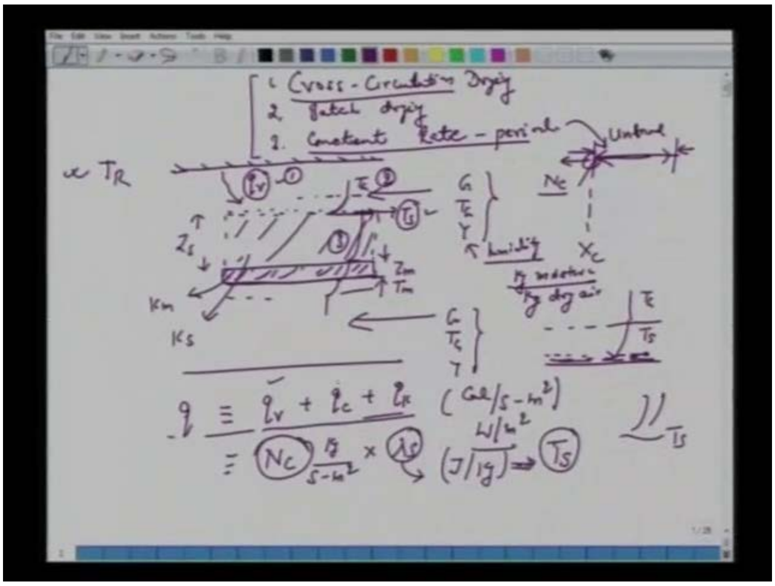

So, we take up this heat transfer aspect in today’s lecture. So, as we said we are studying cross circulation drying, we also have this batch drying and in conjunction with this you have this constant rate period. So, if you recall we are talking of this range where the moisture is unbound. So, this is unbound and this is the rate where we have this X C corresponding to which we have the constant rate as N C, denoted by N C. So, let us see that we have some solid tray made of certain metals. So, let us say put some dimension here let us say thickness is z m this as thermal conductivity given as k m on which we have kept a solid which we want to dry.

So, it is a moist solid to start with this radius this thickness is z s and this has a solid conductivity given as k s. Now, we have the air flow so, it is a cross circulation drying. So, the solid surface both top and the bottom we have this cross flow solid of the gas here. So, let us say the quality is given as say the temperature is T G and the humidity is Y here. So, the Y represents kg of moisture per kg of dry air. So, G, T G, Y all three here and it also comes in side of a solid. Let say here we have a hot plate or some solid surface at a temperature T R from which the solid surface also receives heat by radiation.

So, look at this now we have q r received by the solid surface or this top cover of this, there is may be some thin layer of liquid here we are in constant rate period. So, we assuming that the solid surface top of the solid surface there is a very thin film of water it is a moist solid on the top surface. So, the top surface receives heat by q r since we have G we can expect another conductive heat and transfer coefficients. So, let say the temperature was given as T G bulk and then there is a drop here so, T G to this T S. So, we can have convective heat transfer coefficient bulk temperature is T G air temperature and then there is a drop here.

So, which is top of this surface here is T S so, water evaporates at T S. Only thing is that T S in not known to us we know that radiation temperature T R we know the air temperature which is T G, but the T S is not known to us. We will see that how we can make an estimate or how we can calculate this T S. Now this bottom of this plate which is non it is not weight it is a metal surface here this also receives heat from air. So, there is a convective heat transfer coefficient here also the bulk temperature drops from T G (( )) let us say T m. Now, the heat will also get conducted through this metallic surface. So, there will be another drop here and then another drop here.

So, that solid surface receives heat by the third mode which is by conductions. So, one is by radiation, two is by this convection and third is by conduction of the border. So, we have three components of heat here q equal to q r plus q c plus q k for conduction. And all of this is heat flux here so, we saying this q equal to calorie per second per meter square or we can have watt per meter square so, heat flux. And please note this heat under steady state will also be equal to rate at which moisture evaporates. So, this is kg per second per meter square. So, this is the N C we are talking here that we are in this region unbound which is a constant rate period and the rate of evaporation of the rate of drying is N C.

So, N C kg per second per meter square multiplied by lamda S so, where lamda is your heat of vaporizations. So, we have some joule per kg of solid of course this has to be specified at a temperature T S which is unknown to us. So, looks like will have to make some approximation here we have to make some estimate for lamda S. Latent heat of vaporization does depend up on what is the temperature at which the water evaporates. So, here we said that T S is unknown. So, will have to make some estimate, none the less we have all three components heat transfer heat flux this is equated to drying rate multiplied by this latent heat of vaporizations.

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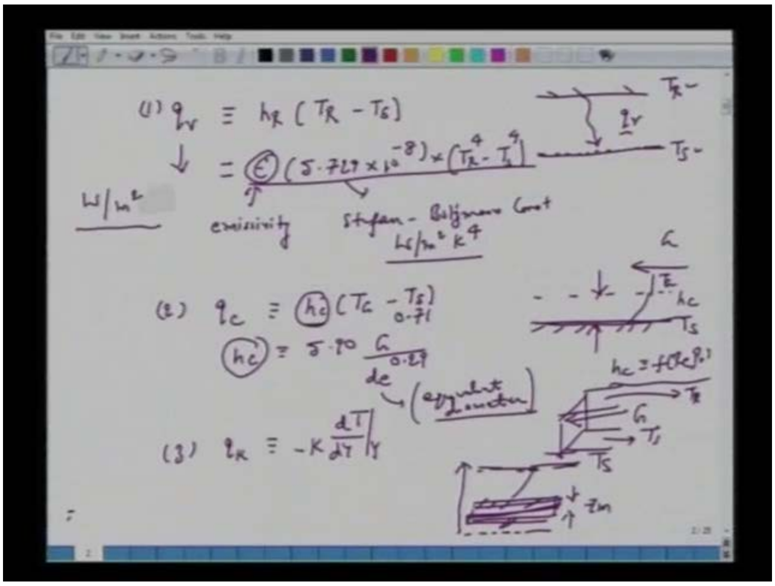

(1) $q_r = h_R (T_R - T_S)$

$= \varepsilon (5.729 \times 10^{-8}) \times (T_R^4 - T_S^4)$

W/m^2

emissivity

Stefan - Boltzmann Const

W/m^2 K^4

(2) $q_c = h_c (T_G - T_S)$

$h_c = 5.90 \frac{G^{0.71}}{d_e^{0.29}}$

(equivalent diameter)

(3) $q_K = -K \left.\frac{dT}{dY}\right|_Y$

So, now let us address each component here q r, q c and q k. So, what is q r you should recall from knowledge of heat transfer this q r is h r. You can write some radiation heat transfer coefficients multiplied by T R minus T S. So, we are talking of now T R and we have this solid surface on which we have very thin film of liquid. So, this temperature is

T S says radiant surface has this radiating heat flux given as q r. So, we have h R T R minus T S. But most fundamentally is that this can also be equated to epsilon where epsilon is emissivity of the surface.

We have a Stefan-Boltzmann constant this number may be familiar to you. So, this is Stefan-Boltzmann constant multiplied by temperature raise to the power four. So, again you go through previous lecture notes on heat transfer or some text book on heat transfer. So, heat of radiation is given as emissivity multiplied by Stefan-Boltzmann constant. Which is also estimated as 5.729 to power minus 8 into temperature raised to the power 4 for radiation minus this T S to the power 4, unit of this number you should be able to recognize that this should be equal to watt per meter square Kelvin to the power 4.So, this q r heat flux is watt per meter square.

So, now, what we have is the first one is q r, second is q c so, heat transfer by convection convective heat transfer. So, we have this top surface which is T S there is a heat flow here G. So, depending up on Reynolds number very similar to mass transfer coefficient which we have extensively discuss in this course there is a heat transfer coefficient given by h c. H c will be a some function of Reynolds number, prandtl number etcetera. So, one has to go through certain correlations reported for a cross flow of flow pass this solid surface with heat transfer coefficient h c. All it reflects temperature drops from T G to some temperature T S here.

So, we have q c given as h c T G minus T S and you said that one has to go through some handouts from for perish hand book to get the best very close correlations. Which reflects on the situation, here we have the solid surface and we have the flow parallel to this. So, one of the very common correlation is given as G c equal to 5.9 G to the power 0.71 divide by d e to the power 0.29, this d e is of course equivalent diameter. So, this equivalent diameter is for different flow cross. So, here in our case we can think of it is cross section where we have flow like this. So, the bottom one here is the solid surface T S and the top one we can think of this T R radiating surface.

So, this is your flow area from which we can get equivalent diameter. We will take up an example where we will address this, third is your q k so, heat transfer by conductions. So, essentially we are applying for a fuel law minus k d T divides d Y at any locations y. So, (( )) we say that we have this metallic surface at the bottom which is none drying.

This will also have a certain thickness so it is a tray which has the thickness we said that z m. So, heat is also transferred from the air so, there is a drop. So, we have same h c it transferred by conductions then we have this solid surface. Then another drop, then it comes to this T S here. So, heat is top solid surface receives heat by conductions from the bottom.

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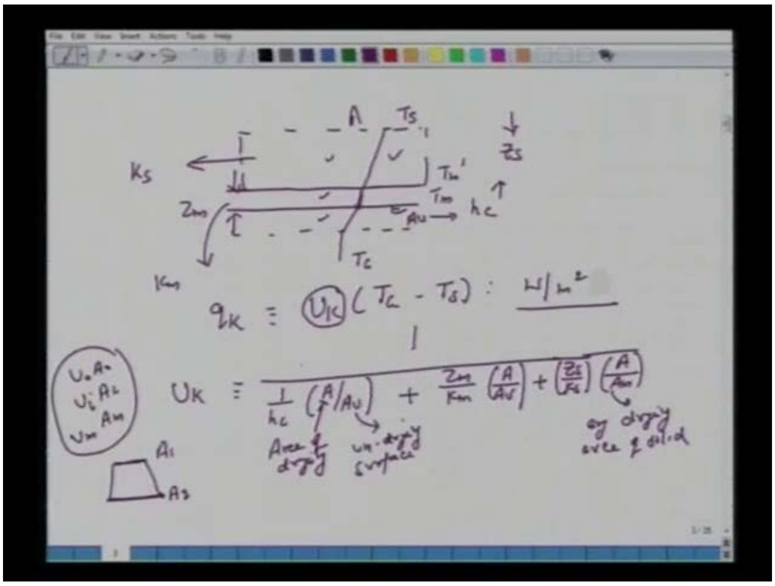

$q_K = \boxed{U_K}(T_G - T_S)$: W/m^2

$$U_K = \frac{1}{\frac{1}{h_c}\left(A/A_u\right) + \frac{z_m}{k_m}\left(\frac{A}{A_u}\right) + \left(\frac{z_s}{k_s}\right)\left(\frac{A}{A_m}\right)}$$

Let us try to draw a schematic of this heat drop. So, let us start with the bottom of this metallic plate, there is one dotted line here which reflects h c temperature drop from T G air let say to this T m. So, this T m is temperature at the bottom of the tray and this tray also has it is own thickness if you said it is z m. So, there is a drop it is a metallic surface so, you will expect very less drop here because thermal conductivity of the metal is generally very high. So, it drops to say T m prime. So, this is the bottom of the solid which we want to dry here. So, generally this has poor thickness k s will be much smaller than k m this thickness of the solid is z s. So, they will be another drop here till we reach T S.

So, this third component q k you can think of if you recall heat transfer they are one resistance, second resistance, third resistance all three resistances. So, you can have over all heat transfer coefficient U K T G minus T S so, this is also watt per meter square. So, heat received by the top surface by conductions and since you realize that they are three resistances one can write U K as a series combination of all these three resistances. This would be equal to 1 over h c A over AU. So, now, we are talking of different areas if you recall from heat transfer. We have U 0 A 0 you have U i A i, U m A m.

So, we are making correction for the different areas it is possible that is some cases you can have the areas like this. So, bottom surface had different area, top surface has different areas. So, for that we have these area corrections, A is the area of drying. This AU is would be area of un drying surface remember there is no evaporation or drying from the bottom of this solid surface. So, there is a tray here. So, this would be your AU and the top would be A of course in this case is rectangular you can say that A U un use or un drying surface is same as the top of the surface.

So, this area correction is one here and none the less this is a general expressions plus heat transfer by conduction so, you have z m divide k m. So, now, we are talking of this resistance here and you have again A over A U plus now we are talking of the third resistance here z s over k s and here we have A over A m. A m you can say that this is average drying area of solid for the same reason that may be this is A 1, A 2 of course in our case all three As may be they depends upon the example or the same. So, we do not have to make this any area corrections after we know all those three.

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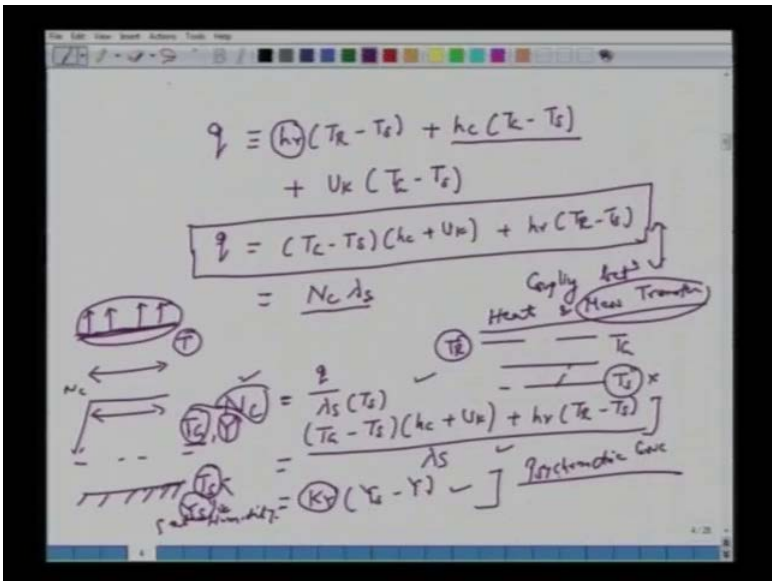

$$q \equiv \textcircled{h_r}(T_R - T_s) + \underline{h_c(T_G - T_s)} + U_K(T_G - T_s)$$

$$\boxed{q = (T_G - T_s)(h_c + U_K) + h_r(T_R - T_s)}$$

$$= \underline{N_c \lambda_s}$$

$$\textcircled{N_c} = \frac{q}{\lambda_s(T_s)} = \frac{(T_G - T_s)(h_c + U_K) + h_r(T_R - T_s)}{\lambda_s} = \textcircled{K_Y}(Y_s - Y)$$

Now, we can add all three heat flux as h r. So, first is T R minus T S. Remember this h r is the heat transfer coefficient for radiations which one can calculate. Based on those, the second expression, which we wrote most fundamentally for radiation in terms of emissivity and Stefan-Boltzmann constant plus h c. Which is convective v transfer coefficient from the top solid receives heat by convections h c plus. We have this conductions U k and from the bottom we are receiving heat by conductions T G minus T S. And, they can be arrange to write as T G minus T S, h c plus U k, plus we have h r and we have this T R minus T S. So, the total heat flux received by the solid surface by three mechanism or three modes of heat transfer of this.

I remember we equated this to N C into lamda because so, this is the where we see the coupling between heat transfer and mass transfer heat and mass transfer heat transfer plays a very major role here. So, what we are saying that whatever is the rate of evaporation. This is control by this unbound region where the rate is a constant given as N C. So, after sometime is going to drop here, we are going to study the first one which is the most simpler the most fundamental rate of drying is the same as rate of evaporation. So, depend upon what is the temperature of the solid surface of course in this problem we said that T S is not known to us, what we know is the T G.

And what we know here is T R radiating surface, T S is not known to us. We will see how we can calculate this so, from this these two coupling of heat and mass transfer. We can now write N C has q by lambda S to say that lambda S is calculated at T S, lambda S is a function of T s. So, there is always some corrections here how much the temperature, latent heat of vaporization does depend upon T S. What is the T S is very close does not vary much sell by few degrees. So, somebody can make an estimate and can do these calculations.

So, N C equal to q by lambda S, q we have written down of the expressions T G minus T S h c plus U k plus h r T R minus T S over this lambda S. And now also recall when talking of this mass transfer just like we have the heat transfer coefficients for over all heat transfer coefficients. Recall from previous lecture we can also write N C as some mass transfer coefficient and Y s minus y. So, now we are saying that this solid surface tops of the solid surface here the temperature is T S and here the temperature is T G of air. So, corresponding to this T S there is Y s relative humidity or saturated humidity corresponding to this T S.

And there is a Y humidity of this air corresponding to this at this temperature T G of course, T G and Y they are unrelated. So, the air has a temperature T G and air has certain humidity corresponding to this T S. We are assuming that the solid surface which is covered with the thin film is saturated with water. So, we have corresponding to this T S there is Y s here saturated humidity at this temperature. So, we have this expression this expression and this expressions k Y again one has to rely on some correlations may be from this we can calculate k Y.

So, depending upon type of the problem the way problem is freeze one can calculate this on that is says that is what we are trying to say here that the rate of the of evaporation. Or rate of drying can be calculated from energy balance and can also be calculated from mass transfer the two are coupled here.

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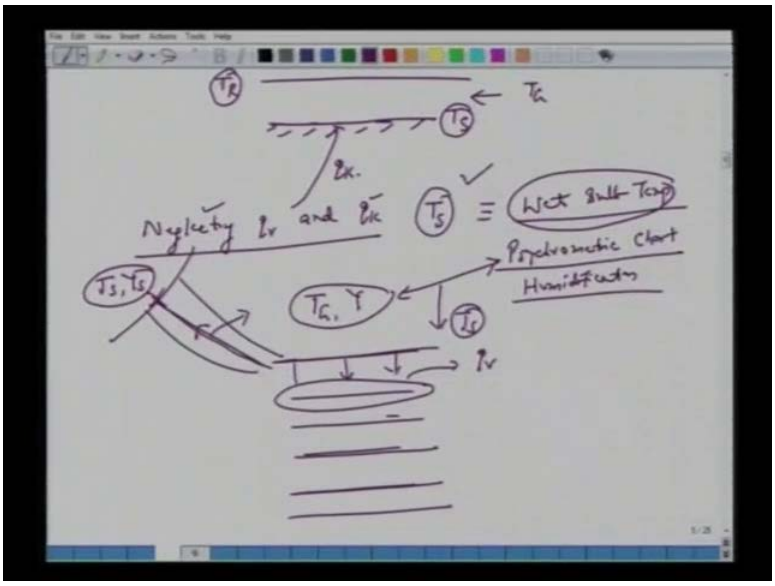

So, if we have say a cyclomatic curve from this we can calculate Y s corresponding to T S will take an example preferable to say this. Now there is one more important thing here we said that the solid surface top of the solid surface which is covered with the thin film of the liquid here. T S is not known to us T R and T G they are known to us. So, the solid reaches this temperature T S under steady state because of all three effects because of radiations. So, it get heated of because of convections and also it receives heat from the bottom by this q k. However if we neglect q r radiation effect and q k, you should be able to appreciate that the solid temperature T S what it reaches under steady state will be same as wet bulb temperature.

So, I am sure you have come across this bulb wet bulb temperature in heat transfer in may be some other course. In which we must have discuss the dry bulb temperature, wet bulb temperature and cyclometric chart. So, maybe you should go through those lecture notes the text book even treybal McCabe Smith, all of them they discuss they have one atleast one chapter on this humidification. Humidification design of cooling tower all of them they fall under this chapter here. So, all we said that if we neglect heat radiations and heat conductions the top temperature will reach.

You can approximate the T S is nothing, but the wet bulb temperature and all it means knowing this T G and humidity from psychometric chart one can calculate T S or one can read T S. One has to follow this adiabatic saturation line the several lines of the psychometric curve find out where it this intersects at T S and corresponding to this we can also read Y s. So, it is a approximations and we are neglecting the other two components of heat transfer. And from the practical point of view all we are saying that if you have trays like this which extract like this.

Where you often the top surface receives heat by q r from some other solid rod or the plates which we have underneath all this trays they may not receive the same amount of heat radiations q r. So, for these tracks which are underneath one can make this approximation to say that the top solid surface has reached the wet bulb temperature. So, that makes our computations very simpler here we will take an example where we see these two effects.

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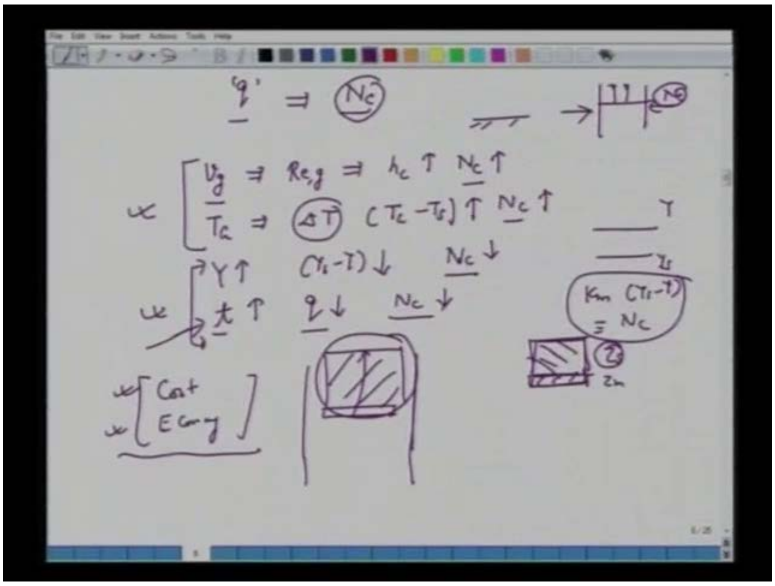

Now, let us see what decides you know different rate flux here which can determine or which can affect the rate of drying because N C equal to q into lambda. So, if the q is large solid surface receive a very high heat flux will have a very large rate of drying. Of course we are talking of this constant evaporation rate constant drying zone where the rate is constant as N c. So, here it all depends what is evaporation rate, what is the temperature here for the solid surface. So, first thing is that it is qualitatively we can discuss here is the velocity of air. Velocity of air is very large we will expect larger Reynolds number, heat transfer coefficient will be very large and given T G we will expect that N C will also be larger.

So, rate will increase rate of drying or rate of evaporation will increase if velocity is large. Similarly, if the air is high temperature T G you will see the heat transfer or the driving force for heat transfer T G minus T S will increase N C will also increase. What will cause decrease well one thing is the humidity if the air is humid then Y s minus Y. So, this is Y s and this is the quality of air Y this will decrease and if this decreases N C will decrease because we are talking of k m into Y s minus Y also same as N C. So, from mass transfer point of view N C will decrease if the air has larger humidity so, how about the thickness, the thickness is a large of the tray or the solid surface.

So, we are talking of z s and z m of course trays are generally small in thickness, but if somebody wants to put more solid here with the view to increase this productivity of drying. So, we have only five trays, but we load it or over load with a very larger amount of solid surface. Then the thickness will be more heat received by conduction will be less now under steady state so, q will be less and N C will be less. So, they are certain factors which increase the rate of drying or rate of evaporations certain parameters certain factors which will which decrease the rate of drying and rate of evaporations.

It is very important the last one which we said here if your very often this is a mistake we make we take a trays and we over load with very large amount of solid surface. With a view to improve power production rate you know in one hour one day we can process a lot of solids. But remember that when we increase the thickness then heat received by conduction is a small because a smaller rate decreases. So, there are always certain cost economy they have to play role here just like any other previous unit operation we did adsorptions at a distillations.

So, they are cost an economy has to play a role and again if you have some design course in your syllabus you will come across some of these aspects on drying or the cost or economy of the drying. We skip that discussion here before we take this example let try to summarize what we have done. So, we talk we started in fact the previous lecture I am here the example because purely mass transferred. The idea was for n type of solid there is a constant there is a characteristic drying curve one has to establish by the experiment.

We did that you know example and there we found at we did some calculation to find how long will you take to try the solid surface that is one. There was no difference to heat transfer, but if you think carefully when we have this constant rate heat drying. So, there the rate of evaporation is same as rate of drying. So, as long as the temperature is fixed, we fix the rate of drying. So, if you have solid surface and initial part of the drying where we have thin film of the liquid here the temperature of the evaporating surface will play a major role.

And how we establish the steady state surface temperature that has to come from heat transfer accepts. So, in that heat transfer accepts we talk of heat radiations from some solid heat plate or we have heat received from the bottom of the plate tray. You also have convective heat transfer coefficient certain Reynolds number so, we put all three. So, we have expression for heat transfer rate we equated to this mass transfer rate by this lambda. Of course lambda is a thermodynamic properties it effects upon what is the surface temperature mind you. Surface temperature is not known to us all we know is a gas temperature.

So, by knowing of course heat transfer coefficient one can make some estimate there how good is your heat transfer convective v transfer coefficients how good is your h r etcetera. So, from there one can make an estimate of T S so, that at least for calculation purpose one can make an estimate for lambda S, latent heat of vaporizations. And one can always go back correct make some iterations to have the improved iteration for the solid surface. So, once we have all three we equated and qualitatively we have discussed. We know in the certain parameters Reynolds number high flow rates a small thickness of solid surface dry air, air should be dry not humid.

All this will affect you rate of drying and mind you if the rate of drying is also decides your critical moisture content right or equilibrium moisture contain actually. So, this is the all we have for this heat transfer what we should do now we will take up an example and we will do some numerical calculations knowing the heat transfer aspect, heat transfer parameters. Or we can calculate N C and how we can find out how long will it take to dry the solid from x 1 to X C atleast. Because beyond that then when you have the capillary effects things become more complicated.

And for this course third year level course we skip those quantity aspects of (( )) then (( )) diffusivity or the rate however, decides your moisture content X C etcetera.

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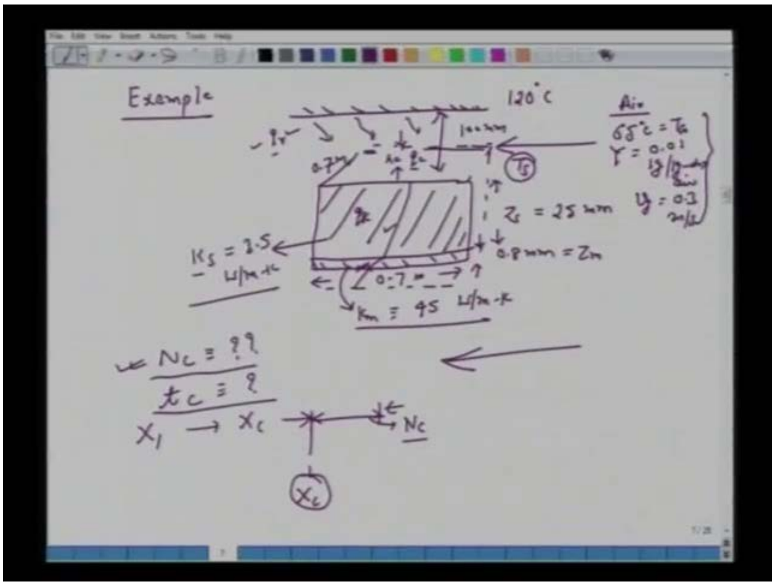

So, let us take this example here so, in this example let us say we have this solid surface at 120 degree centigrade. So, it is a radiating that is a very high temperature and there is a heat radiation q r. We have a tray here let say this thickness is 0.8 millimeter. So, we follow the same nomenclature this equal to z m, m for the plate. On this we have kept k solid say moist solid which we want to dry here. So, not with the same scale qualitatively this z s is let say 25 millimeter. So, one is solid and here we have this plate thickness of this thermal conductivity of the solid surface k s is 3.5 watt by unit meter Kelvin here. So, K m expectedly will be very large 45 watt per meter Kelvin. So, these are the physical properties of solid two solids here.

Let us say this dimension is also given to us this is 0.7 meter and here this is a rectangular slur is also 0.7. So, essentially you have 0.7 by 0.7 and you have this is 25 millimeter so, 0.25 here let us see that let say that distance between the 2 is 100 millimeter. So, why do we require this distance air flows through this, of course air flows from here also. So, when we want to calculate h c we required this d e equivalent diameter of flow so, we further let us put on the specs for air, air has a temperature of 65 degree centigrade.

So, this T G humidity of air is given as 0.01. So, kg per kg of dry air and this velocity of this air is 0.3 meter per second. So, the air flows over this and flow underneath this heat is transferred by conduction from here so, let say there is also h c here like in a previous. So, there is a drop another drop, another drop till we reach this T s. So, again this T S is not known to us solid surface what is the temperature of a solid surface T S will be decide by all three transfer transports. One is q r, one is again there is will be a convective heat transfer coefficient here they will be h c. So, they will be q c and of course from the bottom we have this q k or three which we discussed.

So, question ask here is what is N C how long will it take or what is the drying rate and how long will it take to dry moisture content from x 1 to let say x c. So, we are still discussing this constant drying rate region or the unbound moisture where evaporation rate is or the drying rate is a constant at N C. C is stand for critical moisture content. So, these are the three these are the information given to us. Then from this we have to calculate N C once we know N C we can go back to our previous lecture and we can write down the expression for T C and we can calculate this.

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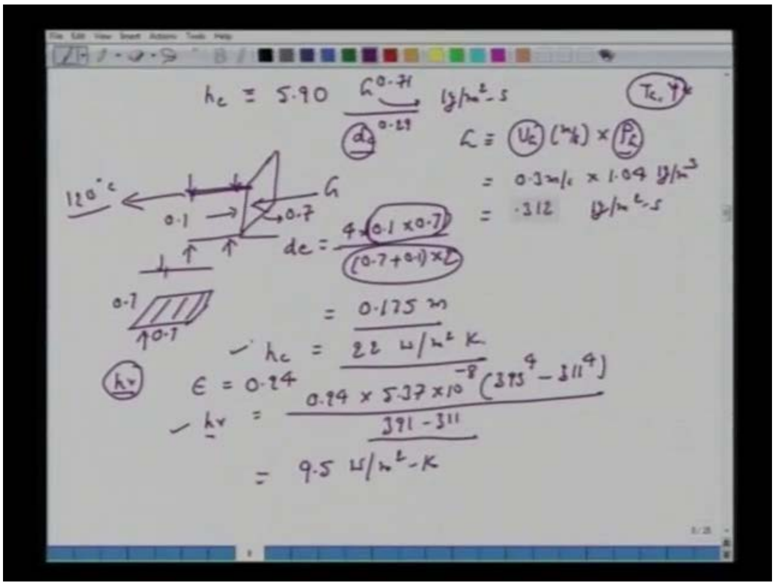

$h_c = 5.90 \dfrac{G^{0.71}}{d_e^{0.29}}$ kg/m²-s

T_G, Y

$G = U_G\,(\text{m/s}) \times \rho_G$

$= 0.3\,\text{m/s} \times 1.04\,\text{kg/m}^3$

$= .312$ kg/m²-s

$d_e = \dfrac{4 \times (0.1 \times 0.7)}{(0.7 + 0.1) \times 2}$

$= 0.175$ m

$h_c = 22$ W/m² K

$\epsilon = 0.94$

$h_r = \dfrac{0.94 \times 5.37 \times 10^{-8} (393^4 - 311^4)}{391 - 311}$

$= 9.5$ W/m²-K

So, let us calculate h c convective heat transfer coefficients we use the same correlation which we had earlier 5.90 G to the power 0.71. This G is given in kg per meter square per second over this d e 0.29. So, what is this G in kg per meter square per second this is nothing, but U G, velocity in meter per second multiplied. So, it is a mass flux multiplied by gas density row g. Now, this U G is of course given to us zero point three meter per second, row G is the gas density. We know the temperature, but mind you there is a moisture also, but Y is given to us 0.01 per kg dry air from a stoichiometries you can calculate the density were mixture of air and moisture.

So, to let say the row G is also given to us or calculated as 1.04 kg per cubic meter. So, this G is 0.312 kg by the meter square per second, what is this d e? We said d e is equivalent diameter of flow. So, here we need the information what is the thickness here for the distance between on the top plate and between the radiating plate and this top of the solid surface through which air flows. So, this d e is very simple here it is a rectangular 4 into cross sections so 4 into 0.1 into 0.7. So, the distance between the two here is given as 0.1 and the solid surface was 0.7 both ways length and this.

So, this all 0.7 here top of the solid surface and the distance here is the plate which is 0.1. So, 4 into 0.10.7 this would be your cross sectional area. All this 0.01 into 0.7 this is your cross sectional area here divided by you will have 4 into this perimeter weighted perimeter so, 0.7 plus 0.1 into 2. So, 4 times this cross sectional area will divide by the weighted perimeter 0.7 plus 0.1 twice of this will give you 0.175 meter. And you put it here direct substitution h c is 22 watt per meter square Kelvin so, approximately let see so, we have h c. Now we have to calculate h r so, h r again we make use of that expression which we wrote in terms of temperature h c.

Emissivity is given as 0.94. So, h r equal to 0.94 multiplied by Stefan-Boltzmann constant which is all s I unit 5.37 into 10 to power of minus 8 T R to the power 4. So, we have 3 9 3 to the power 4, remember this temperature is 120 degree centigrade. So, 120 degree centigrade at 3 9 3 minus this gas temperature as temperature is given as 65 degree centigrade. So, this is 311 to the power 4 divide by we have 391 minus 311. So, h r multiplied by this is same as from your Stefan-Boltzmann equations, if you calculate here h r this is 9.5 approximately watt per meter square Kelvin.

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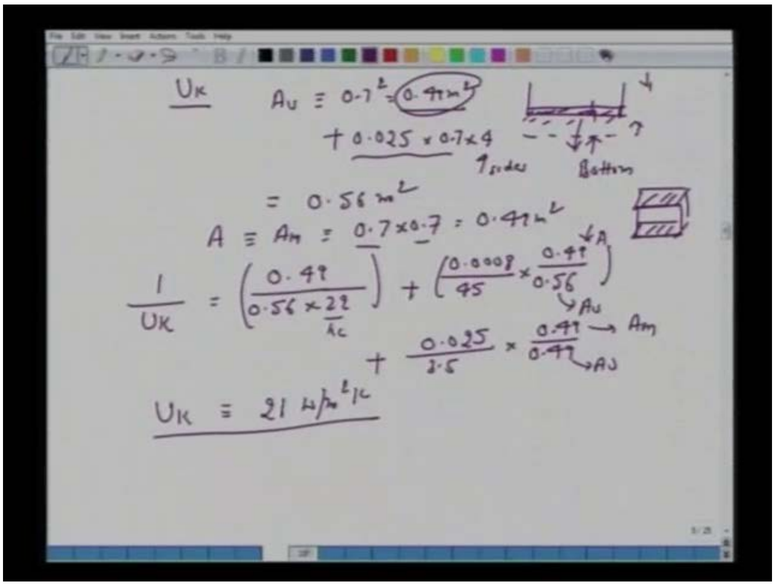

$\underline{U_K}$

$$A_U = 0.7^2 = \boxed{0.49\ m^2}$$

$$+ \underset{4\ \text{sides}}{\underline{0.025 \times 0.7 \times 4}}$$

$$= 0.56\ m^2$$

$$A = A_m = \underline{0.7} \times \underline{0.7} = 0.49\ m^2$$

$$\frac{1}{U_K} = \left(\frac{0.49}{0.56 \times \underset{h_c}{\underline{22}}}\right) + \left(\frac{0.0008}{45} \times \frac{\overset{\downarrow A}{0.49}}{\underset{\searrow A_u}{0.56}}\right) + \frac{0.025}{3.5} \times \frac{0.49 \rightarrow A_m}{0.49 \rightarrow A_J}$$

$$\underline{U_K = 21\ W/m^2K}$$

So, we have calculated h r, we have calculated h c. Let us calculate now it is overall heat transfer coefficient U k so, for that we require say un drying area. So, from the bottom of the plate there is no rate no heat drying here. So, this is your 0.7 square equal to 0.49 meter square plus side walls also there is no drying here. So, you will have to add those two areas. So, this would be your 0.025. So, thickness 25 millimeter multiplied by 0.7 multiplied by all 4 sides. So, this is your 0.49 meter square from the bottom and the 4 sides we have this if you do this 0.56 meter square. Area of drying plus the average mean area of drying it is all the same 0.7 into 0.7 it is a rectangular slab here.

So, 0.49 meter square there is no effect of geometry. So, this all 0.7 here and 0.7 here. So, this is 0.7 into 0.7. So, this is A equal to A m all I supposed to do put in the expressions for U k which is 0.491 area corrected by O.5 6. So, this would be used area multiplied by 21.9 or 22 which you calculated for x c. So, we are adding all three resistances now first resistance is heat conduction heat by convective heat transfer by convections. So, we just now we calculated h c as 22 watt per meter square per Kelvin. So, that is the first resistance here multiplied by the second which is metal so, for the material thickness is 0.8 millimeter.

So, you have 0.008 by thermal conductivity which was very high 45 watt per meter into the area 0.49 divide by 0.56. So, this is area of drying and this is area of un drying here plus you have conduction through the solid surface 25 millimeter thickness 0.025 by 3.5 multiplied by 0.49 divide by 0.49. This one will is becomes your A U, one is A m both of them are the same here. If you do this you will get U k as approximately twenty one watt per meter square Kelvin. So, overall heat transfer coefficient we are calculated U k now go back to the big expressions which we obtain for N C and q s.

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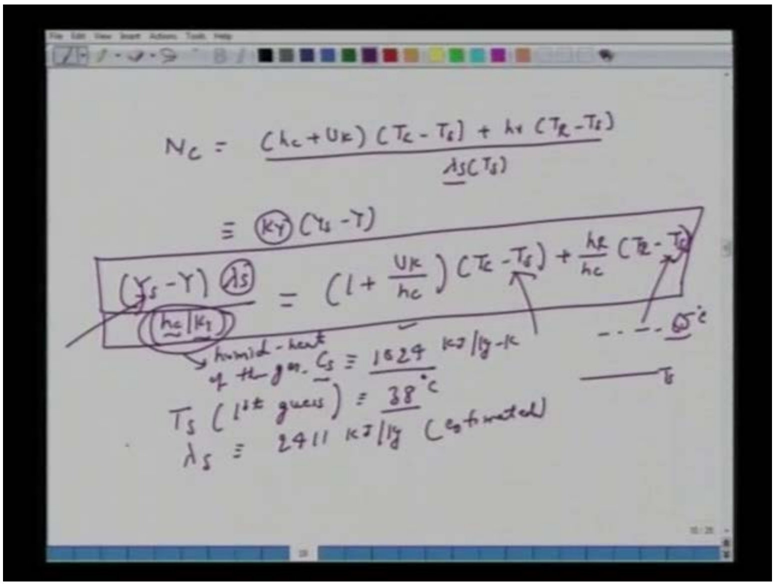

$$N_c = \frac{(h_c + U_k)(T_G - T_s) + h_r(T_R - T_s)}{\lambda_s(T_s)}$$

$$= k_Y (Y_s - Y)$$

$$\frac{(Y_s - Y)\,\lambda_s}{h_c / k_Y} = \left(1 + \frac{U_k}{h_c}\right)(T_G - T_s) + \frac{h_R}{h_c}(T_R - T_s)$$

humid-heat of the gas. $C_s = 1024$ kJ/kg-K

T_s (1st guess) $= 38$ °C

$\lambda_s = 2411$ kJ/kg (estimated)

So, let us re down re write this equations N C equals we had h c plus U k ,T G minus T S plus h r, T R minus T S over lambda S, this we equated to k Y and Y s minus Y. Or notice here this k Y is not given to us, but if you take this two equations and rearrange you should be able to obtain this expressions like Y s minus Y, lambda S over h c over k

Y. Lambda S has to be calculated at T S which is not known to us, but none the one will come back to that later. Let us rearrange this equations like this 1 plus U k over h c, T G minus T S plus h r over h c, T R minus T S. Now I should realize that h c over k Y go back to the again heat transfer this number should be familiar to you.

The ratios of convective heat transfer coefficient over mass transfer coefficients is nothing but humid heat of the gas and this gets a special nomenclature C s. So, C s for this air under that conditions let say is given as 1024 kilo joule per kg Kelvin. So, instead of calculating k Y we realize that h c over k Y is humid heat which we should be able to obtain from thermodynamics we have thermo physical properties of air at that temperature 65 degree centigrade to obtain this number as 102 4 kilo joule per kilo. Now, in this expression everything is known to us except T S and Y s. This T S and Y s, lambda S also is the function of T S.

That means, one has to make an estimate what is the best estimate for T S for which we can calculate lambda S to begin with then we can always come back and iterate. So, the solid temperature here is not known to us, but the air temperature is now 65 degree centigrade not very large temperature let say the first case for lambda is 38. So, T S this is the first guess this is let say surface temperature is 38 degree centigrade. Once we fix T S lambda S we can obtain from the book text book hand out to 2411 kg per kilo joule per kg. So, it is an estimated value at T S. Rest all these numbers are known to us everything except Y S and T S.

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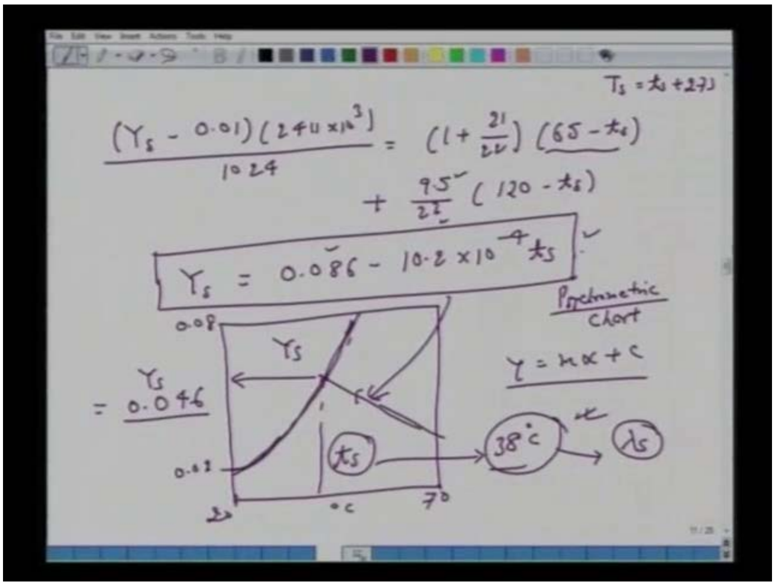

$T_s = t_s + 273$

$$\frac{(Y_s - 0.01)(2411 \times 10^3)}{1024} = \left(1 + \frac{21}{22}\right)(65 - t_s) + \frac{9.5}{22}(120 - t_s)$$

$$\boxed{Y_s = 0.086 - 10.2 \times 10^{-4} t_s}$$

Psychometric Chart

$Y = mx + c$

So, we substitute this number Y s minus Y which is 0.01 then we have 2411 into 10 power 3 to make it below. This is your lambda over X C over k Y which we got 1024 this will equal to 1 plus U k over h c you have calculated 21 over 22, 65 minus T s. So, capital T S is nothing, but small t s in centigrade plus 273. So, the difference will be in Kelvin or centigrade will be the same here plus 9.5 by 22 which is h r and h c, T R and again 120 degree centigrade minus t s. If you do this simplify we have an expression for Y s humidity saturated humidity corresponding to this t s drying surface evaporating surface as 0.086 minus 10.2 into10 power minus 4 t s. So, let us box this. So, now, let us go back to your psychometric chart.

So, this is the way this drying is extensively involved in heat transfer and mass transfer here so, we have this graph here. So, this is available in McCabe Smith or any drying curve any drying book trey bal you mean the book by the data. So, here you will see some number which is humidity 0.02 here you will read the temperature 20 let say seventy degree centigrade. So, this is in centigrade you will see a curve which is your saturated humidity curve Y s. So, Y S verses temperature and we know this curve here Y S equal to which we have calculated in terms of T S, Y equal to m x plus C a straight line. Take the slope take the intercept and draw this line this line will intersect as Y S and this will intersect as T S surface temperature.

So, this is the equation of this line this is saturated humidity essentially you have connected an operating line to this equilibrium line to find this Y S. If you read from the graph you should be read as you should read this as 0.046 and you should read this T S which is 38 degree centigrade. So, we had made a very good guess we are started with 38 degree centigrade just to calculate lamda S. Any other values 41 in the neighborhood of 38 36 37 would have given you the similar result or very close to this. So, lamda S does depend upon the temperature, but plus minus 3, 4 degree centigrade should not make any difference. Any way none the lengths we got the values by intersecting this thermo dynamical curve psychometric chart and this operating line.

Which is from here this gives us Y S and T S once we know Y S and T S this 0.04 kg per kg.

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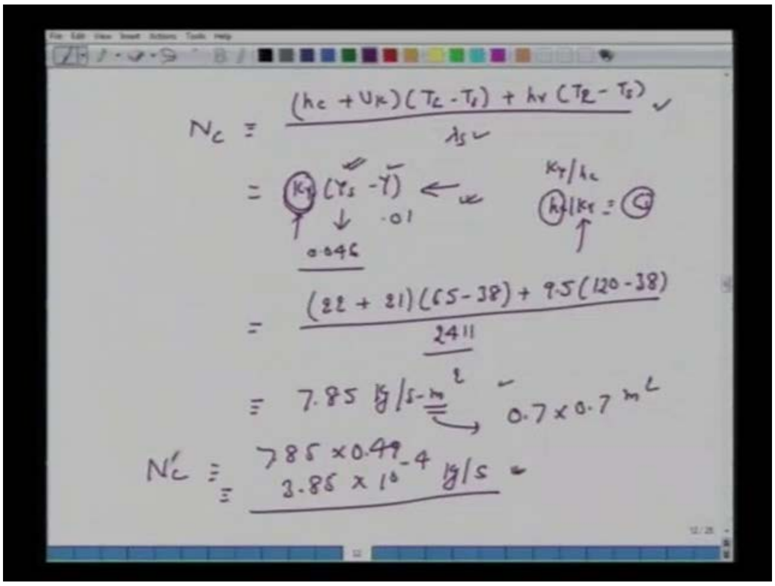

$$N_c = \frac{(h_c + U_K)(T_G - T_s) + h_r(T_R - T_s)}{\lambda_s}$$

$$= \boxed{k_Y}(Y_s - Y) \qquad k_Y/h_c \qquad \boxed{h_c}/k_Y = \boxed{C_s}$$

0.046 $\quad .01$

$$= \frac{(22 + 21)(65 - 38) + 9.5(120 - 38)}{2411}$$

$$= 7.85\ \text{kg/s-m}^2 \longrightarrow 0.7 \times 0.7\ \text{m}^2$$

$$N_c' = 7.85 \times 0.49 = 3.85 \times 10^{-4}\ \text{kg/s}$$

Now we can go back and we can solve for N C all the terms are known here. N c you should be able to go back and write down the same expression which we had X C plus U k, T G minus T S plus h r, T R minus T S over lambda S. Mind you this is also equal to k Y, Y s minus y. Although, we know k Y and we know Y S if you know Y S and Y here, but k Y is not known to us. So, we would not be able to use this expressions we knew you of course ratios of k Y and h c or h c over k Y was given as humid heat. So, if you know this h c we have calculated one can also calculate k Y or we can use these expressions.

So, either use this expression or use this expression to obtain the idea is here is that mass transfer coefficient. We are not calculated directly from any correlations, but from the knowledge of humid heat h c of course we have calculated from the correlation. So, we know the ratios we can calculate k Y we know the humidity of Y which was given us you know which is 0.1 given to us here. And we had this Y s which we had calculated as 0.046. So, if you put these values here we have 22 plus 21, T G minus T S we can still work on centigrade here 65 minus 28 plus h r 9.5.

T R radiation temperature is 120 degree Celsius minus 38 which we have calculated divide by this lambda S which is same as in a previous values which we had 2411. So, from this we can get the values of N C as 7.85 kg per second per meter square of drying surface. What is the drying surface nothing but 0.7 multiplied by 0.7 meter square. So, we have now N C as 7.85 multiplied by 0.49 equals 3.85 into 10 to power of minus 4 kg per second. So, this is the evaporation rate kg per second per meter square or we have a kg per second.

So, this is the example now which we calculated for the N C once you know the value of N C at what rate you are drying it. One can go back to the previous lecture I can find how long will it take to calculate this T C.

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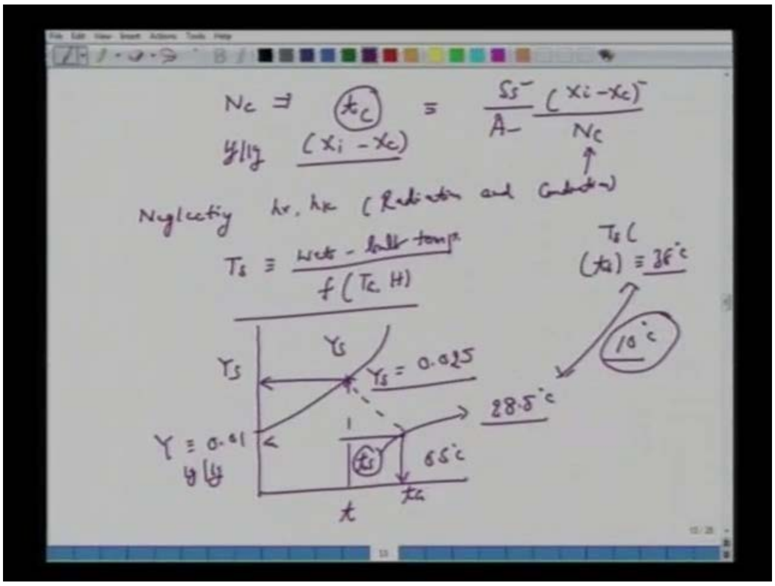

$N_c \exists \; t_c = \frac{S_s}{A} \frac{(X_i - X_c)}{N_c}$

kg/kg $(X_i - X_c)$

Neglecting h_r, h_k (Radiation and Conduction)

$T_s \equiv$ wet-bulb temp. $f(T_G, H)$

So, let us right down the expressions. So, knowing the values of N C we can calculate T C which is from x initial to X C moisture content kg per kg how long will it take one can calculate. S s total amount of solid which you have taken over area x i minus X C over N c. So, once we know the values of N C the rest of the quantities was given one can make this calculations for T C. Now remember this also we said in the just earlier that one can also make an estimate for the surface temperature. If you neglect the heat radiation that means, solid surface say does not receive any heat or as significant amount of heat from radiation. Or from conductions in that case T S surface temperature or evaporating temperature will be same as the wet bulb temperature.

And mind you if you know the dry temperature air dry bulb temperature and the humidity from the psychometric chart, one can follow the adiabatic saturation line to obtain this Y s as well as T s. So, in that case all the calculation for heat (( )) h r or h k radiation and conduction. So, if you neglect this T S is nothing but wet bulb temperature which is just a function of quality of the air. So, if we knew the air temperature and if you know the humidity we can follow this psychometric chart here. So, the same plot which we had here for saturated humidity Y s versus temperature of the air. So, if you know this Y which we know in our case say 0.01 kg per kg we know this temperature here which is 65 degree centigrade.

So, T G and Y follow this adiabatic saturation line of this chart to reach here T S and reach Y s. This value of T S assuming that we have neglected this T S can be read as around 28.5 degree centigrade. So, remember T S what we calculated here or a small T S right this was 36 degree centigrade. So, that tells us that radiation effect because of radiation the temperature is 28.8 to 38. So, difference of 10 degree centigrade is because of radiation and by conductions none the lengths if you neglect those. So, we get 28.8 degree centigrade and we can get Y s as 0.025 kg per kg.

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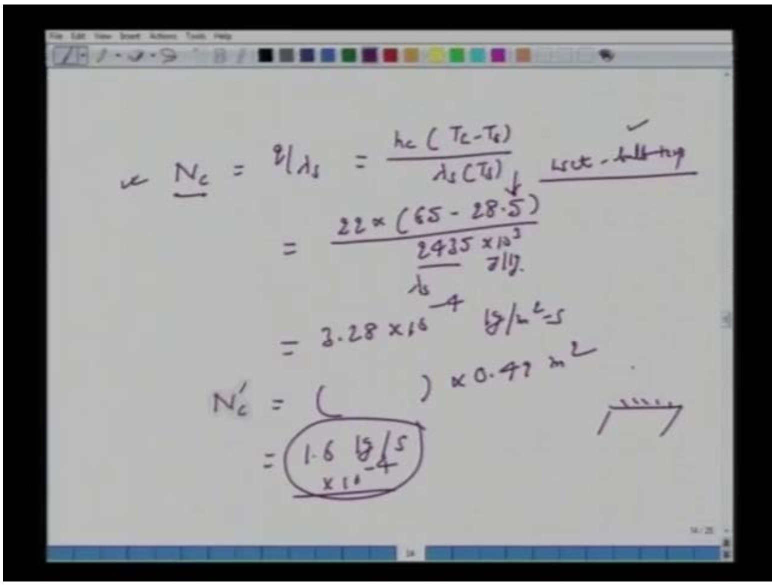

$$N_c = q/\lambda_s = \frac{h_c(T_G - T_s)}{\lambda_s(T_s)}$$

wet-bulb temp

$$= \frac{22 \times (65 - 28.5)}{2435 \times 10^3 \text{ J/kg}}$$

λ_s

$$= 3.28 \times 10^{-4} \text{ kg/m}^2\text{-s}$$

$$N_c' = (\quad) \times 0.49 \text{ m}^2$$

$$= \boxed{1.6 \times 10^{-4} \text{ kg/s}}$$

And then from this we can always go back to calculate N C as q by lambda S for remember. Now, q contains only convections. Convective effect h c T G minus T S over lambda S, h r and h k because of radiation and conductions they are neglected. Now, we know this quantity 22, temperature is 65 degree centigrade. T S just now calculated 28.5 degree centigrade over lambda S. Now again should be calculated based on this 28 degree centigrade again from the literature value you have 2435 into 10 to power 3 joule per kg. So, this is the lambda S calculated at T S which we have calculated 28.5 degree centigrade which was your wet bulb temperature.

So, now, the saturation temperature or surface temperature has reached wet bulb temperature and from this you will get 3.28 into 10 to power minus 4 kg per meter square per second. Again put this area multiplied by 0.49 evaporating area or drying area we have this N C in terms of kg per second as 1.6 kg per second. So, from here one can compare 1.6 multiplied by ten to the power minus 4 kg per second. So, the rate of drying has decreased because surface temperature has decreased. So, this is the one way to calculate make some estimate for N C when we assume that the surface temperature has reached wet bulb temperature.

Otherwise one has to do this comprehensive calculations for h r, h c. So, today’s lecture we would like to conclude or we have said that we have brought heat transfer aspects into drying for mass transfer. You see the direct coupling of heat transfer and mass transfer because we are drying a lot of heat to drive up moisture not only from the solid surface also from the pores from the capillaries. Of course, we said that for this course at this the level we do not make any computations for capillaries. Transfer small (( )) at a graduate level or certain design of course may be you will get a feel of it.

Right now we have whatever calculations we have done that should be sufficient to you know appreciate that what is the role of heat transfer or mass transfer. Next lecture when we meet now we talk of this continuous drying. So, here we have a solid surface and air flows through this now we will talk of a solid surface which moves. So, you have a belt on which you have the solids say minerals. So, belt moves and minerals moves in one direction and you have air which can flow through this or the air can flow in the countercurrent directions. So, you have different type of situations we will take in the spent coupled more lectures to make some simple calculation for drying. Thank you.