**Mass Transfer II**
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**Module No. # 01**
**Lecture No. # 36**

So, in the previous lecture we took up the new unit operation drying. In that case, we said that each solid has a certain characteristics of drying. In other words, if you want to, you know, design a large scale dryer, we have to start with the the solid which we want to dry in the the sand or zeolites or aluminum. The more important here to note was that, for in drying, this characteristics curve, so called the characteristics curve for drying, not only depends upon the type of the solid, all right, it also depends upon the rate of the drying. That means, the operating conditions you know, under which you are doing the drying, that also makes a very significant difference because, we said that the drying is one unit operations, which is accompanied by both heat transport and mass transport.

So, we have to look at, you know, the different conditions for heating say, are we doing by microwave heating or have we exposed the entire surface area to heater, some heater is that the flow through the solids or is there any air flowing just over the surface, all right.

All of these will determine, say equilibrium concentrations some moisture. So, this is slightly different from the previous say adsorption isotherms, when we said that we fix a temperature, then the isotherms given the partial pressure in the gas phase or concentration in the liquid phase, equilibrium concentration is fixed here.

But here, you must have noticed, that there are so many other factors. They also play a major role. So, essentially what we are saying here that, we have to take a sample of certain solid and determine its drying characteristics. So, we will do a very simple experiment; suspend it in air monitor, its weight gain or weight lose. So, drying or wetting. There also we said that there is a hysteresis. So, the rate of drying is different from the rate of wetting. More here important here is that, we should ensure that the operating conditions, Reynolds number or the area exposed for this batch of the solid at lava scale and you know some (( )) plant or the industrial scale is as close as possible.

So, first thing here is establishing the characteristics of drying. We are talking about the batch drying, to begin with. So, we have suspended solid zealots, accurate carbon fibers, charcoals; we have given certain Reynolds number. So, the solid dries and we monitor its weight loss and we get a characteristics curve.

This curve is very important. We must understand, it makes a, you know the basic of your all calculations, as far as a batch drying is concerned. Before that, if you recall, we also had one more characteristics curve; there we plotted partial pressure.

So, equilibrium vapour pressure of moisture versus moisture content. And of course, we dimensionalized the equilibrium vapour pressure by the vapour pressure of pure water.

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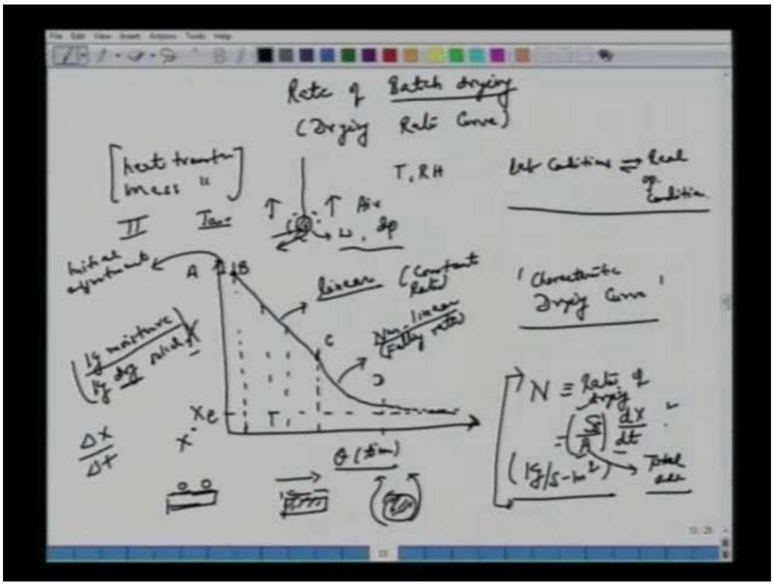


So, that is one type of curve, very important curve. One, in case of drying and the second is this drying rate curve, for this batch solids or batch drying. So, let us begin with this. We are trying to address here the rate of batch drying, all right or we are trying to establish here drying rate curve. We said that, we have to ensure that our lab scale or lab conditions are properly scaled or they are similar to the real operating conditions, under which we want to do this batch drying. So, we are doing here this batch drying. So, what we will do here, so, we have a solid and we monitor this weight loss.

So, there is certain flow rate, air, temperature, relative humidity, initial weight, certain diameter, certain type of solid, everything is right fixed here. So, if we monitor its weight loss, we should expect this type of curve.

So, we have theta, let say time in hour and here, we plot X. How much is the weight loss per kg of moisture is remaining in this solid? So, kg of moisture, per kg of, say dry solid. Here also, we said that the two ways of doing this, one is on the weight basis and another is on the dry solid.

So, we have both heat transfer and we have mass transfer, all right both. So, at certain degree, heat is transported to the solid and mass is getting out of this solid here, which is moisture here. Now, if you do this experiment, we will expect that to begin with, if the solid is cold and this temperature of air is, say very large hot air here, that there will be some initial adjustment time. So, which means, there will be some rate here, say from A to B where, which would be slightly ill defined, in the sense, that air could be colder or solid could be colder than this air or it could be hot, all right.

So here, there is some initial adjustment. Very small amount of moisture will be lost here, so, initial adjustment periods, you can say. Then typically what happens, it is a very typical curve of most of the solids. Of course, there are always exceptions there.

So then, after certain time, very small amount of time, when the solid temperature has reached the air temperature steady state, then there is a weight loss. This weight loss will go linearly like this, for some time, say lets mark it here C, the change in the moisture content is linear, all right. So, after the solid is warmed up or cold has reached the temperature, one monitors very typically this linear change in the weight loss. Then, there is a slow down.

So, let us see, we have till D here D here. So now, we can say that there is a non-linear. We will see this later that this linear curve will represent constant rate constant rate of drying and this non-linear or from C onward, we will call it falling rate; will come back to this later. So, at some rate, this stick reaches non-linearly and then, typically again things gets slow down.

So, we have another decrease, very small decrease till the moisture content has reached say, certain equilibrium values X C or X star. So, this is what we call it a characteristics, a typical characteristics, characteristic with drying curve. Here now, let of course, a different solid will behave differently. This most common that to begin with, one adsorbs a linear, then non-linear and then again, it is non-linear for the rate has slow down here.

So now, if you define like in the previous class, rate of drying as N. So, the most common and practical way of defining this rate of drying would be, so, let say the amount of the solid, dry solid, so without any moisture, let say S, if its area which is exposed for the to the drying is a. So, we can define as d X d T. So, in other words kg per second per meter square. So, that is the rate of drying defined like this. So, you can see that S is initial weight, which is fixed, A is the area which is exposed.

So, either we have a solid like this, slab like this and you want to dry from here or you have a solid like this and you are drying, you have some air flow pass this area is of course, external area; expose area is fixed here. Now, here also we said, that after some time, when the, say the solid is covered with some thin film will be batches that will be developed here. So, of course, the wet area will be different.

But, what we are writing here is the initial total area. So, this area is actually fixed, as per as definition for rate of drying is concerned, it is not the wet area, but, it is a dry area; it is a total area. So, d X by d T, knowing X here, we can do this del X over del T. You can take a small small time step, after we have monitored all this rate and we can calculate this rate here.

So, this is the curve two, second type. Remember, if you had recall, first we plotted relative humidity here, relative saturations right, B bar over B 0 versus x or versus x here. So, that is the number one in drying curve. Now, we have the second drying curve, which we have drawn x versus theta and from this, we calculate n and then again, we plot N versus x.

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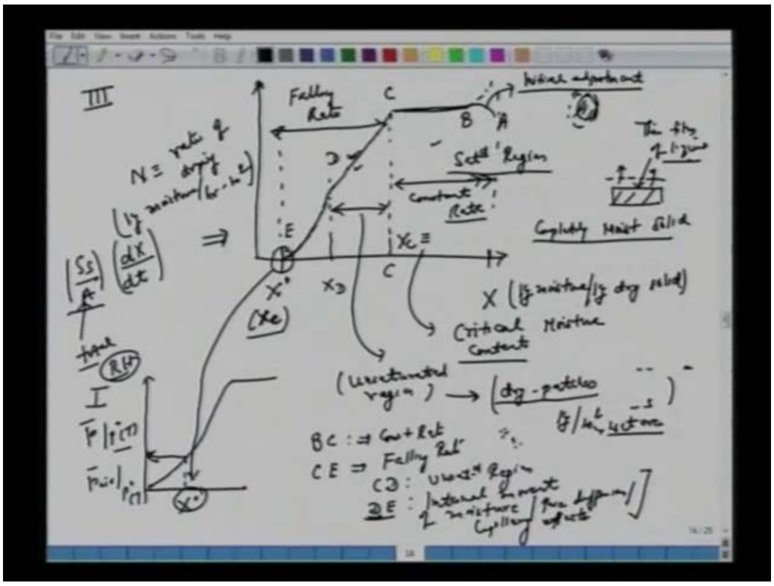


So, this would be the third curve, which is quite important in our context, will revisit them quite frequently. So, once we have x versus theta time, now we calculate n here, which is rate of drying. So now, at what rate moisture, kg of moisture is depleting from the solid surface R over, let say we have meter square. So, that is the unit of this rate here.

Now, we are starting with a very high amount of x, let say that to the extent that solid surface is covered with the very thin film of liquid kg of dry solid. So, that is the unit of this x. So, let us say that we are starting for very large amount of this moisture. So, schematically you can say that we have the solid surface and there is a thin film of liquid or moist moist solid. You can also say completely moist solid to the extent there is a thin film of liquid, all right

So, we can also have a solid which we have suspended and there is a thin film of this solid here or thin film of liquid, excuse me, all right. So, if you recall the first, going back to the previous one, we have linear or there is some initial adjustment.

So, when the solid gets heated or warmed up, you can have, may be like this or you can have like this, starting with a. So, we can call, you know, initial adjustment initial adjustment and this is purely because of heat transfer; solid either is getting heated up or it is getting cold up.

So that, the rate decreases drastically takes some adjustment over a very short time and or it increases or the rate increases. Then, since the x change linearly, d x by d t which we are plotting here, d x by d t, you have S, S over a. So, since x is decreasing linearly, you will expect expect that d x by d t will be constant.

So A, then we start B. So now, the rate is constant till we hit the region, which we call earlier as falling rate curve. So, let us mark this region at C. Why because, X C we will call it as critical moisture content critical moisture content. So, this represents that kg of moisture, per kg of solid till or if their moisture content is greater than X C, rate is constant here.

Why rate is constant? You go back to the previous curve or previous discussions we had, we had thin film of liquids, right. So, just like a constant rate of evaporations, whether you are drying from a solid surface or you have a pool of liquid, all right, it is all the same here, critical moisture content.

Now, after that, so, we can call it, this is a constant rate or you can also call as saturated region. So, these are the different different terminology. All of this, they reflect the same meaning that solid is covered with (( )).

Now, after that we said that X decreases non-linearly. So, the typically what happens here, rate decreases linearly, all right. So, we are plotting d x by d t. Now, the rate will decrease linear.

So, where you say there is a typical drying curve and different solids will exhibit different type of behavior, but, this most common rate is a constant, then rate decreases. So, till here we can say that this is falling rate. So, we have constant rate and now we have a falling rate and here we say, it is a saturated region. Now, we are saying that this region, so, let us mark here, till it falls linearly we will call it, so, C. So, we have x c and d. Let us call give some number here, let us say x d here. So, this is a region which is a part of this falling rate, except, now we are calling it unsaturated region unsaturated region.

So, this is the region. Now, you are start seeing the dry patches. So, some surface has been dried up and some surface still there is a thin film of liquid. Now, you recall also, that it is possible for, in fact, it is like this. That, if you measure this kg per meter square of wet area per second, then whether you are in this region or in this region, the range would be the same, which is same as the rate of typical evaporations.

Since, we are refining to be consistent area, which is the total area; there is a decrease in this. So, as far as the mechanism is concerned, whether the saturated region or unsaturated region, most fundamental, you know, the two mechanisms are the same. What we are seeing here, the decrease is the artifact of this area, which you have chosen, total area. Mind you, the actual area, since because of their patches, actual wet areas are smaller, getting smaller.

So, if you measure per wet area, the two will be the same. So, we have saturated region, unsaturated region, constant rate, falling rate, and go back to this now, because there was earlier when we measure x versus time. There was one more region where the drying rate has gone very very slow. So, you will expect some other rate that will fall like this. When now, it was reached x star or x E, so, this we define; now the solid has reached equilibrium concentrations.

So, this is also, equilibrium concentrations, should go back and recall from our first figure. When we drew p bar over p 0 versus T vapour pressure of pure water and the partial pressure or equilibrium vapour pressure in this, there if you recall, we had drawn like this, typical curve like this, till it reach at some point X star equilibrium concentrations moisture contents, which is in equilibrium with the drying conditions or quality of air, which we said let say p some air over p 0 pure water.

So, this X star corresponds to this X star. It is the same line. Now, this solid cannot be dried below this. It has reached a concentrations, which corresponds to the partial pressure partial pressure which moisture exerts at the surface. That equals your, what about the quality of air relative humidity R H or relative saturation you have in the air. So, now, you cannot go below this.

If you want to go below this X star, you will have to lower, you will have to drier use at drier what will (( )). Now, come back to this this drying curve here. Let us go back, constant rate, falling rate, so, right here, till here it is both are falling rate. So, C to X star, if you call it E for equilibrium.

So that, we can mark this curve B C D E. So, B C is a constant rate, C E is a falling rate and C E has two components. C D, it is a linear decrease. So, rate decreases linear and this we call it unsaturated region. Then, now we have D E; that is more important. Now, here also, we should discuss that what is the mechanism? Here, the mechanism is the same. Whatever mechanism we had here, is the same mechanism, except now, since there are wet batches, so, the area is different. Since, we are plotting with the 'a', total moisture content divide by their total area, will be decreasing, will be smaller here.

Now, here also it is a falling rate. So, second component, except this region, where now internal movement of moisture internal movement of moisture or you can say that, now this is a region where now pore diffusion capillary effects.

See, all these are they reflect the same meaning here, capillary effects. All of them, they start taking place here. That is why we have different characteristics from here to here and from here to here.

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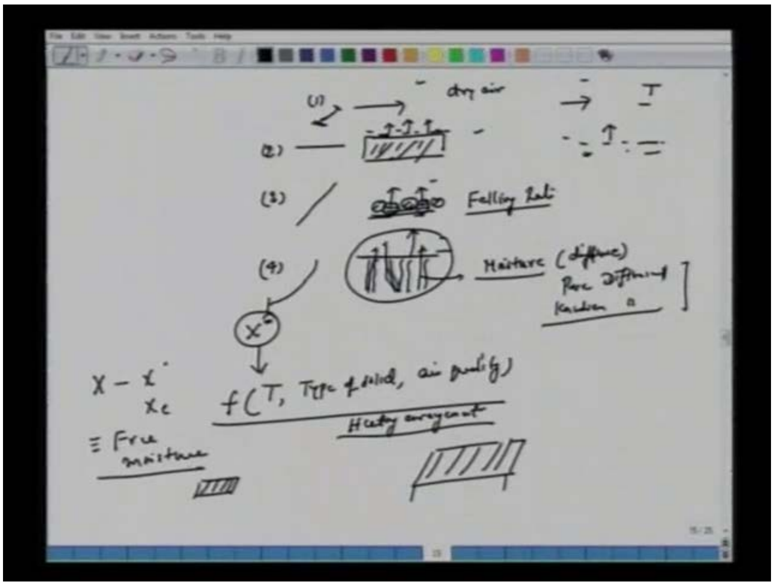


So, what we are trying to say here that, if you have the solid surface, bring in contact with some quality of air, dry air as long as filled with, covered with some thin film of liquid. It is just like a common evaporations. Either you are evaporating from a solid surface or from a lake or from some river, pool of water, the two will be the same. Same rate of evaporation you will expect here and you will expect here.

This water will not see the solid here, all right. So, we have plenty of pool of liquid here. So, the two rates are same as long as the temperature is same and the drying air quality of the drying air is same.

However, when now you have patches, dry patches appears and then, there are thin films here. Here also this rate of evaporation is same as this. The two are the one and the same, that all three, in fact, are the same.

But, why we see a decrease here, falling rate because we are expressing our rate based on the total area. Had we expressed based on the wet area, then of course, this would have happened, since would have been the same here. So, since a total area and this rate decreases because, we have evaporations; moisture evaporates only from here, all right.

Then comes third region, where still it is a falling rate. However, now the capillary forces become a part. So now, we are saying that these solids surfaces are deep solid surfaces, there are capillaries and the moisture has to be supplied. So, this moisture has to diffuse. Moisture has to diffuse by pore diffusion, by Knudsen diffusion. All this we have talked, when we took up the top previous unit operations, adsorption, desorption same mechanism holds good.

The moisture has to be, has to come to the top of the surface, then it will it will be evaporate, all right. Now, we are talking of the hidden moistures. These are the surface moisture, these are the moisture which comes from inside, from within inside. So, we have seen the third type of falling region, where the rate was, you know non-linear here, the rate was linear, and here the rate was constant.

Now, of course, before this we talked of initial adjustment, where the solid gets warmed up or comes to equilibrium with this air (( )). So, one we call in, second is a constant rate, third is a falling rate except linear and the fourth is non-linear falling rate. Till at the end we have reached X star equilibrium concentrations, you cannot dry your solid below this moisture content will be at the most X star. The more important here, this X star is one quantity which depends upon temperature of course, depends upon the type of solid that is one thing, all right.

It also depends upon the air quality. So, look at this equilibrium. This is, you cannot say that truly this is your thermodynamic equilibrium which we have seen in case of adsorption desorption. Here, the drying has to do a lot depending upon the heating arrangement, rate of heating, all of this they decide the heat of rate of heating, the rate of drying, all right.

So, the more important here is of course, the third curve, which we obtain, which is a characteristics of the solid. One more important here is that, we must be very careful that what we do in the batch, in the lab, must also replicate or as close as possible the operating condition on the drying condition on a large, for a on large scale, all right.

So, what we do now, we will like to re emphasize here, that there are three characteristics curves. Traditionally, when you say that characteristics curve, one represents this rate versus x n versus that is very important. But, we must also understand that, we have also have one more curve when you talked of bound moisture and unbound moisture, free moistures, all right. Here also you have X minus X star which is free moisture.

So, X minus X star or X C is your free moisture, all right. So, going back to the previous discussions, now we have three curves. One partial pressure over vapour pressure of pure water versus X, that was the one kg moisture per kg of solid. There we talked about bound moisture, unbound, free moistures, remember the curve we have. Second one is X versus time, that is a true experimental data for that type of solid and that type of operating conditions.

Cross flow, through flow, surprisingly, you know this X star remains that quantity which has not been quantified exactly, you know, for the type of the solids. For the reasons here, that things are so complicated, four diffusions Knudsen diffusion or the heating effects, heat of radiations or heating by microwave heating or heating by conductions to have the flow through, cross through or you just have a flow over the surface equilibrium concentration is a strong function of the operating conditions unlike any other previous, you know, thermodynamic equilibrium we had we talked of absorptions, given partial pressure, how much is a solubility, Henry’s law. So, if you fix a solute ammonia and water, temperature, equilibrium curve is fixed.

Then, we talk about the relative volatility distillations. We have y versus x curve for benzene water (( )) , all right. If you fix the temperature, then we have y versus x is fixed, d x y is fixed. There is no operating conditions here.

Then, we talked of extractions, solubility's immiscible fluids. We fix the temperature, we fix a system and we have this triangular equilibrium diagram A B C. They are distributed by some phase diagram. Then, we talked about adsorptions. We fix a temperature, then moisture content, moisture loading or any solute loading, given the partial pressure in the gas phase or given the concentration in the liquid phase. We have isotherm, Langmuir, freundlich, whatever we have. Here, X star because of this X star, which is strongly depended upon, you know the operating condition. One has to ensure that you have established this equilibrium curve, which is X, which is rate versus X before you design a real system. And, for that you have to choose that the two conditions are batch and the lab conditions or the pilot plant or the industrial scale, they are as close as possible.

So now, we, what we do, we will setup some equation. We like to study, how long will it take given this N versus X curve for certain solid, certain system. How long will it take to dry this solid from one moisture content to another moisture content. So, we will setup the governing equations, then we will like to take an example and we will put some numerical numbers there and we will compute some quantities, all right.

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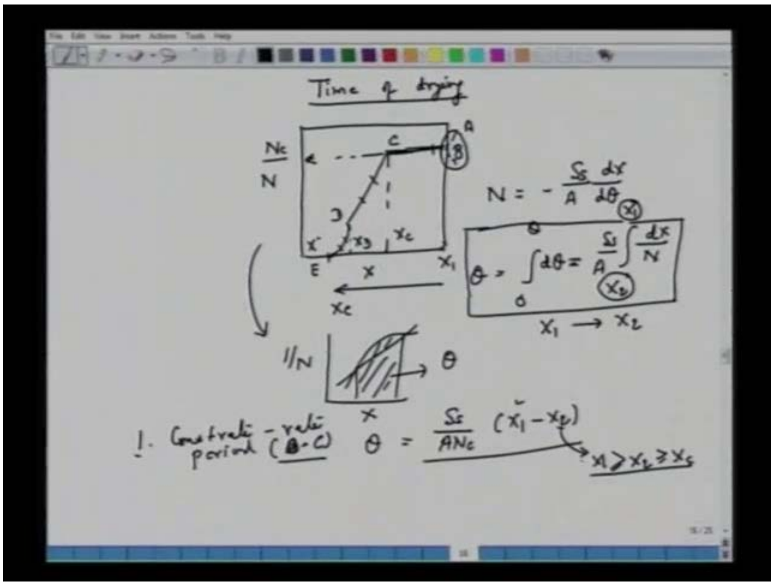


So, let us take this, the second topic is time of drying. So, again we have to start from here. We have N versus x, if you leave aside, you know, this is start initial adjustment. Then, you have p, then, you have C decrease linearly then, non-linearly. So, C D E, you can mark this x C critical moisture. Corresponding to this, we have N c, critical rate of drying, but, notice that this is a flat. So, critical rate of drying is constant between B and C, and then it decreases. So, this is the falling region. Let say, we reach till x t, then we have reach till X star or x C.

So, will follow this nomenclature, B C D E. You took, you can have a dotted line, which remains ill defined, for most of the cases. Initial adjustment, we ignore this time. Here, rate is defined N, as minus S s, the amount of solid dry basis, total area which has been expose for drying over d x over d theta. Theta is the time in hour. So, what is the total time? We just integrate theta 0 to theta, let say d theta equals S s, over ‘a’, since it is a minus sign, here we can say x 2 over x 1 d x over N.

So, we want to dry this solid from x 1 to x 2. Typically, you can have a start from x 1 here; you can go all the way till x e. But here, this x 1 and x 2, you need two quantities in between, may be, they are from the falling linear here, may be they are here, may be x 1 is here and x 2 is all the way till down here.

So, it is a very general expression for calculating this total time of drying, which is integral of this quantity. So, first case is that immediately once you know N, you can find it by area under the curve, from n you calculate this curve, 1 over N, plot x and whatever you have, this we calculate the area under the curve and then calculate this quantity and theta.

But, realizing that the area here, this rate remains constant and the rate decreases linearly, we can have a analytical expressions as well. We can avoid this numerical, you know, integrations, whatever we have, we can have here.

So, the first thing is that, constant rate period. So, we are talking of B C right. We neglected the initial adjustment. So, we have this B C. B C, the rate is constant at N c. So, when you integrate, this very simple integrations, theta will be S s over A, N is a constant. So, N C comes here and you have x 1 minus x 2.

So, this x 2, now, will correspond to actually whatever concentration you have between B and C. So, this x 2 is greater than x 1 greater than x c, but, this is smaller than x 1, all right. So, you understand. So, x 1 and x 2 is any quantities between A and c, x 1 of course, the starting.

So, this is the very simple expression for constant rate period. There is no need for numerical integrations. You read N c from the graph and x 1 to all the way till x c, you can do this calculations. We will come back to this. We will take the example number two comes falling rate period.

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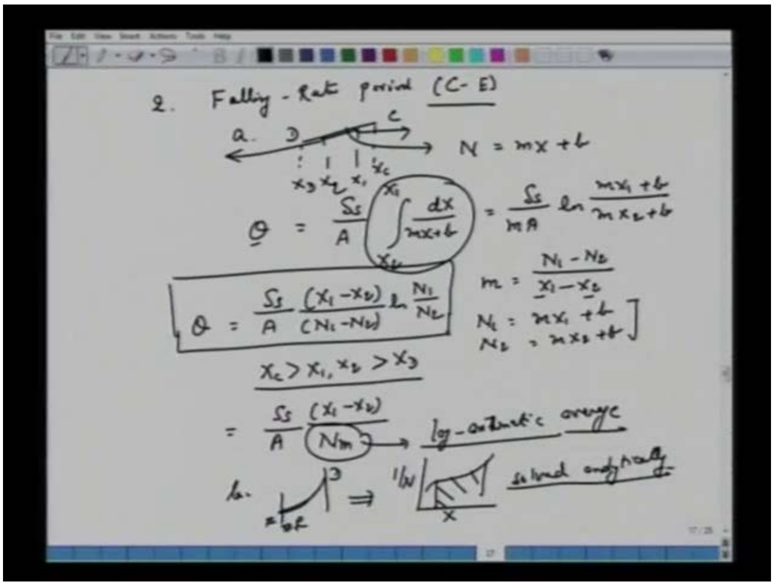


So, falling rate period, now we are talking of C to E, all right. All the way till its falling of course, they are two regions. ‘a’ where the curve is linear starting from critical moisture content, its c to this d, which is x d.

So, here the rate is N equal to m x plus B, all right. You can assume N equal to m x plus B, then you can integrate linearly, you put it back there. Very simple integrations, theta equal to S s over A. We have x 2, we have x 1, d x over m x plus B, which if you integrate, you will get S s. You have m slope of this curve here, ‘a’, l N m x 1 plus b over m x 2 plus B, all right.

You must realize that this m is a slope of the curve. So, any point between 2 N 1 minus N 2, x 1 minus x 2, mind you this x 1 x 2, we now we are talking of N here.

So, now we are trying to integrate between this. This is the slope of the curve N 1 is nothing, but, m x 1 plus b n 2 is nothing, but, m x 2 plus B, all right. You can just put it there, if you like to substitute this m here, to obtain this expression for S s over a x 1 minus x 2, N 1 minus N 2, l N 1 over N 2.

So, now we are talking of x 1 and x 2, which is greater than x d, but, it is less than x c. So, we are in this ratio. So, x 1 and x 2 let us say, this is x 1, this is x 2, this is the rate N 2, this is the rate N 1. So, we got this expressions for linear falling curve. This can also be rearranged to, if you notice that you have l N term here from our some previous context here. This x s over A x 1 minus x 2 and we can call it N m.

What is the n m? You should recall, this is nothing, but, log-arithmetic average, all right. So, now, we have these expressions for this and now the third curve, which is also a falling rate. So, we will call it the region B A, this D C. Now we are talking of this, now D and E till we have reached equilibrium C.

So, from here to here of course, we do not have any analytical expressions. All it means, we will have to integrate numerically. So, one we like to plot one over N and then this x, all right. Whatever curve we have, take that curve, and find the area under the curve to obtain this theta here.

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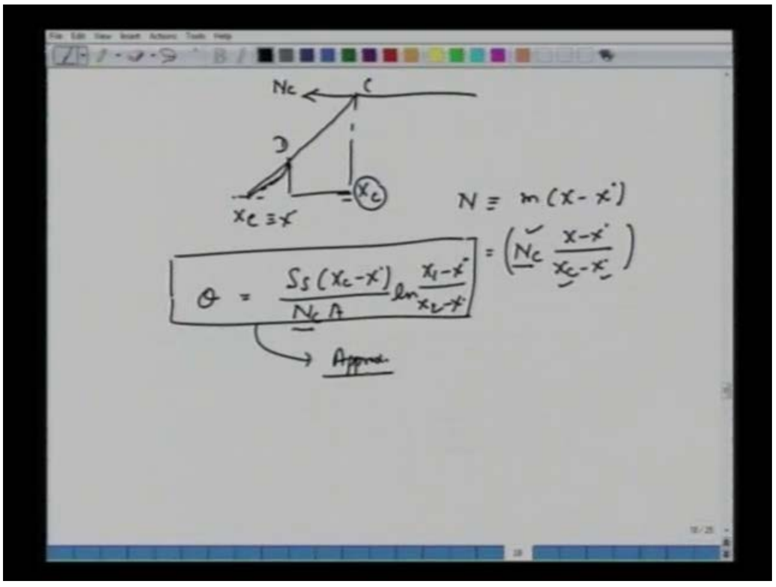

So, the same quantity, which we have here, has to be solved analytically, all right. Very often, what happens that one makes assumptions that, after this constant rate, which starts decreasing at critical moisture content, one makes this assumption that this curve is all the way till x c.

Now, again this depends upon type of solid, where we can say that, well this region is quite smaller than the total region, total theta, or it is possible that there is not much of change here, whenever you have this inflection at d.

In that case, one can make a linear in assumptions, right. From here to here to, say that the rate is an approximations, since m slope of the curve x minus x star. Since, we know this n c, which will be satisfying the equation of this line, right. From, you know or this, whatever you have here, we can say that this is nothing, but, slope is nothing, but, N c x minus x star minus x c minus x star.

So, this another approximations one can do for this N c here. So, if you have N, N c is known, x c is known, x star is known, equilibrium concentrations or x c, same here. Here theta can also be integrated to show that S s, x c minus x star over N c critical rate corresponding to critical moistures into l N x 1 minus x star over x 2 minus x star.

So, it is an approximation. One can make or you can make to the numerical integration as well, all right. So, we have non analytical expressions, as well as these numerical expressions to calculate total time of drying here.

So, starting with this N versus x N versus critical, this moisture content, we identify all the points, constant rate decreasing, then further decreasing till it is x c, one can start with this general example, very basic definition for rate for drying and see which range is constant. We can do analytically analytically numerically or you can make these approximations, you can do the numerical integration as well.

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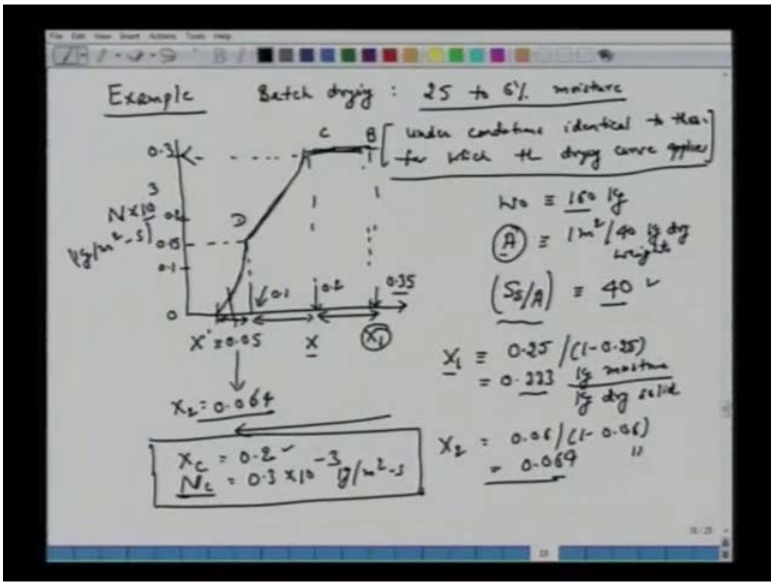


So, with that, now we take an example for this time of drying for a patch system. So, we take this example here, very simple example, very basic. Now, this is about batch batch drying. Problem reads like this, something like this; that we want to decrease the moisture content from 25 to 6 percent, all right.

So, we have to decrease the moisture content here. The more important here, you reads that what condition is this; we want to decrease under conditions identical to those for which the given drying curve applies.

So, what we have been trying to say earlier, it is very important that we understand this. Here, first one has to start with the drying curve, all right. So, this is again very, it is different to what we had, you know, different under the unit operations like adsorptions. There the isotherms is fixed for the system, not for the operating conditions. Here, the problem says that I have I have the solids, it has the moisture content 25 percent, all right. I want to decrease from 25 percent to 6 percent; however, the conditions are the same under which, I have a drying curve available.

So, this drying curve are generated for a priory. It was generated before I want to do design a real system and we are hoping that this drying curve will represent the two conditions which are identical. Of course, it is not practical, you know, doing the experiment in the lab laboratory or in the pallet plant or in the industrial scale.

But, that is the way it is drying is there, you know, it is a more of an engineering drying, that some approximations here. So, may be, we have kept the two Reynolds numbers same. Here, we have the same heating conditions, heating by radiations, same level of radiations convection, same Reynolds number, same heating by conductions. So, that means, we have the thickness of the solid same as what we are going to face there in the real world, all right.

So, with that, so, we are starting with that drying curve. The drying curve is looks like this. So, we have x, we have N, the unit of N is kg meter square per second. This n is reported as N into 10 to the power 3. Let us put some number 0.3. We have 0.2, all right. Let us put a 0.1 here and we have 0 here.

So, you have this B C starting with B. We are ignoring A. The rate is a constant till it hits C. At this level of 0.3, it decreases linearly, it decreases all the way till D.

Let say, this D is 0.15. So, this is a falling linear rate from C to d. Then, it starts decreasing non-linearly, till it goes to all the way till concentration. Here, which is x star and let us put this number at 0.05. From the graph, let us put this number as 0.1. Let us put this number as 0.2 and the first number, here we have this and the graph is 0.3 or maybe, let us put it as 0.35.

So, this is the 0.35 to 0.2. This is a constant. This is your constant rate here, then it is a decreasing rate, linearly decreasing day rate, non-linearly, all these numbers are known to us.

Now, we say that moisture content is 25 to 6 percent moisture. Solid weight is total solid weight is 160 kg to surface area or the area for drying available is 1 meter square per 40 kg of dry weight. So, from this, you should be able to make a guess or make a compute that, what we require is S s, over A total weight that was the expressions for N.

So, the number outside the integral over S s, over A it is nothing but 40, all right. So, kg of total weight, dry weight, no moisture there, per area, so, that is equal to 40. So, this is given to us; total weight is 160 kg, moisture content is 25.

So, the initial moisture content on dry basis, all this is all dry basis right. This is nothing but, 0.25 over 1 minus 0.25. This, we have done dry basis, weight basis earlier in previous lectures. It is always advantageous most of the time to work on this dry basis.

So, 0.333 per kg of moisture, per kg of dry solid. What is x 2? 6 percent moisture, so, 0.06 over 1 minus 0.06, which is 0.064 same unit as before. So, where are we right now, 0.33? So, we are in this range. Very close to this that would be our starting x 1 and wherever we have to go till 0.064, this is 0.1. So, we are going till here.

So, we want to dry this till 0.064, which is our x 2. So, from x 1 to x 2; that means, going to cover all three range, constant falling rate, linear falling rate and this non-linear falling rate, till this locations here, which is 0.064.

So, it is a very straight forward problem here. x c is one quantity we should note immediately because, we are going to use this. So, this x c is 0.2 and corresponding to this x c is, we have N c, which is 0.310 to the power minus 3. So, the graph was reported like this.

So, it is 10 for minus 3 kg per meter square per second. So, these are the two quantities of importance. Important parameters of a drying curve is a critical moisture content that reflects the range beyond which or below which, you will have the falling rate, beyond which, you have constant rate, pool of liquid, etcetera. We have talked those and critical, this constant rate is 0.3 into 10 to the power minus 3 which remains constant between B and C.

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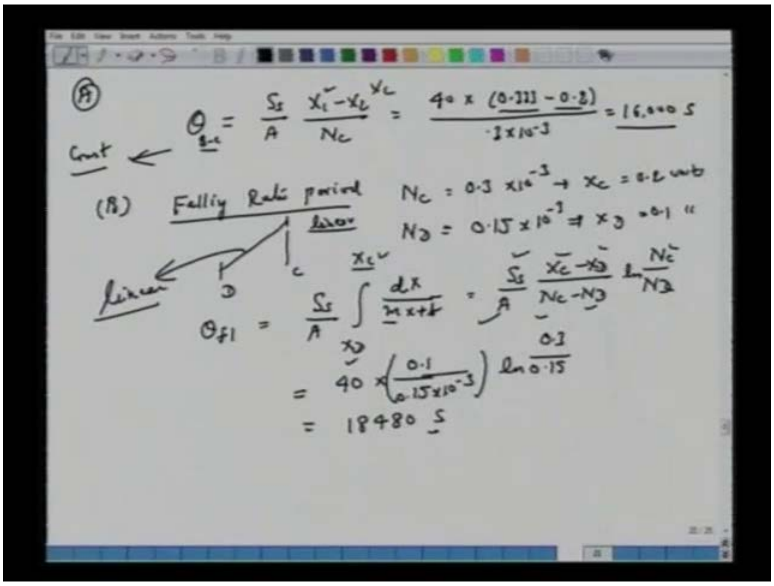

So, with these two, all we have do, we can go back and substitute in our equations which we have developed. You can start from the first principle. You should not take much time to start, to obtain the final expressions, which is x 1 minus x 2 over N c. So, S s over A is 40 x 1 minus x 2.

So, we are starting with 0.333, constant rate goes till 0.2. So, this is the first constant rate we are trying to calculate theta between B and C, all right. So, 0.33 minus 0.22 divide by N c, we have rate 0.3 10 to the power minus 3, we have 16000 seconds.

So, it takes 16000 seconds for the moisture to dry from 0.33 initial concentrations, initial content to theta C or x C, where now, you start to see decrease in the falling rate. in the rate. So now, your heat falling rate period. Already we have said, N c is 0.3 10 to the power minus 3 corresponding to which, x c is 0.2 units you have for both and now, you want to, you have to dry till 0.065. So, you have to go till N d, all right.

So, now we are going starting from C. We go till d, till you have the linear falling rate period. So, this N d if you read again from the graph, this is 0.15. Go back to the previous, here this N d is 0.15 10 to the power minus 3, we have this x d; we can read 0.1 units. So, theta F 1, 1 to denote the first region, falling rate period is same as S s a x d x c.

So, we are decreasing from x c to x d. First, we decrease from x 1 to x c, now, we are decreasing moisture content from x c to x d. We have d x over m x plus b, all of those we integrated; you can go back and integrate here. We can just write down the final expressions. When you substitute the slope of the curve, we have x c minus x d N c minus N d l N N c over N d.

All these quantities are known to us. All you have to do is to substitute S s by A is 40, x c minus x d 0.2 minus 0.1, N c minus N d 0.15 10 to the power minus 3, l N 0.3 divide by 0.15. Put all of these numbers to obtain approximately, say 18480 seconds, all right. So, now we are done with this C to d, where the rate is falling linearly, it is a linear. Here, the rate was constant.

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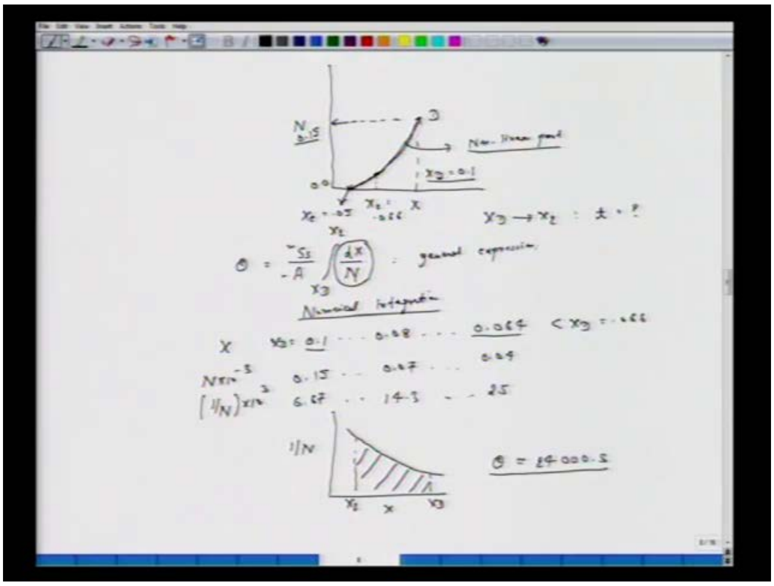

Now, we come to the third rate. Let us plot N versus x for last stage of drying. So, we have this rate curve, drying curve like this, N versus x for the last stage of the drying, when the rate decreases non-linearly.

So, already we did the calculation till d, when the moisture content x d was 0.1. So, this is the non-linear part of the drying curve, the last stage. Corresponding to this d, we have the rate described as or calculated as 0.15. We have been asked to calculate, how long will it take moisture content decreases from 0.1 to x 2, which is 0.066 equilibrium concentration; that means, the rate becomes 0, x c is 0.05.

So, question asked is, x d to x 2, how long will it take; what is the time. Here, we have the general expressions for drying theta equal to S s, over A amount of solids, over the area of drying x d integrated to x 2, we have d x over N.

So, this is the general expressions for drying here. We are discussing here this non non-linear part of this curve right, from here to here. This extended till x c becomes 0.05, at which the rate is 0.

So, to integrate this, you can see, since this is a non-linear part here, we require numerical integrations. So, some numerical integration technique, you take as many as point possible here. So, let say we we have x moisture content 0.1 0.08 decreases to 0.64. So, we have to go till it is below x d, which was given as 0.066.

We are starting from 0.1. We have that, which is your x d equal to this, for this the rate N 10 to the power minus 3 is given as 0.15. These are the data 0.07 0.04. You are supposed to take as many as data point in between, for a very smooth curve here, all right.

So, take as many as points here. Since, we have 1 by N d x, we require 1 by N rate here, all right. Which if you do it, you will get 6.67, 1 over 0.5 will get 14.3 and then, we have this 25 and some intermediate data points. So, all you have suppose to do is to plot 1 over N. We have x here, all right and your trend is like this. You have x d, you have x 2 essentially; this integration d x by N between x d to x 2 is nothing, but, the area under this curve.

So, calculate this area under the curve. Take this as an exercise. Put the values of S a S s to calculate theta. The time taken to reduce the moisture content from 0.1 to 0.066 is approximately 24000 second.

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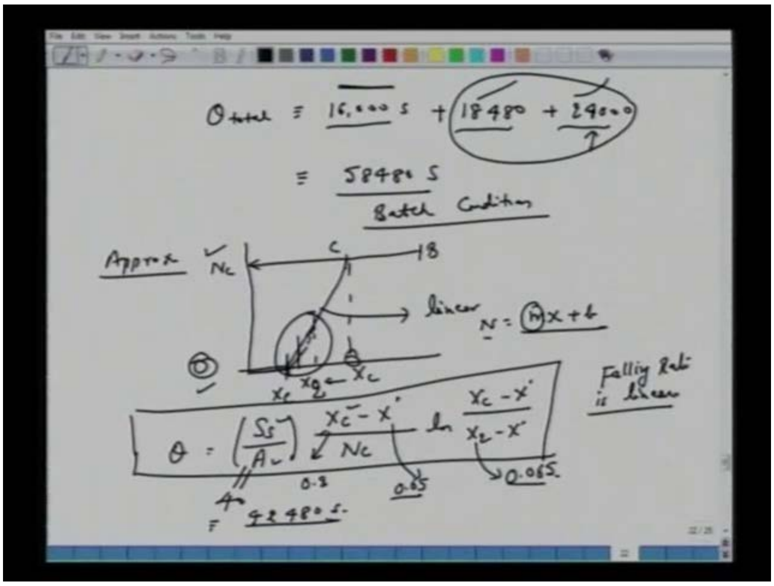


So, we have three, all three times, total time for drying. First, we had number for constant falling, constant rate which was 16000 seconds. So, this is the rate at, this is the time over which the rate which remains constant moisture dries at a constant rate.

Then we have falling rate, but, we have the linear falling rate which was 18480 seconds. The third, which we just now we calculated for non-linear rate. So, this is linear, this is non-linear rate from 24 around 24000 seconds, all right.

So, total time is around 58480 seconds to dry the solid under batch conditions. So, it is a very simple example and here, we have made use of certain analytical expressions or we have made use of certain numerical calculations to calculate the last quantity.

We also said, if you recall that one very good approximation, which can also work. If you assume that from B to C, which is a constant rate, when you hit this critical moisture content then the rate decreases linearly.

So, we ignore this D point here or we are at least, we are approximating that the rate which decrease non-linearly is very close to this, very approximately, we can say that it is a linear throughout. So, we hits till x c so; that means, till x 2 x c to x 2, we can do, we can assume it is a linear. Right here, throughout it is a linear.

So, this is approximations. We also discussed; we can use the expression directly for theta. All you have to do is, now you have the same m x plus b N, N c satisfies this here.

We have x c satisfying here. We know this x c at which the rate is 0, that is equilibrium rate is 0 here.

So, we know this quantity, we know this quantity, we know this quantity here, x c, one can also make use of this to find the slope. What is the slope here? We can go back calculate this n, put in the integration equations, we showed that this number assuming that entire rate of period is linear, falling rate is linear, you have S s x C x star N c, which is also known to us l N x c minus X star over x 2 minus x star.

So, this is another expressions, approximate approximations assuming that entire falling rate is linear. So that, we avoid this numerical integrations, of course, depends upon type of the curve here. May be, in this case, one can do, one has to plot and convince himself that the error is not much here. So, S s by a, is known to us. This is 40, x c is 0.2, x star 0.05, 05 equilibrium moisture contents, x 2 you have to dry till 0.065.

S s by a, the whole quantity was 40. Put all these numbers to obtain, that you are getting this very very close to 18480 plus 24000, the two numbers which is 42480 seconds, all right.

So, in today's lecture, we have taken this example and before that, we had developed analytical expressions to calculate different time of drying for constant rate, falling rate, linear, non-linearly for a batch system. Before that, we said that there are three very important drying characteristics curves. One is partial pressure versus x, then you have x versus time and from that you calculate rate versus x. All three of them have important meanings, in terms of, you know, understanding the mechanism of drying. What is the bound moisture, what is the unbound moisture, what is the falling rate, when does it happen, why it is a linear or de fact of the area, why it is linear mechanism of evaporation is the same in two cases. Then, there is a non-linear curve, where we said that the drying is controlled by capillary forces or capillaries effects. There is a Knudsen diffusivity. Then we talked of this equilibrium concentration, equilibrium content which depends upon, not only the type of the solids, also the operating condition.

So based on, we marked all those regions. All three curves, it should be familiar that play a major role in designing a real system, real drier. So, that is about the batch drying. We had the example. Next time, we, when we meet we talked up, now flow through.

So, we have the solid batch of solids through which, now the drying it, hot air passes through. Here we have the solid and over which, the air falls air flows upon. So, it is a more like a semi batch kind of thing.

The second one is also semi batch. So, all it is a stationary, but, now air will pass through this. Before we taking that, we will also talk, talked about heat transport. How we can, we are heating the drier; remember, we said that drying, we have to give due considerations to heat transport. Solid has to be heated by conduction, by convection, by radiations.

So, we will make use of how understanding, very basic understanding of heat transfer coefficients and convective heat transfer coefficients, radiation etcetera, to address those issues here.