Mass Transfer II

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Lecture No. # 32

In the previous lecture, we discussed breakthrough analysis or the dynamic response in a packed-bed column. We said that there are two ways of doing the experiments on adsorptions or test on adsorptions: one is the batch and one is the flow. Today, we begin with stage wise operation.

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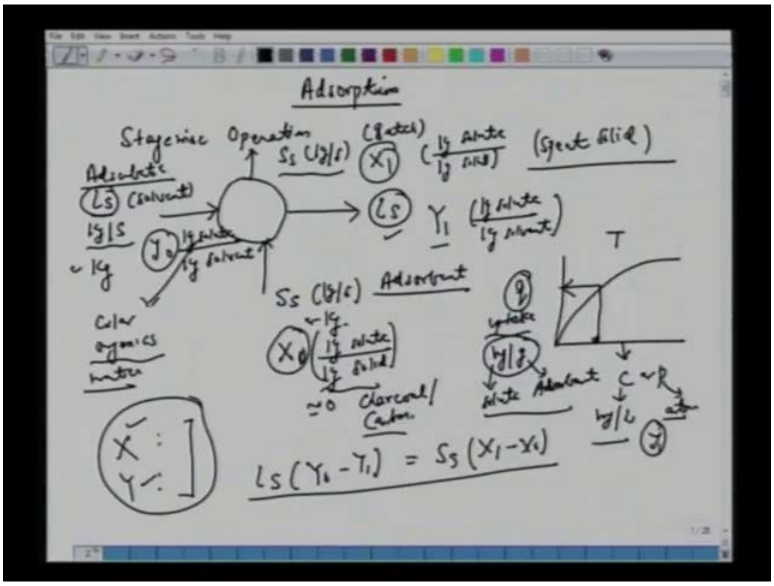

We have adsorption and we are looking at stage wise operation. In stage wise operations, essentially, we have some container, where you bring say the solvent L s if you call it or you have solution – it contains certain amount of solute. Let us say L s is solute free basis. So, solvent – you have say kg per second; you can also have a batch operation; that means we are talking in terms of how many times we can work on kg per second or we can just work in kg. So, it is a question of choosing a proper basis here. You have solvent and say the mole fraction is y 0 kg of solute per kg of solvent. The solute could be say organic, say color or certain dissolve organics in water. Now, you bring in contact with some solid. So, let us say the solid flow rate is S s. Now, this also is again kg per second; or, if you are doing in some batch, here you can have the basis of kg as well.

Now, these adsorbents could be fresh. So, let us say, if this solute concentration in the solid phase is X 1, we have kg of color or the solute per kg of say solid. So, again, here we are defining solute free basis. So, this is X 1 here; this could be 0 – very small quantities. So, you bring essentially a solvent adsorbate. So, adsorbate will be the solute dissolved in this aqua space of some liquid here; you bring in contact with adsorbent. The adsorbent may be fresh or may contain very small amount of the solute. Then, you bring in contact, and finally, you are withdrawing this treated solvent or feed. And, here also, you can have say this L s. L s will remain the same, because you are working on solute free basis. So, assume that the solvent is not getting adsorbed on this solid. So, let say you have charcoal or you have carbon. So, in principles, water will also get adsorbed here, but let us assume that L s remains same here.

Now, this L s (Refer Slide Time: 03:30) will also have certain units. So, you have Y 1, which is also kg solute per k g of solvent. And now, this adsorbent will pick up the solute. So, here you have the flow rate as S s. So, you can reach the solute free basis; amount of adsorbent will remains the same; and, this will have a concentrations let us say here if you call it X 0, let us say this is X 1. So, X 1 will be kg solute per kg of solid. Notice this nomenclature here has given in the textbook X 1 or X 0, but essentially, this is nothing but what we have used earlier also, q uptake. So, how much amount of milligram per gram is the most common unit; how much milligram of solute has been adsorbed per gram of adsorbent. So, that is the most common unit used here. We are using here kg per kg. And, here if you talk of the isotherms, then you have here C or partial pressure of the solute in the gas phase. So, this C would be milligram of solute per liter of solutions. And, here you have this partial pressure say atmosphere.

We can also work on the mole fractions in the gas phase. So, given fixed d, we have these isotherms available (Refer Slide Time: 05:07). So much of C we have q. So, instead of working on this C and q in this problem, let us say we are working on X and Y. So, these are all solute free basis; you know the advantage; we have said earlier also. The operating line based on the solid free mole fractions remains linear; they remain constant. So, that is advantage working on this. So, let us come back to this; we have brought this adsorbate in contact with the adsorbent; we have the treated feed; and, we have this spent solid. So, this is the spent solid. Charcoal has picked up certain amount of solid here. So, if you make a material balance or a species balance, you have L s – Y 0 minus Y 1. So, the concentrations have been brought down from Y 0 to Y 1 by this charcoal, which has picked up. So, let us say this S s (Refer Slide Time: 06:02) is the flow rate kg; it has picked up char color or whatever the organics present as a contaminant X 1 minus X 0.

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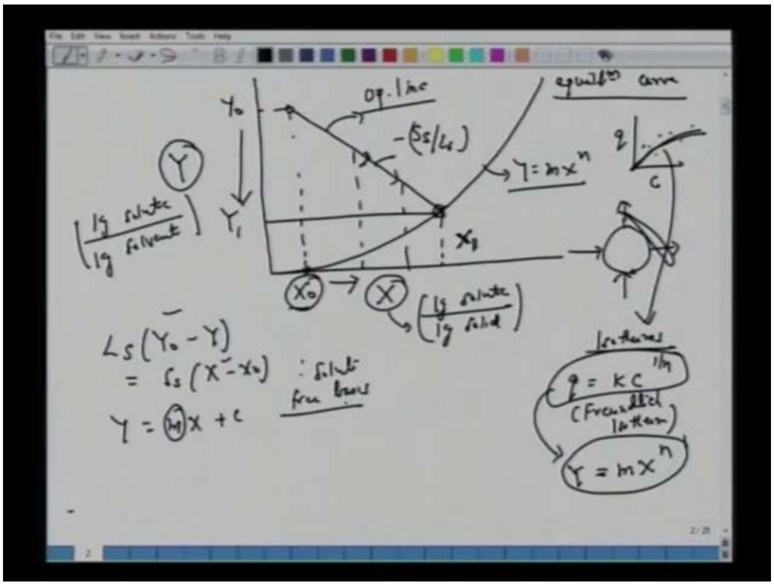

Essentially, if you want to plot on this Y versus X graph, this will be let us say equilibrium curve. So, all we have done – we have converted the q versus c in terms of Y versus X. So, this is kg of solute per kg of solvent. So, we are plotting like this Y. And, X is kg of solute per kg of solid. So, we have only one stage let us say. So, we start with some Y 0; that is a concentration in the liquid phase or the gas phase; very large amount of Y 0. And, you have brought in contact with very nearly pure adsorbents, which does not contain any color or any solutes here. In principle, it will be very small amount here. So, if you look at the balance equation, which we had, L s Y 0 say minus Y equals S s; and, you have X minus X 0. So, Y equal to m x plus c; you can make a notice that the slope will be negative. So, this is operating line equations – should be minus S s by L s – the slope of this curve. So, the concentration of this liquid phase will decrease and the solid phase concentration will increase in these directions till we hit the equilibrium.

If you have the ideal stages and we are assuming that two streams have spent enough time, then this spent solid (Refer Slide Time: 08:16) and the treated feed will have concentrations, which are in equilibrium; or, given by this equilibrium curve. So, essentially, we have come down to X 1. That is the lowest you come down or that is the lowest that is the maximum you can increase solid phase concentrations uptake. And, the lowest you can come down is Y 1; so, Y 0 to Y 1 and X 0 to X 1 here. So, essentially, what we are saying here again that all these isotherms, which we used as q equal to KC to the power 1 over n if you talk of Freundlich isotherms. So, we said that it is the most common isotherm from engineering point of view, where we relate q in terms of C. This can be converted into Y equal to m X to the power n. So, from thermodynamics, classically we have obtained; we have certain data, which we did from the batch experiment; or, the column experiment we fitted with this line; we got the equation q equal to KC to the power 1 over n; we have converted into Y equal to m x to the power n, so that we can plot operating line equations on the same plot. Now, operating line is a straight line, because we are working on the solute free basis.

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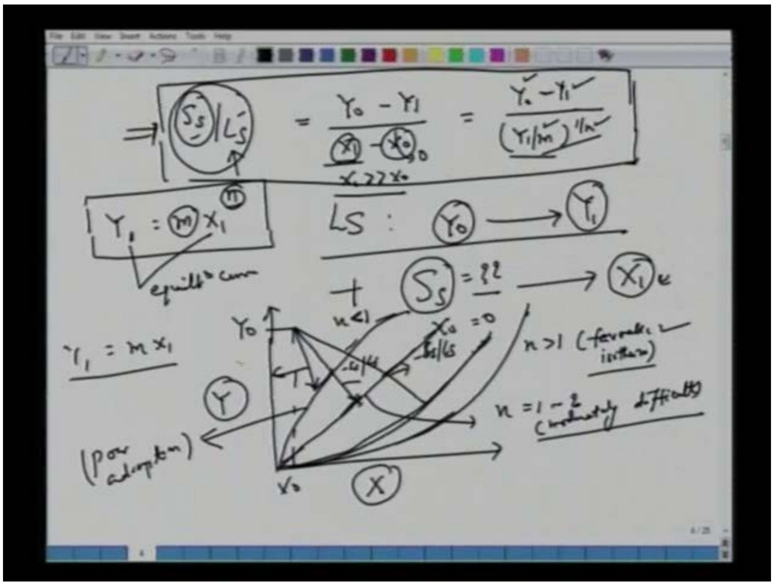

Coming back to these equations, now, you can write S s over L s; so, how much amount of charcoals or adsorbents we required per amount of solvent. This is equal to Y 0 minus Y 1. And, here you have X 1 minus X 0, but let us say X 0 is very small amount – 0. You have fresh adsorbents; or, this amount is much larger than X 0. In reality, as you said, you take the spent solid and regenerate and bring it back to the adsorber. But, in that process, there will be some amount of solute there, but the solute will be much smaller than these concentrations – spent solid concentrations. So, now, you can write as Y 0 minus Y 1. And, if the stage is ideal or you have given enough time, then Y 1 – you can write as m into X 1 to the power n. Essentially, these coordinates of Y 1 X 1 lie on this equilibrium curve. So, we can replace here X 1 with Y 1 over m to the power 1 over m. So, this would be the most simplest or the basic equation or one stage adsorptions.

Let us go back to this equation what is given; you have been given a feed, which concentration is Y 0. We have been asked to bring down this concentration of say color from Y 0 to Y 1. We have been given the amount of this liquid, is L s. To do this, we are bringing in contact with some charcoal S s. Question is how much amount of charcoal is required. This charcoal is fresh X 0 is 0. After adsorption, this concentration will increase; liquid phase will decrease; solid will increase to X 1, but we are not that much interested in knowing this – how much is the amount of X 1 here. So, that is why we have replaced X 1 in terms of Y 1 and other parameters; the properties of these isotherms – slope of the curve and the power index here. So, Y 1 is known; Y 0 is known; m, n come from this equilibrium curve equations; we can find out these ratios of S s over L s per thousand kgs of liquid, how much amount of solid is required from (( ))

Now, look at this slope of the equilibrium curve; we have Y versus X. And here, we have m as a slope and n is the power of index. Now, if n can assume different values; that means, when you plot Y versus X, isotherm may be like this. So, when you plot like this or if you obtain the data like this, all it means n is greater than 1 and you have a very favorable isotherm, because this keeps a very large capacity; very large amount of Y can be removed by this solid. Look at the slope of the curve; it is monotonically increasing – very fast rising curve; that means, if you start with Y 0, it says certain X 0; slope of the curve is like this, which is minus S s over L s. Now, n is 1; that means, you have Y 1 equal to m X 1 straight line; you will have the slope like this – straight line. And, this is n equal to 1; generally, n equal to 1 to 2; it is known as moderately difficult separation for adsorptions. So, n is greater than 1; everybody favors larger values of n; the curve will be more and more increasing – very sharp rise here – n equal to 1. In that case, slope or the operating line will be straight line except it will end here.

Similarly, we can have n less than 1. So, this is n less than 1. And, it says it is very poor adsorptions. It is very difficult to remove often solvent phase, because look at the slope here – keeps on decreasing here. In this case, you will have the slope like this (Refer Slide Time: 14:16). So, we have three cases depending upon type of isotherms – Y versus X; n greater than 1; n equal to 1; n less than 1; n greater than 1 – it is the favorable isotherms. The very more detailed difficult look at the slope S s by L s. Here also it is minus S s by L s; here also it is minus S s by L s; for the same amount of L s, you can see from here, you require a very large amount of solids.

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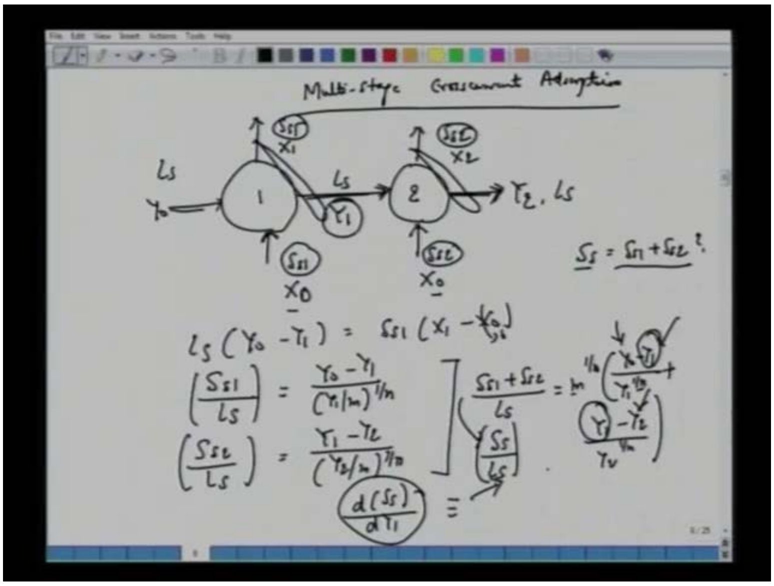

$S_s = S_{s1} + S_{s2}$?

$L_s(Y_0 - Y_1) = S_{s1}(X_1 - X_0)$

$\left(\frac{S_{s1}}{L_s}\right) = \frac{Y_0 - Y_1}{(Y_1/m)^{1/n}}$

$\left(\frac{S_{s2}}{L_s}\right) = \frac{Y_1 - Y_2}{(Y_2/m)^{1/n}}$

$\frac{S_{s1} + S_{s2}}{L_s} = m^{1/n}\left(\frac{Y_0 - Y_1}{Y_1^{1/n}} + \frac{Y_1 - Y_2}{Y_2^{1/n}}\right)$

$\left(\frac{S_s}{L_s}\right)$

$\frac{d(S_s)}{dY_1} =$

Now, we take multistage. This is the single stage here. Now, let us take if we have multistage crosscurrent adsorption. You have stage 1; you have stage 2 L s Y 0; S s 1 – first stage – say is X 0 here. Spent solid concentration is X 1. This is now the Y 1; again bring in contact with the solid S s 2. Say you have X 0 here; same concentration of charcoals, fresh adsorbents; and, after treatment, S s 2. And here, you have X 2; and the treated solvent is Y 2. Flow rate L s will remains the same throughout, because all you have solute free basis. So, L s is same here. Similarly, S s 1; S s 1 is same here; S s 2 and S s 2 remain unchanged; only the mole fraction changes. We will be working on the solute free basis; again for the stage 1, you can write a balance here – Y 0 minus Y 1 equal to S s 1 X 1 minus X 0; let us say s 0 for simplicity; so, fresh adsorbents. This equation you can write S s 1 – how much amount of solid is required in a stage 1 over the amount of solvent treated will be equal to Y 0 minus Y 1 over X 1. But, X 1 you can write as Y 1 over m to the power 1 over n. So, Y 1, X 1 are in equilibrium; Y 2 X 2 are in equilibrium.

Similarly, we can write down this balance for the second equation – second stage. We have S s 2 over L s. This would be equal to Y 1 minus Y 2 over X 2, but X 2 – we can write as Y 2 over m to the power 1 over n. So, what you want to do here –we may like to optimize total amount of solid; S s equal to S s 1 plus S s 2. How much is the total amount of solid? Should we use the larger amount here or should we use smaller amount here? So, if you add the two amounts here, one can write S s 1 plus S s 2 over L s. This will be equal to – if you add these two and rearrange, one can show that you will get m to the power 1 over n Y 0 minus Y 1 over Y 1 to the power 1 over n plus Y 1 minus Y 2 over Y 2 to the power 1 over n. So, one can optimize total amount of solid, is now, S s over L s; how do we optimize? L s is fixed here; Y 0 is fixed; Y 2 is fixed; n is fixed; m is fixed. So, intermediate is the Y 1. So, all it means what is the total. First stage should be in use equal amount; 50 percent of S s or should be 40, 60 or 60, 40. That will decide what happens to this intermediate concentration. So, essentially, one can differentiate mathematically d of S s over d of Y 1; Y 1 is intermediate. We can differentiate these equations to obtain expressions for Y 1 in terms of the known quantities. Before that, let us plot graphically what we are trying to do here.

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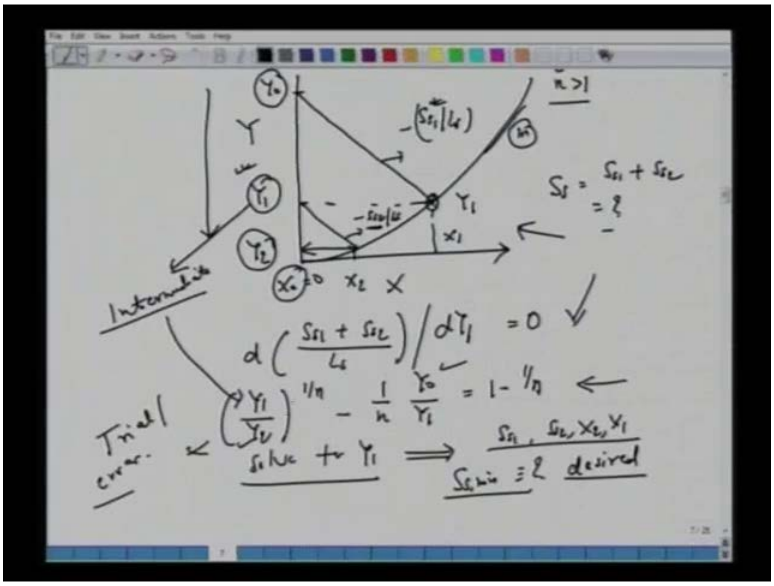

We have this X, Y equilibrium curve here. Let us say we are using n greater than 1; favorable isotherms. Y 0 is fixed; let say X 0 is 0; so, fresh adsorbents. If we know the amount of solid in the stage 1, we can take their operating line and we can reach these intermediate concentrations Y 1. So, this would be Y 1; this would be X 1; this will be the slope of the curve as minus S s 1 over L s. So, if you know this S s 1, you can draw this operating line. This intermediate concentration will be fixed at Y 1; unfortunately, we do not know. Let us assume that if this Y 1 is known, then from here, now, I go to the second stage; take a different slope – minus S s 2 over L s to come down to this desired level of Y 2; for which we have this X 2. So, again, go back to the previous question we had – Y 0 is fixed; Y 2 is fixed. We have been asked to reduce these concentrations. X 0 is fixed here. So, these three quantities are known here.

N is known; slope of the curve, m is also known to us. We would want to know how much the total amount of solids is; S s equals S s 1 plus S s 2 equal to 1. So, that is what we said that this Y 1 is an intermediate quantity. If you fix any of these variables; suppose you fix Y 1, you can go back to this; you can draw this line; then, we can calculate from the slope how much the amount of the solid required is. Once we know Y 1, we know Y 2. From here again Y 2 and X 2 are in equilibrium – last stage; we fix this point; you connect this; you get S s 2 here. So, any of these three quantities: Y 1, S s 1 or S s 2; if you know we can complete this discussion. Mathematically, we go back to this; differentiate this equation S s 1 plus S s 2 over L s over d Y 1 equal to and set to 0. Do this as an exercise to obtain the expressions for Y 1; intermediate this quantity here over Y 2 to the power 1 over n minus 1 over n Y 0 over Y 1 equal to 1 minus 1 over n; solve for Y 1.

Once you solve for Y 1, you can calculate all S s 1, S s 2, X 2, X 1 everything either graphically or by this differential equation, one can solve to optimize how much is the minimum amount of S s min required. What combination actually which will give you desired; not the minimum amount the desired amount of solid, so that we obtain the Y 2 concentrations. And actually, when you solve this, looks like this iteration here; Y 0 is fixed; Y 2 is here. So, one has to solve this by trial and error or some numerical technique to obtain these values of Y 1. Then, you can go back and put the equation for the first stage and the second stage to obtain the other quantities.

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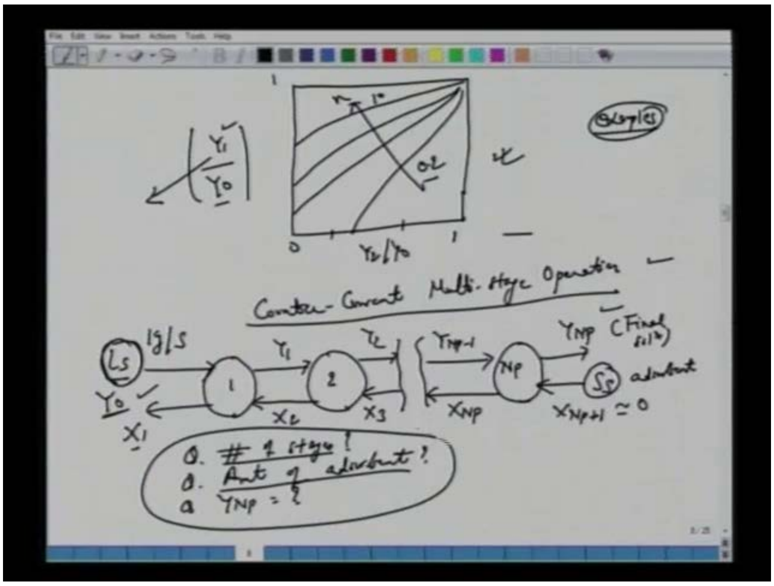

In some of the textbooks, they have also reported the solutions of Y 1 over Y 0 for different values of n. For example, Treybal reports in graph very similar to this; 0 to 1, 0 to 1. There you have different type of curve here for different values of n. So, let us say n equal to 10; very high favorable to very small amount of n, 0.2, which is non-favorable. So, for everything, they are reported here and some log plot; from this also one can calculate this Y 1. So, either graphically or already available plot like this or by mathematically, one can solve for this crosscurrent multistage adsorptions.

Now, we begin with the next topic, which is countercurrent multistage operation. Here also, so far, in the crosscurrent, we will have some certain examples in the next class to address how much amount of solvent is required to bring down this level from say Y 0 to Y 1; or, given the number of stages, what is the final concentrations in the treated feed or how much is the amount of uptake here. All those we will take the example in the next class. So, let us continue with this countercurrent multistage operation, which is also very similar to what we had in case of extractions. So, we have stages like this say stage 1, stage 2 and the last stage N P. So, you have solid or you have say solvent L s; essentially, you have feed, but again you want to work on solute free basis. So, it is a kg per second; mole fraction is Y 0; solute free basis kg of color or kg of adsorbate per kg of solvent. And then, you have nomenclatures Y 1; you have say Y 2 and have a break here; the last stage N P. So, this would be Y N P minus 1. And, the treated feed will have the composition mole fraction Y N P. So, this is the final solution or final solvent. And, to do this, you have done a broad S s adsorbent.

This concentration is X N P plus 1, (Refer Slide Time: 25:44) which could be 0; phase adsorbent or approximately 0. So, we have this X N P. Stage 2 – this would be X 3; stage 1 – this would be X 2; and finally, the exit concentration mole fraction is X 1. And, we can always ask several questions like how many stages are required; different type of questions – how many stages are required given this L s, given this S s from this level to this level; or, the question can be asked – amount of adsorbent. So, one has to look at the degree of freedom, what is given, what is not given; or, given this number of stages, amount of adsorbent? What is the final concentration one can achieve in the treated solid solvent? So, different questions can be phrased or rephrased in different manners.

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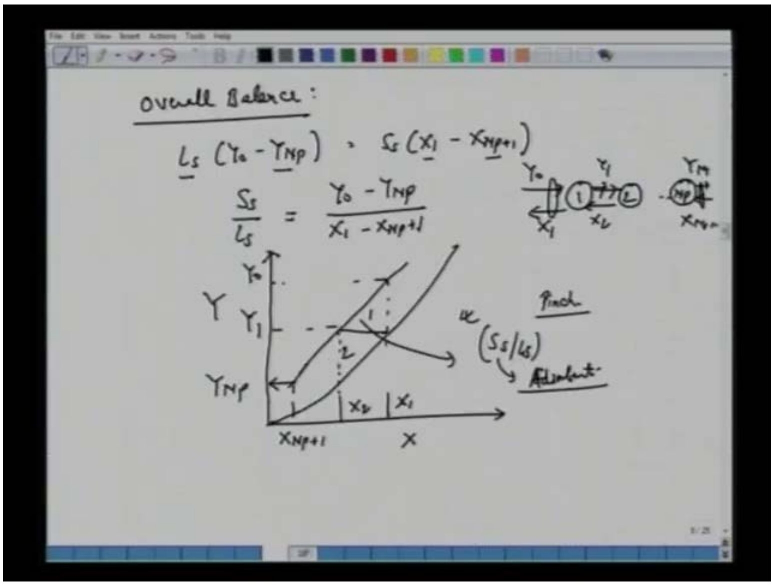

Overall Balance :

$$L_s(Y_0 - Y_{NP}) = S_s(X_1 - X_{NP+1})$$

$$\frac{S_s}{L_s} = \frac{Y_0 - Y_{NP}}{X_1 - X_{NP+1}}$$

Now, how do we do this? We have the similar treatment – first is, one has to make overall balance – overall material balance or overall species balance. So, there you have L s Y 0 minus Y N P equals S s X 1 minus X N P plus 1. So, this is overall material balance. And, from this, one can obtain S s by L s – an expression in terms of the four boundary mole fractions X 1 minus X N P plus 1. We can plot or we can explain graphically again Y versus X. Let us say we have this equilibrium curve like this; we have now the operating line. So, what is the operating line? This connects the bulk phase concentrations Y N P X 1 or X N P plus 1. So, starting from overall balance, if you have this is Y 0; so, we are talking of now, stages 1, 2. And, at the end, we have X or N P. So, Y 0, which is here; and, we have X 1 here. So, 2 phase concentration; 2 streams – y 2.

And now, we have this X 1. We bring down to the level of X 1 here (Refer Slide Time: 28:19). Now, the last – here you have X N P plus 1, which could be very small. Let us say this is X N P plus 1. So, what you obtain here will be this Y N P. So, that is the meaning of this operating line equation, some more material balance Y N P. And then, you have all these stages; we assume that X 1 equilibrium with Y 1. So, go to this Y 1; you have this now Y, Y 1. Once we know Y 1, then from the operating line, we can get X 2. So, this is X 2. And so forth, once we know X 2, we can further construct to obtain the different stages: stage 1 and stage 2, etcetera. And, if you recall, now, in the beginning, we said in the class of absorptions, or whenever you have this countercurrent stage processes, there is a risk of pinch. All it means operating line; wherever it intersects, there will be a pinch; mass transfer will be 0; and, we require infinite number of stages. So, from here, this will give how much amount of solid is required – S s over L s, but one has to also determine minimum amount of adsorbent.

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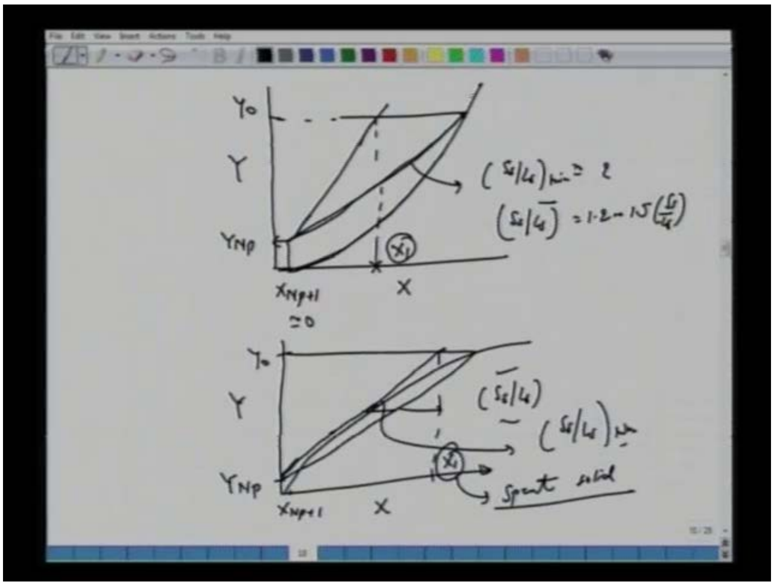

All we have to do is redraw this curve say let say Y versus X and that treatment is same; we have given Y 0. We know the last stage Y N P, corresponding to which we have solid phase concentration X N P plus 1, which could be very small or very 0 here. And now, to find the minimum amount of adsorbent, we connect this. So, that is a straight line, which will give us zero driving force here. We can obtain S s over L s minimum amount of solvent. From this, we can obtain… Then, we can have the operating S s over L s; let us say 1.2 or 1.5 times the minimum amount. So, we can construct this to obtain the spin concentrations on the first stage, which is X 1 here. And then, we can reconstruct the different number of stages required once we know this operating line and once we know this equilibrium curve here.

Again here one has to be careful with the shape of this isotherm. Let us say if we have isotherms like concave downward, then this is Y 0. And, let us say this is very small amount here X N p plus 1. And, we are treating this fluid or this feed till Y N p. If you draw this line here, then this S s over L s will give us some amount of S s – amount of adsorbent required. But, then you have to be careful that there is prospective of things becoming tangent; that means, you take this point (Refer Slide Time: 31:42) and find the tangent; or, wherever it crosses, it touches this point here; that means, one is supposed to calculate this slope here. This slope will give us the minimum amount of adsorbent required. So, this is also very similar to the case of absorption when we have different type of solubility curve and then you get this x 1, which will give you the maximum concentration of your spent solid. So, whether you have concave downward or concave upward, one has to do some inspection to see the curve and see where it makes a tangent here.

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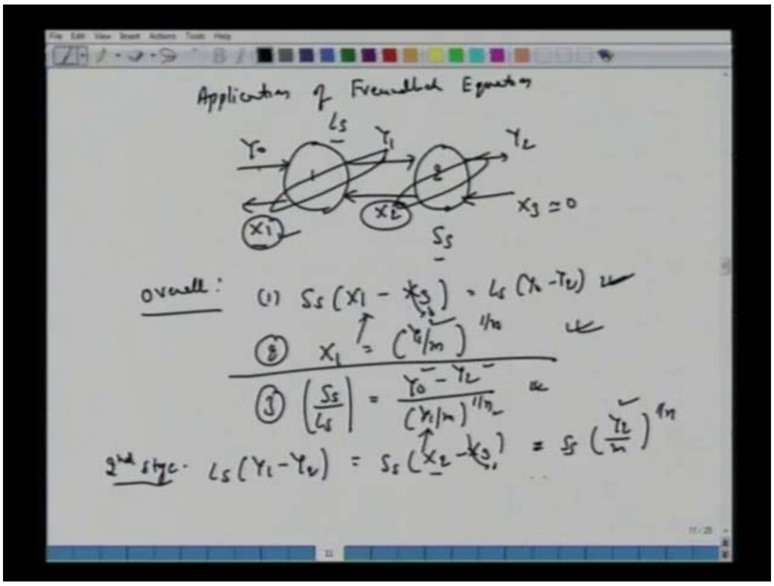

Application of Freundlich Equation

overall: (1) $S_s(X_1 - X_3) = L_s(Y_0 - Y_2)$

(2) $X_1 = (Y_1/m)^{1/n}$

(3) $\left(\frac{S_s}{L_s}\right) = \frac{Y_0 - Y_2}{(Y_1/m)^{1/n}}$

2nd stage: $L_s(Y_1 - Y_2) = S_s(X_2 - X_3) = S_s\left(\frac{Y_2}{m}\right)^{1/n}$

Now, we move on to this and see how we can apply this Freundlich isotherm equation. What we will do now, we will set up the equation and… So, we have two stages for simplicity stage 1 and let us say we have stage 2. And, we have this countercurrent operation. So, Y 0; we have X 1; this is Y 1, Y 2. Here we have X 3, which is nearly 0. So, X 2 here; amount of this adsorbent remains the same as S s – solute free basis; and, the amount of solvent here is a fixed, which is L s. So, let us write down the expressions; overall we can make a balance say S s X 1 minus X 3, which is 0; L s Y 0 minus Y 2. So, whatever is lost here this picked up in this solid as an uptake. But, notice X 1 and Y 1 are in equilibrium. So, let us write down X 1 as Y 1 over m to the power 1 over n.

Similarly, we can write down for the third stage, for the second stage as S s over L s equal to Y 0 minus Y 2. So, what we are writing right now is same equations. If we substitute X 1, let us say, in terms of Y 1 in this equation, and then, we can write S s over L s as Y 0 minus Y 2 over X 1, which is Y 1 over m to the power 1 over n. So, equation 1 – this operating line; equation 2 – equilibrium curve – the two together has given as this equation. The ratio of S s over L s; Y 0, Y 2 are known to us; Y 1 over m to the power 1 over n. This Y 1 is not known; it is intermediate concentrations. Similarly, if you write down the balance for let us say the second stage, then you will have L s Y 1 minus Y 2 equal to S s X 2 minus X 3; but, X 3 is again 0. So, we have done the overall balance; and now, we are writing this balance for the second stage. So, here we can write S s X 2. Again X 2 – we can write in terms of Y 2 over m to the power 1 over n. So, X 2 is in equilibrium with Y 2. So, all we have done, we have replaced solid phase concentration X 1 and X 2 in terms of corresponding Y 1 and Y 2. So, we have this second independent equation; the two together – again you have L s and you have S s. Here also we have S s and L s.

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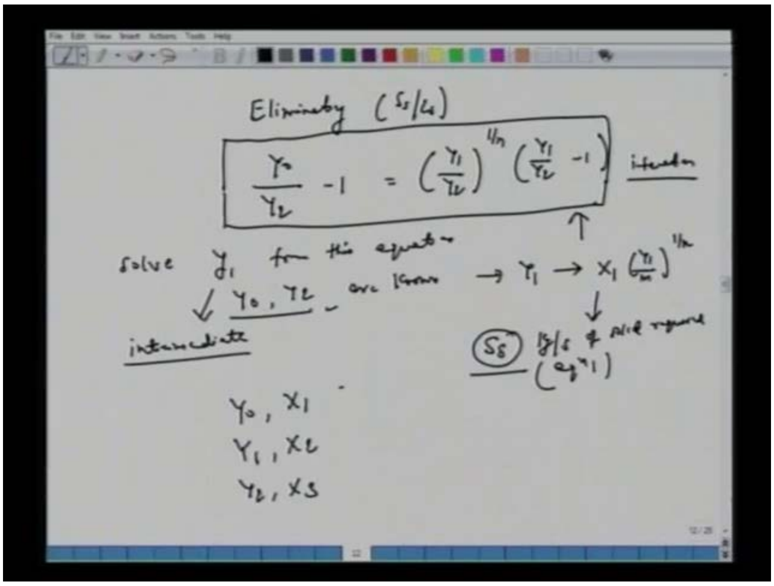

Eliminating (S_s/L_s)

$$\boxed{\frac{Y_0}{Y_2} - 1 = \left(\frac{Y_1}{Y_2}\right)^{1/n}\left(\frac{Y_1}{Y_2} - 1\right)}$$ iteration

solve Y_1 from this equation

$\downarrow$ Y_0, Y_2 – are known $\rightarrow Y_1 \rightarrow X_1 \left(\frac{Y_1}{m}\right)^{1/n}$

intermediate

$\boxed{S_s}$ kg/s of solid required (eq^n 1)

Y_0, X_1

Y_1, X_2

Y_2, X_3

One can substitute S s over L s to obtain… So, if we are eliminating S s over L s, we can obtain this equation Y 0 over Y 2 minus 1 – should try this as an exercise Y 1 over Y 2 to the power 1 over n Y 1 over Y 1 minus 1. So, let us box this equation. From here, now, we can solve Y 1. So, solve Y 1 from this equation. In general, Y 0 – inlet concentration of the liquid to be treated; and, the exit concentration Y 2 are known. So, from Y 0, Y 2, we can solve for Y 1. Once we know Y 1, we can solve for X 1, which is nothing but Y 1 over m to the power 1 over n. And, once we know X 1, we can go back to the equation 1 and we can solve for S s kg per second of solid required to treat this feed. So, that is the usual procedure. Going back to this equation, now, this requires some iteration here. From the iterations, one can solve Y 1 – the intermediate concentration. So, this Y 1 is nothing but intermediate concentration. And then, from this number, we get Y 0, Y 2 – are known to us; we get X 1. From X 1, we can go back to the equation 1 and from there we can get this solvent… Now, let us mark all these points for 2-stage contactor; say Y 0 we have; X 1 we have. Go back to the previous schematic for 2-stage contactor – we have Y 1, X 2; and then, we have Y 2 and X 3.

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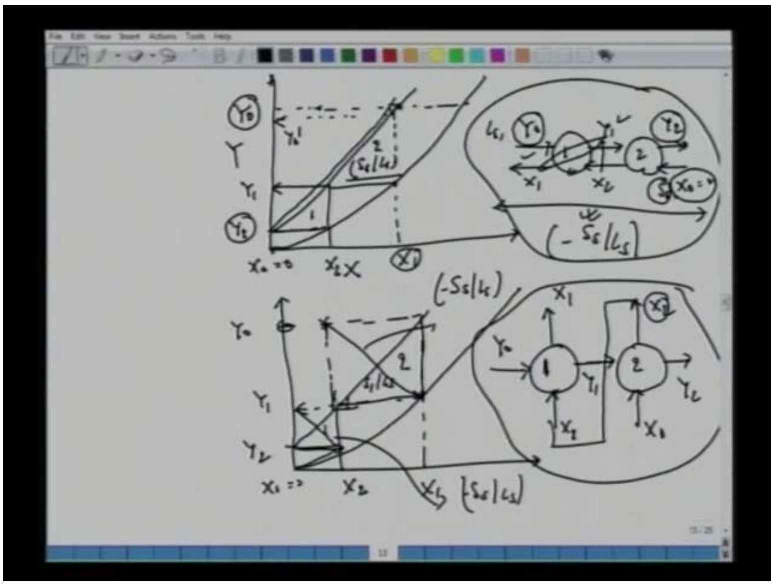

Let us mark these points on the graph; or, we can solve graphically as well. So, we start from here; we have this Y versus X; let us say this is the equilibrium diagram what is given to us. So, we have stage 1, stage 2; incoming stream – Y 0, Y 1 and Y 2. And, we have S s. So, we have X 0. Let us say X 0 is 0; we have X 2; we have X 1; and here we have L s. Let us say this is Y 0; so, Y 0 is here. Let us mark these points first. And, Y 2 is given to us; X 0 is given to us. So, starting from Y 2 and X 0, let us say this is Y 2 and this coordinate here is X 0, which is 0. Now, we know this is Y 0 here – much larger than Y 2. So, this concentration is known to us. But, notice here that we do not know the flow rate – this S s here.

In other words, the slope of the operating line, which is S s over L s minus; and, you get a slope for countercurrent stage processes; we do not know the amount of S s. In other words, this problem is very similar to the previous method, which we had. We try to solve analytically this required situation. In general, suppose we have this countercurrent; we have this slope here G s. So, we have S s over L s. So, if you have S s over… we know the slope we can draw this line (Refer Slide Time: 39:43). But, in general, we do not know what the amount of solvents is. So, one has to do some iterations say start with Y 2 and Y 2 is in equilibrium with X 2. So, Y 2 is in equilibrium with X 2 here. So, this would be the stage 1. Knowing X 2, we can find out what is Y 1 from the operating line. So, this is Y 1 and this would be stage 1 (Refer Slide Time: 40:06). So, we have the stage 1 here.

Once we know Y 1, then Y 1 and X 1 are in equilibrium. So, we have this X 1 here. And, once we have X 1, then from this operating line, we should get this Y 0. So, look at this (Refer Slide Time: 40:23). If we do not have correct estimate for slope here for the amount of solvent S s, then we get some value, which is smaller than this; that means, one will have to adjust the values of slope to obtain exactly Y 0. So, in that case, going back to this slope, one will have to adjust the slope here. So, it requires some iterations to obtain this two stage process; where we have brought down the concentration of feed from Y 0 to Y 2. And, the solid phase concentration has increased from X 0 to X 1 by two stages: stage 1 here and we have a stage 2 here.

Now, there is another approach here. Suppose we have – let us redraw this graph here for two stage countercurrent process. So, here we have Y 0; (Refer Slide Time: 41:12) we have Y 2. And, let us know we know the amount of solvent here. So, we have… From the slope, you get X 1. This is S s over L s. And, here we have this Y 1; and, from the operating line, we have this X 2. So, this X 0 equal to 0; we said that we have stage 1. So, here we have this – let us say this is stage 1; and, we have this stage 2. Now, recall – we said that in the previous class that this countercurrent stage process can also be considered as series two crosscurrent stage here. In other words, look at this – Y 0 and X 2 are brought in contact. So, let us put Y 0 and X 2 like this. Schematically, we write like this. We have two output streams: Y 1 and X 1, which is in equilibrium. So, now, we have Y 1 and we have this X 1. Now, this y 1 goes to the second stage and comes in contact with X 0. So, now, we are representing the same stage processes by cross current. So, this Y 1 comes to this X 2.

And now, we have two streams leaving Y 2 and X 2. But, notice this X 2 goes back to the first stage. So, that means, this stage – we can write like this. In other words, this two stage countercurrent process is equivalent to two stage crosscurrent process, where this stream X 2 (Refer Slide Time: 42:55) is brought back to the first stage as incoming stream. In other words, from this graph, one can also do the same analysis assuming that it is a crosscurrent process. So, starting from say Y 2, Y 0, X 2… So, this is Y 0 and X 2; this coordinate here. Now, you take a negative slope for crosscurrent. For countercurrent, we have S s over L s. And, for this cross current, if you recall from a stoichiometry balance or a species balance, now, you can draw the slope with line as S s over L s, except this is not negative.

And similarly, here we can have this X 2 (Refer Slide Time: 43:39) and we can draw the second slope of the curve as S s over L s; the amount of solvent is the same as the previous case. In other words, these are the first stage; this is the second stage. Now, we have put them as a crosscurrent. Two methods are same and they must give you the same result. Starting from X 0, now, we start from first stage; we bring down to this level here Y 1. Now, we bring in contact with this X 2; Y 1 here to bring in contacts with now X 2 here; to bring back to the second stage we will have this Y 2 and X 2. So, Y 1 and X 2 again are in contact here to bring back this concentration level at Y 2. So, one can also do this analysis; you have different interpretations whether you have a countercurrent method or you have a crosscurrent method.

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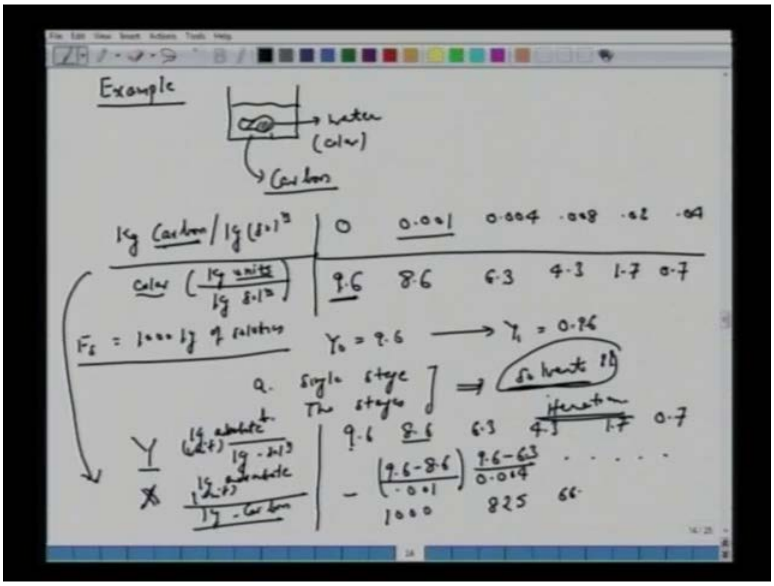

kg Carbon/kg (soln)	0	0.001	0.004	.008	.02	.04
Color (kg units / kg soln)	9.6	8.6	6.3	4.3	1.7	0.7

F_s = 1000 kg of solution $Y_0 = 9.6 \longrightarrow Y_1 = 0.96$

a. Single stage
b. Two stage] ⇒ Solvent ?

iteration

Y (kg adsorbate / kg soln)	9.6	8.6	6.3	4.3	1.7	0.7
X (kg adsorbate / kg carbon)	–	$\left(\frac{9.6-8.6}{.001}\right)$ 1000	$\frac{9.6-6.3}{0.004}$ 825	66·	. . .	

Now, we take an example of one stage and two stage adsorption process. So, what we have in this example – we have water, which contains some dye or color; and then, bring in contact with charcoal or carbon. So, the equilibrium data the weight is given is kg carbon – how much amount of solid is required per kg of solutions. So, the way this experiment was done – we start with certain colors. Now, the color will also have certain units say kg some units; that means one has measured this color say by spectrophotometer UV rays. So, it has certain units. So, kg units per kg of solutions; and, to remove this color, we have brought this carbon here. So, how much amount of carbon is required per kg of solutions. The data is given like this. We begin with the solutions as

9.6 units of color. So, this is the first data. You are starting with water, which contains 9.6 amount of color.

Now, in this, you add 0.001 kg of carbon. So, the basis is let us say, we have the feed and we start with 1000 kg of solutions. So, per kg of solution has 9.6 units and then you bring in contact with 0.001 amount of carbon per kg of solutions. Now, because of adsorptions, the color is now reduced to 8.6. You add some more; you have now 0.004 at equilibrium (Refer Slide Time: 46:18). Now, the color unit is 6.3. Similarly, you have this data here point 0.008, 4.3, 0.02, 1.7; you have 0.04 and here you have 0.7. So, the question is starting with 9.6. Y 0 is let us say 9.6; we want to reduce to Y 1 equal to 0.96. And, we have single stage and we have two stages. So, here stages are given; we have been asked to find out how much amount of solvent is required. So, this is a very previous case. Whenever you have been given the stages and you are asked to find out solvent the solution will require iterations. This is in general; you must have noticed in the earlier class also. But, if the amount of solvent is given, then how many stages are required; then it is a straightforward problem.

Here we will see that graphically or analytically how we calculate the amount of solvent. So, the first thing here is that we must convert this unit the way you have done earlier. So, we have this (Refer Slide Time: 47:33) Y and X; where we write Y as kg of say adsorbate. So, adsorbate is color here. Either kg or we can have some units. So, here we have color; so, adsorbate per amount of solution. So, that is the Y. And here, we have kg adsorbate, which is again certain units of color per amount of carbon. So, this is the solid phase X; and, Y is the liquid phase.

Now, from this, we are supposed to convert into this. So, notice this is (Refer Slide Time: 48:17) 9.6, 8.6, 6.3, which is same as this; you have 9.6, 8.6, 6.3, 4.3, 1.7, 0.7. So, the first you leave it like this. So, now, 9.6 has become 8.6 after we added 0.001 kg of carbon; that means the amount adsorbed is 9.6 minus 8.6 divided by 0.001. So, this would be the first data point which is in equilibrium with this liquid phase. Similarly, we have 6.3 here. So, starting with 9.6 minus 6.3, we brought in contact with 0.004 – amount of kg – we have this number here and similarly, etcetera. All these numbers if you calculate, you have 1000, 825, 663, 395 and you have 223.

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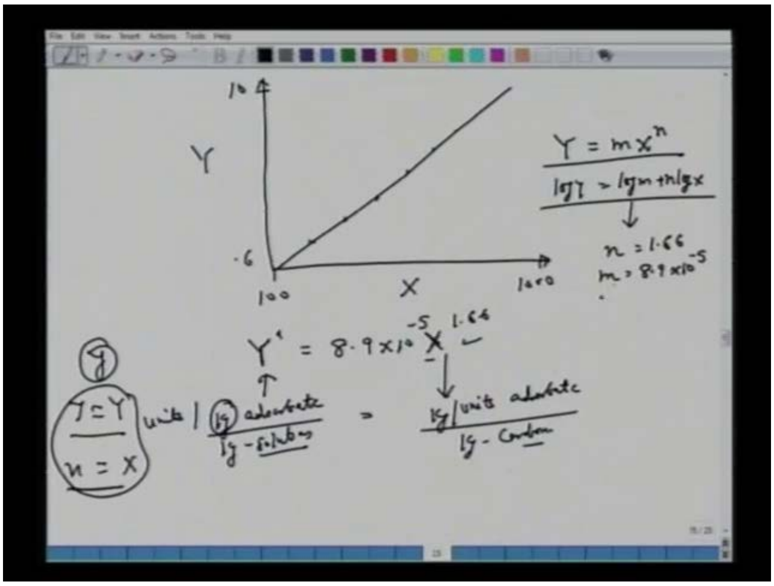

Now, you have Y versus X diagram, which we can plot; and, since the variation is lot, it is not a bad idea to plot on log-log scale. So, if you plot on log-log scale 100 and here we have 1000; 0.6 per Y the lowest units and here we have 10. So, essentially, we plot this data and fit this Freundlich isotherm. So, if you take a log here log y equal to log m plus n of log x. So, on log-log plot, this line will be a straight line. So, essentially, you have the data points, which you have fitted by the Freundlich isotherms. From this slope and intercept, one can calculate; you will see that n power is 1.66 and m is 8.9 10 to the power minus 5. So, now, you have an isotherm, which you can write as Y star to represent equilibrium 8.9 10 to the power minus 5 x 1.66. So, you have kg of adsorbate, which is color here; per kg of solutions. So, you are writing here kg, but this is nothing but some units of color.

Similarly, here you have this x, which is again some kg or you have units of adsorbate per kg of carbon. And also, notice that when we write solutions and here this should have been a small y (Refer Slide Time: 51:08). Essentially, this nomenclature capital Y and capital X – we use for solute free basis. But, here it is a very small amount of solutes. So, approximately, y has been approximated as capital Y; x as capital X. One can still work on small y and small x, but without much of improvement in the accuracies. So, whenever we have the dilute solutions, we have seen in the case of absorption also that in general, one can still use capital Y and capital X and get a straight line for both equilibrium as well as the operating line. So, after this, we get nm; we have the isotherm

– Freundlich isotherm. Now, we can start making the species balance or we can draw the operating line.

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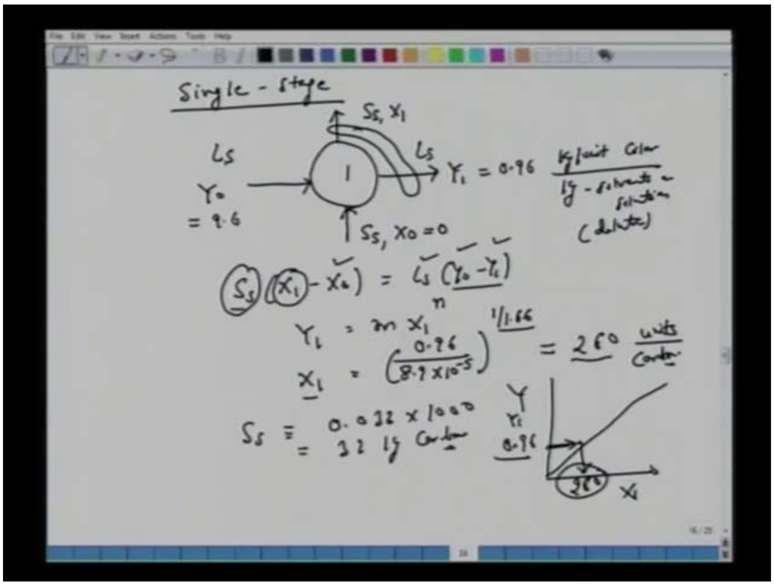
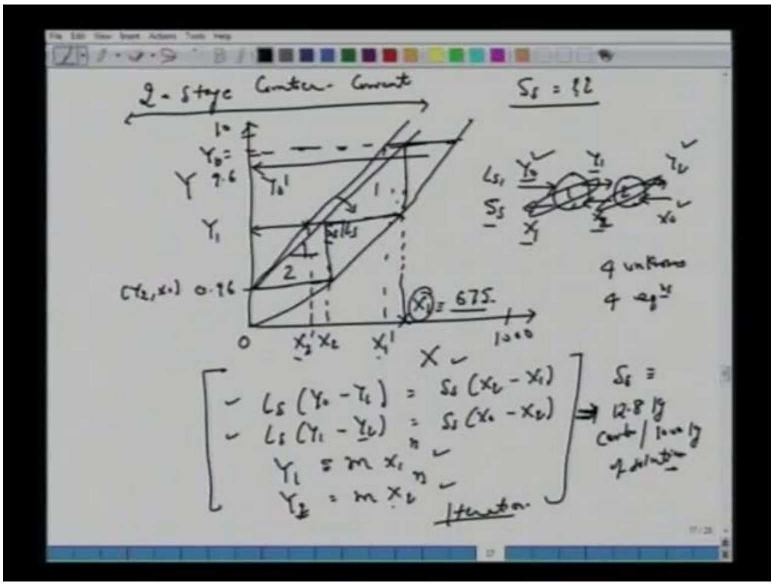


Let us say that we have single stage. So, it is a very straightforward problem now; we have stage 1; we have the solutions say L s; either you put it kg per second or just kg. This contains color, which is 9.6; bring in contact with certain amount of solid carbon, which is X 0 here. X 0 is 0. Let us say we have pure carbon, pure charcoal. And after treatment, we have S s and this unit is X 1 and this solvent is reduced; its flow rate is L s. But, the concentration is now Y 1 equal to 0.96 kg or unit of color per amount of solvent; of course, solvent or solution – assume that it is a dilute system. And, we notice that X 1 is in equilibrium with Y 1. So, it is given by the equilibrium curve and we just write down a simple material balance here; species balance – you have S s X 1 minus X 0 equal to L s Y 0 minus Y 1; S s – we have to find out. X 0 is 0 here. L s is given; 1000 kg of solvent we have; Y 0 is given to us; Y 1 is given to us. We have 2 unknowns here – S s and X 1. But, you must realize that X 1 is in equilibrium with Y 1. So, we have Y 1 equal to m X 1 to the power n – Freundlich isotherm. So, X 1 equal to Y 1, which is 0.96 over 8.9 to the power 10 to power minus 5 to this power 1 over 1.66. So, that is the n.

And if you do these calculations, you will get 260 units. So, again, this X 1 is 260 of mass; this color adsorbing this carbon solid phase (Refer Slide Time: 54:05). And, S s will be equal to 0.032 – X 0 is 0 here; into 1000 – this is your 32 k g of carbon. So, this calculation can also be done here – X1 minus X 0; X 1 we have calculated. Now, we know L s; Y 0 minus Y 1; we can put this equation to obtain S s equal to 32 k g of carbon. Notice here that if you plot down the graph, then Y 1 is point 9 6 here; if you read from directly from the graph, you should be able to obtain this 260. So, this is X; this is Y (Refer Slide Time: 54:49). And here, without solving, all it means one can… or if we have plotted the graph, we can read from the graph to obtain these values of X 1; still for Y 1.

(Refer Slide Time: 55:07)



So, more important now this second stage – let us say that we are using 2-stage countercurrent process. Now, we have 2-stages countercurrent. Again, recall, we just now said that if the stages are given and we are asked to find out how much is the amount of solvent, solid, solute here, then it requires iteration. We have seen in the in the earlier slide as well. So, let us see we can solve graphically as well as we can solve mathematically. Let us plot the graph first for our understanding. So, we have X; we have Y; we have this equilibrium curve. Now, let us say it is a linear. So, here we have 0 to X. We can also plot on log-log to make it linear line here.

Let us say this is 0; this is 1000. Now, here we have 10. So, what is given here is two stages. Solution enters here with Y 0, which is given to us; Y 1 is intermediate; we do not know. Y 2 is given to us – 0.96. X 0 is known to us – pure charcoal; X 1 is intermediate – stage 1, stage 2 – counting from left to right and here we have this X 2.

So, X 2 is now unknown to us; X 1 is now unknown to us. L s we know – 1000 kg of solutions; this solid we do not know. So, these are the 1, 2, 3, 4 unknowns. So, you would be expecting we require four; so, four unknowns. So, we require four equations. We can go back and we can put down all the equations here, for example, L s Y 0 minus Y 1 first stage equals to S s X 2 minus X 1; second stage L s Y 1 minus Y 2 equal to S s X 0 minus X 2. So, these are the two material balances; or, species balance for two stages. This is X 2 (Refer Slide Time: 57:19); and, this is X 1. So, Y 1, X 1 is in equilibrium. So, we have Y 1 equal to m X 1 to the power n; Y 2 and X 2 are in equilibrium. So, we have Y 2 equal to m X 2 to the power n. So, four unknowns and four equations.

We can solve this to obtain S s as 12.8 k g of carbon is required per 1000 kg of solutions as in comparison what we had earlier 32 kg per carbon per 1000 kg of solution. So, 2-stage is much more efficient, but more important here is that when we say that we are going to analytically, we must realize that this is not a straightforward; it is a non-linear equation. So, in general, we require some type of iterations here, because if you want to substitute Y 1 in terms of X 1, which was this intermediate here, you go back to this X 1, but S s is not known. Then, you are supposed to replace Y 2 in terms X 2 in the second equations, divide the two equations, etcetera. You will get non-linear equation; you can solve maybe by iterations. So, this iteration business should also appear on the graph here (Refer Slide Time: 58:38). So, as we have seen in the earlier also let us say Y 0; so, Y 0. And here, we have Y 2. So, these are the two points known to us; X 0 is known to us. So, we can start from here.

Let us say Y 2, which is 0.96 and this is 0 here (Refer Slide Time: 58:55). So, this coordinate is Y 2 X 0. Now, we know Y 1 – incoming concentrations, which is 9.6, but we do not know the amount of the solvent; all we know there are two stages. So, if you know the amount of the solvent, you can draw line, curve like this, which is positive slope. For counter current, this should be equal to S s over L s, but we do not know; that means we require iterations. Let us say we have made a good guess that all it means – this is Y 0 here; for this Y 0, this concentration must be X 1. So, we do not know. So, all we know there are two stages. So, starting from Y to X 0, now, let us say Y 2 is in equilibrium; we have Y 2 X 0; so, Y 2 is in equilibrium with X 2. So, we have this X 2. X 2 extends to the operating line; we get Y 1; and, Y 1 X 1 are in equilibrium; we will get this X 1 exactly here. So, this is stage 1 and this is stage 2; or, we do the counting from here; this is stage 1; this is stage 2. So, notice that if you have made a good guess, then this will reach exactly at X 1. And, this X 1 will be shown whatever we have got from the previous 260, etcetera, which we had obtained from here.

But, notice here this X 1 – if we do not know the slope, then it might require iteration; that means, one has to go like this and then see whether we get the correct values of Y 0 or X 1; all here we know is only the X 0. So, this point is not known to us. So, when we start from here… This stage is not given to us. So, notice that if suppose we make it first0 case like this, (Refer Slide Time: 1:00:52) then we have this operating line. Now, X 2 is moved here; let us say X 2 prime. And, this X 2 prime and Y 1 – now, given this, we will get some other X 1 prime here. So, this requires some iterations; we may not be able to achieve the same Y 0 what we have obtained. Maybe will be first iterations; we will have Y 0 prime here. One has to adjust the slope again back to this to obtain the correct values at exactly two stages, where we have this X 1 here. If we put all these numbers here – X 1, X 2, etcetera; say X 1 will be 675.

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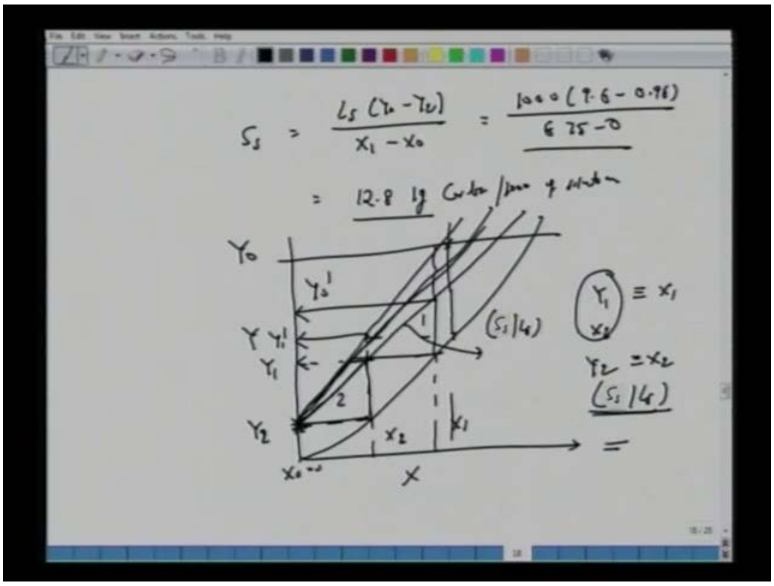

$$S_s = \frac{L_s(Y_0 - Y_2)}{X_1 - X_0} = \frac{1000(9.6 - 0.96)}{675 - 0}$$

$= \underline{12.8\ \text{kg}}$ Carbon/1000 of solution

So, from the slope, here also, one can obtain the values of S s, which is nothing but L s Y 0 minus Y 2 over X 1 minus X 0. So, that is the triangle now you have to taken. L s is 1000; Y 0 is 9.6 minus 0.96 over 675 minus 0. This will give us 12.8 kg of carbon per 1000 kg of solutions. So, again, we want to make a note here that if the stages are not given to us, then it requires iterations starting from Y and X. First, you are supposed to do is, take the equilibrium curve and mark this what we know (Refer Slide Time: 1:02:19). So, Y 0 is known here. All we know is Y 1 and X 0, which is 0; that is it. All intermediate values Y 1, X 2 – after the first stage, they are unknown to us. All we know Y 1 is in equilibrium with X 1 and Y 2, which is the exit concentrations; this is Y 2 – this is in equilibrium with X 2. So, first stage if we draw starting from here, say we get X 2… Now we do not know the operating line, because we do not know the slope here, which is S s over L s. Let us say, arbitrarily, if we make a slope like this; if this is a correct slope, then from this X 2, we should obtain this Y 1. So, this is one stage here. So, this would be starting from the end. So, this is the second stage.

Now, this Y 1 is in equilibrium. So, we have this X 1. But, this X 1 now should give us Y 0. So, if we extend this, now, this should give us Y 0, which we do not have – Y 0 prime. So, look at the error here. All it means we are supposed to now increase the flow rate. So, G s by L s; the slope is S s by L s. So, now, we increase the amount of G s. So, slope if you increase; and, let us say if you increase like this, now, restart the calculations from here – Y 2 X 2. Now, you go to these different levels of X 2 (Refer Slide Time: 1:04:03). Now, you go to this level here. Now, we have Y 1; come back to this equilibrium; take this extend here. Now, again if you extend, this has just exceeded Y 0. So, that means somewhere in between we require this slope. So, this requires iterations either you do graphically or if you do it analytically. We have seen, now, there is non-linearity in the equations. However, if the number of stages are given; if this amount of solvent is given, then we can draw this line and then we find out how many stages are required to achieve this Y 2. So, this was the example we have taken for single stage and for the double stage – second stage adsorptions. Again, we said in the previous lecture also that one can solve graphically or one can solve analytically. Analytically, when you solve, you require some iterations; if you solve graphically, then one has to adjust the slope of the graph. So, you have the similar type of iterations.

Next lecture, when we meet, we will take some more examples on this adsorption either for a single stage or multistage countercurrent or crosscurrent before we start this packed bed column, which is a continuous packed bed. So, we had two cases: one is stage wise and one is a continuous column packed bed.