Mass Transfer II
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Module No. # 01
Lecture No. # 28

So, in the previous lecture, we took an example on multistage cross-current, right. And the idea was there that, we showed that multistage cross-current; it performs better than a single stage, and in the sense that if you fix the total amount of solvents, then multistage cross-current will give much lower raffinate concentrations treated feed than, the single stage or you know if we have more number of stages, then amount of solvent required we will be lesser in case of multistage than in this single stage, if we fix the raffinate concentrations.

So, that was the main example of course, we made a material balance, and we also said that one can work graphically as well as one can work by solving the equations. So, if that when you solve the equations, then you look for a relation between y and x for the equilibrium. So, in that case, you have additional equations.

So, generally what happens, thermodynamically all these datas are available in the tabular form instead of the equations. So, it is wise, it is better, more convenient to work graphically, plot all this y-x data for the equilibrium, and then beyond that, one should do both; try to plot graphically with the with the slope which you calculate or try to solve analytically to see that you are reading the correct values, you are marking the correct coordinates.

So, it is recommended that you try to be familiar in both you know both ways; graphically as well as analytically solving using the equations. Today, we take up one more example on this cross-current multistage extractions, except what do we do here, we do we take a different type of systems. So, earlier we had A plus B plus C and we had to rely on this ternary diagram or equivalent to that ternary diagram, we work to the rectangular coordinate of course, today we will take up an example where we say that the two are immiscible liquids. So, you can go back and previous class notes where we had that that type of systems.

So, we have A B C, say A is a feed, and it contains some C. We bring in contact with the solvent, but the solvent is immiscible with the feed. So, it is only the C which is getting distributed between A and B. So, you can realize that or you can appreciate that this is a simpler example, all right. So, here we will have the same analysis, and we will see that much simpler from of this equilibrium curve should be you know should suffice.

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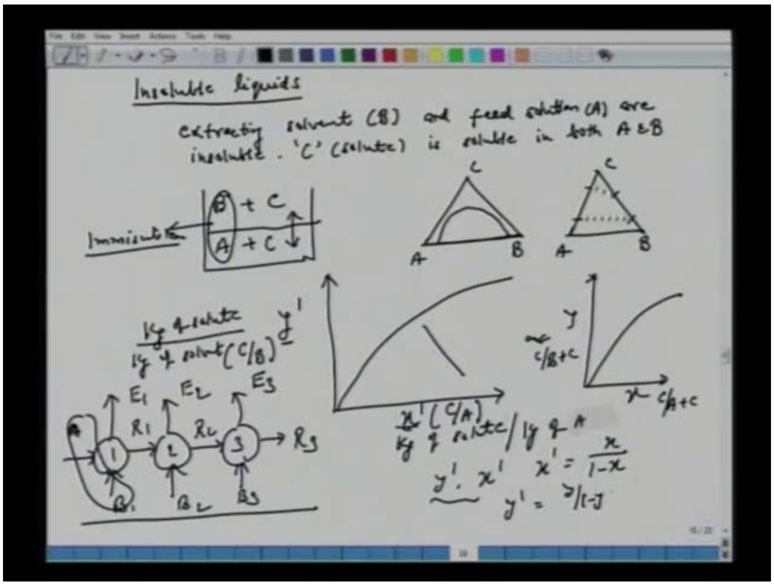

Insoluble liquids

extracting solvent (B) and feed solution (A) are insoluble. 'C' (solute) is soluble in both A & B

So, let us take this example on insoluble liquids. ((No audio from 02:59 to 03:07)) So, what we are trying to say here, you know you should go back and see the previous lectures on extraction. Extracting solvent which we call B, and feed solution ((pauses)) A are insoluble, ((pauses)) are insoluble and C which is the solute is soluble in both A and B.

So, in other words, if we have feed here A, and we bring in contact B solvent, then at the equilibrium, we will have plus C here and plus C here. So, its C is the transferring components. A and B are immiscible or they are insoluble liquids.

So, recall in earlier cases, we worked on this type triangular graph like A B C like this, but now we are saying that A and B they are insolubles. So, we do not have any intersection here. So, generally if this is C at A and B, you take A and C, then there will be some phase diagram intersections like this. In other words, this much simpler case and equilibrium curve like Y versus X should be sufficient.

So, we are saying that X is C over A, C is dissolved in A, and here we have Y; C over B. So, the notation is the similar, you know convention we falling the similar conventions. So, we are potting Kg of solute, per Kg of solvent, and here also we have Kg of solute, per Kg of A, right. So, we have this Y versus X this diagram should be sufficient and here of course, we can have the cross-current multistages. So, feed A is brought in contact with the solvent say stage one some B 1. We have extract and we have this raffinate and we have different amount of solvents B 2, B 3, and we have treated sorry extract phase solvent and we have the treated phase here with raffinate R 3, R 2, R 1, all right

Now, since A and B they are insolubles, then you should also appreciate that instead of working on Y-X, one can work on Y dash and X dash. So, in other words, it is better to work on X over 1 minus X. So, here may be C over A, this is like this. So, this will also work or Y versus X if we write C over A plus C and Y as C over B plus C.

So, either of these two equilibriums diagram should work, but again if you recall in our previous discussion on absorption as well, if you work on solute free basis, then you make advantage of linear operating curve, all right. So, of course, this is the most fundamental A more fundamental, then this then we like to convert this Y X to Y prime and X prime working on this solute free basis.

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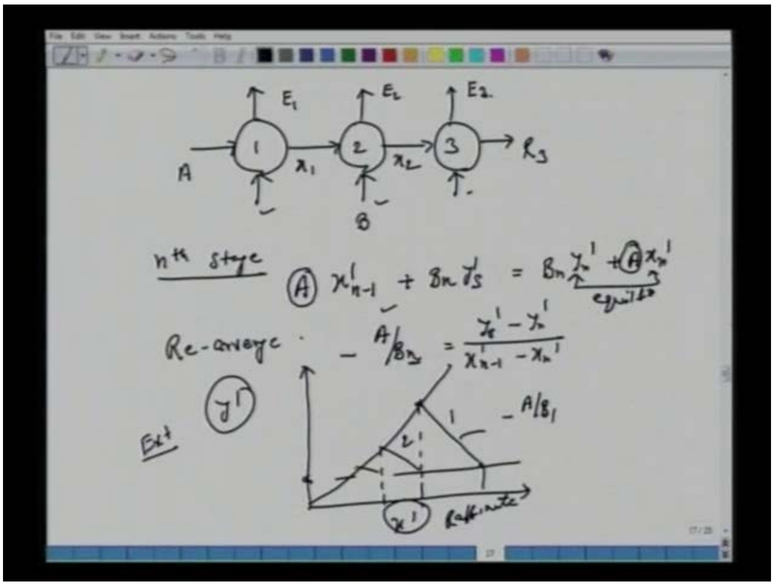

So, we had several examples in the past. We will like to work on this. So, again going back to these stages here, so, if you have cross-current, we can redraw. So, let us say with three stages. ((No audio from 07:17 to 07:25)) So, 3 2 1, A gets in here, then we have R 3, and we have this solvent getting in and we have extract E1, E2, E3.

So, if you make a balance on say let us say nth stage, all right. So, we can write A X prime. So, now, we are working on solute free basis because we realize that A does not get transferred into this extract phase. So, it is better to work on X dash. So, we have X dash n minus 1. So, this is X 1, this is X 2. So, at the nth stage, incoming is X dash N minus 1 plus the amount of solvent in the nth stage is BN. This may have some solute Y dash S. This will be equal to BN YN prime plus AXN prime and YN and XN; they are in equilibrium.

So, we can rearrange this equation to make it A over B N. We had y dash dash. Notice that A remains the same; for the same reason that A and B are insolubles and amount of solvent could be different in different stages. So, we have this B N. So, A over B N; amount of feed to the ratios amount of solvent can be rearranged to written to write like Y at S dash prime minus Y N dash over X dash n minus 1 minus X N.

In other words, now if you work on this equilibrium curve with Y dash and X dash, and if you have this equilibrium curve like this, say they have there is some amount of solvent. So, this is Y dash and X dash; extract phase and the raffinate, then starting with this, say certain amount of feed, if you take this slope minus A over B 1, you have the equilibrium, get down here, then take another slope here and get down here, take another slope etcetera.

So, that the stage 1, stage 2, stage 3, we can find out how much amount you have brought down in the raffinate phase starting from here. So, this is based on your extractions. Here we have saying that we have insoluble liquids. A and B they are insolubles. It is C which is getting transferred to A and transferred to B.

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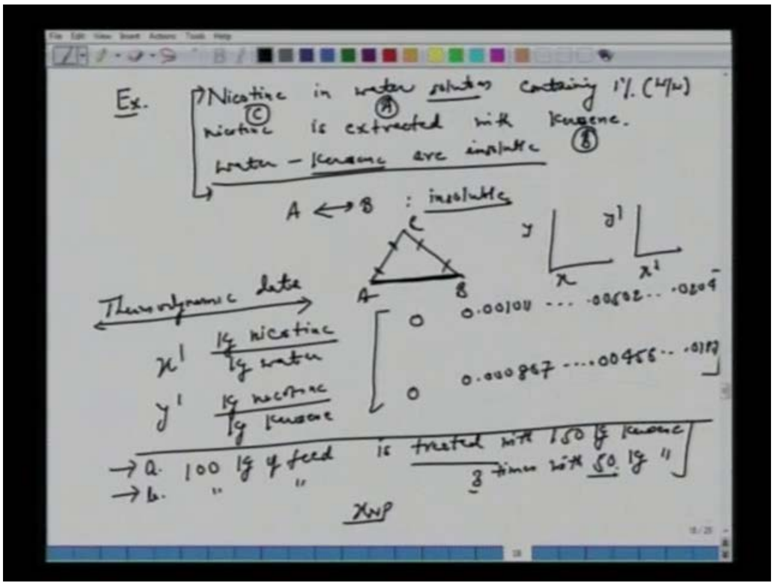

Ex. Nicotine Ⓒ in water Ⓐ solution containing 1% (w/w)
nicotine is extracted with kerosene Ⓑ.
water – kerosene are insoluble

A ↔ B : insoluble

Thermodynamic data

x' $\frac{\text{kg nicotine}}{\text{kg water}}$ 0 0.00101 ... .00502 ... .0204

y' $\frac{\text{kg nicotine}}{\text{kg kerosene}}$ 0 0.000807 ... .00456 ... .0187

→ a. 100 kg of feed is treated with 150 kg kerosene
→ b. " " 3 times with 50 kg "

XNP

So, let us take this example. So, numerically we can take this example here. Let us say nicotine. So, nicotine in water. Water solutions for; that means, we are saying that nicotine and this water consist of nicotine plus water the solutions, nicotine water solution containing one percent weight by weight. Nicotine is extracted with kerosene. ((pauses))

So, and we make a note that water and kerosene are insoluble or considered to be insoluble. So, this is an example of the previous section, we discussed earlier also. So, nicotine from the statement, you can make out its C solute, water; so it is A, and the solvent kerosene is B or S, but A and B; they are insoluble. So, in this case, they know we said that this type of diagram will also work except that this becomes unnecessary because A and B we do not have any phase here.

We can have phase A and C and C and B. So, if you take a solution of A and B and at all fill settle, we can have some separations here, right orbiters here. So, such cases, we look for Y versus X, that is sufficient or Y prime over X prime; solute free basis. So, what we do you can we require first thing is that thermodynamic data for solubility.

So, again you are its recommended, you go visit some site or make use of certain hand book on thermodynamic data for this nicotine, water and kerosene. This system you will you will have phase diagram or you will have the phase data like this X dash, Y dash, you have Kg of nicotine, Kg of nicotine here, per Kg of water and we have Kg of nicotine, per Kg of kerosene. So, you have 0, you will have some data say 0.0001,011, may be 0.00502, 0.0204. Corresponding to this, 0.000, 807, 0.000456, and 0.0187. So, some this data are available in the literature, some text book on thermodynamics or Treybal Mccabe Smith or the book by Dutta, you know you can go through those and collect this data.

The problem here is that it says, if you take 100 Kg of feed, so when we say feed, its water and nicotine, and it is treated with 150 Kg of solvent which is kerosene here. So, this is step 1. Step b says same 100Kg of feed is treated three times with 50Kg of kerosene. So, the total amount of the solvent is the same 150Kg. So, one time you do it one time, and second time you do it three times. Take the raffinate again treat with 50Kg, take it the second raffinate treat with the 50Kg, and the question asked is you know similar questions; what is the raffinate concentration x from the third stage NP in the two cases, which is more effective? Using 150Kg one time or 50Kg three times.

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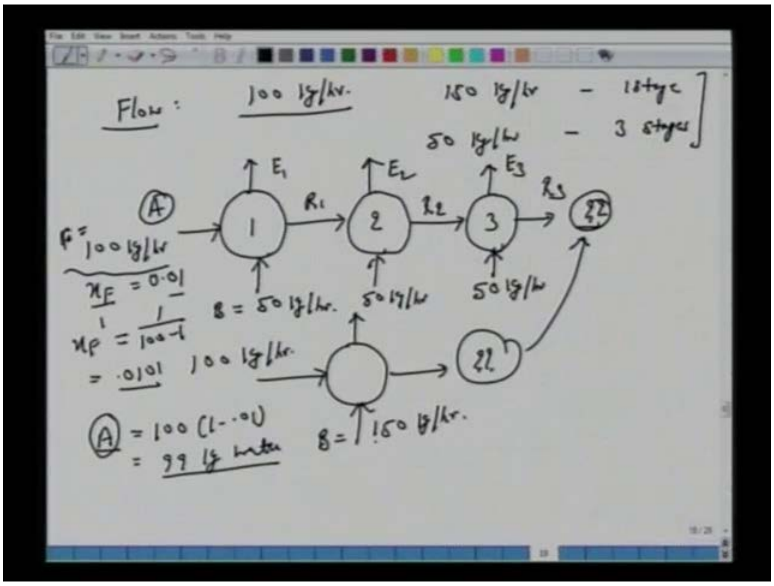

Now, mind you, this is looks this is a batch problem, but previous lecture we also said that there is no difference between this problem and if we have the flow problem where we say that well we bring 100Kg per hour or per unit time in contact with 150 Kg per hour in one stage or we bring 50 Kg per hour in three stage three stages.

So, our mathematical analysis will be the same whether it is a batch problem or it is a flow problem. So, although the problem was asked for the batch, we can treat this as a flow problem, and we can use a similar you know schematics which we used.

So, we have this feed A bring in contact with the solvent stage 1, treat this take the raffinate r 1, bring in contact with the second stage, treat this R 2, bring in contact with the third stage same around 50 Kg, we have 50 Kg per hour, all right. And how much is your raffinate concentrations R 3 here. So, these are the extract E1, E2, E3, this is feed is 100 Kg per hour. So, f it contains one percent. So, X f is 1 percent or 0.01 weight by weight Kg per Kg, all right. So, idea is the same; whether you have three stages or you do it three times.

And of course, we will compare this with just one stage where I bring it like this, bring it like this, and treat with 150 Kg per hour. So, same feed 100 Kg per hour brought contact with this, we will like to compare this concentration with this concentration.

So, let us start, say the feed is 0.01, as we said we should work on solute free basis. So, Xf prime is 1 percent. So, this is 1 over 100 minus 1. So, this number is 0.0101, slightly different from here. And A, this is A which is pure water. So, we are saying that A is now 100 into 1 minus 0.01. This equal to 99Kg of water. So, generally we make a mistake here, this F is 100Kg per hour. You should read the statement correctly, solution is 100Kg per hour, but the water is 99Kg per hour.

Because we are working on this quantity Xf dash, if you work on Xf, then we should work on f, but right now we are working on solute free basis. So, it is A is like this, B here is 150Kg and B here is 50Kg each.

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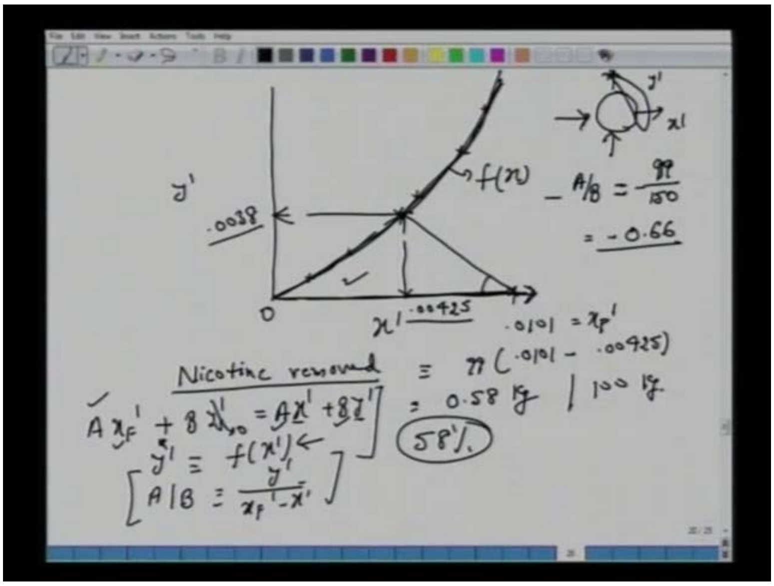

So, let us see this. So, what we do first is we plot y versus x dash. So, let us say this is the diagram. This is equilibrium curve, and now we have should mark this. So, what is the feed? Feed is 0.0101. So, first thing is that when you plot this X dash Y dash, you must when the datas are given, there is no need to go beyond you know say 0.0101, you should scale properly this we have said so many times earlier that from the diagram, you should choose the largest feed concentrations which is available here because all concentrations will be smaller than this so that you can scale up properly starting from 0.0101.

So, go back to the equations, the first operating line equations. We have the slope of minus A over B n here of course, is one stage. So, the slope of this operating line is minus 99 that is water divided by 150 which is minus 0.66. So, this is Xf or Xf prime dash. With this, take a slope of minus 0.66.

So, let us say this is slope, tan theta or this divide by this except this is negative here. So, take a slope the scale, draw this curve, read this data here. This is 0.0038 and read this data here which is 0.00425. I said earlier you can work mathematically writing down the equations material balance for 1 stage, species balance for one stage, two incoming streams and two leaving streams with the constant that the two leaving streams; they satisfy this equations or this coordinates their coordinates Y dash and X dash. They lie on this equilibrium curve.

So, you can solve mathematically as well, if you know the equilibrium of if you know the equation for this these data, but we said that generally all these datas are in tabular forms. So, it is more convenient to work on this graphically, all right.

So, from this we read 0.0425. So, how much nicotine we have removed? ((No audio from 20:55 to20:59)) nicotine removed is 99, that was a feed and we started with 0.0101 minus 0.0425. So, we have 0.58Kg per 100Kg, all right. So, we can say that we have removed 58 percent of nicotine. Keep this number in mind. Again you know working graphically or writing down the equations, we have AXF dash prime; solvent was pure. So, we have Y dash dash 0, then we have treated raffinate X1 prime that is B Y prime, all right.

So, A is known, B is known, Xf dash is known; these are the two unknowns. We require one more equations. So, we have Y dash and X dash in equilibrium. So, they satisfy this equation, say let us this is Fx. So, we have this equilibrium star for this. So, two unknowns two equations, one can solve to obtain A over B which is Y dash over Xf dash minus X minus. So, same as what we have done graphically. Should be able to get the same results if we know the equations of this line here per hour ok.

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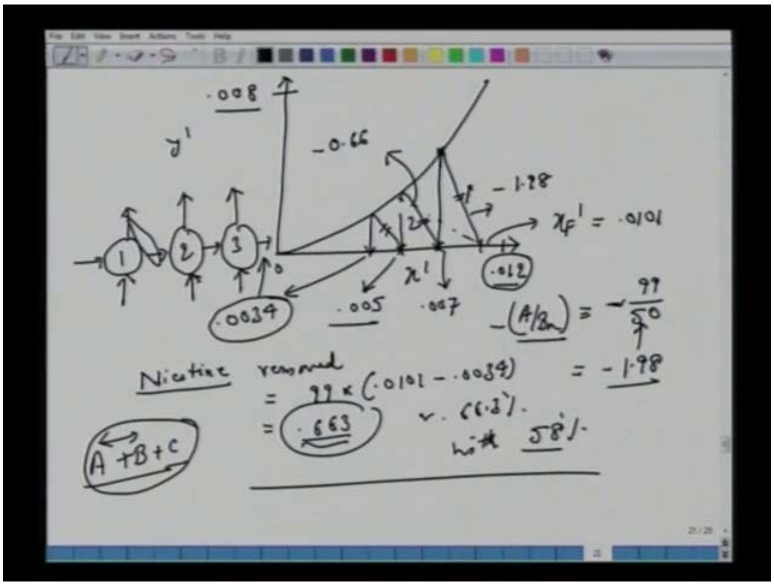

Now, let us go to second part where we are saying that now we are using three stages. So, now, we have three stages. Now we are going to use 50Kg each. So, the total amount of solvents is still the same; 150Kg per hour. So, we have this equilibrium curve, Y dash, X dash and here may be maximum go till 0.012, here may be you mark till 0.008. So, right now you have the idea that everything you should be able to address between this scale of 0 to 0.012 and the scale of 0 to 0.008 for Y dash.

So, we start from again this is Xf dash which is 0.0101. Now you have to find out A over B N, all right. Here this all this amounts are equally distributed 50Kg per hour. So, now, we have minus 99 over 50. See that is the difference here, instead of 150, now we have 50. Therefore, the slope of this new curve is minus 1.98.

So, it is less, you will expect that this is your now less incline. So, we have say let us see this is with slope of minus 1.98. Earlier may be you had some other slope here with minus 0.66. This was the previous example where we had the slope of minus 0.66. Anyway we do not need this right now.

So, we draw with the slope of minus 1.98 and here we have this equilibrium. So, we are talking of stage one, bring in contact with this from this; two are in equilibrium. So, we reach here. Pure solvents; so its 0. Come to this stage. If you read this number, this should be 0.07. Again from here, take the same slope because amount A by B and remain the same. So, draw a parallel line to this.

So, this line and this line; it is in parallel line. So, read the raffinate concentration the second stage. So, now, we are here, stage two, read this number as 0.005. So, first stage we have reduced the concentrations from 0.012 to 0.007. Second stage lets write down one stage, let us write down second stage, we have brought down to 0.05. Again take the same line same slope and plot here. You should be able to read this number the third stage as 0.0034.

So, now we have the third stage and this raffinate concentration is 0.0034, all right. So, what we answer here, how much amount of nicotine we have removed? Amount of nicotine removed? We had 99 Kg of feed, we started with 0.0101 minus now we have brought down to the level of 0.0034 to obtain this number is 0.63 or 66.3 percent removal.

So, compare this number 66.3 with earlier case we had 58 percent removal. So, we see that advantage here using three stages multistage, instead of single stages, all right. And this is also very similar to earlier example which we took for cross-current, but we had ternary system A plus B plus C.

Here all we said that the similar cross-current examples, but we have A and B which are insoluble. So, the idea you know remains the same, multistage is preferable than the single stage, all right. Either you say that you have you want to bring down the to the level of feed concentrations from one to another concentration

So, for the same raffinate concentration amount of feed is smaller. We require lesser amount of solvent in case of multistage or if you want to keep the same amount of solvent, then one can show that the raffinate concentration in case of multistages will be smaller than the single stage.

So, these are the two examples we have used for cross-current. And now we should take another example on counter-current multistage. So, that was also our previous discussion lectures. Now we have also stage wise except now the feed enters from the left and solvent is entering from the last stage, counter currently. So, they do not see each other, and we have the treated feed; raffinate, and we have the extract solvent phase leaving the first stage. They also they do not leave each they do not see each other, all it means they are not in equilibrium. Every stage inside yes the two leaving streams they are in equilibrium if you consider them to be ideal stages and here I can will have the equilibrium diagram. Now let us take an example that all ABC; they are partially soluble. So, we have this ternary phase diagram.

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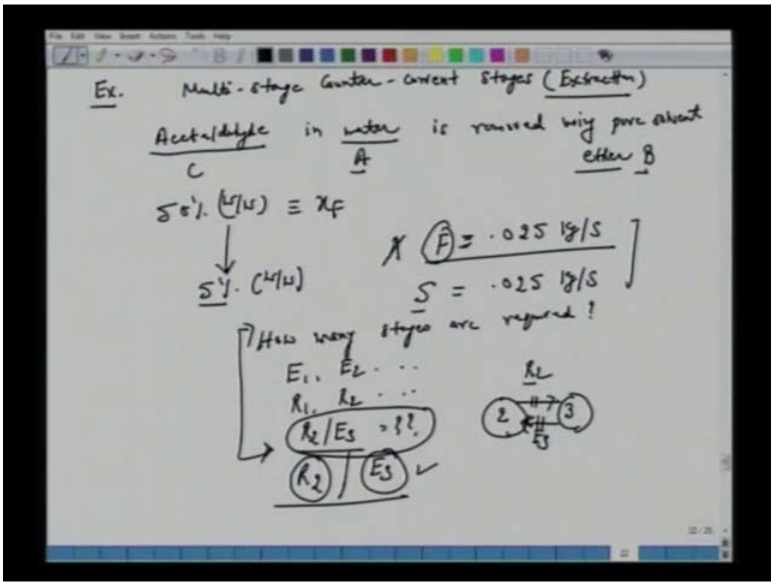

Ex. Multi-stage Counter-current Stages (Extraction)

Acetaldehyde in water is removed using pure solvent ether

C A B

50% (w/w) ≡ x_F

↓

5% (w/w)

F = .025 kg/s

S = .025 kg/s

How many stages are required?

$E_1, E_2 \ldots$

$R_1, R_2 \ldots$

R_2/E_3 = ??

R_2 | E_3 ✓

So, let us this example here. So, this example on multistage countercurrent ((pauses)) multistage countercurrent stages or we have this extractor. We are talking of extractor an extraction, that is a unit operations. And let us say we have the system acetaldehyde. So, acetaldehyde in water is removed or is to be removed using pure solvent which is ether in our case or isopropyl ether.

So, from the statement, this is C; solute impurity, we have water A feed, and we have the solvent which is B or S we use. And problem statement is that solute is 50 percent weight by weight. We bring down. So, this may be Xf and we have been asked to treat this to bring down to the level of 5 percent weight by weight.

Solution flow rate. So, this is A or F is 0.025 Kg per second. So, this is the solution feed rate and the amount of solvent; that is also let say that S or B here is also 0.025Kg per second. So, we are treating this feed with equal flow rate of the solvent. And then we can ask the question say how many say feeds how many stages are required, how many stages are required? That is one question. We have been also we asked to say mark all these lines, what is the extract flow rates E1, E2 etcetera in this in the different stages. You have been also asked to find out how much is amount of the raffinate leaving each stages. We have also been one more small questions that find the ratios R2 over E3, all right.

So, when we say R2 over E3, this is second stage and this is third stage. So, we are falling the same similar nomenclatures. R2 is this stream which leaves originates from the stage two and E3 is the stream which originates from third stage.

So, we are talking of molar flow rate or kg flow rates of R2 and E3; the stream which leaves the second stage and this stream which arrives stage number two from the stage number three. So, what is this ratios here. So, essentially we have to determine all the compositions, composition of every streams leaving all the stages and their flow rates.

We will address that this one in the last what is R2 over E3. Do we have to calculate separately R2 and E3 and then take the ratios or there is a way to just calculate the ratios?

So, it is very important that we also understand, we read the problem carefully; it says that what the ratios of R2 are over E3; that means we do not have to necessarily calculate the values of numerical values of R2 and E3 separately. All we have to do is to take the ratios, all right. So, these are things we have to calculate in this example. And again we have been given equilibrium data diagram.

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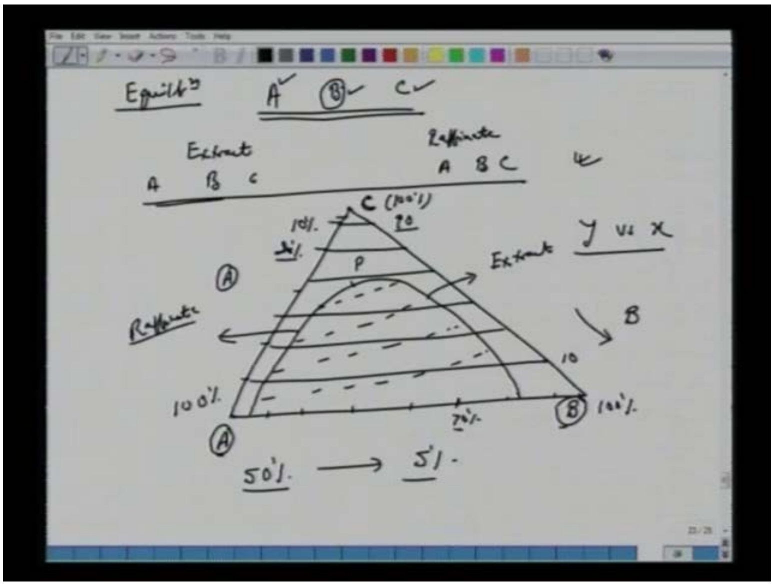

So, again it is recommended you again do some literature survey, go through some physical hand book of data for three compositions, three components A B and C, all right. For acetaldehyde, for water, and then you have this ether isopropyl ether; it is a well common you know very common system of ABC and most of the text books Treybal Mccabe Smith or if you do google search or some any physical hand book on they will give information on A B and C.

So, you will have extract and you will have the raffinate and you will see A B C here. You will see A B and C here, all right. You can then you have to draw, you have to take this type of triangular graph, equilateral triangle; this also available in most of the book stores or again from the internet, you have to download this graph this type of graphic.

Sometimes it is available that this equilibrium diagram is already tabulated, already plotted so that you do not have to plot this equilibrium diagram. So, for us, it is for this lecture, we start with this. It is available to us. A, we have this B here, we have this solute C here, say acetaldehyde, A is water and we are using the solvent isopropyl ether and we have this equilibrium phase diagram like this.

So, this is marked like this; A is 100 percent. So, let us say this is 100 here, then we will have 90, 80, 70, 60, etcetera. So, this is let us say here it is 10. So, this is 100. So, if you look at, this is 10 percent, 100 percent, all right. So; that means, without saying this is A actually what they are writing here is a amount of A decreases in this directions, amount of C decreases in this direction. So, 100 percent C is here.

So, when we write 10 percent C here, then this is your A. C would be 90 percent here. So, if this is A is 50 percent or this is 20 percent here, so, if this is 20 percent here, then C would be 80 percent. So, you will see the lines like this. So, we have talked about this in the very first lecture. Similarly A to B; this is 100 percent here and A is 0 A is 100 percent here.

So, if in this direction, B increases. So, we can also have level like this. So, when we say 70 percent, this is 70 percent of B here. A would be 30 percent. Similarly some levels here let say this is 90 here let say this is 10 here. So, C increases in this direction. So, its 10, 20, 30, and we have 90, then we have 100, and B increases in this direction.

So, not necessarily in the graph you know, one has to understand that A in which directions A B and C increases or what are these numbers reflect here. So, first thing is this, and then on this graph also you will have tie lines. So, this is your Y versus X. ((No audio from 35:43 to 35:50)) So, essentially you also you have a data Y versus X which you have marked and which you have connected here. So, this is extract and this side; its raffinate, somewhere here you will have plate its P.

So, this is we are starting from here. So, instead of tabular diagram, we are starting with this graphical equilibrium phase diagram. And we will like to mark those points where how many stages are required starting from 50 percent of feed to get down to this 5 percent.

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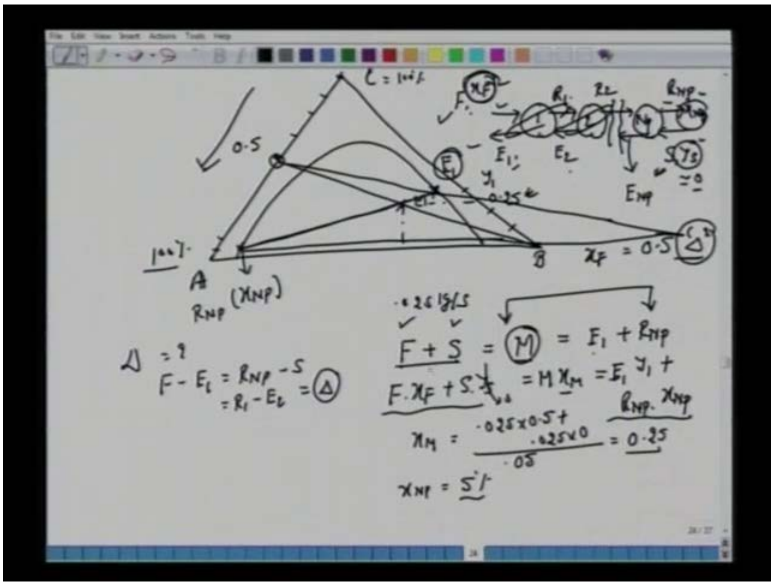

So, let us again re-plot this phase diagram qualitatively, C A B. You have this phase diagram. So, feed let us mark the first point which is... So, this let us say this is stage one, stage two, let us take arbitrary stage or last stage N P. So, you have feed getting like this. Let us put a break here and then we have treated feed and we have countercurrent, this is solvent S, this would be RNP; nomenclature N P stands for the stage from where this raffinate phase originates. So, this should be extract ENP, all right.

So, this should be E1, this is F. So, this would be R 1 and R2 etcetera. So, this is E2 again, and similar corresponding weight fractions. Of course, this S is Y S; it is a pure solvent. So, this YS is 0 and here we say that feed Xf, this composition Xf was 50 percent. So, this is 0.5. First thing we should do we should mark this point. So, this is 0.5 which will correspond to 50 percent. So, on this is scale, we said 10, 20, 30, 40, 50. So, this should be 50 here. Similarly this will increase to make this 100 percent. So, A is 100 percent, you should be careful with the scale. So, it is represents amount of A as you go along in this direction. So, 0.5 we mark. So, this stream composition we have marked, we know the solvent is 0; Y S So, this is 100 percent pure solvent. So, there is no solute in this. So, this is B here, we can connect this point, all right.

So, if you know the amount of F, we know the amount of solvent. We know this composition, we know it is compositions. Essentially mathematically what we are writing here is F plus S; we are writing this giving rise to some mixture M of course, this mixture M will also be equal to E1 plus RNP. So, we are making over all material balance. F plus S two incoming streams and two leaving streams E1 and RNP, they are miles apart. They are not in equilibrium at every stage, there is equilibrium. These two leaving streams over all, they are not in equilibrium like this and we know it is compositions F Xf plus S to Y S of course, Y S here is 0, will also be equal to M XM; so, hypothetical mixing mixture here M.

See they are they do not see each other, unlike you can make a balance per one stage where you can say that two streams see each other. So, F is here, S is. So, this M is hypothetical point here, physically it does not adjust graph of course, we can mark this M and X. This will also be equal to E1 into Y1. So, this Y 1 plus RNP into X N P.

So, knowing this coordinate here 0.5 and 0 and the amount of solvent S and F, F is given to us, s is given to us, right, the same amount 0.025 Kg per second. Both mathematically as well as graphically, we can find the mixing M. So, let us say this is m here or X M. So, M location is here, and if you do this calculation here XM, this is nothing but 0.025 multiplied by 0.5 plus S is also 0.025, but solute is pure, it is a pure solvent. So, it is 0 here multiplied F plus S 0.05 we have 0.25.

So, when you read this 0.25, is nothing but 25 percent. So, C is here is 100 percent; that means, this number which you are going to read, this would be between 20 and 30, all right. So, this would be exactly this is 20, 10. So, 0, 10, 20, 30; this would be 0.25. Solute increases in this direction till is 100 percent.

So, you mark this point, all right. Now XNP is also given to us, this XNP is 5 percent, very small amount of solute in the treated raffinate concentration to raffinate here. So, again go to scale of A C, A is your feed; raffinate and C is here is the solute. So, C is 100 percent here and we are looking for 5 percent, then you have to get down to this level, may be this is 10. So, we are going to read 0.05 here.

So, this is RNP or XNP, all right. So, when you know this M, so now, we are talking of the second set of equations. So, this plus this, we know the M, connect this M here to mark this E1, all right. So, this is what we are applying this mixing rule here; F plus S and E1 plus RNP. We can also write down this E1; this equations with this second set of equations. So, either you do mathematically or graphically except you must realize that whatever XNP or whatever raffinate comes out of this stage, it is in equilibrium with this phase. It is not in equilibrium with this phase.

So, this line still lies on the equilibrium curve. This point also here lies on this. So, this E1 or Y1; this phase is in equilibrium with this with the raffinate R1, not with this RN, but the two points lie on the equilibrium. So, when you are connected with this, this is not the tie line, this E1 and RNP; they are not a tie line. They are still operating line. So, we have E1.

Now what how do? We go ahead now. What we have to do here? We should realize that we are looking for del, difference point; the difference between the streams F and E1. We are discussing earlier R1 and E2, R2 and E3 etcetera, or RNP and S, etcetera, F minus E1, RNP minus S1, now it is a del point, difference. So, difference is conserved. F minus E1 is RNP minus S or R N1 R1 minus E2 any stage, this is constant, del.

So, that is the advantage of working this del here. So, this E1, if you extend and you extend this line here going through 0, you will get del point. So, it is very similar to our distillations where we located the difference point del D for the rectifying sections and del B for the bottom sections stripping sections.

So, here now we are working now on difference point. So, first we locate M mixing; F plus S mixing, then we obtain this E1 which is 0.25 here, all right and then when we connect this two and extend, now we have located del point, all right. So, in fact, now based on this four locations, one can also show, we will you should be able to obtain when you get E1 and when you get E1, Y1 as 0.25. You should be able to obtain E1 as

0.037Kg per second. Y1 from the graph, we have obtained as 0.32 from graph because it is of the equilibrium phase diagram.

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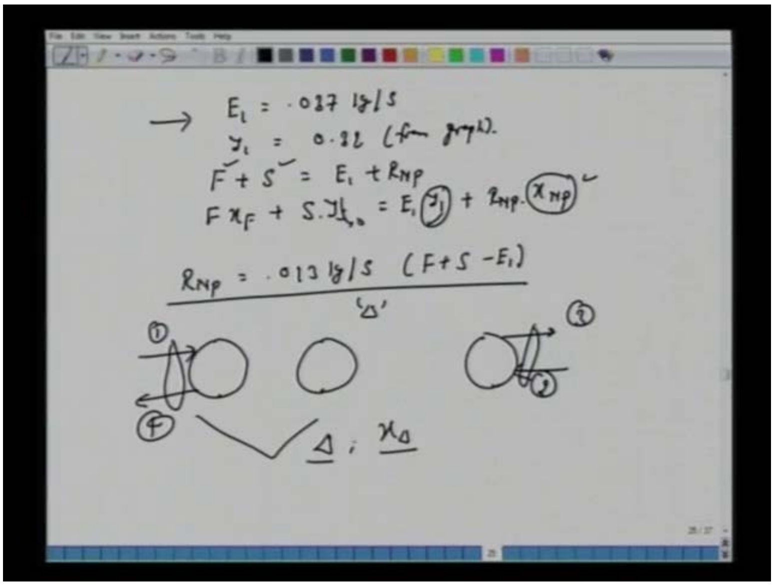

$\rightarrow E_1 = .037$ kg/s

$y_1 = 0.32$ (from graph).

$F + S = E_1 + R_{NP}$

$F x_F + S \cdot y_{s,0} = E_1 \boxed{y_1} + R_{NP} \cdot \boxed{x_{NP}}$

$R_{NP} = .013$ kg/s $(F + S - E_1)$

'Δ'

So, we are all we are writing here is again F plus S as E1 plus RNP and F Xf plus S YS which is 0 here equal to E1 Y1 plus RNP into XNP. So, we know F, we know S, all right. We know Y 1 from the graph, we have read XNP is given to us. So, two unknowns two equations to obtain E 1 0.037 here, and RNP as 0.013 Kg per second, all right. RNP is nothing but F plus s minus E1.

So, this also you know we make a note of all these four streams and then we have marked this del points. So, del point is now known to us, overall compositions of all two stream leaving streams; they are known to us.

So, we have stages like this. In the last stage, 1, 2; we know its composition and flow rates. We know its compositions and the flow rates 3 and 4. Then we have we know the difference between the two, that we know del, molar flow rates and this will also have a composition which you call it X del. This too we have also been located on our rephrase diagram. Then we can determine how many stages are required.

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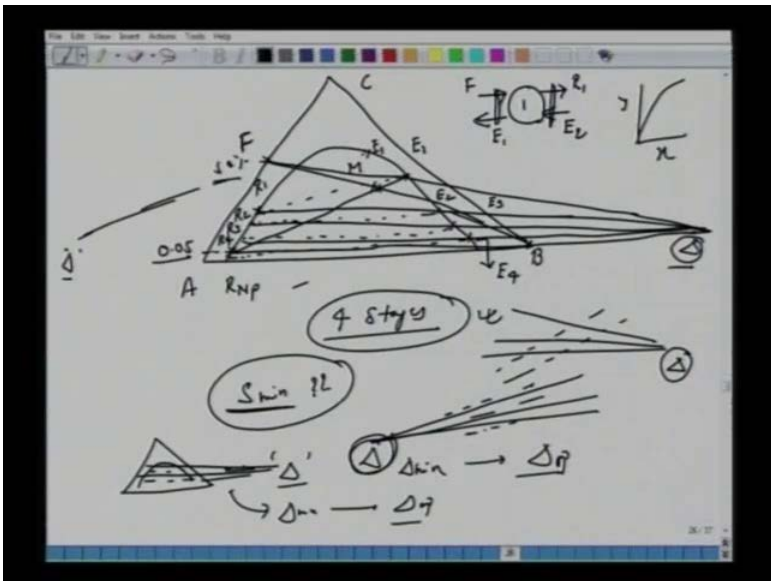

So, we will draw or we will go back the same equilibrium curve. Let us re-draw, C A B, phase diagram, started with here, pure solvent we connected with this straight line, we have mixing point M, F, we have B, this is M, we know this XNP; it was given to us 5 percent. So, this is 5 percent, this is 50 percent. So, this is RNP. We connect this to obtain E1, last stage, all right, this E1 and now we are connecting with this to obtain our difference points. So, you have to take extend this till we locate del. Now we have now start from here first stage two leaving streams E1 and R1 in equilibrium. Take this line, now you draw tie line. So, we have this R 1; connect this R 1 with the del point.

So, when you connecting it mathematically, you are saying that again this R1 and E 2; difference between the two is constant. Same as what we had for the feed and E1. So, the difference is mole kg flow rate is same. So, we have R 1, this intersects say E 2, connect, and find the tie line; this tie line need not be parallel. You have to read the equilibrium curve for Y versus X. So, we have R 2. Mark this point; R1, R2, E1, E2, connect this. We have now E3, take another tie line. We have now R3. Connect this to del, mind you all the equilibrium curve tie lines are dotted and all the operating lines; they are solid lines.

So, we have following the same conventions we had for distillations. So, this point here is now E4. So, E1, E2, E3, E4; and again when this time when you take a tie line, you will see that this has gone below this 0.05 percent. This feed was 50 percent. All it means, we require 4 stages, 3 complete and one partially, essentially for practical purpose, we say that we require four stages. This tie line has gone below this 0.05 is coordinately. So, we require four stages.

Now, one has solved graphically, but you know for the sake of the discussions, whatever treatment we have done graphically, one can also solve mathematically. Of course, mathematically we require the equations for such equilibrium curve Y versus X, etcetera that is one point you have here.

Second point you want to address here how about this R min or S min? We remember in the last class, so, let us recall in our previous case, we talked of minimum amount of solvent required, and when do we require this minimum amount of solvent, that time we recall tie line or equilibrium curve coincides with the operating line. The same situation we had in case of distillations. The two lines coincide then we are required infinite number stages and driving force goes to 0.

So, here question also comes, is there any impossibility of operating line coinciding with this equilibrium curve? Look at the curve again, tie lines are downward slope like this, and luckily del point is here, operating line goes like this. So, there is no way, these operating lines going to coincide with this tie lines.

So, here such questions of minimum amount of solvent or the risk of you know any tie line becoming coinciding with this operating line does not arise. Had the del point on the left had side so that we have the tie line like this, and also the operating line connecting to this left hand side del, now there is a risk of this operating line coinciding with this equilibrium curve. In this case, yes we can have del min, then take 1.5 or 1.2 times to make it del operating, alright.

And when this happens, when the del point goes to the left or when it goes to the right, you should recall, it depends upon the amount of solvent. If you have chosen here this E1 is here, but if the E1 falls here because of different amount of solvent, alright, or this mixing point goes like this, then there is a possibility that this E1 and F extend, this may go back to the left hand side, alright.

So, of course, in this example, all operating lines we have the del on the right hand sides, tie lines are like this or in some cases its possible, say this different system when the operating line is on the right, del is on the right hand side, and we have all operating line going like this, but the tie lines are also on downward. So, there also we should be careful of obtaining this del min first, and then getting this operating line, right.

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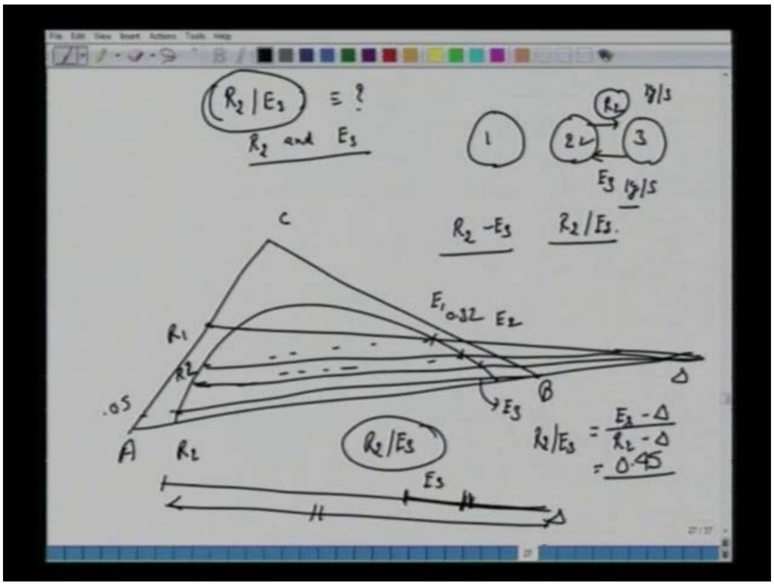

Now, let us go back to the previous case, we were the example which we are discussing here, when the question asked was how much is the amount of R2 over E3, ratios of R2 and E3. So, it is the last part, alright. So, if you recall, we said that do we have to determine the values of R2 and E3 separately or we can find the ratios of R 2 and E3 directly.

So, question here is R2 and E3 which is talking of the second stage 1, 2, 3; this is the raffinate R2, and we are talking of E3; extract arriving from the third stage. What is the ratio. This is kg per second and kg per second, alright. We know the difference R 2 minus E3. We have been asked to find out R2 over E3. How do we do this very quickly and very conveniently? Again we said this equilibrium diagram previously which you have plotted. If you have the phase diagram like this, this is A, this is B, this is C, you had feed, it had intersected say E1 at 0.32, we had extended this here, the solvent; this raffinate was at 0.05 pure solvent, we had extended this here, you get this del, this was E1, take the tie line, this gives R1, connect here this gives E 2 tie line, this is R2 and now another operating line which connects to this to give us E3.

So, what is R2 over E3? What is the amount of raffinate leaving this stage two and what is the amount of extract arriving here. So, what is R2 over E3? So, we have a straight line like this del, somewhere here we have this E 3, and somewhere here we have this R2. So, applied lever rule, R2 over E3 equals this length of the segment R per R2, excuse me this becomes E3 segment del; so this segment, over E 3, so we have taken R 2 dash del.

So, take a scale and measure this. It is all linear here. So, this is the scale, measure this and measure this length without any calculations. Measure this length and measure this length. The ratios should be able to obtain 0.45, alright. Of course, if you go very mathematically regress without realizing, we can obtain this ratios, you again you will start with some material balance, you will say that R 2 plus R2 minus E3. This is equal to del. This is also equal to f minus E1, etcetera, alright.

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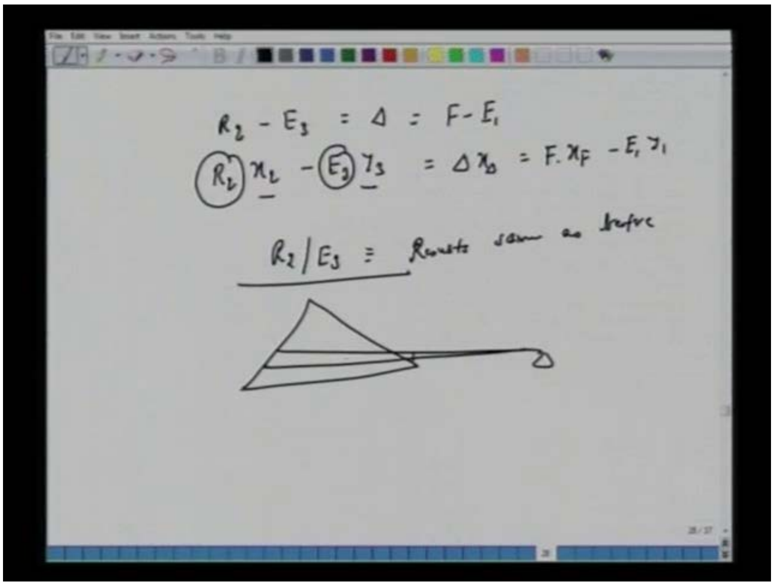

$$R_2 - E_3 = \Delta = F - E_1$$

$$\textcircled{R_2}\, \underline{x_2} - \textcircled{E_3}\, \underline{y_3} = \Delta x_\Delta = F.x_F - E_1 y_1$$

$\underline{R_2 / E_3 =}$ Results same as before

Then you will write R2 into X2; you will read from the graph, E3 into Y3; you will read this value from the graph. It will also equal to del into X del, the coordinate of this difference point, this will also be equal to F into Xf minus E1 into Y1. Then you of course, you can obtain the values of R 2 and E23 unknowns two equations, then you can take these ratios two you will should be able to convince yourself that you are getting the same results as before.

But here we have made a point that if the question asked is only the ratios, then we should make use of this lever rule any operating line. So, connects this, the ratios of any points here between these two segments. They are in the same proportion as the amount of two streams here. So, this example what we have taken the last example is on multistage counter current, and all such statements, all such examples related to multistage countercurrent, they rely on this location of del; difference point. This is very similar to again our distillations which we did earlier. We always try to locate for del D for the rectifying sections and del B for this stripping sections.

So, here once we have the difference point difference between the two streams, then we can start connecting with two streams which are in equilibrium two leaving streams from any stage, mark those on the coordinates, connect to the del point that is the operating line, again find the new stages, find out how many stages are required till we go below this raffinate concentrations. Problem can also be re-phrased; given the raffinate concentration or given the number of the stages, what is the finite heated concentration. There also one should always look for this del D.

We can have one more example where there is a possibility that once we discussed here a tie line becoming or coinciding with this operating line. So, essentially we are looking for del min; what is the minimum amount of solvent. So, that example we will take in the next lecture where first we locate the del min; minimum amount of solvent which gives us infinite number of stages which requires infinite number of stages.

Once we locate del min, then you take 1.2 or 1.5 times larger amount of solvents. Once we have exceeded this, then any amount of solvent will give you finite number of stages, alright. So, that is the example we will take in the next lecture.

Thank you.