Mass Transfer II

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Lecture No: # 26

So, in the previous class, we started this single stage contactor. So, the idea was essentially to show that entire this extraction consists of two parts - two units. One is the mixing, where you allow the feed and the solvent to come in contact, and then you have a settler, where you separate extract from this raffinate.

So, we made a material balance for the mixer as well as for this extracted units or the settler units. And then, we marked the raffinate point, extract point and we show that we can apply the mixing rule as well as the line falling on the tie on the tie lines for the coordinates on the tie lines. So, we will just go over this very shortly, once again, before we start now switch to the multistage contactors.

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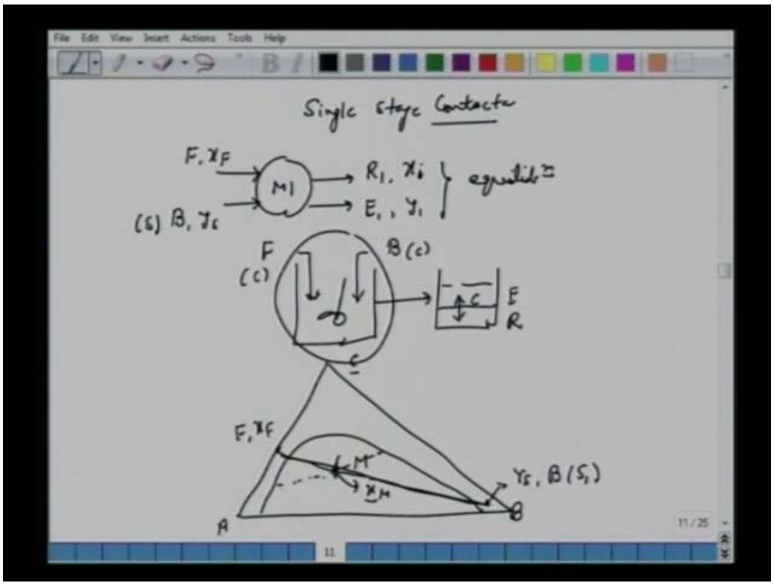

So, what we had earlier was in the previous lecture single stage contactor. So, we have M1 alright and here we mix we bring feed x F in contact with solvent, let us call it B or you can call it s sometimes with y s, and then they are in contact, then we separate, and we have this raffinate phase with the coordinates as R 1 and x 1, and then we have this extract with E 1 and y 1. And we say that if it is an ideal contactor, then they are in equilibrium. So, essentially here we said that we must realize that this consist of two different units. So, the industrial, you will see, there is a mixer where we bring this feed in contact with the solvent, and so this contains both F or B; B may be pure, but B may contains some amount of c, F contains some c, B allow them to mix it, then there is a settler equation.

So, when settles in their two phases: one is extract, one is a raffinate. And this is a transfer of c co-solute from one phase to another phase. And then we marked on the different coordinates on this equilateral triangle. So, we have A, B, and we have C here; thermodynamics tells us there is a phase diagram, we start with F. So, which is x F here - this is mixed with certain amount of solvent, may be solvent is not pure and this has contains some amount of solute c. So, when we mix this feed with the solvent, say B here or s 1 - we can call it for first stage s and then 1. Then we can apply the mixing rule to find out the exact location of this x M. So, concentration of solute in this mixture. And then, so that is about the first stage; then you have settle, and then you have this equilibrium. So, then look for that tie line which passes through this M. So, the same amount is now getting distributed. So, one time this material balance is applied along this line connecting between F and B, you can apply the tie rule to exact to determine the location of this x M, and then you have an another line which is the equilibrium line - tie line going through this x M. So, you have extract which is an E 1 here and you have this R1.

So, in essentially if you write down this material balance, you can say that you have F plus S 1. So, S 1 is the amount of the solvent which we are using as B here; this equals M 1 and this will also equal E 1 plus R1. So, this is one unit and this is the second unit.

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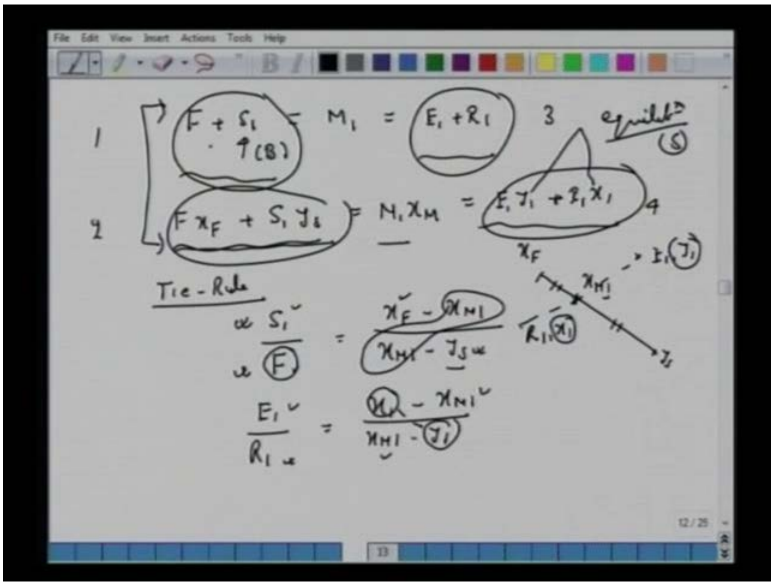

1 $F + S_1 = M_1 = E_1 + R_1$ 3

2 $F x_F + S_1 y_s = M_1 x_M = E_1 y_1 + R_1 x_1$ 4

Tie-Rule

$$\frac{S_1}{F} = \frac{x_F - x_{M1}}{x_{M1} - y_s}$$

$$\frac{E_1}{R_1} = \frac{x_1 - x_{M1}}{x_{M1} - y_1}$$

Similarly, we can write down this species balance or the solute balance, we have F x F plus S 1 y s, then you have M 1, then you have x M, then you have separation between extract and we have this raffinate. So, then you have this mixture here, and then you have this settler here. And then, we can apply this Tie-Rule, belong you know, both operating lines as well as this equilibrium lines. So, this should be the operating line, you here you just mixing it, and then you are separating it. So, you have this equilibrium line. So, one can find we can write that S 1 by F - the amount of solvent added to this feed will be equal to x F minus x M1 over x M1 minus y s.

So, essentially go back to the diagram, we have connected this x F to this y s. And we have located this x M1, 1 means first stage. So, essentially the most general case would be that feed is given to us, we are adding certain amount of solvent, feed has certain composition x F, and solvent has certain composition of solute here y s.

So, from here we can get x M - mathematically we get x M here or from this segment, you know this ratio to this ratios, but we given by S 1F we can also locate x M1. So, we can do geometrically or we can solve for x M1. Once we have x M1, then we are looking for a tie line, which passes through this, to give you this E 1 and R 1.

So, we can write similarly E 1 over R 1 as x 1 minus x M1 over x M1 minus y 1. So, idea here is that - this is your y 1 and this is x 1. So, once we find a tie line which passes through this x M, x M1 is already known from here, then y 1 and x 1 they are fixed. So, this is known to us; this is known to us; x M1 is also known to us, they are with equilibrium. So, we can find out what is the ratios of extract and raffinate form here and here.

So, the problem can be you rephrase different ways. The idea here is that you must not forget that whatever balance we are making it for total balance or the species balance, we are doing it twice; one for the mixture and one for the settler; one for the mixture and one for the settler. On top of this we are saying that x 1 and y 1 they are in equilibrium. So, this extra constant we apply for this ideal stage. So, essentially we have one equation, we have second equation here, third, fourth and we can also say that we have the fifth equation extra constant for the equilibrium, if it is the ideal stage alright. We will talk about the efficiencies and inefficiencies in the later class.

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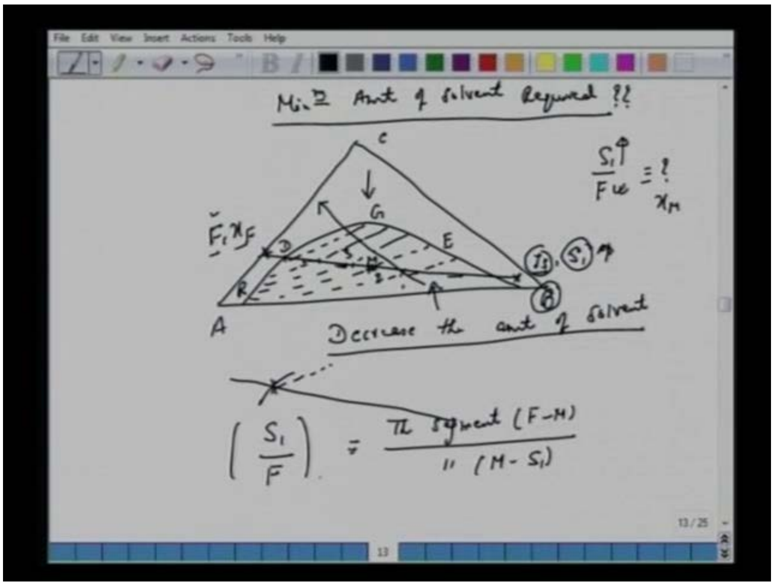

Now, we will address here, the minimum amount of solvent required. So, you should re-call try to compare, what we had in case of absorptions, when we said that what is the minimum amount of solvent required to absorb certain amount of gas from a gaseous phase; certain amount of solute say sulphur dioxide from gaseous species.

So, similarly here we will like to address, what is the minimum amount of solvent required for an extractions? So, here we will address again geometrically, try to understand this, before we put down some equations here. So, we have A, we have B, we have C, we have again this phase diagram alright. And let us take a general case where we have this feed, and let say solvent is not pure, we have this y s. So, this is y s or we can say S 1 alright. And here, we have F and we have this x F, then we have this mixing.

So, this tie line passes through this. We have now extract E and we have this R or even an R1. Now, let us monitor, the locus of this point. What happens, if increase the amount of the solvent alright. So, if increase the amount of solvent, you will expect that this line this x M will will move towards this x F B. Mathematically also you can write down the the expression for S 1 into F for a given amount of F, if i increase S, then let us look at the locus of x M alright. This will give you the same result by increasing this amount of solvent, this line will pass through this. So, then you can you will take another tie line which passes through this to give you another extract another F.

Let us take this second case; where now, we decrease the amount of solvent. So, now we want to decrease the amount of solvent. Now, when the it decrease the amount of solvent, now this line will pass through this. So, now we have another tie line, essentially we have this locus of this mixing point moves in this directions. So, the question is we are asking here, to what extent we can decrease this amount of solvent. So, follow this locus, now this x M1 may be 2, may be 3 here, and then there will be a limit, so we increase here, then we have reached almost on this intersections - let us call it D here.

So, you can make out that - that the last or the minimum amount of solvent will be decided by the tie line which passes through this D point. So, the intersection of this feed line which we can draw like this, and wherever it intersects with this equilibrium curve. From that you take a tie line and pass through this; this will give you the minimum amount of solvent required. Because that is your phase, this is the heterogeneous region this is the heterogeneous region, here you have this homogeneous regions. So, at the most you can go to the extent of reaching this D which the intersection of the line connecting with the feed and this solvent here.

So, once you fix the mole fraction or the quality of the solvent, this coordinate is fixed; once you have fixed, the quality of the feed this point is fixed, we can connect with the line that this is the straight line, now the only the given amount of F, if you keep on increasing the amount of solvent. The x M has to fall on this on this straight line. So, your mixing point in moves in this directions, till it reaches D from here, you find a tie line which intersects this equilibrium curve at G. So that means, we can write now S 1 over F. This is nothing but in general the segment F - M.

So, let us take a general F, M here. So, of how much is the amount of solvent is nothing but this ratios of F - M divide by the other segment which is M and S 1. So, this is the segment this by this; that is the amount of S 1, F. So, what is S 1, F minimum.

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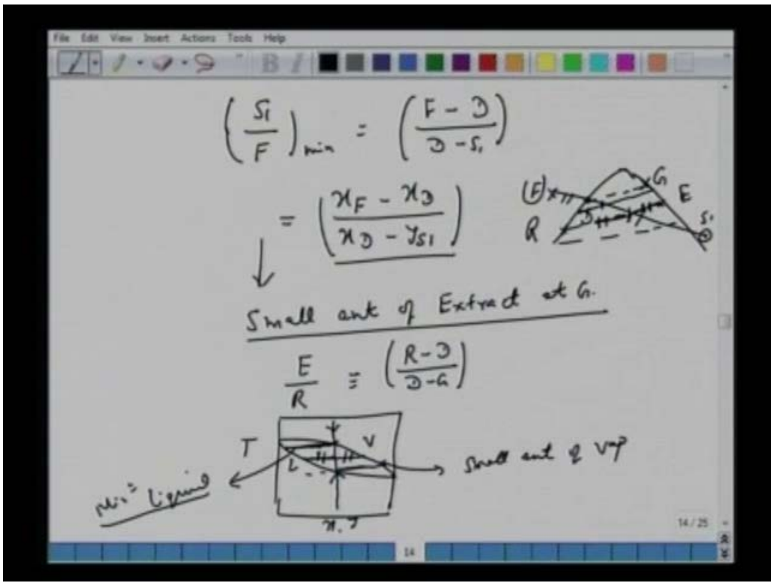

$$\left(\frac{S_1}{F}\right)_{min} = \left(\frac{F-D}{D-S_1}\right)$$

$$= \left(\frac{x_F - x_D}{x_D - y_{S1}}\right)$$

Small amt of Extract at G.

$$\frac{E}{R} = \left(\frac{R-D}{D-G}\right)$$

Small amt of vap

Min^m liquid

So, what is S 1 over F minimum, wherever we have this intersection of tie line going through D and G. So, we have F - D over D – S 1. So, that is the segment alright. So, may be you can again draw this phase diagram, and we are saying that this is the line start originates from F, ends with solvent S 1, and we have reached this this locations that is the limit where we have this G and we have this E here. So, this will give you the minimum amount of solvent. So that is the ratios F - D over this D - S 1. This ratios divide by S. That is the amount of S 1, F per minute. And this will also be equal to by mixing rule x F. So, we have this x F here minus x D that is x D you can read from the graph, divide by x D minus you have this y s1 coordinate of this point. So, this will give you minimum amount of solvent required.

Now, we should also make a note here that - at this limit where you have the tie line crossing through this D and G, you have very small amount of extract. So, this all coordinates which you had here in several tie lines, this was your extract and this side you have raffinate. So, here we have reached an amount, now this when when the tie line passes through this mix, again we find the ratios of extract over raffinate as the ratios of this over this. So, in this limit where there is no segment left here, all it means that we have very small amount of extract at this point. Because this is a very hypothetical case whenever you want to apply a tie Tie-Rule or Lever-Rule, we have to have certain intersections of this feed line with this tie line.

So, at this limit we can say that very small amount of extract at this at G. Because we have to apply this Lever-Rule at least this has this line has to be inside this phase diagram. So that is the limit actually. Because we can also write that E over R at any general point is nothing but R - D over D - G

So, any other line, you know, we can think of this ratios to this ratios alright. One should also compare this, you know with with this absorb distillation column, if you re-call from your previous knowledge on that we had this T x y diagram, and we said that - if we take certain binary mixtures of liquid and heated. The first bubble is formed at this bubble point. At this locations of course, we have very small amount of vapour form. So, when you want to apply the tie Lever-rule at least you have to be settlers inside. In that case, you can say that the amount of liquid over vapour is the ratios of this segment to this segment. Here, it is a limit equation. In here, varies you can say that small amount of vapour is formed

Similarly, if you start from here and condense it, then the first location you can say that we have a... this is a Tie-Rule you know, you can say that very small amount of liquid is formed. So, that is a limit in case minimum amount of liquids. We want to apply a Tie-Rule or Lever-Rule you have to be inside this phase diagram. So, similarly here, in this case also, this case we can say that very small amount of extract is formed. Of course, as you go along this and we have a large amount of extract formed. So, the ratios is given by feed compositions and whatever you have intersections of this this line with the raffinate, this side of the phase diagram and the other side of this phase diagram. So, similarly, other let us look at other scenarios where we have where we increase the amount of solvent.

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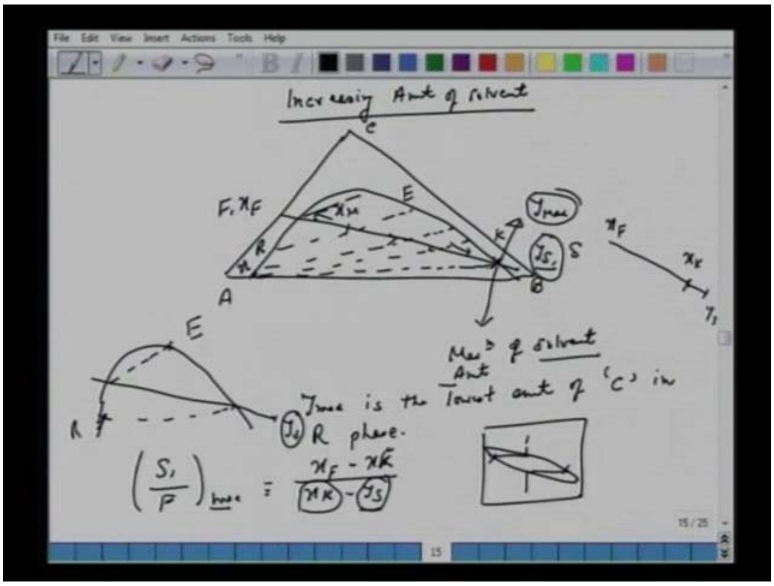

Increasing Amt of solvent

Max

Max^m Amt of solvent

y_{max} is the lowest amt of 'C' in R phase.

$$\left(\frac{S_1}{F}\right)_{max} = \frac{x_F - x_R}{\boxed{x_R} - \boxed{y_S}}$$

So, in this case, we will address that this is very similar to what we had in case of absorptions where we had this spent liquor concentration increasing or decreasing with the increasing or decreasing amount of the solvent.

So, let us look at the other scenarios where we increase the amount of the solvent. And let us monitor the locus of this mixing point. So, we have A B C, this is the phase diagram, we have the feed and we are adding this solvent here. So, we have this y s, S you can call it, then we have this F, then we have this x F, let say we have this x M here, we have this tie line which goes through E and then this goes through R here. Now, if you keep on adding more amount of solvent, then this location this x M will move in this directions. So, you have situation like this. Then you have moves here, then you have another tie line which goes through this. So, what is the limit again you can expect that at this locations when you have the tie line insist through this; this gives you another limit.

So, here you can say that this is the maximum amount of solvent maximum amount of solvent. You can add here. But more important here is to note that this composition, suppose we have, some let us put a new name, suppose we have this y 1 here or y max here alright. So, this y max is the concentration is the lowest amount of c in raffinate phase. So, now what we are trying to say here that that as you go along this then, the amount of raffinate is very small here alright. So, when you go to this extant this, you have some coordinate here, and you have some coordinate here alright. So, these compositions cannot go below, the equilibrium concentrations of what we have in case of y s.

So, if you want to further lower this, it is quite obvious that we have to take a solvent which has a smaller amount of solute, than this what we have y s here alright. So, in in this case, we have seen that you know, when we in change the amount of the solvent, the coordinate moves in this directions and in this directions. So, we have essentially two limits. So, if the phase here and this line is fixed; one limit is given by here, and another limit comes from here alright. Here, we have very small amount of extract and here we have very small amount of raffinate.

So, this is what we are trying to compare with the T y x diagram that you have two limits, where we are very small amount of liquid is formed, and when you go from this very small amount of vapour is formed, here and here. So, these are the two corresponding points we are trying to discuss here alright. This concentration, this concentration is always greater than this y s. So, similarly here we can write that S 1 over F max, we can write at x F, let us call this point as k. So, x F minus x k over x k minus y s. So, we are talking of this line here, this is x F, here this is x k and this is your y s. So, we can apply the mixing rule to answer what is the maximum amount of solvent, which will give this composition of x k alright which is larger than this y s. So, if you want to have larger this concentration of the solvent or we want to in further decrease this, we have to decrease this y s. You should able to monitor, you know how does this x M x M moves and what are the two limits, we get here and we get here alright. And the finally, we should we must say that whatever we have done on the triangular diagram, this is this can also be done on this rectangular coordinate system.

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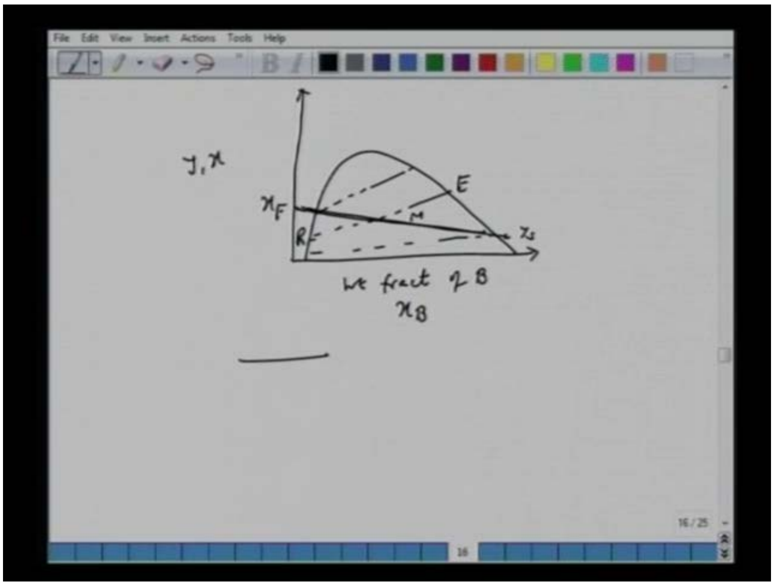

So, if you re-call that said that we can also plot y x versus weight fraction of B. So, we have x B and here we have y x or x y, then we have this is the phase diagram, so, instead of triangular diagram, this coordinate system also works and if you make if you work on this type of triangular diagram, this type of coordinate system, then also we have the same procedure of doing our calculations. We have mark this x F or y s. Let us say we mark x F and we have the solvent y s. So, we can do the similar argument that this is this coordinate will give you minimum amount of solvent, and here we will get the maximum amount of solvent, in general, anywhere in between we can have M, x M, we apply the mixing rule between this and this, then we find the tie line which goes through this and we have this extract and we have this raffinate alright. So, this is the discussion we had for this simple stage.

Let us try to understand a re-call before we take up these multistage or double stages alright. So, in the single stage, we are bringing feed, and we are bringing solvent, to in contact then we have mixing, we apply this mixing rule, then we how them to settle, so, we have extract and we have this raffinate.

Now, we bring the second stage. Now, the second stage, again we work on the raffinate and we connect this raffinate to this solvent. So, we have similar story that - first again the second stage also consist of a unit which is a mixer and then we have a settler. So, every time you have six equations alright. What are the those those six equations? We have one for your material balance for both mixture as well as for you have a settler, then you write down this species balance for both mixture as well as for the settler, then you have apply this Tie rule. Tie rule will apply which passes through this mixing curve which gives you two coordinate systems.

So, these are the mathematically these are the general equations which should be applying for every stage - multi-stage alright. And the more important here is that we should be able to mark those coordinates on this triangular diagram or this rectangular diagram. So, let us take now this multi-stage cross-current extraction.

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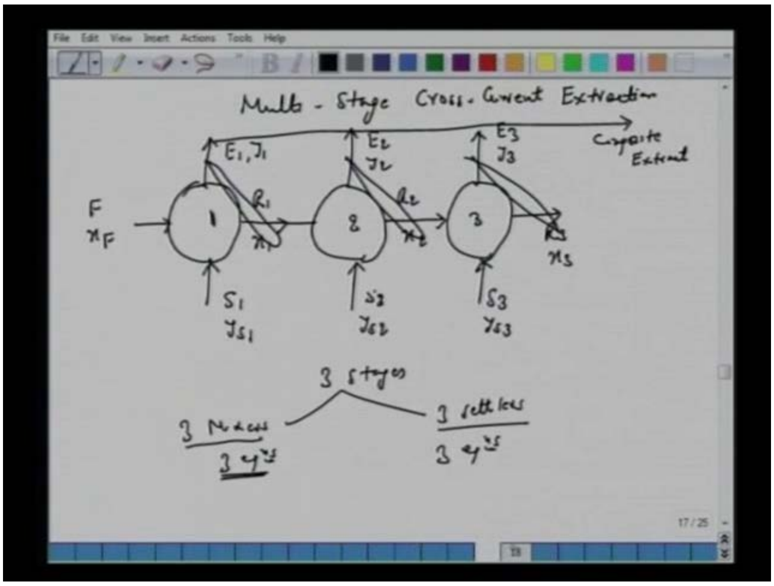

So, we have multi-stage cross-current extraction. So, now let us take one stage, we have second stage and we have this third stage. So, we add feed F with the composition x F. We add solvent say amount is S 1 and this composition is y S1. So, we have this extract E 1 and let us say composition y 1. So, we have this residue or the raffinate R 1 and x 1. So, we have molar flow rate some F kg per seconds or you have moles per second, this comes to this stage. So, one shall we consist of two units, we talked it is a mixture and then we are separating. And if it is an ideal stage, than these two streams are in equilibrium. Bring this to the second stage, where again you can add different amount of solvent with some different composition y S2 or or we can say that y S2 is same as y S1, then you have this E 2 and then you have this y 2.

Similarly, this treated residue or the raffinate R 2, x 2 plot to the third stage, where you add some amount of solvent is 3 again with some composition y S3, you have this raffinate phase E 3 with composition y 3, and then you have R 3, composition x 3, one can also mix all these three and collect, you know some composite extract, and every time the second stage or the third stage the leaving streams are in equilibrium, if you say that these are ideal stages.

So, we have three stages. Essentially, there are three mixtures and there are three settlers. Each mixtures has three equations alright; one mixing rule for the one equation for the mixing, another for this for the settler, then you write down the species balance for the mixture and for the settler, and then also you have one for tie lines, you know this extra constant saying that y 1 is in equilibrium with with your with the extract on the raffinate phase. So, each three mixtures, you know we have three equations, settlers also you have three equations. And let us now plot on this equilibrium curve on this triangular diagram.

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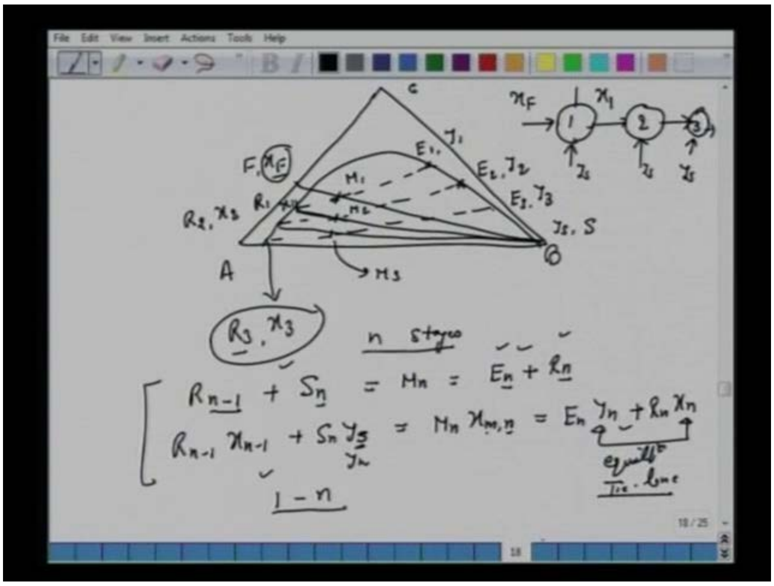

$$R_{n-1} + S_n = M_n = E_n + R_n$$

$$R_{n-1} x_{n-1} + S_n y_S = M_n x_{M,n} = E_n y_n + R_n x_n$$

So, we have A, we have B, we have C here, let us say this is the phase diagram. Let us say this is the feed F and we have this x F. So, as usual we connect with say some S, this could be pure solvent or we can have some amount of solute c here as y s. Let us assume that y S1 equal to y S2 equal to y S3. So, when we add this y s certain amount of solvent S, then we say we have with this mix x M1 here. If we find the first tie line, we have find the tie line which passes through this, we have extract E 1 and we have this x 1 which is like raffinate and here we have this y1. So that is the one mixing; one mixture or one settler we have. Now, what we are doing it that follows this schematic or the arrangement from one we have brought to this second stage. Here again, we had the second solvent.

So, from here now R 1 x 1 is here, now we connect this, let us say with the same amount of solvent alright. So, earlier we add the feed composition was x F, now we have brought down this composition to x 1. So, this was x F, now the weight fraction of the solute is x 1. In this add the second solvent, now we have this M 2. Again find a tie line which goes through this. So, we have E 2 and we have this y 2. Similarly, here we have this R 2 and its composition is x 2.

Now, it goes to third stage. So, again from here connect this to this same coordinates. If we assume that the solvent y s y s and the third stage still you have the same y s. So, again you apply the mixing rule, now may be we have here we have M 3, find the tie line which goes through this. We have extract E 3, composition is given as y 3 and here you have now R 3 raffinate with the composition x 3. So, that is your product alright. Essentially, we have reduced the weight fraction of the solute from x F to x 3. That is how raffinate, and for that we have added three times or we have added the solute in three stages, and the extract composition changes from y 1, y 2, y 3. Of course, one from one can collect all this and you can have the composite withdrawn.

So, in general, now we can talk of this, you know, equations here. Now, if you have in general say n stages; it is not difficult to write down this balance. So, overall balance for the nth stage. So, if suppose it is nth stage, you are adding, you have a raffinate coming from the previous stage, so, R n minus 1 if you are adding a solvent to this nth stage of S n. This will be equal to M n mixture, and then you will have this separation given by this tie tie line E n plus R n. So, we are following the same nomenclatures that suffix n now signifies or denotes the stage from where that stream is originating.

So, raffinate was coming from previous stage which is n minus 1, solvent comes from there is fit to the nth stage. So, we have a S n and E n plus R n. Similarly, if you make a species balance. So, we have R n minus 1, X n minus 1 plus S n y n or you can call it y s which is constant in our case. So, generally you have solvent one solvent which is fixed which has a fixed quality for all the stages. So, you have S n y s or you can have S n y n as well make it general, this will be equal to M n, when you have X m, n; m for the mixing and n for the nth stage. This will you also be equal to E n y n plus R n and X n, only thing you means it is note that y 1 and X n they are in equilibrium. They are given by this equilibrium curve for that tie tie line.

So, this extra constant here, one equation, you have second equation, third, you have fourth and you have this fifth stage, this tie line. So, similarly we can write down this set of these algebraic equations for all the stages from one to n to do our calculations. Again, finally we make a note that whatever we have done for this equilateral for this triangular diagram, you can also done for this rectangular coordinate system.

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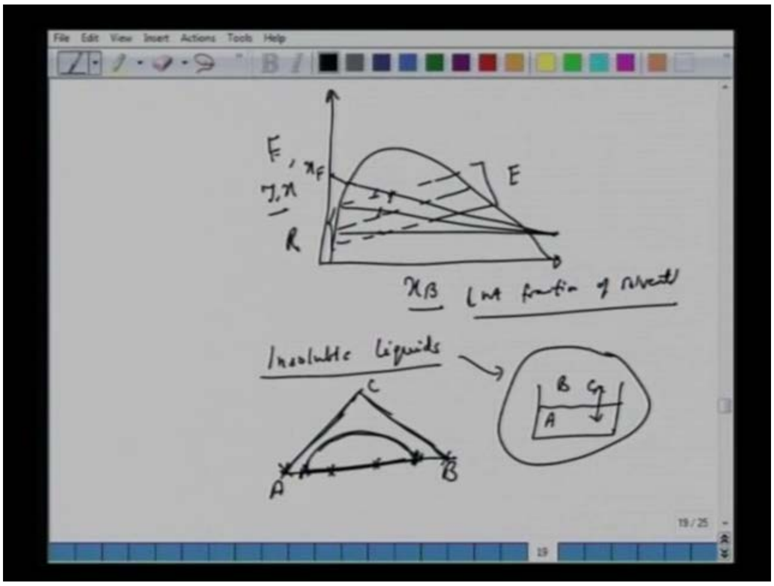

So, there also again you have this y and x positions here. We have this x B weight fraction of the solute, you will have phase diagram and you can do the same thing there also, take this F and a take this S. So, we have with x F or we have this F f here, find M and do like this. Then you have then you find, connect with this alright. Then you have another mixing M here, find another tie line going through this connect with this x, you have another tie line going through this etcetera. We can do this all extract and this is all this is raffinate. So, one can also apply this, one can also make use of this - rectangular coordinate system where we have this x B weight fraction as, this is a weight fraction of solvent. So, either equilateral triangular triangular diagram or triangular graph or this rectangular graph both systems will work here.

Now, let us look at different system, say we have insoluble liquids. So, so far what we said earlier that we have the system where two phases; this this was A, this was B, this is partially immiscible. So, that is why we got this type of phase diagram. So, C is completely soluble in A, C is completely soluble in B. Now, we are saying that and A and B they were partially miscible. And now, we are saying that let us look at this system where A and B they are completely insoluble alright; completely insoluble C.

So, earlier if we take a mixture here and we separate they will separate in this two cases, this phase diagram. If you take here, they will also separate in this two weight fractions, but now, if A and B are completely immiscible, then we do not have any phase diagram here, we do not have any equilibrium line intersecting this A B equations. All it means, we are looking at this type of system, where we have A and B completely insoluble and we have C which is getting transparent rules. So, re-call that this is much simpler case here and in this case we can have a much simpler phase diagram; just like represented in earlier first few lectures when we talked about the different cascades.

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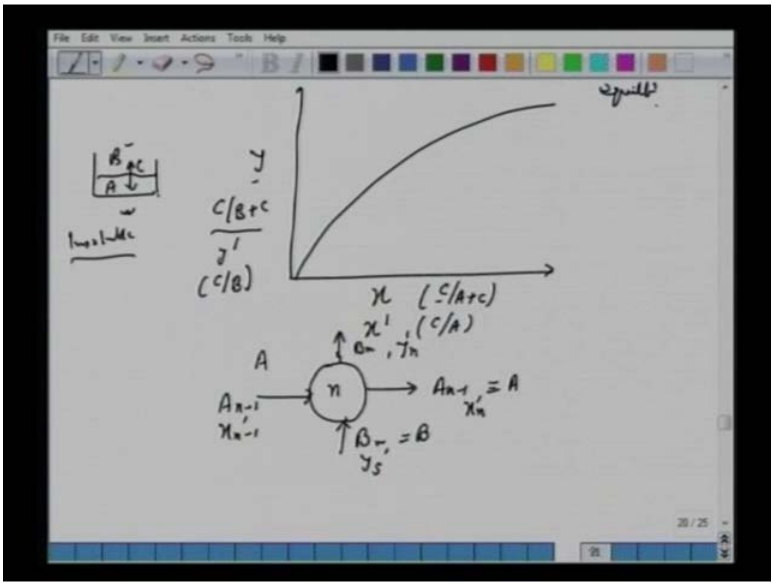

So, in this case y versus x will work very nicely alright. So, instead of working on this now equilateral equilateral triangular, this graph now we can work on this coordinate system. So, you must re-call that and you must realize that in this case when we say that A and B they are completely insoluble. So, it is only C is getting transfer. This case is very much similar to binary component discussion, we had earlier when we did this cascade systems.

So, in in other words, we can have C over B plus C. So, this is the weight fraction of C in the extract phase and we have this C over A plus C, weight fraction of C in the raffinate phase, one can also write Y prime solid free basis C over B and we can have C over A for X prime.

So, all we are trying to say that for such type of system, when A and B are completely insoluble liquids. This type of equilibrium diagram y versus x will be sufficient. So, in this case also you know, if we start, say certain phase, certain feed. So, let us take that we have nth stage and there you bring A add with solvent B alright. So, you have a raffinate and you have this extract here.

So, if you suppose you want to work on this solid free basis for this B n will remain as B n, if you are adding at A n minus 1, this will remain as A n minus 1 here. So, this all solid free basis A n minus 1, A n minus 1 this remain the same. Let us say that this is A, let say this is same as B. We have the quality of this y s, solvent, solid free basis. So, prime n solid free basis. We have this feed composition x n minus 1 prime here. And so, forth we have this x n for the nth stage, and this is your y n nth stage or y n prime or x n prime.

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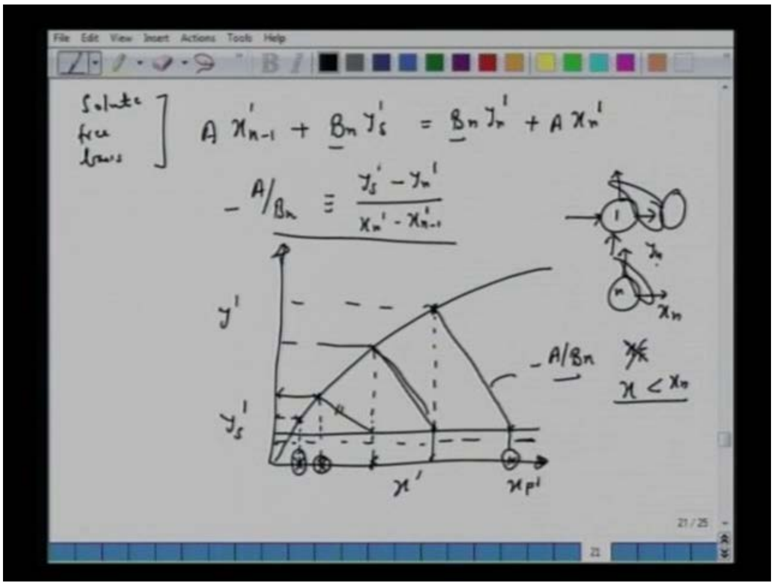

Solute free basis

$$A x'_{n-1} + \underline{B_n} y'_s = \underline{B_n} y'_n + A x'_n$$

$$-A/B_n \equiv \frac{y'_s - y'_n}{x'_n - x'_{n-1}}$$

So, we can write down this balance for A x n minus 1 plus B n y s this will be same as B n y n prime. So, notice that B n does not change, because we are working on solute free basis, plus we have A x n. So, this is simple species balance which you have made across that nth stage. And then, we can write at A over B n, you will see, why we are writing like this, minimum minimum minute A over B n will be equal to y s prime. So, it will makes a prime here for solid free basis, y n prime over x n prime alright; x n prime here minus x n minus 1 prime.

So, essentially you can you can go back to this y versus x diagram or y prime over x prime, suppose these are phase diagram and this is say the solvent. So, this is the quality of the solvent is here is fixed as y as prime. Now, the first stage, let say that we start with certain x F prime. So, this x F prime is brought in this contact with this y is solvent. So, we have this coordinate on this it is very easy to make out that now you have a operating line with a slope of minus A over B n. Take the slope, now at this line which as here alright. This stage - first stage we adding the solvent and we have the raffinate and extract phase they are in equilibrium A and B are completely insoluble. So, we can write this this type of one diagram will be sufficient more discussions.

So, these streams are equilibrium, it takes all this way, till it intersects here that is the first stage. This stage you can brought in contact with the same quality solvent, when you take difference in the slope of minus A by B n, if the amount of solvent is same in the all the stages, then this slopes will be - these lines will be parallel.

So, again come back to this where here, come back to this. So, essentially you go down from except all the way to this whatever concentrations you have. You must not notice that you cannot go below alright. So, this is all your equilibrium line. So, when this operating line it intersects with this equilibrium compositions. So that means, suppose is the last stages this y s is an equilibrium with this x here. So, all it means we can write and that no raffinates. So, this is the raffinate here. So, this is the raffinate - solved raffinate, raffinate, raffinate and here we have this extract, extract, extract. So, no raffinate of concentration smaller than that what you will get in this extract phase - solvent phase will be is possible. That means, suppose in some stage you reach here.

So, what is the composition of this raffinate it is an equilibrium with this extract phase. So, you cannot go below this which is an equilibrium with this. So, that is a limit of your treatment. That means, starting from x F prime you cannot go below this. The last stage composition of whatever is extract phase in equilibrium with this raffinate.

So, if this last stage n n, this is y n, this is x n, x n is an equilibrium with y n. So, x F and cannot be or that this treatment of the last raffinate cannot go below this x n. So, that is the limit here. To go below this again you must realize that you have to go below this; you have to lower this quality of your solvent here. In that case is possible that we can go below this whatever was a desired in our earlier case.

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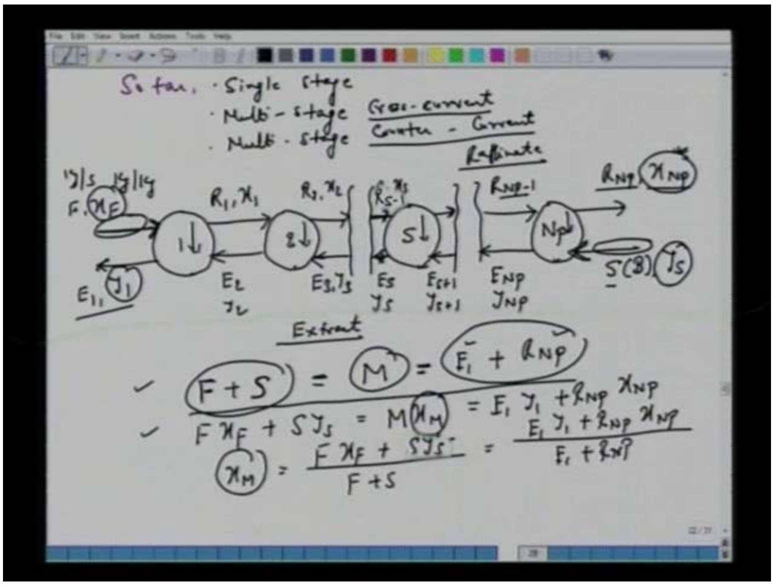

$$F + S = M = E_1 + R_{NP}$$

$$F x_F + S y_S = M x_M = E_1 y_1 + R_{NP} x_{NP}$$

$$x_M = \frac{F x_F + S y_S}{F + S} = \frac{E_1 y_1 + R_{NP} x_{NP}}{E_1 + R_{NP}}$$

We did single stage, then we did multi-stage cross-current and now, we do the last one here multi-stage counter-current. So, essentially what we have, now we have multi-stages would the mixing is cross-current. So, let say we have stage 1, 2, and then we have general stage here S, and then we have last stage NP number of plates or stage is NP. So, now the feed enters at flow rate F; so, the composition x F. So, this feed is again k g per second, x F is k g per k g all weight fraction. And then, we have the solvent - pure solvent which enters in a counter-current. So, that is what we are doing multi-stage counter-current. So, we have S now which is like B your solvent and this may have some mole fraction let say y s may not be 0.

So, now we have the feed. So, the numbering would be now R1 raffinate, raffinate, residue, mole fraction, weight fraction x 1. So, we have now R 2 and x 2, and let us take a break here. So, let us put a break here, and then we have this raffinate R s and x s. So that is the specs for this stream which enters the stage S, and then again let us take a break in the last stage, we have R NP minus 1 and this treated and the raffinate is now R N P and the weight fraction is x NP.

So similarly, we have the same nomenclatures here this would be extract. So, E NP, because it originates from the stage number N and we have y NP. So, for this stage S this is E s plus 1 - this is counter-current it originates from the next stage and we have y s plus 1. Similarly, this would be E s by this is E s sorry. So, here we have this should be E s and we have y s, and then we have this should be it is originates from E 3. So, E 3 y 3, you have E 2 y 2, then we have E 1 y 1. So, this feed enters from here and it is a counter-current with the solvent S or B with the weight fraction w s y s, and we have this treated effluents. So, the composition of this raffinate or the feed after the nth stage is x NP, we have brought down the concentration form x F to x NP. And this solvent concentration will increase for a small quantity y S to y1 as this component is transferred every stage.

So, look at the nomenclatures we are still following the same convention. The number and the weight fractions has the subscript which denotes the stage from where it as originator. So, that is a multi-stage counter-current. Unlike in the previous case multi-stage cross-current, we have to add the feed separately. So, this is a raffinate and downstream we have extract stream. So, up you write raffinate here we have this extract.

So, now let us make this overall balance here F plus S. So, two overall feed enters here and S enters here alright. Then, you have M; so, it is mix ting. So, you can always write F plus S as M which is over all x E 1 plus R NP. So, this is over all balance F plus S equals E 1 plus R NP to entering and to leaving same as M. Similarly, we can write F x F species balance S y s equals M x M equals E 1 y 1 plus R NP x NP. So, overall material balance, and the species balance. The physical meaning of x M is both ways based on the feed to entering this stream F x F plus S x s equals F plus S and this will also be equal to leaving streams E 1 y 1 plus R NP x NP over E 1 plus R NP. Of course, you must understand that this M or x M does not have any physical entity on this on this in the system.

The two streams F and S which enter they do not see each other. Similar, it is to leaving streams E 1 and R NP they leave from the two ends. So, this M is only hypothetical points, the mathematical points, signifying summation of two streams F plus S, E 1 and R NP or you have weight corresponding weight fractions.

So, here also you must note, now we follow the same nomenclatures - that all operating lines connects say y 2 versus x 1 or y n plus 1 versus x n. So, let us go back to the previous schematics which we have R 1, you have Y 2 and x 1. That is the material balance will give you operating lines – E 3 and x 2 operating lines. And if it is ideal stages then x 1 y 1 will be an equilibrium, x 2 y 2 will be an equilibriums and so forth.

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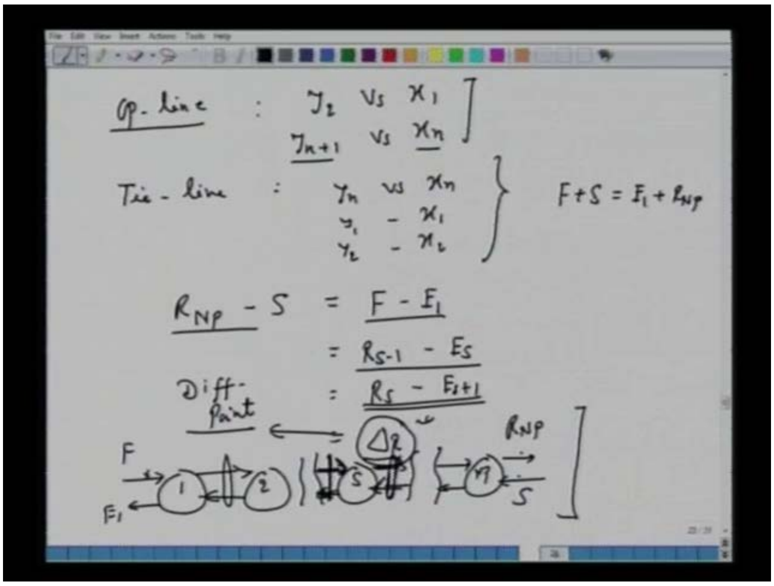

So, y 2 versus x 1, y n plus 1 versus x n, they come from the operating line and the equilibrium line or the tie line, if you have ideal stages, then you have y n versus x n for y 1 is an equilibrium with x 1, y 2 is an equilibrium with x 2.

So, we are following the similar convinces which we followed in the previous in two operations, then we had a distillation stage wise. More here important is that, note a difference R NP minus S. So, we are writing it from the previous balance which we made we wrote F plus S equals E plus R NP which means R NP minus S equals F minus E 1. And this will also will equal to any general stage R S minus 1 minus E S or you can have R S minus E S plus 1. So, the difference is constant, and this should remind us that we have a difference point here del R which is very similar to what we did in case of distillation column when we mark the difference between the two stages.

So, go back to the previous schematics which we can redraw here, if you have the stages stage 1, stage 2, feed goes in this directions and we have the extract going this directions. So, this is S stage and we have two streams getting in like here last stage we have NP. So, the difference between any two streams, here molar flow rates of that two streams, any way it is a constant given by del R.

So, we have R NP. So, this is R NP, this is S, R NP minus S is same as F minus E 1 or it same as for stage S, this is R S, this is E S plus 1. So, the difference between the two is del R or ahead of this you have R S minus 1 raffinate, and you have this E S plus 1. So, this difference is also constant. And that is what we are denoting it by del R. So, this is very equivalent to you know previous balance which we did for distillation column to identify this difference point.

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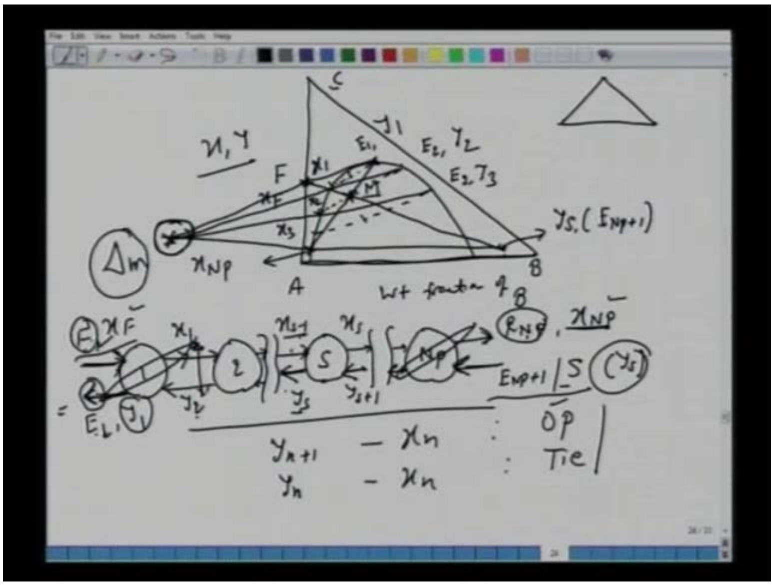

Now, let us mark these points one our equilibrium curve. Let us start with rectangular coordinate system; that means, we have we are working on this A, B, C, we have plotting weight fraction of B on x axis and on the y axis, we have weight fraction of raffinate and extract x and y. And we have this equilibrium curve which is like this.

So, instead of equilateral graph we can also work on this rectangular. So, start again with with the feed. So, let say this is F and feed does not contain any B. So, we have this coordinate marked here at x F on this A C line. And let say the solvent is composition is y s. So, this y s it is same as you know represents E NP plus 1 extract entering the last stage. So, again you can actually go go back to the previous schematics for different stages alright. So, we have general stage here S and we had interruptions here like this, like this, like this. So, it is stage S, it is a last stage, feed comes like this and solvent goes like this, counter-current.

So, stage 1, stage 2 we have this feed F and we have this extract for generating from 1. So, this is E 1 and this is E NP plus 1 which is same as S we said molar flow rates. This has the composition, let say y s very small amount of solute c here. It is possible that solvent is pure, in that case y is can may lie on this line A B, let us take E S E alright. So, this stage is a raffinate. So, this would be R NP, and this is x NP, and here we have f xF and this is y 1. So, this is y s, this is y s plus 1, this would x s minus 1 and x s same nomenclatures.

So, let us start with F here and S here. So, we have we know F quality of this feed and the quality of this strap or the solvent which being used here. You know this, we can connect, we can find out M x M here. So, this is F M here, again we said that this feed is here, and solvent is here, they do not see each other, this M is only a hypothetical point, it is a mathematical point which is present the composition of the mixing of F feed with the solvent which is M here alright. And we also know that E 1 and we know this R NP. So, these points - these two points can also be marked alright. So, essentially we have treated this solvent from x NP we have got down this level to this x NP.

So, let say x NP is somewhere here, x P. So, x NP is an equilibrium with this y NP. So, let us mark this point here. And similarly, if we know E 1 extract here, we can mark this point here, but in more important note this here that this line operating line has to pass through this. So, this is E 1 and this is y 1. So, again please note here that F and S 2 entering streams, and we have E 1 and R NP to leaving streams, they do not see each other in the previous like cross-current, but so that means, this M what we are writing down here is only a mathematical point, does not exist anywhere on this system.

So, F y s M they will lie on one straight line, and the same this M point will also lie in lie on the straight line connecting x NP and y 1, x NP here and y 1, from the mixing role. So, we have marked the 4 exit streams here 1 2 3. Now, we can start drawing the construction n for the stages. So, we realize here that E 1 y 1 is an equilibrium with x 1 ideal stages.

Now, with this E 1, we can draw a tie line which say let say goes through this to give us x 1. So, we have marked x 1 here. How do we proceed this x 1 and now we have y 2. So, now, we are talking of this operating points how do we reach this point. Just now we said that difference between any these two streams F and E R NP, and S E NP plus 1 or x 1 y 2 x s minus 1y y s or x S y s plus 1 is a constant. That means, if we draw these two lines and extent, and these two lines F 1 this F which we have drawn here and E 1, they will meet at point which we call it del M.

So again try to understand, we are trying to locate this point here. Or we are trying to say that difference between F and E 1 or any two streams x S minus 1, y s is a constant on the material balance. So, once we locate F E 1 we can extent and once we locate y S and x NP, we can extant to operate this del n points; once we have the difference points now, we can start constructing out of stages we have y1, so now, we have x 2, how do we get y 2, again we connect from here this x 1, again one has to connect to this operating line del point here.

So, if you connect this and extent, you will get E 2 corresponding to this y 2; take this draw the equilibrium curve to obtain x 2; correct with the del point, and extent you have E 3 and y 3. And then similarly, you take stage equilibrium here, take this equilibrium here, you get your x 3 etcetera. So, your stage 1, stage 2, stage 3 and so forth till you reach this stage.

So, here we are try to understand our methodology. So, we have the same story y n plus 1 and x n we are connected with the operating line, and y n and x n they are connected with tie line or equilibrium curve, and how do we obtain this, we look for this del m - difference point; difference point how do we get we realize the difference between the molar between the weight fraction, weight flow rates of any two streams are in this conserve is a constant

So, knowing F and E 1 extend here, and knowing y s and R 1 here, the difference between the two the constant extant you get this del M point; once you get del M point just like a distillation column, start with E 1, you get x 1 by tie line connect x 1 with I 2. So, this is del points extent you have y 2, find another tie line, this will gives you x 2, x 1, x 2, x 3 etcetera keep on constructing with this line here.

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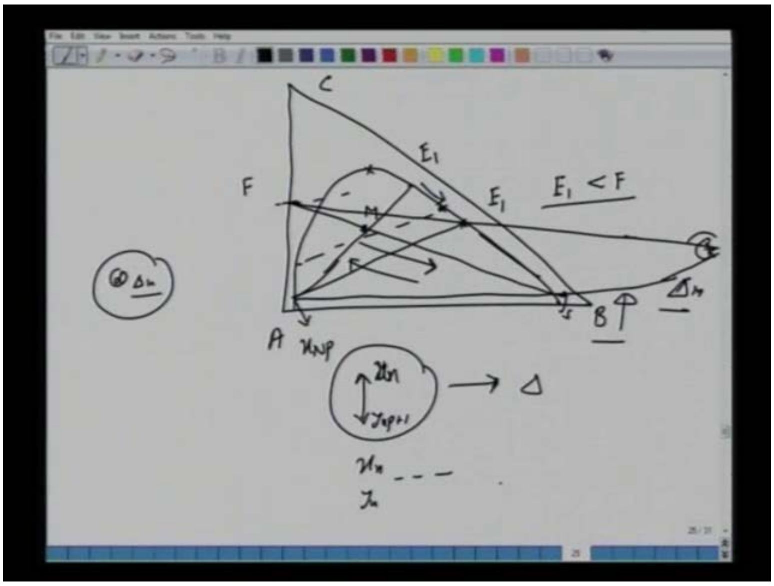

Just to conclude here, you can also have this difference situations, where the del point can lie on the other side of the graph which means, suppose you have equilibrium curve like this; suppose feed is here, and if you add and this is solvent and extend this you have mix, if you add more amount of solvent, you should be able to make out that this location of M will move this directions, because you are taking more and more solvent, larger amount of solvent here. So, this point will move in these directions.

And then, if you knowing this, suppose this is your x NP, if you extend this this, will give us E 1. So, as M increases, you can see that E 1 will decrease, this location of this point will move like this. Earlier, we had E1 here, now it move to this move to this as increase the amount of this solvent. So, it is possible that if an extent F M E1 and extent R NP and y N 1, M extend and connect somewhere here as a del m points.

So in this case, all we are trying to say that E 1 is less than F, the previous case we have less amount of solvent E 1 was greater than F, write this location of E 1 was higher than this F, here it is weight fraction of this E1 is a smaller than weight fraction of feed. So, we have locations here. So, that the two points when you extend, they increase del D. So, whether this side or this side, del M our principle methodology is the same, the difference between the two streams, y NP plus 1 right and x n is given by this del points. And x n and y n they are related by this tie line. So, anywhere if you extract, you draw a tie line is given by y1x1 or y2x2 or y3x3 etcetera, and when you connect with the operating line that gives you y NP plus 1 x n.

So, it is whether the del m point is this way or this way, it all depends on the amount of solvent locations of A, it is A; it does not depend upon what type of equilibrium curve we have, it all depends upon the amount of solvent which is added. So, M move this way, extract location moves this way or upward if you decrease the amount of this solvent

So, today's lecture we conclude here, and the next lecture when we start, we will address the minimum amount of solvent require or what is the maximum amount of solvent very similar to we have the treatments for the cross-current or in fact, in the distillation column or absorption columns. And then, we will take some example on these three stages and we have three types of arrangement we have; one was single stage the most simple, then we had the cross-current stages where we added the solvent feed solvent at every stage, and then we have this counter-current stage. So, we will take these examples in subsequent classes on extraction very good.