Mass Transfer II

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Module No. # 01

Lecture No. # 22

So, in the previous couple of lectures, we took examples on distillations. So, we had essentially two methods McCabe-Thiele method and Ponchon-Savarit method. In today's lecture, we also take one more example on McCabe-Thiele method; however, with slightly different objective. So, you must have realized that most of the times we have made use of graphs. So, it is essentially by graphical method, we have we have addressed like you know, how many number of trays are required or what is the heat load in the reboiler of the condenser.

In today's lecture, we take one more example; so, this example will help you or we can solve analytically, you know writing those McCabe-Thiele operating lines, and see, how we can substitute one variable from one equation to another equation. So, this example is again on McCabe-Thiele method, but this will give you slightly different angle or we can write a programming code on how we can solve algebraically by writing some equations here.

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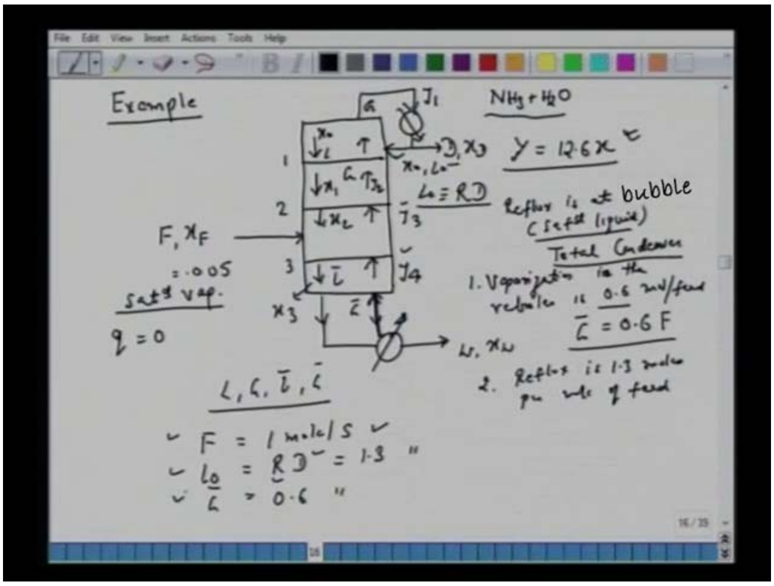

Reflux is at bubble (Sat^d liquid)

Total Condenser

1. Vaporization in the reboiler is 0.6 mol/feed

$\bar{G} = 0.6 F$

2. Reflux is 1.3 moles per mole of feed

$F = 1$ mole/s

$L_0 = RD = 1.3$ "

$\bar{G} = 0.6$ "

So, let us take this example on McCabe-Thiele method. So, what we have here, for simplicity let us say that we have only three plates. So, most of the times you had earlier when you were asked to find out the number of plates, but here we have been given that let us say, we have three ideal plates.

So, idea here is to solve non-graphically. So let us say, the plate number 1, 2 and 3; feed is sent between 2 and 3. So, it is a general case, you can have more number of plates and feed can be, you know, inserted anywhere. So, let us say the flow rate is F and mole fraction is x F; let us say, we have ammonia plus water system. So, it is aqueous solution of water. So this feed, let us say the composition, this feed is 0.005 and where we have it is a saturated vapor.

So, if you recall immediately there, when you say it is a saturated vapor, the feed is saturated here, we say that q is 0. So, that is your q line, and let us say, the thermodynamically we have this equations y equal to 12.6x over the concentration range of our interest here. So, we are trying to make simplify here things here, that we have only three trays; we have this equilibrium line, which can be returned by this equations - that is one; feed enters between second and third; and let us say, that reflux is at bubble point. So, it is a also a saturated liquid and we have total condenser.

So essentially, this vapor on the top plate comes to a total condenser; here this top product flow rate is D composition x D, and we have this reflux to the composition x 0, flow rate L 0, this is y 1 here, this is vapor flow rate is G. So, we have the same nomenclature, this is x 0, we have L l l 0 equal to L, vapor leaving from here, we have liquid dropping from here. So, this composition would be x 1 and this is top plate one. So, this is y 1, this would be y 2; and in the top, we have, it says vaporizations. So, about this reboiler, the problem statement reads - vaporization in the reboiler is 0.6 mole per feed.

So, we have this liquid, which comes to the reboiler, we withdraw the product W x W and part of this vapor or the vapor is sent back to this reboiler as a reflux here. So, that is given here, in the reboiler this flow rate the vapor is just as so that, we have this G bar. So, this is your G bar, which is 0.6 times of heat flow rate, 0.6 into F.

So, if this third plate here, you can say that this composition is y 4, this vapor flow rate is G bar and what liquid triples here is L bar. So, we are following the same nomenclatures L bar G bar for the stripping column; and L and G for these rectifying sections. So that the stream which is arriving at the plate third has a composition y 4 and this should be equal to x 3. So, this is again x 2 and what here we have again this y 3.

So, with the same nomenclatures, now this problem, other than this, we have been given this, of course, equilibrium data and we have been asked to calculate this other quantities, remaining quantities let us say, all the mole fractions in the columns for the liquid phase and the gas phase and the different flow rates which is L, G, L bar and G bar. So, how do we address this? Now, if you notice that here we have given a number of plates, and the feed flow rate is not given, but the other data are given like, what is the vaporization rate in the reboiler? So, mole fraction is given except. So, first thing is that we will like to solve graphically; so, we have this algebraic equation given for this equilibrium line, you are supposed to setup the operating line equations for both top column and the bottom column.

So, before we do that, let us see what are other quantities, which we can determine from the information or the data, which is given to us. Now, the feed flow rate is not given to us; that means we can choose a basis. So, with the basis of F as 1 mole per second flow rate, let us see if we can compute some other quantities.

So, F we choose as a basis 1 mole per second; now, other quantities which should be interesting here is one more data is given here, the reflux is saturated liquid, total condenser; number 2 - reflux is 1.3 moles per mole of feed. So, when this is D here, your L 0 which goes back as the reflux is R into D, if R is the external reflux ratios; here, reflux is 1.3 times per mole of feed. So, when we choose F as 1 mole per second, L 0 which is R into D, neither D is given nor R is given, equals 1.3 same units, moles per second. Now, what is given here is G bar reflux - vapor reflux - which is 0.6 times F. So, we have this G bar as 0.6 moles per second; so F L 0 and G is given to us. Now, let us let us see if we can compute the other quantities

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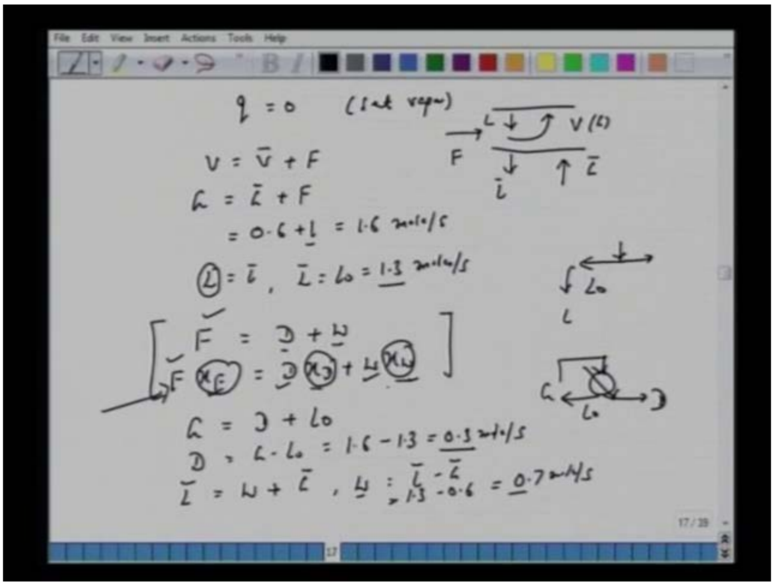

$q = 0$ (sat vapor)

$V = \bar{V} + F$

$G = \bar{G} + F$

$= 0.6 + 1 = 1.6$ mole/s

$L = \bar{L}$, $\bar{L} = L_0 = \underline{1.3}$ mole/s

$$\begin{bmatrix} F = D + W \\ F x_F = D x_D + W x_W \end{bmatrix}$$

$G = D + L_0$

$D = G - L_0 = 1.6 - 1.3 = \underline{0.3}$ mole/s

$\bar{L} = W + \bar{G}$, $W = \bar{L} - \bar{G} = 1.3 - 0.6 = \underline{0.7}$ mol/s

Now, when we say saturated vapor, then we have q equal to 0; that means at the feed plate which is between 2 and 3, where the feed enters saturated vapor, this goes with your vapor V, but G, and this is your G bar, liquid is L here - liquid flow rate, molar flow rate - and here we have L bar. So, that means saturated vapor, whatever V the top column we have, equal to whatever arrives V bar plus F, for the following the same the same nomenclatures F G equal to G bar plus F. So, G bar is reflux which is sent from the bottom column, from the reboiler here, which is 0.6, and we have one feed, we have chosen as the basis, we have 1.6 mole per second.

Now, q is 0; that means saturated vapor, L equal to L bar; remember L is same as L 0. So at the top column, reflux is saturated liquid, which is L 0. So, when it enters the column, we have L equal to L 0; that means, we that means, we can write L bar as L 0 equal to 1.3 moles per second, which is given to us.

Now, let us see based on this what else we can do; we have the overall balance F equal to D plus W, F is known, but D is not given, W is not given; F dot x we can write the species balance, D x D plus W x W. So again here, x D and x W that composition of the distillate and the bottom products are not given here, x F is of course, given here. So, to looks likes from these two equations, we are unable to determine the rest of the quantities.

However, look at in the bottom column or the top column here we have G which arrives to the condenser equals D plus L 0. So, the D or the vapor from the top plate, and it arrives here as the G flow rate, and this is your top product D, and we have the reflux L 0. So, we can calculate top product D as G minus L 0, G 1.6 and L 0 we have calculated one point it is given 1.3, which is 0.3 moles per second. So we got D.

How about for the bottom column? We have L bar a split into 2 W plus G bar. So, W equal to L bar minus G bar; L bar we said same as L 0 1.3; G bar is the reflux, which is sent back from the reboiler as a reflux vapor, which is 0.6, 0.7 moles per second. So, essentially now we got all the quantities; now if you write, substitute in these equations, F is known, D we have calculated, W we have calculated, x F is known, but x D and x W they are not known to us. So, this equation is still we are not in position to calculate it.

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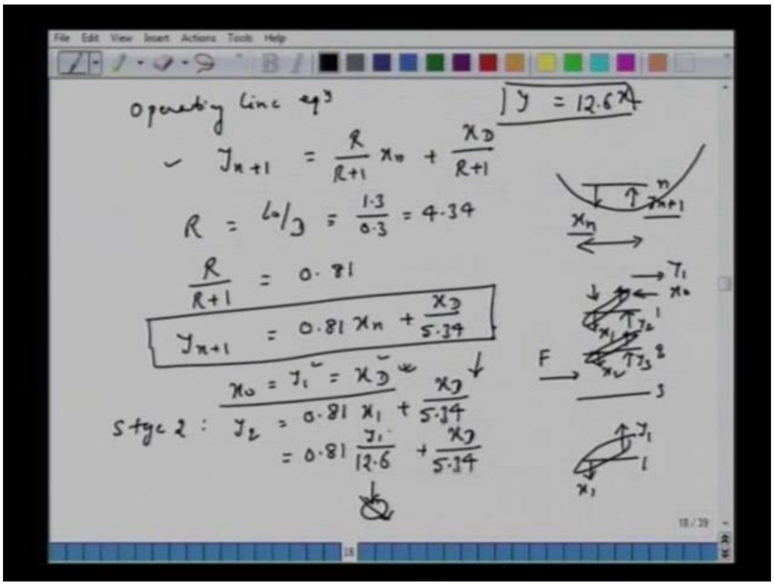

operating line eq^n

$y = 12.6x$

$$y_{n+1} = \frac{R}{R+1} x_n + \frac{x_D}{R+1}$$

$$R = L_0/D = \frac{1.3}{0.3} = 4.34$$

$$\frac{R}{R+1} = 0.81$$

$$\boxed{y_{n+1} = 0.81 x_n + \frac{x_D}{5.34}}$$

$$x_0 = y_1 = x_D$$

Stage 2 :

$$y_2 = 0.81 x_1 + \frac{x_D}{5.34}$$

$$= 0.81 \frac{y_1}{12.6} + \frac{x_D}{5.34}$$

What else we can do? Now, we can write down the operating line equations. So, for the top column, we have a general expressions, y n plus 1 equal to R over R plus 1 x n plus we have x D over R plus 1. So, if you can choose another plate here, the vapor which arrives here has composition y n plus 1 and the liquid which leaves the nth plate has composition x n. So that is what operating line does, connects y n plus 1 and x n, by this equation which was obtained, if we recall by taking this envelope region one around this condenser and this plate nth to have this mutual balance.

So, we know the reflux ratio now, R equal to L 0 by D, 1.3 is given to us that reflux is point 1.3 mole per molar feed has sent as a condenser over D, which we have calculated as 0.3, make it 4.34. So, we know the slope of this operating line as R over R plus 1 as 0.81. So now, we can substitute y n plus 1 equal to 0.81 x n plus x D R plus 1 to make it 5.34. So now, we have got an equation, which can give us the composition of one is stream, if we know the composition of the other streams. So, which is 1 and 2 here, we have this plate here 3, feed enters between 2 and 3.

So, this is y 2, and this is x 1. x 1 this is the total condenser is same as y 1 and same as x 0. So, since a total condenser, we can write x 0, which goes to this as a liquid as a reflux, same as y 1 and same as your top product x D, but we do not know this x D, but we can write down the equations or we can connect y 2 and x 1, and we can connect y 3 and x 2, the two streams are in equilibrium; two streams are in equilibrium. So, let us write down this equation for a stage 1 or a stage 2; stage 1 if you recall, if we can write down this equations, the way it is all x 0 y 1, x D is the same, that will will be same as if you substitute here for x 0 and y 1, we will get the same result.

Let us do it for a stage 2. So, when we do it for a stage 2, y 2 equal to 0.81 x 1 plus x D over 5.3. So, x D is not known to us, nor x 1 and y 2; but remember for the plate this x 1 is in equilibrium with y 1, plate 1, and we have given the equilibrium line equations. So, we can substitute x 1 and y 1 about 12.6. Remember the equilibrium line equation is given as y equal to 12.6x. So general expression here; so, since x 1, y 1 they are on the equilibrium line we can substitute here to obtain this equation x D by 5.34; but what is y 1? Remember this y 1, which arrives from the top plate to the condenser, we have this total condenser y 1 equal to same as x D. So, we can substitute y 1 here with x D; simplify this equations, that is y 2 equal to 0.25 x D. So, if you substitute, now we have another expressions which relates x 2 x D to y 2; now, let us again try to see, what we want to do here.

First thing is that McCabe-Thiele method traditionally, you draw the two operating lines and find the number of trays; now in this case, we know the slope R by R plus 1, we know all the vapor flow rates L and G, but we do not know the composition of the top product and the bottom product.

So, you would not be able to draw the operating lines and mark these stages here. So, we are doing analytically, if you want to write down a program, there also you can write down this equation of the operating line, you can do the coding, and you can put a loop in all the for all the stages for the top section, for the bottom sections. So, here our strategy is that we have one equations analytically, F D plus W, we know the flow rates and when we write down the species balance F x F as D x D and y, and bottom W x W, but we do not know x D and x W. So, we have just a one equation, we are looking for one more equation.

So, what we do, we start from x D, x D is same as x 1, x 1 is in equilibrium with y 1; then, you connect y 1 and x 2 till you reach arrive at bottom. So now, you establish one more equations with between x D and x W. So now, then you will have two unknowns and two equations, and you can solve for x D and x dot W; once you know the compositions, now you can mark all the compositions on your graph or in your answer book. So, here we are trying to solve analytically, why because we do not we cannot solve graphically, because the compositions are not known to us.

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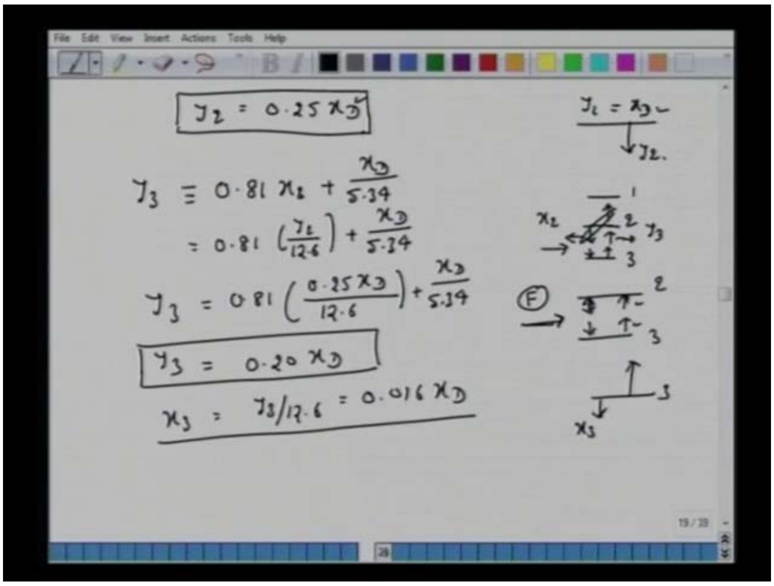

$$\boxed{y_2 = 0.25\,x_D}$$

$$y_3 \equiv 0.81\,x_2 + \frac{x_D}{5.34}$$

$$= 0.81\left(\frac{y_2}{12.6}\right) + \frac{x_D}{5.34}$$

$$y_3 = 0.81\left(\frac{0.25\,x_D}{12.6}\right) + \frac{x_D}{5.34}$$

$$\boxed{y_3 = 0.20\,x_D}$$

$$\underline{x_3 = y_3/12.6 = 0.016\,x_D}$$

So, let us continue with this, we have written, now we got the expression starting from x D, which is same as y 1, because the total condenser, now we have connected this to y 2.

Now, let us do it for the third stage. So, stage 1, stage 2, now we have this stage 3; mind you this feed enters here. So, this compositions and this composition; what is this composition? This is y 3 And what is this composition? It is x 2. Now still we can write down these expressions between x 3 and x 2. So, this y 3 and x 2, the composition here do not change. So, in this plate between 2 and 3, if you recall, we establish q line; what was the q line? q line we had energy balance; whatever energy is brought in here, we had energy balance between the two streams of vapor here and for the liquids here. So now, what we are trying to do here, that we are extending this operating line for the stage, just underneath it. So, we started from x D, then we had this got it into y 2, now we write down the expression for y 3 and x 2; y 2 and x 2 they are in equilibrium; y 3 and x 2 they will be given by the operating line. So we extend, we write down the same expressions; now for y 3, y 3 equal to 0.81 x 2 plus x D over 5.34, but again 0 x 2 this is stream is in equilibrium with this stream y 2. So, we can substitute from the equilibrium curve y 2 over 12.6 plus x D over 5.34.

So, now we can write y 3 0.81, y 2 is given in terms of x D. So, we can substitute here 0.25 x D over 12.6 plus x D over 5.34. So, now we have another expressions y 3 as 0.20 x D. So, this y 3 is unknown to us; once we know y 3, we can calculate now x 3, so this is third stage and this is composition here is x 3. So, x 3 will be equal to y 3 divide by 12.6 equal to 0.016 x D.

So, let us try to understand what we are doing here again; lets discuss here; we have this feed which enters between stage 2 and stage 3; now this is a mixing zone and we made use of energy balance, if you recall we have the total balance F what enters and two streams L and L bar, and we have G and G bar. So, we had we wrote down this, go back to your lecture notes, we wrote down this overall material balance, and then, we wrote down this energy balance or the enthalpy of all the five streams: two liquid and two vapor streams with the feed here, then we got this q line.

Now, what we are trying to say here that the compositions of the vapor arriving at the stage 2, underneath 2 is same as the composition of the vapor which leaves a number is stage 3. So, we wrote down this operating line for rectifying sections all the way till y 3 and x 2; x 2 is a composition of the vapor liquid, which arrives from the stage 2. So, we could write down the balance between y 3 and x 2; now, we are in the stripping sections. So, we have to now begin with the stripping line equations. So, we are trying to substitute x D in terms of y 1 total condenser, then y 1 x 1, x 1 to y 2 etcetera, till we have one more equation between x D and x W; then, we can make an overall material balance or a species balance, we will have two unknowns and two equations. So, let us continue with our balance with the stripping sections

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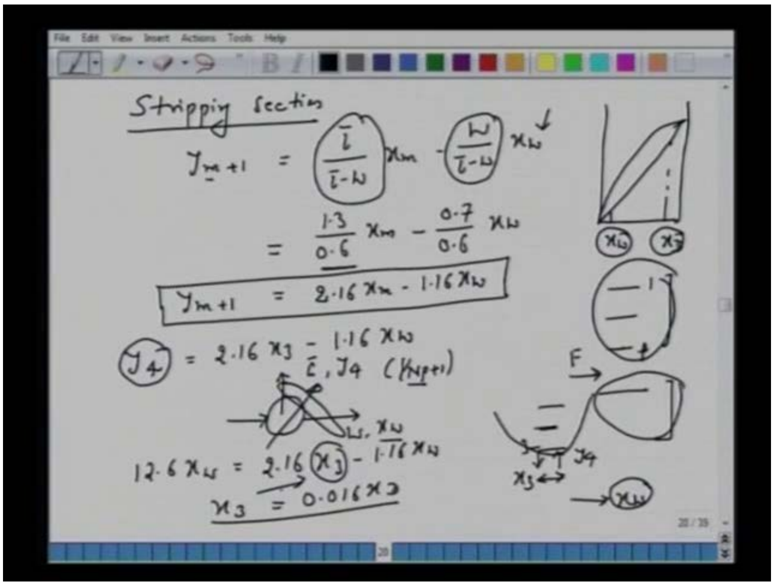

Stripping section

$$y_{m+1} = \frac{\bar{L}}{\bar{L}-W} x_m - \frac{W}{\bar{L}-W} x_W$$

$$= \frac{1.3}{0.6} x_m - \frac{0.7}{0.6} x_W$$

$$\boxed{y_{m+1} = 2.16 x_m - 1.16 x_W}$$

$$y_4 = 2.16 x_3 - 1.16 x_W$$

$\bar{L}, y_4$ (y_{m+1})

$$12.6 x_W = 2.16 x_3 - 1.16 x_W$$

$$x_3 = 0.016 x_3$$

Now, the equation of stripping line, stripping section, is y m plus 1. So, we are using now m, instead of n; we have L bar by G bar, where G bar we write in terms of L bar minus W x m minus W over L bar minus W x W. So, again we know all this quantities here; only thing we do not know is this mole fraction x W. So, that is what we said in McCabe-Thiele methods even if you know the slopes, but if you do not know the compositions of the two products, you cannot draw this graphically. So, neither x D nor x W are known to us.

So, we have to solve analytically and these equations can always be programmed for every stage. So, you have realize that all the stages gave up the feed, and all stages underneath this feed where it enters, you can write down two equations: one equation will hold good for all this, another equation will hold good for this. We can also write down this one equation for this q line. So, that is the way you have to do it; if you want to do by programming, here we are trying to write down this equation for every stage above this and every stage below this feed F.

So, we have these general expressions, let us put the number, you know, L bar by G bar which is 1.3, and G bar we have calculated as 0.6 x m, actually 0.6 was given in the suppression in the reboiler is 0.6 moles per feed. So, we choose F as 1 mole per second, we have this G bar is 0.6 minus W, W is given as 0.7. So, we can substitute here as 0.7 L bar minus W is same as 0.6, we have this x W. So, we can write y m plus 1 as 2.16 x m minus 1.16 x W.

So, now let see, now how we can apply; so we have three plates 1, 2, 3 and the 4th plate - theoretical plate, it is from the reboiler. So, we have this vapor composition which is y 4, and the liquid composition arrives from here is x 3. So y 4 and x 3, they will be connected by this operating. So, now we can write y 4 equals 2.16 x 3 minus 1.16 x W; what is y 4? y 4 is in equilibrium with compositions x W. So, when you have the reboiler, the liquid arrives here, we were drawing the product W and x W, and part of this liquid we have reprising it as G bar with the composition y 4 or remember y n plus NP plus 1, where NP is a last stage. So, we have three stages; so, this is y 4 and the two streams are in equilibrium. So, y 4 we can write in terms of x W as 12.6 x W equal to 2.16 x 3 minus 1.16 x W.

But remember this x 3 also we calculate from the previous equations which we had or operating line for these rectifying sections, where x 3 was calculated as 0.016 x D. So, go back to the previous slide here x 3 is calculated in terms of 0.016 x D. So, we started from x D slowly by writing these equations for every stages, we have expression x D in terms of x 3. So, and that we can make use here x 3 equal to 0.016 x D and we can substitute here to obtain another equation.

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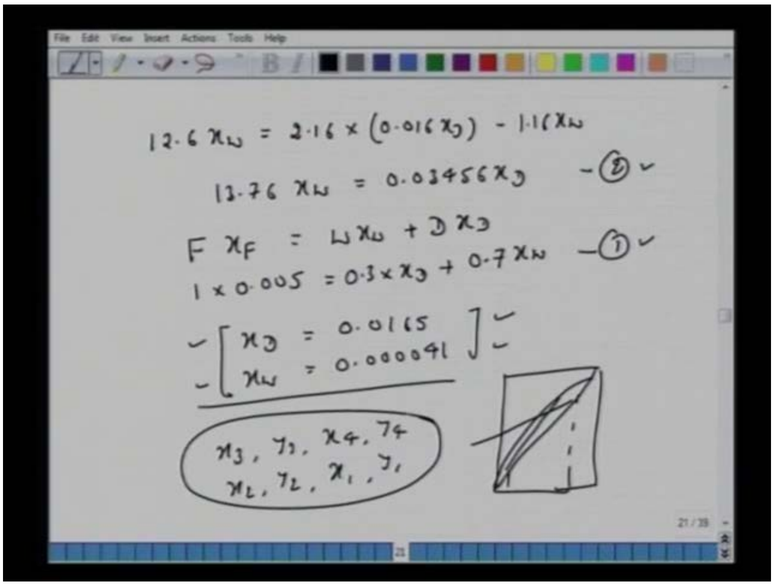

$$12.6\,x_W = 2.16 \times (0.016\,x_D) - 1.16\,x_W$$

$$13.76\,x_W = 0.03456\,x_D \quad - ②$$

$$F\,x_F = W\,x_W + D\,x_D$$

$$1 \times 0.005 = 0.3 \times x_D + 0.7\,x_W \quad - ①$$

$$\left[\begin{array}{l} x_D = 0.0165 \\ x_W = 0.000041 \end{array}\right]$$

$x_3,\ y_3,\ x_4,\ y_4$
$x_2,\ y_2,\ x_1,\ y_1$

12.6 x W equal to 2.16 multiplied by 0.016 x D. So, here we have substituted the values of x 3 in terms of x D minus 1.16 x W. So, now what we get, we get one more relation between x W top product and the bottom product, but x W equal to 0.03456 x D. So, this is the second equation here.

The first equation we had F x F W x W plus D x D, F is 1, x F was given mole fraction as 0.005 compositions; W we calculated as 0.3 or D we calculated as 0.3 into x D plus W we have calculated 0.7 into x W. So, we have this was the first equations and now we have the second equations, one and two if we substitute, we should be able to get to the substitute here to show that x D equal to 0.0165 and x W equal to 0.000041.

So, we have got two composition unknown quantities here x D and x W; now, we can go back, mark this point on this McCabe-Thiele plot, we can draw the operating lines the q lines etcetera, here. So, idea here is that once we get x D and x W, now we can do the back calculations to obtain x 3, y 3, x 4, y 4 or x 2, y 2 and x 1, y 1.

So, let us summarize this, the example, which we have done here; the idea here is to solve theoretically, of course, graphically cannot be used, because we do not know the compositions. So, our strategy was to write down the operating line equations for both the top section and the bottom sections, we start from x D, and see how we can substitute x D in terms of y 1, then y 1, x 1 they are in equilibrium; once you know x 1, you get y 2 you get. So, make use of this tie line and the equilibrium line, what we have been doing graphically on the McCabe-Thiele method, same thing we are doing by the equations. So, we are substituting one variable to another variable till we arrive at stripping sections, where we write the another equation for the for the operating line, and slowly so gradually, we do the same exercise for the last stage, so that, we have one more equations in terms of x D and x W; we have two unknowns, two equations we can solve.

Now, there are two ways of looking at this, as we said earlier, one is of course, to solve analytically, algebraically if you have to set up all the algebraic equations then, there is no need for the graph. The second is that we can also do this programming code if you know Fortran or you can know C, you can have, you know, you can write down this operating line equations for every stages, you can define a variables like y, for every stage x, for every stage moral flow rates are constant. So, you can put some loop 2 i equal to 1 to number of stages for a top column. Similar exercise you can do for the bottom column - bottom column bottom section of the column; do i equal to 1 to how many stages, that many stages you have for this bottom column and one can calculate, solve by writing all these equations, you can also call some numerical programming to solve set of algebraic equations. So that can also be done.

So, this is the way most of the engineering commercial software they work, when you have the equilibrium line equations give equilibrium line given by some equations and then, you have two operating lines and you have the q lines, all this four can be used in the programming code to solve set of unknown quantities. So, that is our example.

Now, what we do now as we said earlier, we take we discuss very common distillation column, which is based on what do we call open system. So, if we have two components A and B, out of which one is water, so, aqueous and you have another non aqueous system; and if you want to distill, say, the bottom product is your rich in water and top product is rich in some other organics, it is a very ideal system, no isotopes; in that case when you have the vapor reflux sent back to the distillation column, it will be rich in water. So, if you have that type of situations where your bottom reflux, the vapor reflux, is highly rich in water, say, 0.999 percent less volatile component here; then, there is no need to put physically vapor this reboiler you can have a steam, you know available from nearby plant neighborhood that can be send as a reflux.

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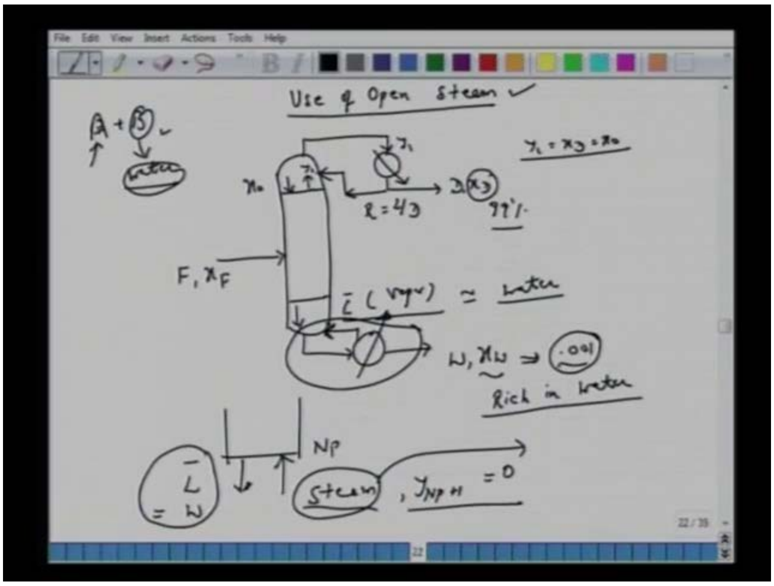

So, let us continue with this a new topic here - Use of Open Steam. So, what we have here is a distillation column, we have stages here, feed is sent somewhere here, we have the composition with the flow rate F and x F, we have the top product we drawn, then we have this condenser. So, this is the product here D and x D; let us say, it is a total reflux and reflux is saturated liquid. So, we have this reflux ratio R equal to L by D, this goes to this top plate here.

So, we have this x 0; this is y 1 which is same as this y 1. So, y 1 equal to x D equal to x 0; now the system which we have here is let us see, A plus B, A… B is let us say less volatile is water, and A is more volatile, some different compound component here which is going to be distill as a top product. So, you will expect that is x D, let us say, it is a very highly rich in A, let us say it is 99 percent; now, the bottom product, liquid which arrives for that bottom plate to this reboiler, typically this will be rich in the other components B and very poor in this bottom product.

So, what we will do here, this liquid will be reboiled part of this vapor, part of this column liquid will be sent as a vapor reflux as G bar, and here, you have this bottom bottom product W and x W. So, this x W will be, let us say 0.001 mole fractions. So, it is very poor in this A, but rich in water. So, this reflux vapor, which is sent back to this distillation column from this reboiler is nearly water.

So, if you have this type of situations, where one of the two components, the less volatile is water, and your bottom product is highly rich in water here and very poor in the top product, you will expect that the reflux will also be very rich in water; in that case one can do away with this arrangement of reboiler, and essentially, what you will have a bottom, this this is a bottom tray here, what reflux you will send back will nothing but pure steam.

We can say that composition of y is, say this number of plate is NP, this y NP plus 1 is 0; approximately 0. So, whatever product you are getting L, L bar is nothing but your bottom product. So, this what, we are calling it as a open system; we have the bottom product, bottom column, in which whatever liquid we are withdrawing is highly rich in water. So, the reflux which is supposed to be sent from the reboiler is nothing but steam, pure water, we can this steam may be available from some nearby plant. So, from there we can take a steam and send back to the distillation column where there is no reboiler we call this as an open system.

Now, how do we address this type of situations, whatever based on whatever we have learned in this course - in this McCabe-Thiele method. So, McCabe-Thiele method essentially, what we do, we write down the species balance or the material balance for the top column and for the bottom column. So, you have two lines - two operating lines - they intercept you have this q line, then you calculate the number of plates; we can also have a Ponchon-Savarit method where you recall you have HB saturated vapor, composition enthalpy and saturated liquid enthalpy. There you locate difference points such as top x D and the bottom as x W, and then, you mark your tie lines and the operating lines. In this system what happens when there is no reboiler? So, we will have to make certain modifications in both McCabe-Thiele method as well as in Ponchon-Savarit methods. The idea is that how we can address starting from the first principle or by making some analogy on both the methods Ponchon-Savarit methods and McCabe-Thiele methods. So let see, what we do here.

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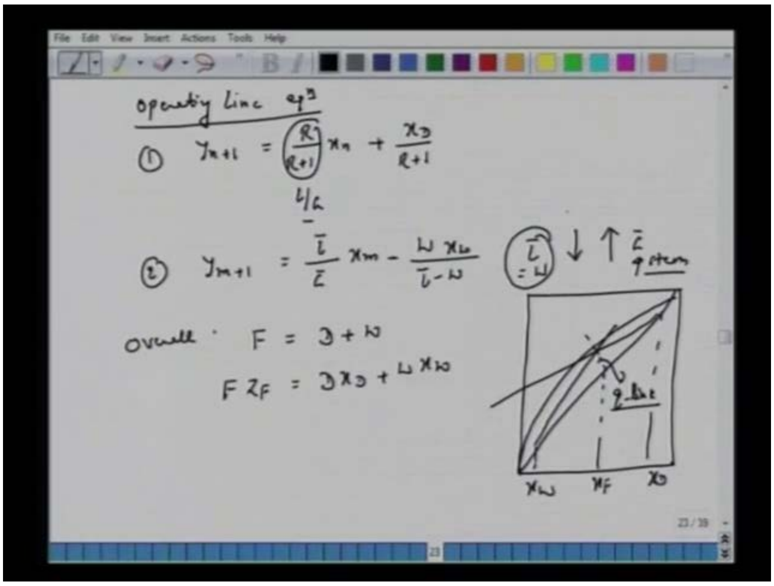

operating line eq^n

① $y_{n+1} = \boxed{\frac{R}{R+1}} x_n + \frac{x_D}{R+1}$

L/G

② $y_{m+1} = \frac{\bar{L}}{\bar{G}} x_m - \frac{W x_W}{\bar{L} - W}$ $\quad \boxed{\bar{L} = W} \downarrow \uparrow \bar{G}$ steam

overall $F = D + W$

$F z_F = D x_D + W x_W$

So, let us write down the operating line equation, which is y n plus 1 equal to R over R plus 1 x n plus x D over R plus 1. So, R by R plus 1 is nothing but L by G. So, that is one equation; second operating line equation we had for this stripping sections, which we wrote as y m plus 1 equal to L bar over G bar x m minus W.

So here, we have actually W x W over L bar minus W; overall we wrote as F equals D plus W and we wrote F z F equal to D x D plus W x W. So, we have this McCabe-Thiele method where this is x D, this is x W. So, we draw this operating line, we have another operating line here, and wherever it intersects x F, we have this q line. So, now, in our case when there is no reboiler, and we have direct steam, and we have the bottom product L bar, which is nothing but our product L W. This is G bar here, which is nothing but the steam flow rate, how we can modify this approach and do the same calculation what we have been doing earlier.

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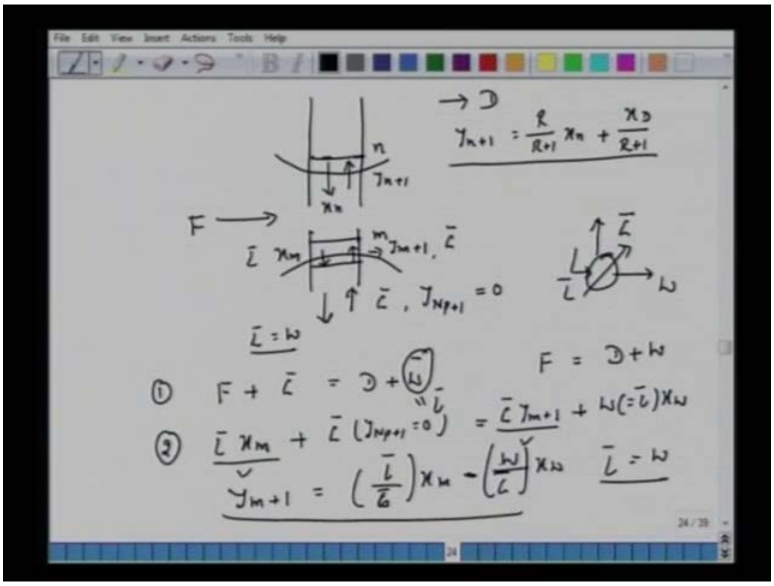

So, you must realize that our the operating line equation for the top section will not change, which means if we take this envelope underneath this plate number n, we have y n plus 1, and we have this x n. So, this equation is unchanged, R over R plus 1 x n plus x D over R plus1. So, this is unchanged or for the bottom column, now we have this mth plate. So, what we have done here is…So, this is mth plate and we took this envelope like this. So, this steam which arrives here is y m plus 1,the stream which is arriving here is x L, this flow rate here is L bar, this flow rate here is G bar, and this liquid L bar, we know that this bottom product W and the vapor which is nothing but pure steam is G 1.

So, we have this y, just we said in NP plus 1 equal to 0; so, there is no reboiler. So, what happens to our balance here overall balance F plus G bar? So, this is F, F plus G bar equal to D plus W bar; mind you this W bar is nothing but L bar. So, compare this to the earlier case where we had F equal to D plus W; now, we have F plus G bar, that is what enters equal to the top product D, which leaves and L bar or W. So, this is new equations we have; let us make now species balance here. So, when we say operating line here either for the top column or for the bottom column is nothing but the species balance for that region. So, when you make the region balance here; now L bar, x m, this is what we have here, two streams, stream which enters this region or envelope which we have drawn here plus G bar into y NP plus1, but that is 0.

So, these are the two streams equal to what we have here is G bar y m plus 1 plus W or L, which is nothing but your L bar into x W; see, compare this to the previous case, where there was one product which was W here, then when we wrote the balance here, then we had L bar x 1 this term here, we had this term here, plus we have the last one. But now, what we have here, this compositions is still 0 here. So, if you simplify this, you will have the equations of operating line as y m plus 1 equal to L bar over G bar x m minus W over G bar into x W.

So, now let us compare with what we had earlier and what we had now, previous operating line equations which we wrote, we had L bar by G bar and we had W by G G bar, but L bar was not same as W. So, we had the flow rate L bar arriving to this reboiler, we had this bottom product which was W and send back as G bar, here we are saying that there is no reboiler. So, this product L bar is nothing but the product (( )). So, this is a new equation where L bar equal to W. So, we see one modification here, in the overall balance and we see this balance difference equations which is format with the same except that L bar is same as W bar L. So, let us see what all we can model or how we can plot of this McCabe-Thiele method.

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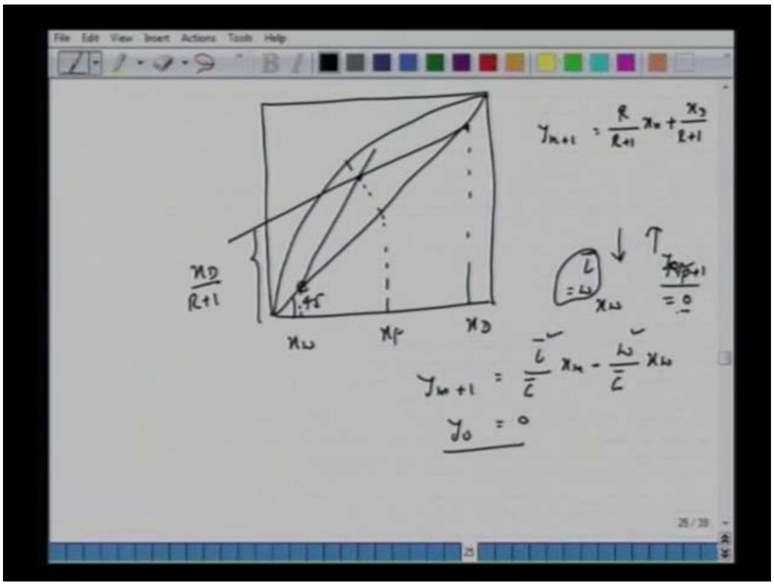

$$y_{n+1} = \frac{R}{R+1} x_n + \frac{x_D}{R+1}$$

$$y_{m+1} = \frac{\bar{L}}{\bar{G}} x_m - \frac{W}{\bar{G}} x_W$$

$$\underline{y_0 = 0}$$

So, we realized that equilibrium curve is the same and the operating line equation for rectifying section is the same. So, but the equation which we had for the top y n plus 1 equal to R over R plus n x n plus x D over R plus 1; this point x D also satisfies 45 degree line, which means at this point you can take a slope or intercept of this x D by R plus 1, we had this operating line. So, this x D equal to y D if you substitute here, you will get this 45 degree line.

In earlier, earlier case, similarly, we took this x W here, and we took the slope to connect here, this point here which you extend, we connect this to x F will give you q line, but notice here that now what we have at the bottom is this L bar, which is same as W, has the composition x W, but the vapor which arrives as a composition 0. So in the previous case, again, when you wrote down the expressions for the operating line as y m plus 1 equal to L bar over G bar x m minus, you had W over G bar, where x W, we said that when you substitute x W here, then this line will again satisfy, will will will also lie on this 45 degree line. So, y m plus 1 will also be equal to same as x W, that is why we extend this to this point.

But now, L bar is same as W, L bar is same as W; that means, when you put x W, you will get y 0 equal to 0; that is what we have here. So, this is y NP plus 1 actually, which is 0 here.

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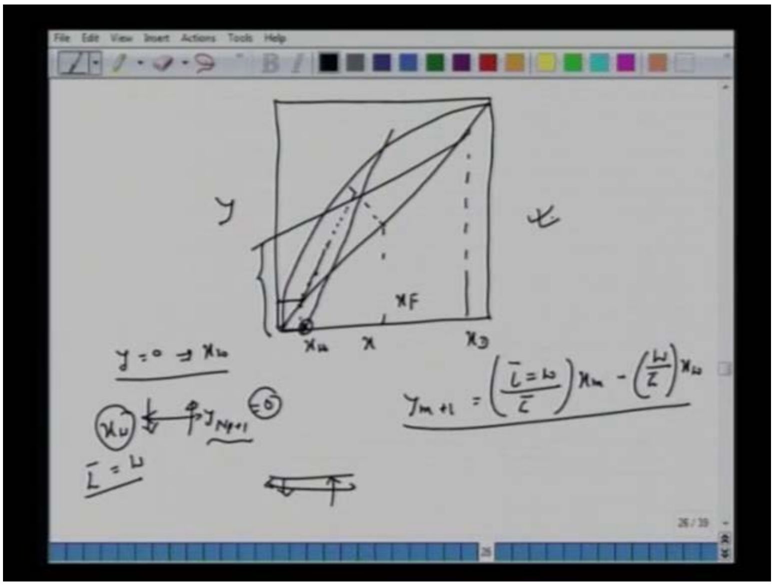

So, how this what happens to that the points here, now they do not lie on this 45 degree line; now, what you will have? x D connect with the slope or the intercept whatever you have, but now y equal to 0 corresponds to your x W. In other words this is your y NP plus 1, which is 0, pure steam, and the liquid which trickles down as the product is x W, has a composition x W, this flow rate L w L bar is same as W here. So now, we have this point x W right here, but no more we have this y equal to x W, because now we have different equations which, in which we substitute x W, you will get y equal to 0.

So, this is the point here; with this point if you take the slope of L bar by G bar plus y m plus 1, lets rewrite here, L bar which is same as W over G bar into x m minus you have W over G bar, and here you have x W, in this if you put x m equal to x W, you will see that since L bar equal to W, you have 0 here. So, now with this and with this slope or with the intercept - negative intercept - now, you have to connect like this; this will now correspond to your x F or the q line. So, notice the difference here instead of connecting from here, now you have connected with this plus x axis where y equal to 0, because this what operating line connects two phases, phase which arrives here, liquid which leaves here, operating line connects like this.

So for the bottom tray, you have y NP equal to 0, because pure steam and x W is your bottom product composition which is known to us, we mark the point here and now we connect to obtain this intercepts. So, these would be the two new operating lines; remember again instead of starting from here, you have to start it from here, because earlier x W also y x W equal to y W also satisfied this operating line equations. Now, y equal to 0 and x W this satisfies this equation. So, we have modified this McCabe-Thiele method.

Now, again you should go back and carefully see that this open system or open steam system, whatever you are calling it here, is nothing new; new in the sense that our approach remains the same, you write on this operating balance equations or a species balance equation for this bottom envelop, the sections underneath the feed, to realize that there is a modification in the overall material balance and there is a modification in the species balance for this stripping sections here, and then, modification comes from, you know, realization that we do not have a reboiler, L bar is nothing but the bottom product W and x W which is the compositions of the steam, which is leaving the distillation column has corresponding vapor phase compositions which is 0, because it is a pure steam. So, that coordinate does not lie on the 45 degree line that coordinate lies on the x-axis from there you have to connect to the operating line. So, otherwise all the three steams are the same, you can start from the top or from the bottom and can construct the trays here.

Now, the same argument should also could for Ponchon-Savarit methods; remember in Ponchon-Savarit method also we located the two difference points, that is (( )), one is x D del x D and 1was del x W. So, we had Q dash, leaving steam has Q dash energy, and the leaving steam at the bottom has Q dash. So, all we do at the top, x D will remain the same, Q dash will remain the same; however, this x W or Q double dash which we had in Ponchon-Savarit methods will be now different. So, let us locate in what way now varies that the where does that coordinate lies and how we can calculate this point.

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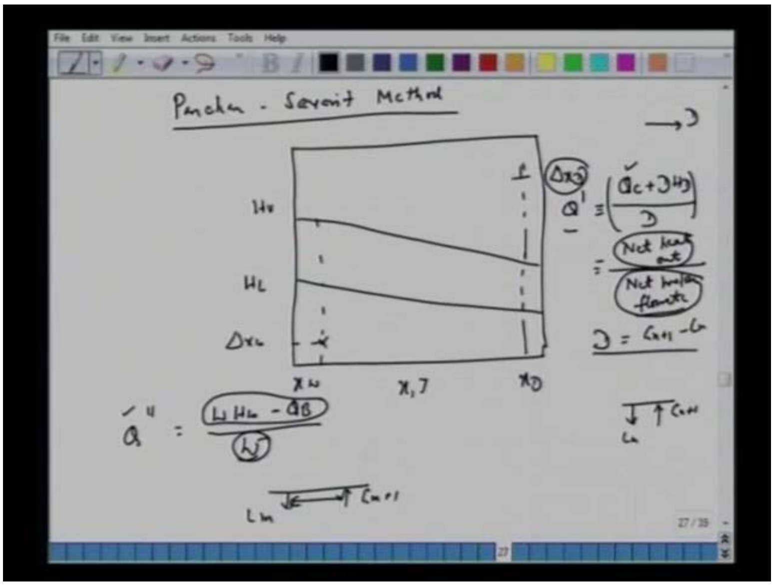

So, when we do this Ponchon-Savarit method. So, what we had earlier when we had the reboiler, we had the two saturated vapor and saturated liquid enthalpy curve; then, we said that corresponding to this x D, say, in case of total reflux, we mark this point del x D which corresponds to the coordinate Q dash, which is Q C condenser load plus DH D.

So, a hypothetical stream, leaving the system with a condenser load and the top product divide by D. So, that was our Q dash, which is nothing but net, we said net heat or energy out over net molar flow rates, net molar flow rates. So, what is this net mode molar flow rate? It is nothing but D, the D equal G n plus 1 minus L n. So, at any plate the vapor flow rate G n plus 1 and the liquid leaving L n, the difference is same as your top product. So, this is net molar flow rate, net heat out, because of course, the stream leaving is D, but we also included this condenser load D Q dash.

So, we mark this point; similarly, you same thing we did each for the bottom product where we had this x W and we said that q dash corresponding to this del x W point equals WH W. So, that is the enthalpy leaving the system plus your minus you have this Q B you are given energy here divide by this W; so, same definitions net heat out divide by a net molar flow rates of W. So, for any bottom trays we have G m plus 1and the liquid leaving is L m. So, the difference is same as W here. So, the definition for Q dash and Q dash the same, the net heat, heat out or net molar flow rates; now what happens in our case when we have this open system.

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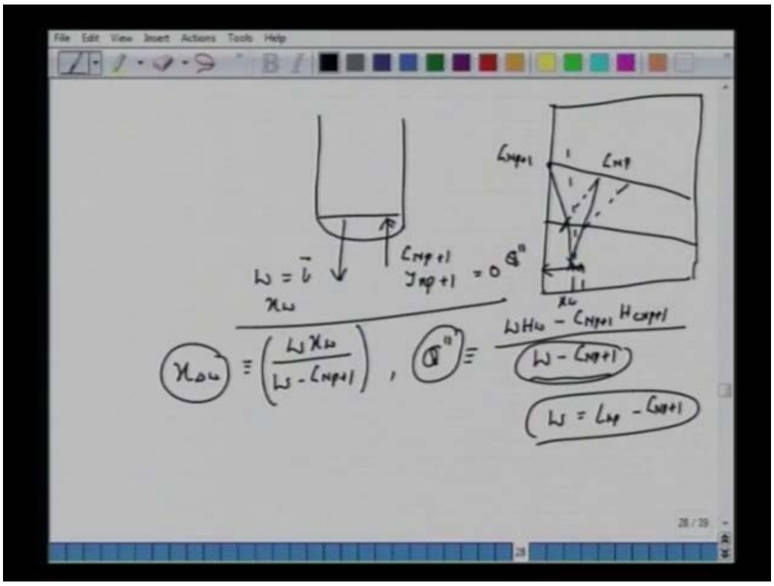

$$x_{\Delta W} \equiv \left(\frac{W x_W}{W - G_{N_P+1}}\right), \quad Q'' \equiv \frac{W H_W - G_{N_P+1} H_{G_{N_P+1}}}{W - G_{N_P+1}}$$

$$W = L_{N_P} - G_{N_P+1}$$

So, we have this bottom plate, from the bottom trays we have W, which is nothing but L bar, composition x W and we have this vapor which is G NP plus 1with a composition y NP plus 1equal to 0, it is a pure steam. So, the top locations in the Ponchon-Savarit method, if you redraw here will not change, what will change here is x W.

So, we have to locate where is this new location of this x W; so what was the definitions for x del W, can whatever is leaving from the bottom product, which is nothing but W x W over the net molar flow rates, which is W minus G NP plus1. So, this equations, when you put this 0, gives you same as what we are before; now the compositions have of this hypothetical stream, which is leaving the system is W x W by W minus G NP plus 1.

So, this is this stream will also have a modified heat energy leaving, it should be WH W; so, this is the energy leaving, heat, leaving the bottom plate minus what we had here is G

NP plus 1 H G NP plus1. So, this would be the enthalpy of this steam - saturated is steam - which goes under in to the column, net molar flow rate is now W minus G NP plus1. So, look at difference, we said that bottom W earlier was G NP plus 1 minus or it was actually, W was difference of L NP minus G NP plus1, but now we have the net molar flow rate as W minus G NP plus 1, because we have this pure steam.

So, all we will do this composition x del W and Q dash now changed here. So, what x W here is here, will not be different we have mark to this new del x W and new Q dash. So, we get a new coordinate here, new difference points, and then, we can do the constructions what we had earlier remember this G NP plus has y equal to 0. So, we locate this G NP plus 1 here; if you connect with this, with this x W, now you will get new coordinate from where now you can say that this is in equilibrium with G NP, and then, we can keep on constructing this and you can draw this diagram.

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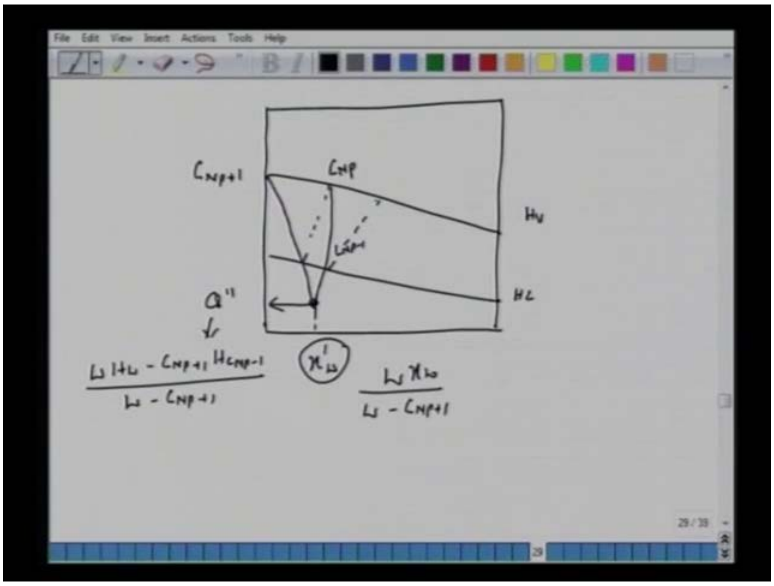

So, let us just redraw this Ponchon-Savarit method graphically; we have H V, we have H L, and we got this new data point, which is Q dash, and we have this x dash new new mole fractions here, right in the we had the calculation for x W as W x W W minus G NP plus 1 net molar flow rates and Q dash is also net total enthalpy leaving minus G NP plus 1 H GNP minus 1 over W minus G NP plus 1. So with this now we can connect, this is your y NP plus 1 equal to 0. So, we have G NP plus 1 on this axis, we connect with this new del point; once we have this, then we can construct our new tie line, which will correspond to G NP, again we connect here with this which is L NP minus 1 and so forth we can construct here.

So, this is the discussion we had for this open system; now, again try to look at this problems slightly different way, that what we are trying say here that in we can still apply the same approach what we have learnt for McCabe-Thiele methods and Ponchon-Savarit methods based on material balance or the energy balance. So, here, now we have to look for the new difference data points; if you apply for Ponchon-Savarit methods, Ponchon-Savarit method top product has not changed, quality has not changed, except at the bottom you have a new difference points. Why? Because now we have we do not have a reboiler; now we have one stream which enters the top, enters your region or the envelope which you have drawn, and the bottom product leaves as L bar, which is same as W. So, you have to slightly modify, you have to make use of same definitions for your del D, x W, Q dash or Q double dash, except you have to ensure that you are putting, inserting a new numbers for your net flow rates. In McCabe-Thiele methods, we also have the same approach, now we wrote on the operating line balance till we realized that the compositions y equal to 0 corresponds to your x W, earlier when we did not have open system, we had the closed system, we had only one product which was leaving the envelope of our calculations and there we had x W equal to y W falling on this 45 degree line from which we connected.

So, again I should go through the text book, and see, if we can solve very simple example here. So that concludes today’s lecture.