Mass Transfer II

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Lecture No. # 21

In the last class, we started our discussion on mccabe-thiele method, and we said that this this method is simpler than the previous one. And we had one very regress assumptions that molar flow rates of liquid, and molar flow rate of vapor, remain constant in that respective sections. And then based on that assumptions, we we have derived the operating (( )) line equations for the stripping sections, and for the rectifying sections. And we said that the two operating curves on y versus x diagram or linear straight line equations they intersect at certain points.

Then we talked of, now we have to make, what do you call a balance or species balance or energy balance on this speed trace. So, we will talk about, what do we call q line? Essentially, we start with again y versus x diagram, we have x D top product compositions, and we have x W bottom product compositions, and we had one operating line for the rectifying sections, straight line having some intercept, and slope. Similarly, we had another equation for stripping sections - a straight line, a slope, and intersect - and intercept. Now, what we will do in today’s lecture? We will obtain the equation of feed line, what do we call q line here.

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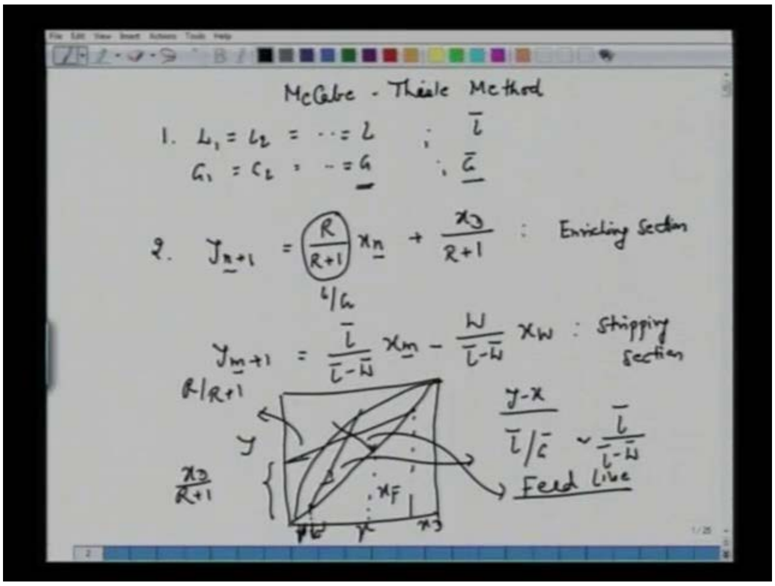

So, with that introduction let us, get into this mccabe-thiele method, and complete our discussions of drawing the operating lines on y versus x diagram. So, we continue with this mccabe-thiele method, the first thing just note down that just we have all L1, L2 same as L, we have all G1, G2 same as G. And similarly, we have L bar, and we have G bar for enriching section, and for stripping sections. And we had operating line equations y n plus 1 as R over R plus 1, where R is the reflux ratio x n, thus x D over R plus1, and we did say that R over R plus 1 one can show as L over G. So, this is the equation of enriching section - operating line enriching section. Similarly, we have straight line equation for stripping sections. Here, we have L bar, and L bar minus w bar x m minus W over L bar minus w bar x W. So, n signifies the number of plates in enriching sections, and m signifies plate number in stripping section, and if you draw these two operating lines on y versus x diagram.

So, we have this 45 degree line, and we have y versus x equilibrium curve even from (( )) etcetera. We mark this x D top product; we mark this x W as a bottom product. Then the equation, of two lines is a straight line here, we can intercept x D over R plus 1, and the slope as R over R plus 1. So, we have this equation here. Similarly, we can write down, we can draw the equation for the stripping line. So, that this also a straight line with the slope here, given as L bar over G bar or L bar over L bar minus W bar, and with some intercepts etcetera, and we said that this line passes through 45 degree line, where y equal to x, and similarly, this line also passes through y equal to x line where we have y equal to x D what we do? We like to do now, that we have this feed composition x. So, we are interested in finding the equation of this line that is called feed line, and feed line is obtained, and if you make a species balance, and we make energy balance at the feed tray.

Let us, do this introduction of feed so, we are looking at this plate at which feed enters. So, just above this plate feed plate, we call it; we have F molar flow rate feed, and it has composition Z F. We have the liquid deriving from the top plates we have H L liquid, and let us call it f minus 1. So, f is it stands for or signifies the number of the feed plate and then we have G, and we have H G f. So, it originates from f s, and we have here L bar. So, notice this change in the molar flow rate, because of this f produced L bar, and we have H L f, and this vapor which arrives as molar flow rate G bar, it has specific enthalpy H G f plus 1.

So, we are trying to make species balance or mass balance on this feed plate. Now, this feed can also have, it could be a mixture of liquid and vapor. So, that is also possible feed could also be saturated feed at the bubble point or this feed could also be saturated you know bubble at the dew. So, more combinations so, several combinations combinations are possible one most general could also be G F separated in feed is separated into vapor. So, some vapor compositions, and we have H G F specific enthalpy for this vapor phase, and it is a mixture of G F plus L F with x F, and we can have H L F. So, several combinations for feed is possible.

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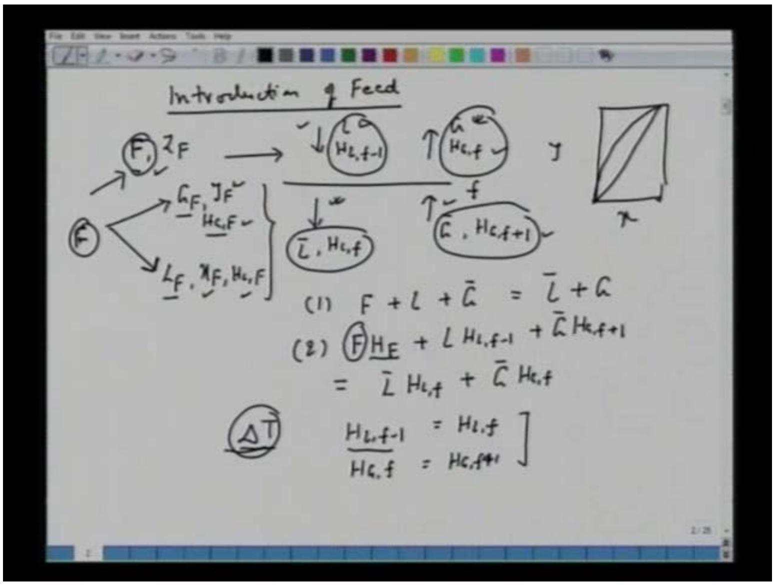

Let us start with this general case where the feed consist of vapor as well as the liquid with the composition y F, x F, H G F, and H L F, as the specifications. So, what we do now is the balance. So, F plus L plus G bar. So, F L G bar arriving at this plate F will be same as L bar plus G the two streams leaving this stage in mccabe-thiele method, we said that energy balance is not required, but this is the only place where one has to make this energy balance enthalpy balance just like we did in case of (( )) methods otherwise our discussion for mccabe-thiele method is almost complete with y versus x diagram this is the only place of where we make this energy balance. So, let us write it F we have H F plus L H L f minus 1 plus G bar H G f plus 1. So, all the energies of incoming streams 1, 2, and 3, we mark will also be same as the n total energy leaving that plate F. So, we have L bar G bar H G. Now of course, we said that F can also be decompose into G f, and L f. So, we will come back to this later let us just work on this total molar flow rates of the feed as f, and this has a total enthalpy specific enthalpy as h f which can also be written in terms of H G F, and H L F, etcetera.

Now, we make one assumption here that since the temperature drop across two conjugative trays is not very large is not very significant means one can assume that specific enthalpy H L f minus 1 specific enthalpy of the liquid arriving from plate F minus 1 is same as H L F specific enthalpy of stream leaving this plate F of course, you must understand the total energy of this feed is different from this what assumption we are making (( )) only thermodynamic, that is specific mole calorie per mole for this tray; and this tray is almost same here, because the temperature drop between the two plates are not, you know very two conjugative plates are not very significant-very small. So, this one is assumption for similarly, H G F same as H G f minus 1 or H G F plus 1 here.

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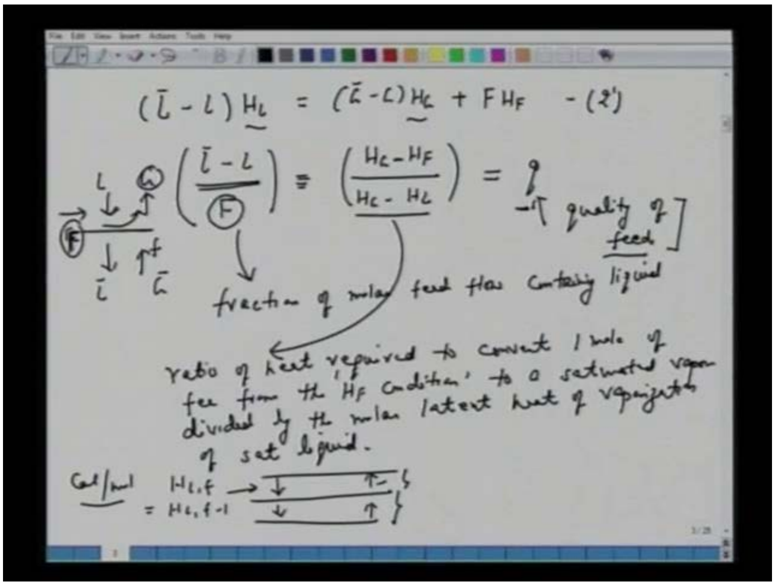

So, this specific enthalpy is the same of course, the enthalpy of this stream total energy of this stream, and total energy of this stream will be different. So, this is thermodynamics assumptions, we are making realizing that the temperature difference between the two conjugated trace is not much. So, with that assumption one can write this equation as L bar minus L H L so, this will equal to G bar minus G H G. So, saturated enthalpy of the liquid for that temperature for the feed tray, and the saturated enthalpy of the vapor-vapor of this saturated vapor for this feed tray temperature is H G plus F H F so, all f minus 1 or f plus 1; we have approximated for this specific enthalpy here. So, the second equation is written like this. So, may be two prime here, and then equation first, and two q bar, we can write as L bar minus L divide same as H G minus H F over H G minus H L.

So, we have these ratios. So, let us look at this feed plate we have F, we have L, we have L bar, we have this G, and we have this G bar. So, all we are writing here L minus L bar over F is shown to be equal. So, this is your molar flow rate the ratios is shown to be equal to the difference between H G minus H F same ratios of H G minus H F same as H G minus H L whatever we have this saturated enthalpy for the liquid phase, and the vapor phase, for this feed tray. So, we have this ratios we call this ratio as a q.

So, this is first time we are using these nomenclatures; we calling it quality of feed. We will see this significance of this essentially, this first term: if you recognize all it means fraction of molar feed flow containing liquid. So, we should be able to recognize that this q will assume different values depending upon quality of the feed. Whether this feed is hundred percent saturated liquid or hundred percent saturated vapor or it consist of some components of liquid, and vapor. So, just the discussion if feed is hundred percent saturated with respect to this temperature here, liquid as a liquid then no component is vaporized here. So, whatever L bar is, we have L plus F q will be 0. Similarly, if entire feed is saturated vapor then entire vapor will get into this we have q equal to 1. So, this q is called quality of this feed, if you go by this definition here H G minus H F over H G minus H L. One can also write that, it is a ratio of heat required to convert one mole of feed from the H F condition from the H F conditions to a saturated vapor.

So, we are saying that how much energy is required to convert feed convert one mole of feed from H F to H G divided by the molar latent heat of vaporization of saturated liquid. So, this is H G minus H L; and of course, you know let us reiterate our assumptions that, we said that the temperature across these two trays is not very significant to the extent that we can make- we can assume that specific enthalpy here of these two streams not the total energy, specific enthalpy of these two streams or the specific enthalpy of these two streams arriving, and leaving they are the same. So, we wrote all those H L F same as H L F minus1. So, this is calorie per mole not the total calorie, because molar flow rates are different here of as the feed is introduced.

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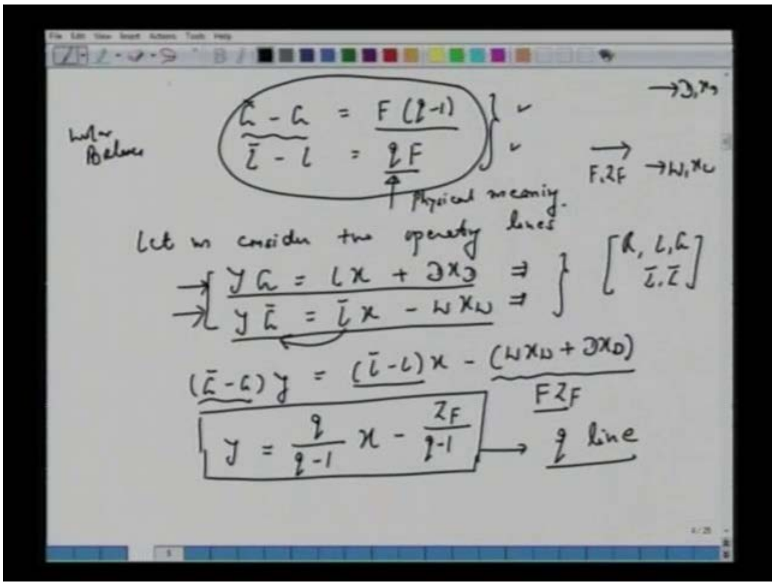

So, there is a difference for definition for G as a consequence now that definitions we can write G bar G as f q minus1, you can also write as L bar minus L as q F. So, it is the same one and the same. So, this is the in these are the equations from which we can give physical meaning of q here quality of heat q assuming 0, q assuming 1 etcetera. We will come back to this bit later; let us, say that now we have an equation which you in terms of q quality of the feed which is given to us. So, we can write down these expressions or these expressions. Let us, consider again two operating lines. So, now we want to draw this q line, we will see, what is a q line?

So, we have the equation of the operating line in two sections y G equal to L x plus D x D. Notice, you know the same as you know all we have done we have written as y equal L by G plus we had D by G. So, we have written different. So, this is the operating line equation for energy sections, and L by G is same as R by R plus 1, etcetera or we have here x D over R plus 1. So, it is different way of writing this. So, this is an operating line equation for enriching sections, and we can also write operating line equation for stripping section, as y G bar equal to L bar minus x minus W x W. So, again here one can write or we earlier we wrote y as L bar by G bar. All we have done here multiplied by G bar, and we have W over G bar. So, different ways of writing the two operating line equations: either in terms of reflux ratios or in terms of L, and G or the L bar, and G bar for stripping sections. So, you should be familiar with you know different ways of writing it.

So, if you have these two equations: now if we subtract you can. So, that G bar minus g into y. So, look at this G bar minus G, this is what this is how we are going to get q line equations G bar minus G same as L bar minus L x minus W x w plus D x D. You should be able to recognize immediately, this W x w plus D x D is nothing, but F Z F. So, we this is what we had F Z F and we have the top product D x D or y D, and here we have w and x w. So, this term is written like this; that means, and G bar (( )) we can write f q minus L bar minus L. We can write as q f, and this term, we are writing as Z F. So, as an exercise just takes a minute to rewrite this equation as y equal t o q by q minus 1 x minus Z F by q minus 1.

So, this is what we call it q line this is q line here. So, what is this q line one can also interpret from this way that this intersection of the two operating line curves. So, we started with the first line this equation for enriching sections; second equation for stripping sections; subtract make use of this balance nothing, but a molar balance for the feed plate G bar minus G will be f q minus 1, L bar minus l is q equal to f q into f if we have saturated liquid q is zero if we have saturated liquid then q is 1 excuse me, all it means L bar will be equal to L bar plus f. So, everything is liquid here. Similarly, if you have saturated vapor then you can say that f equal to 1 etcetera and can entire vapor, which it is converted into G. So, we have this equations for q line which we like to plot on y x diagram.

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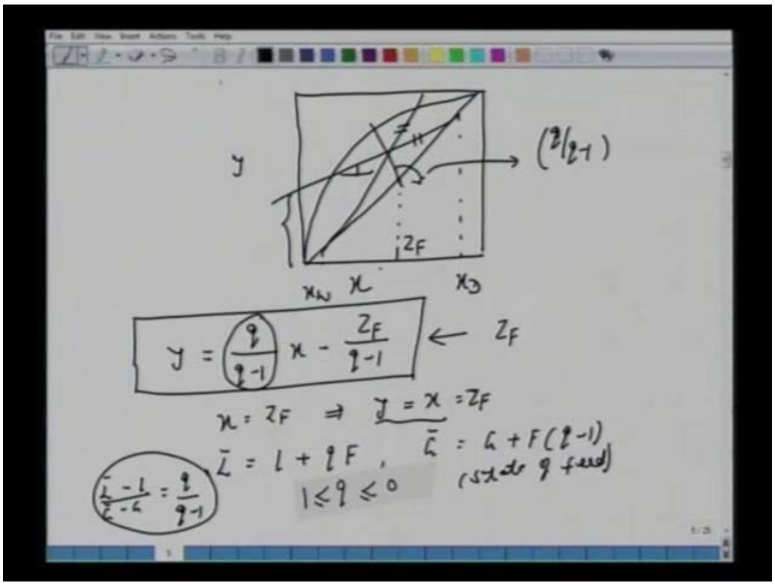

So, we have this y versus x diagram, and we have this forty five degree line, we have this equilibrium curve y versus x, and we start with here as usual x D. We have a straight line for the operating curve with the slope with the intercept etcetera start from x w we take another operating line here. Now, we want to mark this feed line q line. So, go back to these equations which we should write here, again y equal to q over q minus 1 into x minus Z F over q minus 1. And here, like in the previous case, if we substitute x equal to Z F we should able to get y equal to x, all it means y equal to x equal to Z F all it means Z F line coordinate this also satisfy satisfies this operating line; that means, from Z F you draw with this intercept with this slope of q minus 1, q over q minus 1 or intercept of minus Z F q minus 1 should be able to convince that this line will be passing through this.

So, this is your Z F bar, you come to this 45 degree line, and takes this slope of this line as q over q minus1; this line will pass through the intersection of operating curves this, and this, because we obtain this line. So, as an exercise you should be able to draw the graph, and convince yourself, that this q line with this slope will pass through Z F as well as intersection of these two operating lines. So, there is no need to this intercept of Z F q minus1 negative to draw this operating line although this is possible. So, all we have you know we had L bar as L plus q F, q is 0. L bar equal to L everything is vapor phase, and we have G bar as G plus F q minus 1. So, if q is 1. So, everything is G bar equal to G;

that means, which is saturated equilibrium. So, q can take any values 0, and 1. That is, why it is called quality state of the feed? It decides state of feed, and from this. If we divide, then you should be able to get L bar minus L, G bar minus G. So, that this is your q over q minus 1 slope of this q line, what this mean?

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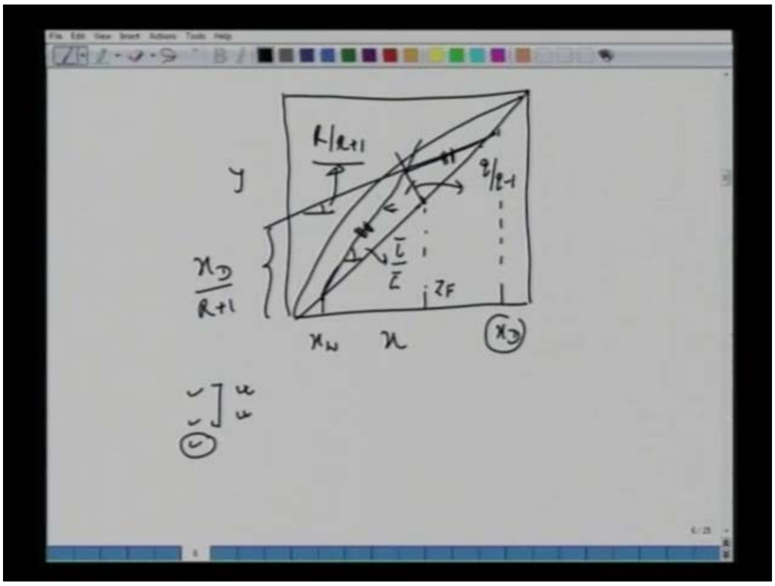

If I redraw this curve equilibrium curve y versus x 45 degree line, and now, what I do as thought with x D take the slope of R by R plus 1 or intercept of x D over R plus 1. And then, I take this now, Z F to draw this q line with the slope of q over q minus 1. We know this q line intersect this 45 degree line where we have Z F equal to y. So, this point you just draw with this slope. This will intersect here, if connect this x W. I have the operating line equation for this stripping sections or it means to tend we have three lines here equation of enriching section equation of operating line for this stripping section, and we have this q line. If we now do you can find the third.

Suppose start with x w draw this slope as L bar over G bar, I know this operating line intersects 45 degree line at x equal to y equal to x W, and then, I connect with this operating line, and from the intersection. If I connect this to 45 degree line at x D should be able to obtain the operating line equations with the intercept of x D over R plus 1 with the slope of R over R plus 1. So, this all expected, because we have three equations: the third is intersection of the operating line for the stripping section, and the enriching section. So, fix any two: you get the third; you fix these two; you get the third; you fix these two, and connect with this ZF you will get this q over q1.

So, let us sum up, what we have done here. So, going back to this again mccabe-thiele method, we work on y versus x diagram; then we have x D and we have this x D. So, from x D, if I draw an operating line, and with this q line. I draw this, you know Z F, I draw this q line wherever it intersects from there. If, I connect with x W, you should be able to convince yourself that this is the operating line equation for your stripping sections; similarly, if you start with the q line intersects; you know draw this operating line for a stripping sections wherever it intersects, you connect to this x y equal to x corresponding to x t should be able to convenience yourself. Now, you have obtained the operating line of this enriching section.

In other words we just three equations knowing two will give you the third; now let us, see what are the values we talked about this q as quality of feed; you can assume a value between 0 and 1. So, you have a saturated liquid at the bubble point, we have saturated vapor at the dew point two extreme cases or it can be a feed which consist of liquid plus vapor. So, what happens, if q assumes different-different values or what happens if the quality of the feed is different or varies from one end of saturated liquid to another end of saturated vapor or in fact, you can have supper heated vapor you can have sub cool liquid below the bubble point and greater than your key point. So, we will like to draw different-different types of q line here for our betterness understanding. So, we will draw again this y versus x curve. So, every time we have this mccabe-thiele methods we have y versus x like in ponchon-savarit methods; we had H x y x type.

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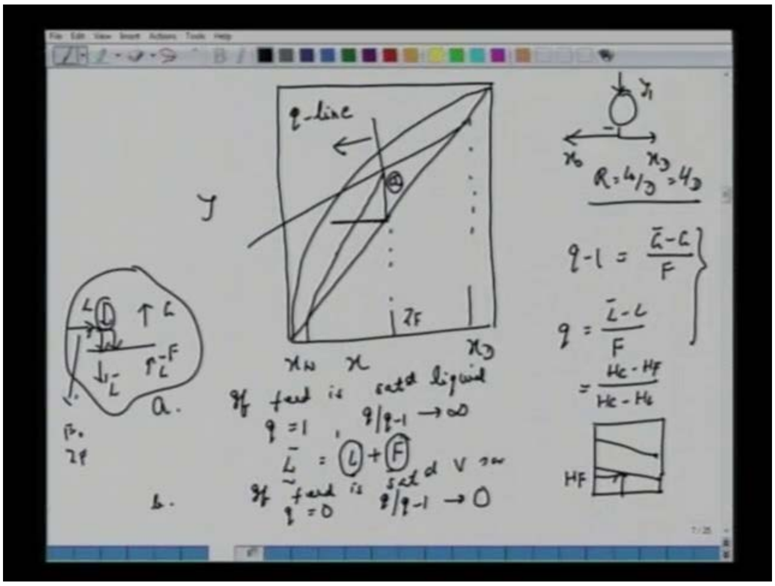

So, we have we have x D here and we have x w. We understand you draw two operating lines any where q line will be fixed. So, what we do here let us assume that we know the reflux ratios from the total condenser let us assume it is the total condenser. So, the vapor phase composition y 1 same as x D same as what goes to the top plate x 0. So, all three are the same total condenser we know the reflux ratios are as L 0 over D or L over D so, with this x D; now, we draw this operating line. Now, let us assume case one if feed is saturated is liquid. So, if the feed is saturated liquid now again you should invoke on the definition for q, which we had earlier q as L bar minus L over F from the material balance, and from your enthalpy balance or energy balance we had H G minus H F over H G minus H L.

So, if the feed is saturated liquid you know it is all at bubble point q is 1. So, we are talking of q as 1, because now we have entire liquid. So, this is the feed plate here, and this feed enters. Here we have the vapor, we have the liquid L bar, we have L we have G, and we have G bar. So, everything is saturated here saturated liquid. Then q is 1 all it means of course, q over q minus 1 this will be infinity, and entire L bar is L plus f. So, L equal to q F plus L bar, q is 1; L bar is L plus x. So, entire liquid was saturated entire feed was saturated. So, whatever L we had liquid flow rate from the top column entire liquid goes into this. So, this is schematic of your first situations, where we are seeing that feed is saturated liquid q is 1, q over q minus 1 is infinity. So, now, let us mark Z F. So, this is a feed composition. So, this is your F this is your Z F talk of this enthalpy H x y diagram straight line we are talking of this liquid which is right here this is your H F. H F is same as your H F.

So, saturated liquid L bar equal to L plus f slope of this equation you know slope of this operating line q line is not infinity q is 1 which means now we have this q line going vertically up. So, this is your q line. And in that case equation of this stripping section line will go will connect like this. So, this is the case one for a saturated liquid. Second case we will expect is all feed is saturated vapor. So, if all the feed is saturated vapor we are saying that q is 0, because similar to this equations we also have q minus 1 as G bar minus G over F. So, keep all this two equations together. So, now we got the equations of q over q minus1 it will divide this. We obtain this line now, the feed is saturated q equal to 0 or it means q over q minus 1 is infinity sorry this q over q minus 1 is 0, which means from here if we draw this line Z F line; then, this line will go horizontally.

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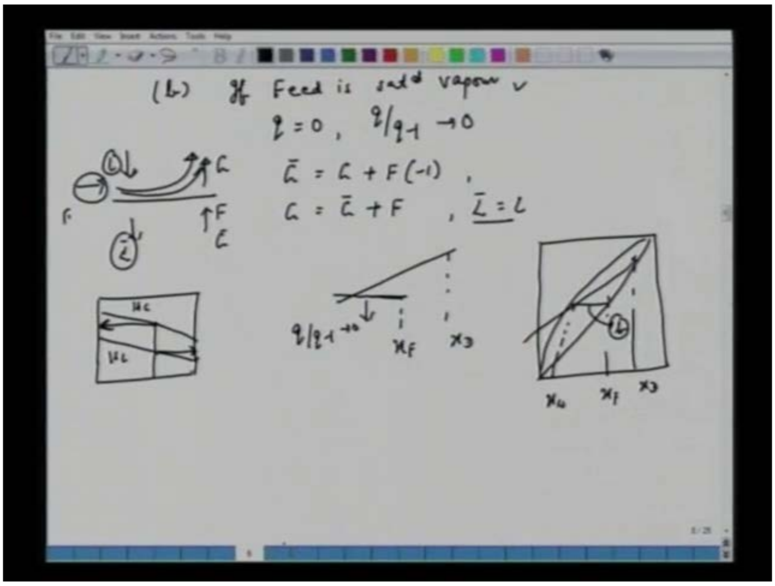

So, let us redraw the case b, where we are saying that, if we have this feed is saturated vapor. So, q is 0, q over q minus 1 is zero which means G bar equal to G plus F, we have minus 1 or we are saying that G equal to G bar plus F. So, let us go back to this feed plate feed is saturated vapor. This is L this is L bar all vapor get's into this vapor phase all feed get's into the vapor phase. So, we have G, and we have this G bar, you can also write that G equal to G bar plus f or L bar equal to L. That is what you will expect? If everything is a vapor here vapor phase saturated vapor.

Then all vapor will get into this vapor phase leaving this feed plate, and the molar flow rate of the liquid will remain the same L equal to L bar. Now, if you draw this on this q line calling all we are saying that we have this operating line is fixed starting form x D, and at x F or Z F, mole compositions. Now, we have horizontal line like this. So, this is your q line q over q minus slope is 0. So, we have horizontal lines or on this y x y diagram. We can draw like this is x D, and if we have this x F. Then we have this horizontal line q over q minus 1, and intersection, if you connect with x w will get this stripping line equations.

So, we have two situations here feed is saturated vapor, and feed, is saturated liquid we can also, and this. If you go back to this H x y diagram, all we are seeing that now the feed is here this is the H G curve, this is your H L curve, and feed is saturated the first case, feed was cold saturated liquid here, and the feed was once again case number d is going the feed is saturated vapor.

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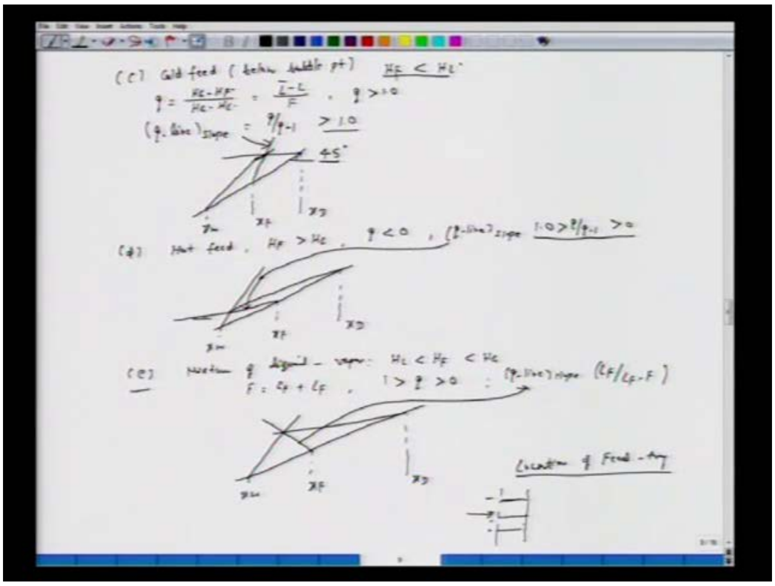

Let us, take second case c call it cold feed. So, what we are discussing here is feed, which is whose temperature is below bubble point or we can say that H F is less than H L. So, going back to the definition of definition for q we have q equals H G minus H F over H G minus H L which is from material point material balance L bar minus L over f q is greater than 1.0, because H F is smaller than H L. So, that means, the slope of this q line which is q over q minus 1 will be greater than 1.0; So, if we plot let us say this is forty five degree lines, this is x D, this is x f, and we have x w. This operating line is slope of the operating line is greater than 1.

So, it will be something like this, we have one operating line, and the second operating line which will intersect like this. So, this is your q line, this is you can write this is your forty five degree line on your graph. The second case or the d case will be hot feed. So, what we are trying to say here that H F is greater than H G. So, it is a vapor phase here which means going by the same definition q is less than 0, and the slope of this q line will be q over q minus 1 this will be greater than zero you can see yourself, and but less than 1.0.

So, in other word in this case if we plot if we take again this forty five degree line let say this is again x D, this is x F, and here we have x w. Now, the slope is greater in zero positive, but it is less than 1, this q line will have a slope something like this alright with operating line connected to this, and this. So, this is your q line this is a q line here with the slope q by q minus 1. The last case could be mixture of liquid, and vapor. So, essentially we have H F less than H G, but it is greater than H L. So, we have mixture of both vapor, and liquid feed, consist of we can say G F vapor, and the liquid. In this case q is greater than 0, and is less than 1. And one can show that, this q lines slope will be nothing, but L F amount of liquid in the feed over F minus F in this case the forty five degree line is like this x D, we have x F, we have LF, this x w, and if we draw this line this operating line is something like this with the operating q line is like this with the operating line like this. So, this is a q line. So, these are six cases a to e; we have discussed in this lecture, what we should discuss? Next is the location of feed tray. In other word, what locations what trays which the feed should be enter. So, we have the trays, like this at what location the feed should enter this distillation column.

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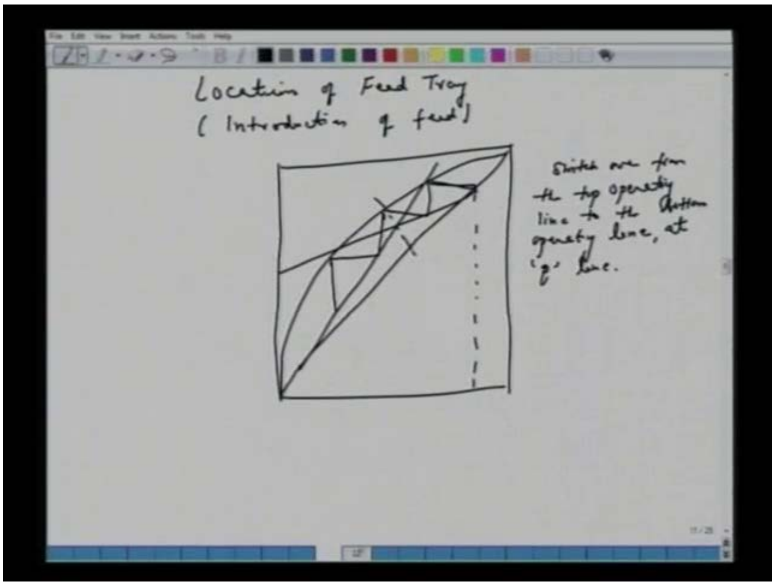

So, we have this location of feed tray. So, essentially we are looking for introduction of feed. So, in this case let us take one of the conditions one of the situation where we have let us say this is y x diagram, and this is the two operating line, and this is the q line. So, to draw this stage again we start from x D the top distillate composition. So, we go to this equilibrium curve we come down to this operating line top plate of the top column then we cross to this. So, once we cross this q line, then we are supposed to switch over to this second operating line the operating line for this stripping sections. So, essentially here we switch over from we switch over from the top operating line to the bottom operating line which is operating line of rectifying section stripping sections. Once, we go at you know q line. So, once we cross this q line then we are supposed to switch over switched on to this second operating line.

So, and the second case would be we can delay the feed entry. So, we are trying to say here that we have this equilibrium curve like this let say we have this operating line for the top column, and we have the operating line for the second column, then start drawing this tray go to this operating line, and this is a q line. So, we are trying to say here that, we can still we can still continue marching on the top operating line, and then we can come down to this second operating line. So, what we are trying to say here that in this case we have this late entry if you like to expand this here. So, we have the two operating line we have this q line here, and if you start in from say x D come to the equilibrium curve. Then even, if you have gone past this q line one can keep on marching on this.

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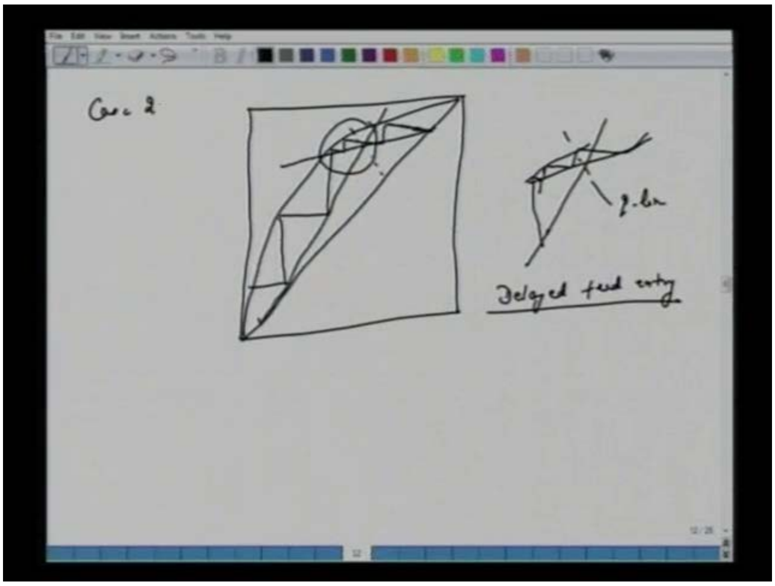

So, this is equilibrium curve till after sometime you know you can reach almost the point of pinch, and then you can come down to this second operating line. So, here we have delayed feed entry. So, it is the case two: where we have delayed feed entry, and then we can have-also have, what we call early entry. So, if you again draw this y versus x diagram. So, let say you have y x diagram like this start from x D.

So, x D like this start from you know x W; you have second operating line like this is the q line, there is a possibility that you can have early entry, which means we go like this come to the first one come to the second, but now before the q line one can switch down to this second operating line. So, even if you have you could have delayed here say at stage one, stage two, and we could have delayed to the third stage, but now we can switch to the second operating line even before this q line. So, here we have this early entry.

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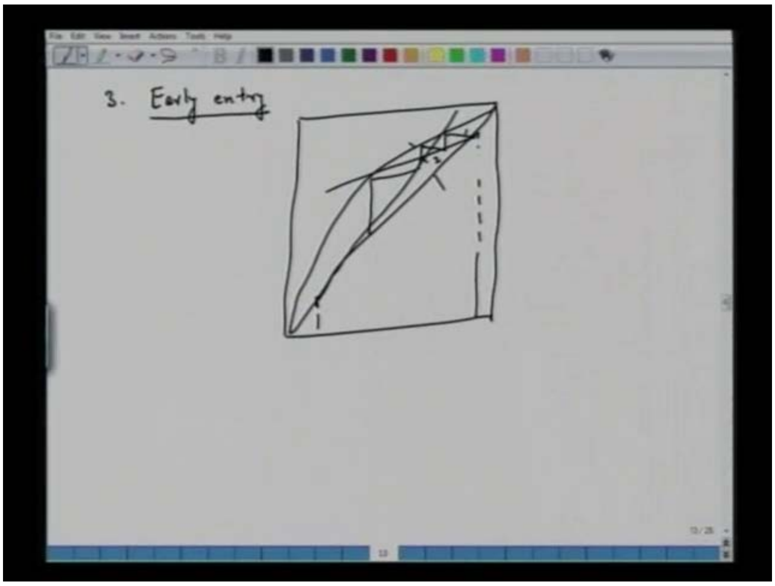

So, let us make a point here that the three ways; suppose, you have a new discussion column which you want to design here. Then, there are three possibilities of locating this feed at what feed tray we should have the entry. In one case even before this q line one can switch down to the operating line number two for this stripping column, in one case we can have the early entry. So, that is the where you know we have the q line start from drawing the trays stages we have the options that before q line or after q line in some cases. It is possible that you switch down to this operating line for this stripping sections.

But the most optimal number of or minimum number of plates one can show that you will get it where you have the opportunity the first opportunity to cross this q line. So, all it means that even if you have not violated this enthalpy balance or this species balance or the total material balance; there is a flexibility of adding feed say between number plate three to 5 in 1 case one can switch to the second operating line at the plate number third. In one case you have the options for switching at number five. In one case, you have the options at q line; you know just going pass the q line at plate number 4. So, one has a luxury of entering the feed between 3 to5. So, as we said earlier that mostly you know for a new discussion column; one would like to have a minimum number of plates. So, in that case it happens that one should try to switch down from operating line for the rectifying section to the stripping sections at the first opportunity, where it goes past this q line.

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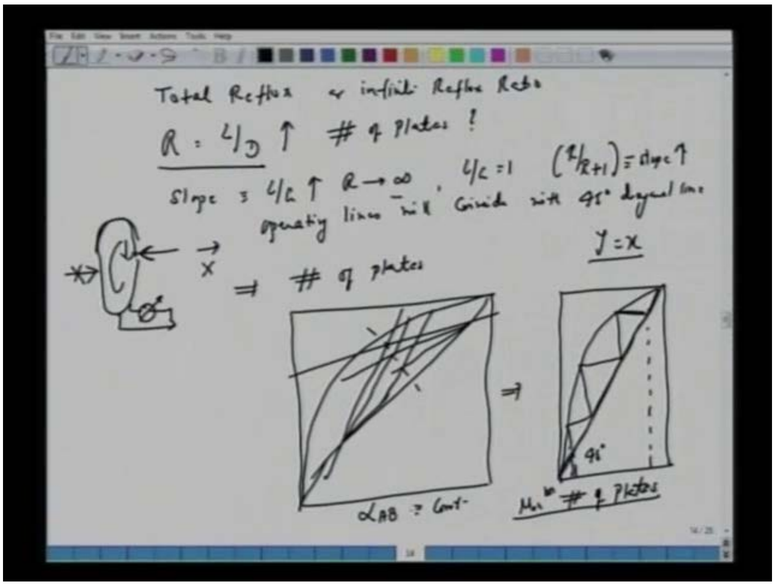

So, these are the three different ways of entries: now, what we do here just like in the case of (( )) we try to find out the minimum number of plates corresponding to the total reflux or the infinite you know reflux ratios, and the second case when we had this minimum reflux ratios when we have the finite number of plates and infinite number of this condenser load or the boiler load. So, these are the two different situations we will address to so, what we are trying to say here is condition one then we have this total reflux or what we call infinite, finite reflux ratio all we are trying to say that if a reflux ratio is defined as R equal to L by D.

We will like to monitor the slope of the curve with increasing L by D, and see what happens to the number of plates here. So, the slope of the curve is L by G all it means that when we take the limit R tends to infinity, we have L by G equal to 1. So, you should be able to show that that the reflux ratio is nothing, but slope of the curve is nothing, but R over R plus 1. So, when you take the limit R tends to infinity the slope becomes one all it means the operating line will coincide will coincide with forty five degree line diagonal line.

So, physically all we are trying to say here that when we have this discussion column, and we keep on increasing these reflux ratios relates you know whatever we have infinite reflux ratios. So, there is no top product withdrawn here in that case we have very large amount of liquid flowing through this even at the bottom product entire vapor will has to be sent back to this discussion column to the reboiler. So, essentially this feed has to be switched off, and we have this liquid entire reflux circulating with in this discussion column of course. It is a hypothetical case, but you know these conditions will give us minimum number of plates. So, the idea is that we should see geometrically or graphically what happens to the locus of intersection of the two operating lines or the slope of the two operating lines. So, if we have this y versus x curve, and we have two intersection of this operating line; then, if you increase the slope. So, when we are saying that reflux ratios is R over R plus 1, and we increase the reflux ratios.

So, the slope increases. So, R equal to R plus 1 is a slope or L over G slope increases. So, which means this is the q line takes the q line. You will see that, now we have-we are tracing the curve like this slowly, and slowly. Slope is increasing the two operating lines they approach forty five degree lines till at infinite reflux ratios when R is infinity we have R over R plus 1 reaching one this equilibrium curve here two operating lines are coincided this forty five degree. So, in this case construction of number of stages all we have operating line you do the same thing what we did earlier from starting from x D go to y 1 write path you know x 1, y 2 ,etcetera since you have the operating line coinciding with this you get minimum number of plates minimum number of plates. And in this case since operating line is now y equal to x; one can obtain analytical solutions for minimum number of plates, if we have certain assumptions like relative volatility of a over b is constant.

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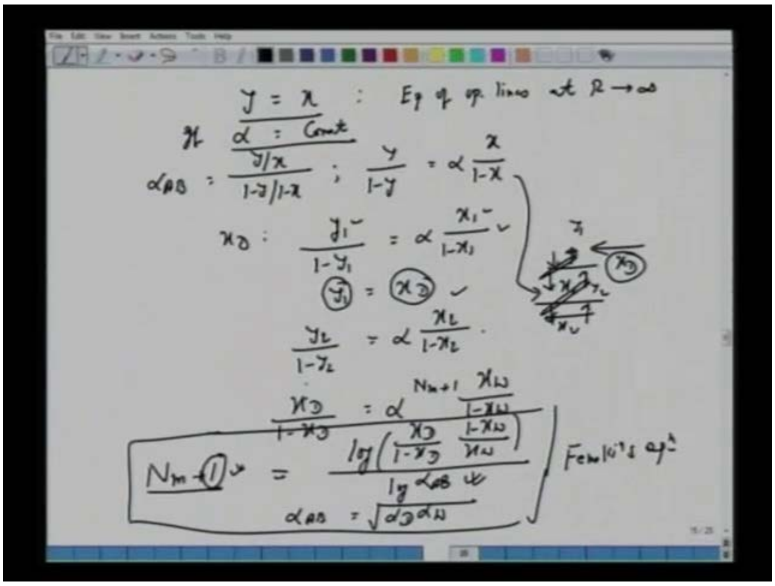

So, in this case one can do some calculation here. So, what we want to do here is that we y equal to x. So, that is equation of operating line equation of the operating line at infinite reflux ratios, and if we assume relative volatility alpha is constant, and recall the definitions for alpha A B as y over x over 1 minus y over 1 minus x. So, we can write y over 1 minus y as alpha x over 1 minus x, and starting from x D one can start substituting you know equilibrium curve, and operating line alpha. So, we have x 1 over 1 minus x 1 where we are obtaining this y 1 from x D. So, starting from x D we have y 1 that is the operating line, and once y 1 is in equilibrium with x 1. So, we have the situation that this is x D, and the top plate we have y one and we have this x 1.

So, by making use of operating line and then this equilibrium curve we can have set of equations like y 2 over 1 minus y 2 alpha x 2 over 1 minus x 2. Similarly, I can write here x D over 1 minus x D equal to alpha keep on substituting one equation to second equations to have N m plus 1 x w over 1 minus x W. Since, we can obtain analytically the expression for N m plus 1. So, one signifies here the reboiler which we have at the bottom column. So, N m plus 1 minimum number of plates at infinite reflux ratios where the relative volatility is constant one can obtain an analytical expressions. Log x D over 1 minus x D here one minus x w over x w over log alpha A beta, and for beta alpha A B generally taken as a geometric ratios of top product at the alpha for and for the bottom product, and this equation is well known, and known as well-known as Fenske's equation.

So, as an exercise you can try this start from your top column you know x D then y 1 x 1, then you have y 2, x 2 in equilibrium. So, every time you write down this equilibrium curve equations for this and for the operating line; you have every time you write in this y equal to x. So, with these two equations successive substitutions one can obtain an expression for minimum number of plates including this reboiler as the ratios of known quantities like top product, top composition, bottom composition, and the average the relative volatility.

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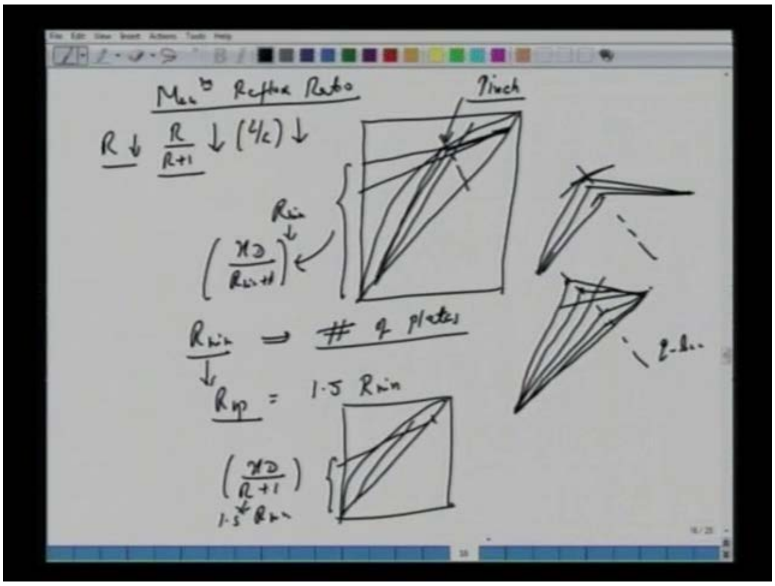

So, this is, what we have for the total reflux similarly we have the second case when we have this minimum reflux ratio. So, now, we will do the same thing when the reflux ratio decreases then the ratio also the slope of the operating line also decreases L over G decreases. So, here again we will do the same thing from y over x diagram, we will monitor a slope of two curves. So, the general case would be when we have the section like it is equilibrium curve. Now, when you increase, decrease that reflux ratio the slope decreases or L over G decreases we can make out that now the point will move upward. So, this is try to extend here and large then you have this q line, and two operating line intersects, here at minimum reflux ratios or as the reflux ratio decreases then slope will decrease. And this point will move upward till it makes pinch with the equilibrium curve.

So, if you take this line here line this here we have this pinch, where the driving force is zero. So, corresponding to this if you extend this will give you minimum reflux ratio. So, you have essentially slope of this curve R min, it comes from intercept from the intercept you get x D over R min plus1. So, this is the intercept from this one can calculate minimum reflux ratios. So, at these minimum reflux ratios, you have infinite number of plates. So, here the number of finding this minimum reflux ratio the maximum reflux ratio is the same you start with fix the q line you know the composition of the two curves. So, we have n say general operating curve like this. In one case you must have noticed that, slope increases like this till, it makes a coincides with this 45 degree. In one case, it is moving upward like this till it makes a pinch here.

So, that is argument for minimum reflux ratios and the maximum reflux ratios from this R min generally one can define a number of plates, but the operating reflux ratio is generally typically it is 1.5 times R minimum. That means, again you choose this the ratios R p equal to 1.5 times, and one can reconstruct the equilibrium curve. So, now we have this intercept as R plus1 where this R is 1.5 times the R minimum corresponding to this pinch curve. In this case there is one small thing to discuss here that we can also have a some non ideal situations in which y versus x diagram may not be necessarily concave down ward all the time; which means there could be you know equilibrium curve like this.

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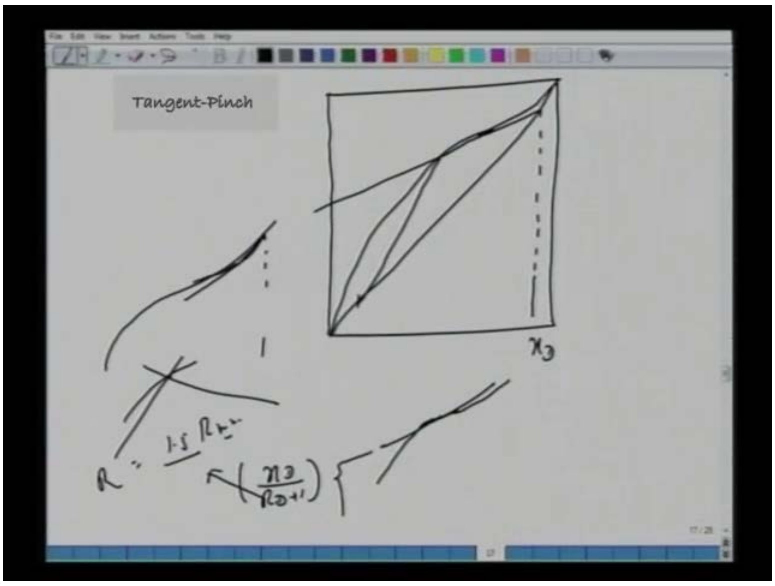

In this case, if we start from x D looking for this minimum reflux ratios, it is possible that this operating curve can make it tangent much ahead of this pinch here. So, on this in this case, just like similar to this what absorption curve, we have one has to draw this pinch here make a tangent wherever it makes a pinch with the operating curve. So, if we have a slope like this and you are starting from here. Then look for the tangent with this equilibrium curve; unlike in the previous case, we said that let us make an intersection of the two operating lines at this equilibrium curve. So, in this case you know one has to see that where this operating line it makes a tangent with tangent with this equilibrium curve. So, that is the point where once should take this R minimum. So, if you take intercept this will give us X D over R D plus 1. So, from here again we take 1.5 times, etcetera R operating equal to R minimum. So, very obtain when we have this pinch. Then one has to be a bit careful, and look for this shape of the curve, where the operating line can make a tangent. This is very similar to what we did in case of absorptions.

So, today's lecture we will like to summarize that now essentially starting from q line we can q line, and one of the operating lines. We can draw the second operating lines or we can take the two operating lines, and can draw this q line. So, out of we have three operating lines equations; one is a q line which came from the… or which we obtain from the energy balance. And we have the two operating lines which we obtained from species balance knowing the two, the third can be found then we can do this construction of the trays.

And we said that as a general practice the feed entry is always or very often it is at that locations where we have the first opportunity to step down from the operating line in for the enriching section to the operating line for the stripping sections just past this q line. Otherwise, there is always an attitude there is a luxury where you can have an early entry or when you can have late entry, but that gives you larger number of plates; then what we will get in case of optimal line. So, then that is the one thing then we had a two general scenarios of infinite reflux ratios, and we had the second one; where we had the minimum reflux ratios which requires infinite number of liquids. And in this case, all we have to do is to monitor the slope of the two operating lines as the reflux ratios increases or decreases.

So, when the reflux ratio increases we saw that these two lines: operating lines coincide with this forty five degree line, that time you get infinite number in minimum number of plates, and if the constant. If we have the mixture you can assume that, you have a constant relative volatility one can get analytical expressions for minimum number of plates we call it Fenske's equations. So, that equation you should be able to try on your own as an exercise all you have to do is that operating line is not simplified y equal to x. And then you have equilibrium curve in terms of alpha relative volatility between y, and x. So, you can have successive substitutions use operating line then equilibrium curve then operating line equilibrium curve; one can obtain these Fenske equations. In case of minimum reflux ratios, we try to decrease the slope in which case the two operating lines makes a pinch or intercept at the equilibrium curve, where the driving force is 0. Then we can obtain, we can see that we will get infinite number of plates. So, there we get intercepts of this operating line or the slope also that will give you R min generally distillation column we have 1.5 times very similar to what we are in case of absorption column.

So, now we get the new reflux ratio based on the new reflux ratios, we have the new operating lines, and then we can find a number of trays number of plates. We also said that, when very you know peculiar situations, where we have these isotopes or we have non-ideal solutions. Then one has to watch out for the operating line making a tangent with this equilibrium curve at some point other than, where we have this intersection of two operating lines at the equilibrium curve. In that case, we get this R minimum at slightly smaller reflux ratios. So, from that we get R min again we take 1.5 times we construct the y versus x diagram with the equilibrium curve, and the appropriate operating line find the number of plates some stages. So, this today it is the end of this lecture. Next time, when we meet we will have one example based one mccabe-thiele method. Thank you.