Mass Transfer II

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Module No. # 01
Lecture No. # 20

So, in the previous lecture, we had discussion on Panchan-Sevarit method, so the method based on enthalpy balance and very rigorous balance. And then then based on those that method, we had couple of examples, where we calculated the number of plates, condenser load, boiler load etcetera.

Today lecture, we take up a new method known as McCabe and Thiele method alright; this method is actually the approximations, so you have certain assumptions made in this method, but it turns out to be that this method is more popular; more popular in the sense that most of the industrial discussion column can be designed or we can do the calculations for condenser load or the re-boiler load or the number of plates etcetera, quite with a good reasonably accuracies. In other words, if you apply Panchan-Sevarit methods and we get say a number of plates say, 20 or 30, then this method will give a very close results say, 31, 29 etcetera. So, although there is a there is an approximation in this method, but it turns out to be that for most of the purpose for most of the purposes this method is a reasonable, good working one.

So, before we get into this mathematical prospect of it, let us try to understand that what assumptions we have in this method. So, if we recall, we made energy balance or enthalpy balance, we made a species balance and we made a total molar balance; we took a sections in enriching sections, we took a section in the stripping section and we took we made this balance over that envelope. Now if you recall in this case or in the previous case, the liquid flow rate it changes right, so from one plate to other another plate, the molar flow rate of the liquid will change.

Similarly, vapour which goes flows from one plate to another plate, the molar flow rate of the vapour will also vary. What assumption we make here is what we call equimolar overflow and vaporizations; that means, we assume that l, the molar flow rate in the enriching section and L bar molar flow rate in this stripping sections remains the constant.

So, L 1 to L 2 to L 3, L 4 in one section; and L 1, L 2, L 3 or L bar 1, 2, 3 etcetera in the stripping section, they are constant. Similarly, the vapour phase, vapour molar flow rates in both the sections are in constant.

So, physically what happens here? If you have five moles of vapor reaching a certain plate, then one moles suppose condenses to the liquid phase, then another one mole gets vaporized to this vapor phase; so, that is the reason. So, whatever we have del x change in the molar flow rates of the vapor, because of the condensation, its same as the molar flow rate of the liquid cause by vaporizations. So, that that results in this assumption that L in one section or L bar in the section or G in section and G bar in another section, remain constant.

So, if you make these assumptions, based on certain you know thermodynamic calculations or thermodynamic assumptions, then we will see that there is a much easier method to calculate the number of plates or to do the calculations for condenser as well as a re-boiler. So, as I said earlier, although the sounds a bit rigorous assumptions thermodynamically turns out to be that it is not be far from the reality - one; and the second, it results in a very considerable simplifications, calculations; and third, the results which we get approximated results per number of plates, it is not very far from what we get from this Panchan-Sevarit methods

So, in other words, if you count for certain efficiencies in the column etcetera, then it is possible that we get the two results very close. So, we look at this McCabe-Thiele method in this lecture; and see where we are making these assumptions, and how does this assumption result to simplifications.

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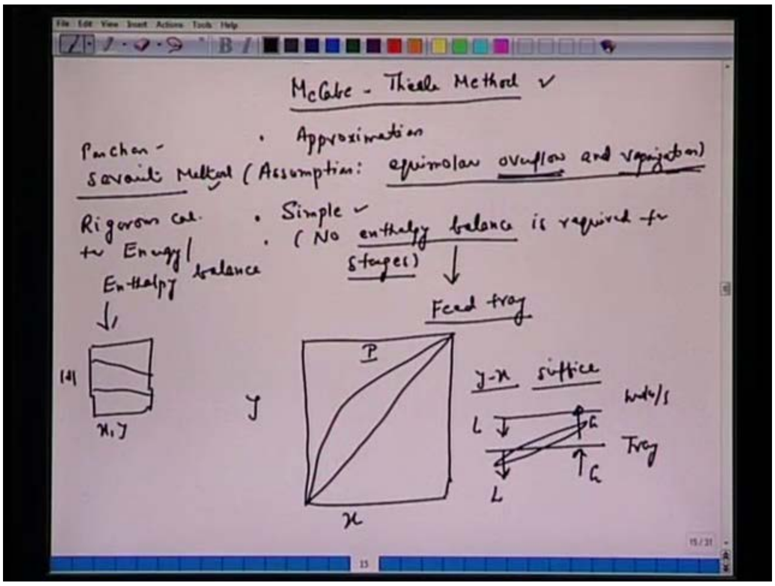

McCabe - Thiele Method ✓

- Approximation

Panchan - Savarit Method (Assumption: equimolar overflow and vaporization)

Rigorous cal. for Energy/Enthalpy balance

- Simple ✓
- (No enthalpy balance is required for stages)

Feed tray

So, let us start with this McCabe-Thiele method. So, we make a note that of course, this method has an approximation. So, what is the approximation in this method? We assume or there is an assumption that assumption of equimolar overflow and vaporization. So, we will discuss about this. We are assuming that liquid flow rate essentially overflows; so from one tray to another tray, the molar flow rate remains constant.

Similarly, when we say equimolar vaporizations, then the vapor flow rate equimolar flow rate of the vapor from one tray to another tray is a constant here. So, we will continue with the discussion later. This method is simple; simpler than the previous method which we had panchan known as sevarit. If you recall this method, this method has a very rigorous calculation for energy balance or we had this enthalpy balance.

In this case, no such you know, enthalpy balance is required on required for a stage or for stages. So, you can see the method this quite simple here, and we do not require any enthalpy balance alright; of course, we have some minor calculations for heat tray. So, any balance will we will do, it is only for the heat tray. So, you can see that this method has a although very rigorous assumption as it reads here, but it results in a very considerable you know, simplicity. So, essentially all the time or most of the times, you would be referring to this y verses x equilibrium diagram. So, this is the foundation and the basis of your McCabe-Thiele methods. Unlike in the previous case Panchan-Sevarit method, we focus considerably almost of the times on H x, y diagram, where we had this

H G and H L curve. But in this case, in this y verses x diagram for at certain temperature or for a certain pressure, whole square; so, we have this… This will suffice.

So, of course, in this assumption McCabe-Thiele method, we have the similar story that on a tray, the vapour leaving the tray and the liquid leaving the tray, they are in equilibrium. So, this assumption of the two streams in equilibrium still holds good, it is a McCabe-Thiele method or it is a Panchan-Sevarit method. Except here, we make certain assumptions that this flow rate L is same as what we have getting here. Similarly, what vapour phase, we have G here - molar flow rate, moles per second is same as same as a tray underneath this right. So, what is the basis of this assumptions, how does it evolve, from where do we get? Let us focus here.

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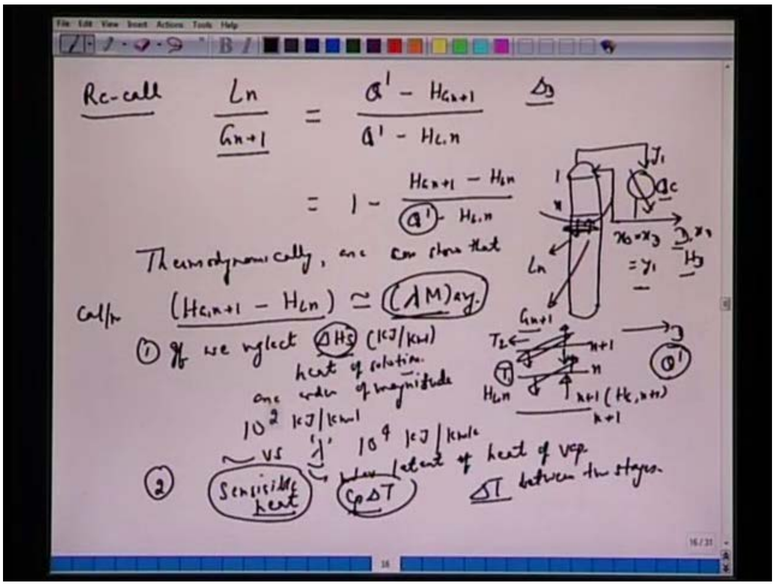

Re-call $\frac{L_n}{G_{n+1}} = \frac{Q' - H_{G,n+1}}{Q' - H_{L,n}}$ Δ_D

$= 1 - \frac{H_{G,n+1} - H_{L,n}}{\boxed{Q'} - H_{L,n}}$

Thermodynamically, one can show that

Call/r $(H_{G,n+1} - H_{L,n}) \simeq \boxed{(\lambda M)_{av.}}$

① If we neglect $\boxed{\Delta H_S}$ (kJ/kmol) heat of solution

one order of magnitude

10^2 kJ/kmol vs 'λ' 10^4 kJ/kmole latent of heat of vap.

② $\boxed{\text{Sensible heat}}$ $\boxed{C_p \Delta T}$ ΔT between the stages.

If you recall in Panchan-Sevarit method, we had and an expressions for L n over G n plus 1. So, the liquid which is leaving the tray n and the vapor which is arriving from n plus 1 or reaching this nth plate, the ratio of that two mole of flow rate, you wrote in terms of Q dash enthalpy of the vapor phase n plus 1 and we had Q prime over H L n alright, which we can also write as 1 minus H G n plus 1 minus H L n over Q prime minus H L n.

So, let us draw its schematic of section column, and let us try to recall that we had the situations, vapor coming here, we had condensation let us say assume total condensation, we withdraw the product D, and the liquid goes back to this column as a reflux. So, we had this x 0 same as x D alright and we have the same as y 1. So, we have the total difference; and we said that this q prime is nothing but the enthalpy of this leaving the stream, taking into account for this Q c. So, it is hypothetical stream with the molar flow rate of D carrying a energy Q dash. So with that, we made this, we came out with a expression, we said that we choose any plate here n, then L n, and we have this G n plus 1. The difference between the two molar flow rate is same as d and the difference between the energy of the two streams L n and G n plus 1 remains same as Q dash. So, we had this differential point given as del D. So, this is true with any envelope, which we choose around the plate number n counting from the top. So, we came these expressions and we can write like this.

Now, what happens the thermodynamically, thermodynamically one can show that H G n plus 1, so the enthalpy of the vapor arriving from plate number n plus 1. So, this is what we have here H G n plus 1, so the plate n and we have plate n plus 1; H G n plus 1 minus H L n, the enthalpy or a specific enthalpy moles per calorie per mole of the vapor of the liquid leaving this plate L n, which is H L n is constant. One can show that this number is approximately constant at as lambda m average. So, these assumptions (( )) we make here in this McCabe-Thiele method now every plates which we have in a stripping section or in the rectifying section, now we have the stream leaving vapor and the liquid in the equilibrium. And the difference between the total energy of these two streams is same as Q dash.

What we are saying here is that specific enthalpy H G n plus 1; see the temperature varies from T 1 to say T 2 or T n plus 1 to T n alright, and this vapour and the liquid which are leaving they are saturated with respect to this temperature. Similarly, this vapor and this liquid which are leaving the plate number n gets saturated with the temperature T 1. So, although there is the temperature change, but it does not change much you know a lot. As a consequence, the difference between the specific molar enthalpy of the two streams, one the arriving here and one leaving here is approximately can be shown thermodynamically equivalent to the latent heat of vaporizations multiplied by the molecular weight average.

So, although the molecular weight changes, why because the species mole fraction changes along the column along the trace, but the product of this lambda and m approximately they remain constant. Why this happens if we thermodynamically, if we neglect, if you recall from previous calculations, we had the contribution from heat of solutions right if you neglect heat of solutions, which is actually true in the sense that kg per mole per kilo mole of this quantity numerically, what we call heat of solution is actually much smaller. So, it is a one order of magnitude. So, if you look at any handbook or thermodynamic table, you will see that this del H s heat of solutions is one order of magnitude for example, 10 to power 2 kilo joule per kilo mole in comparison, if you compare this with a lambda latent heat of vaporizations.

So, although latent heat of vaporization will change with the temperature or with the compositions etcetera the more or less this quantity is very large 10 to power 4 kilo joule per kilo mole, so this molar latent heat of latent heat of vaporization. So, first thing is that this heat of solutions one is much smaller than the latent heat of vaporizations; and number two on the same note sensible heat, sensible heat or specific heat, due to the specific heat you know C p del T etcetera this is also very small. If you neglect temperature drop del T between two stages, two conjugative stages, we can a neglect these two quantities in calculations of this H G n plus 1 minus H f to show that we have a quantity lambda m, which is can be considered to be more or less constant. So, with these assumptions of H G n plus 1 minus H L n being constant, we can see here that the ratios of L n over G n plus 1 can be assumed to be constant alright.

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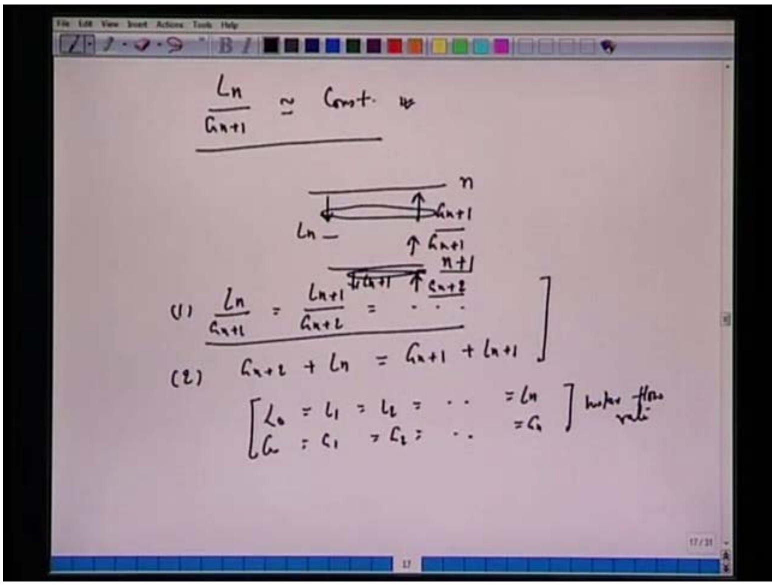

$$\frac{L_n}{G_{n+1}} \simeq \text{Const.}$$

(1) $\frac{L_n}{G_{n+1}} = \frac{L_{n+1}}{G_{n+2}} = \cdots$

(2) $G_{n+2} + L_n = G_{n+1} + L_{n+1}$

$$\left[\begin{matrix} L_0 = L_1 = L_2 = \cdots = L_n \\ G_0 = G_1 = G_2 = \cdots = G_n \end{matrix}\right]$$ [illegible]

So, what we have here? We are saying that the ratios of L n and G n plus 1 it is more or less constant alright; that means, if you look at one stage here n, and we have this molar flow rate of vapor G n plus 1, and the liquid which is leaving this plate L n, the ratios of these two L n and G n plus 1 is a constant ratios, this comes out from energy balance in and in that energy balance. We said that the difference between H G L n plus 1 and H L n specific enthalpy or energy here is a constant. From this, we come to this conclusion that the ratio is a constant.

Now, if you recall we can move, we can make a molar balance between one plate and another plate say n plus 1 here; if you do this we can also write that L n G n plus 1 it means the same as L n n plus 1 over G n plus 2 etcetera. So, this is G n plus this is G n plus 1 here, so we have the tray the steam which arrives here is G n plus 2, then we have this L n plus 1, so this ratio is also constant. In other words we have these conditions here L, the ratios of liquid and vapor is constant - number one; number two if you look at any plate here we have G n plus 2 plus L n. So, if you look at this plate here n plus 1, molar flow rate of vapor is G n plus 2 and the liquid which arrives here is L n. So, the same as the molar flow rate of leaving streams, which is G n plus 1 plus L n plus 1. With this one and two, one can show that L 0 will be same as L 1 will be same as L 2 and G - will be same as G 1 will be same as G 2 will be same as G m. So, this is what is McCabe-Thiele method (( )) molar flow rate is a constant.

Let us try to understand what we saying here. So, what we started with Panchan-Sevarit method alright; we said that energy balance, enthalpy balance holds good the way we had. The difference between the two streams reaching or leaving the plate same as Q dash that is ok. Then we said that we came out with the expressions with L n and G n plus 1. So, the molar flow rate of the liquid leaving the plate L n and the molar flow rate of the vapour, which arrives at the plate n from n plus 1. We took this ratios in terms, we express this ratio in terms of H G and H G n plus 1 alright H L, H L n and H G n plus 1.

Now, with that Q dash is fixed, we came to the conclusion from thermodynamically we said that the difference between the two quantities, specific molar heat H G n plus 1 and H L n can be shown to be approximately equal to the latent heat or vaporization multiplied by molecular weight. So, this quantity also more or less limit constant why because we very often we neglect the heat of solutions, it is one order of magnitude in comparison to latent heat of vaporizations. And look at the thermodynamic table given the text book tray ball or some physical hand book thermodynamic hand book to see to convince yourself that this heat of solution is very often negligible.

Similarly, it happens that sensible heat you know recall your knowledge of a thermodynamics or a stoichiometry at a point, sensible heat is also very often is negligible in comparison to the latent heat of vaporization; which means from one tray to another tray although the temperature changes, it turns out to be that the difference between a specific enthalpy of a two streams, we are talking all the time you know H G n plus 1 and H L n more or less this remains constant. Now, if this is constant, one can show that the ratios of molar flow rate L n over G n plus 1 is a constant.

So, we have two trays n and n plus 1 or one and two. So, we talk in terms L n L 1 G 2 then L 2 G 3 alright, we can always take the ratios of any plates in say you know, rectifying sections. So, if it is a constant, the ratio is constant, and then you make this total balance at one tray you have two flow rates you know, one liquid gets here another liquid leaves here. So, we have L one and then we have L 2. Similarly we have you know, G 2 and G 3 you make a balance for this two the ratios, one can show as I leave this is an exercise to show that even L 1 individually L 1 L 2 L 3 will also be same; similarly, G 1 G 2 G 3 will also be same. So, this is the foundation of your assumptions (()) assumptions it sounds McCabe-Thiele method.

So, so far whatever we did it in Panchan-Sevarit method, remember we had a suffice L 1 L 2 L 3 L 4, G 1 G 2 G 3 to identify the source of this two streams or from which plate it originates. So, the plate a n, we said we have L n and we have G n. Some other plates we said G n plus 1 and L n plus 1, but now because of this you know simplifications we do not have to drop this suffix, because all L(s) are same, all G(s) are the same. So, it is a very rigorous assumption, but it has certain thermodynamic foundations, because of certain numbers quantities like heat of solutions latent heat of vaporization, sensible heat from this one can show that you know and this approximation is not free far from the reality.

It does happened the raise temperature does not change much in the sense the two conjugative temperature of trays or two conjugative trays are more or less, they are very close. So, we can make this assumptions to result in the quantities like the ratios of molar flow rates of liquid and vapour is a constant, then you make a molar flow balance around the plate for stream to show that even the individual molar flow rates is liquid and the vapor are the same. So, with that assumptions, now we continue, and we see that thus similar whatever we got you know for these rectifying sections should also be holding good for stripping sections.

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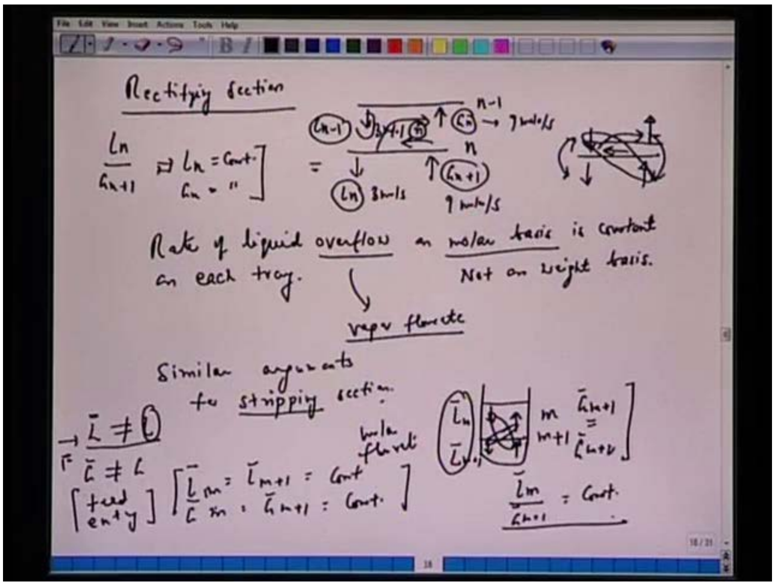

Rectifying section

Rate of liquid overflow on molar basis is constant on each tray.

Not on weight basis.

vapor flowrate

Similar arguments for stripping section.

So, what we did was we had this rectifying section, let us note down what so rectifying section, we took a plate here n, we had this vapour G n plus 1 and the liquid L n. Here we have the plate say n minus 1, we can also chose n plus 1 here. So, we had this G n and we had this L n minus 1. Now we are saying that L n minus 1 is same as L n and G n plus 1 is same as G n; physically, what does it mean? If you have nine moles per second let us example let us see here, this is also nine moles per second, if we have say three moles per second of the liquid here, you also have three moles per second. So, how does this possible, physically all it means that whatever vapor has arrived here say x number of mole or n number of mole condenses, this will also result in n moles of liquid vaporizing from here, so that the molar flow rate is conserved.

So, we so, what we had so far was the ratios L n over G n plus 1, then we made a material balance to convince ourself that with this as as good as L n(s) are constant G n(s) are constant. So, physically what is happening? We have nine moles or ten moles per second of vapor arriving here to this plate, which comes in contact with this liquid. So, we look at this plate vapour comes here and the liquid comes here, we have the pool of the liquid two streams, when they come in contact the vapour will condense and the liquid will vaporize.

So, if we have five moles of these streams condensing to this, we have five moles of liquid vaporized as a consequence, the molar flow rate will remain the same of this two guy and these two streams. So, here it is very important that we understand the meaning of, the physical meaning of, physical interpretation of this McCabe-Thiele method. We are saying that molar flow rates are constant for the liquid and the vapour, but physically also it should be consistent, no with your mechanism here. So, what is happening here that whatever vapor arrives at certain plates, it comes in contact with the liquid from the next higher stage, when they come in contact the two streams, then if we have x number of moles condensed right, then we have x number of moles vaporized. So, that entire molar flow rates of the vapor as well as the entire molar flow rates of the liquid they remain same that is the basis of your McCabe-Thiele method.

And in the word we can write here that rate of liquid overflow on molar basis is constant is constant on each tray of course, first we note that some molar basis not, we can a make a note here that not on weight fraction basis or weight basis. So, liquid over flow as well as in the vapor flow rate is constant on each tray. We can do this similar argument, we can have these similar arguments for stripping sections right. So, even in the stripping section if we chose a plate, say you know we gave the nomenclatures say m and we have m plus 1 counting from the top, then one can you know, we have the same argument here that liquid which arrives here.

So, if we take a liquid which arrives to this plate and the vapor which arrives to this plate, when they come in contact, we have equal vaporizations. So, whatever number of moles of vapor condenses to this, it gives rise to the same number of the liquid, so that the physical meaning of this these two streams are still in equilibrium, so that whatever flow rates we have if we denote by L bar, and if we denote here say L bar m and here we have L m bar m plus 1, the two will be the same and molar flow rate will be the same. So, similar argument here for the vapour G bar m plus 1 and here we have G bar m plus 2, the two are the same. So, we can also write here L bar, so bar here it means this molar flow rates for this rectifying for the stripping sections of course, we must make a note that L bar is not same as L molar flow rate in the two sections are different, why because we have in between the feed entry.

So, they will be we will talk about this later in details, but L without any bar here it represents rectifying sections and L bar represents the molar flow rate for the stripping sections. So, L bar 1 or L bar 2wo right or m plus 1 m etcetera are constant. Similarly, we have this G bar m same as G bar m plus 1 equal to constant right of course, L bar would not to be same as L and G bar will not be same as G, because of feed entry D 2 feed entry. And here we have the same argument you know, starting for the first principle one can say that L bar m over G bar m plus 1 will also be constant.

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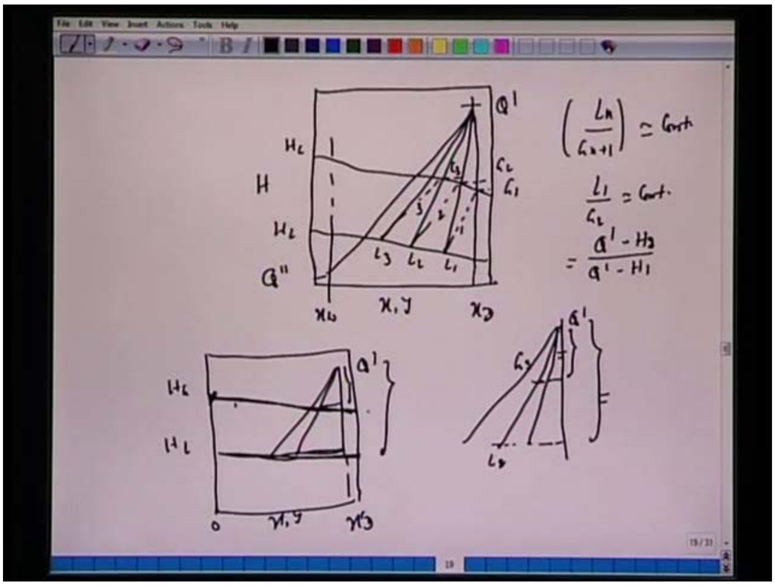

So, now let us see what we have here in this H x y diagram of course, we said that now will can find our discussion in this McCabe-Thiele methods on x y diagram, but here also we try to understand the physical meaning of this there is of two molar flow rates being constant. So, if you draw an arbitrary curve for H G and for H L, then we have something like this h G and h L, recall we this vertical line going from this x D, let us assume your total condensation we have this Q dash, this line with through and we add another line here which is Q double dash we have x w.

So, if you draw all these operating lines like in the previous class, so first we had the one stage, we had the second stage, now similarly we had this another stage here. So, the stage one, stage two, stage three and all of this line will connect here to make this an operating line. So, the first number we put it as say G 1 or H G 1, then we have G 2, then we had G 3. Similarly here we had L 1 L 2 L 3. So now, we are talking of this ratios and arbitrary ratios of L n over G n plus 1. So, we said that this is approximately constant.

Now, when it happens here, so all we are saying that say L 1 over G 2 this is constant; and why this was constant, because we said that all this quantity is same as Q dash minus H 2 or H G 2 over Q dash minus H 1. So, if you mark all this points here, look at you know one vertical line in this and another vertical, another this line right here for say G 3 and here we have this L 2. So, this what we are trying to say here that this is Q dash Q dash minus this enthalpy. So, this this is Q dash minus G 3 right or this enthalpy of this H 2 here.

Similarly, we have Q dash minus H 2 Q dash minus H 2 is the (( )). So, the ratios of this these two segments is a constant for any stages, which we can draw here. So, what happens you know on the geometrically essentially, we have the H x, y diagram and suppose we have this linear enthalpy between between the enthalpy of the pure components, so 0 1 if you draw a linear lines between G and H. In other words, we assume that the enthalpys of vapour and the liquids are linear so and parallel. So, they are linear and the parallel, so the distance between the two is a same and we draw this line in which we have drawn as for Q dash we are starting from x D and we take another line here in arbitrary line one can show that the ratios the segments which we are talking here this segment and this segment will remain the same.

So, even here we are trying to say that this is also consistent with the enthalpy, if if the enthalpy lines for the vapour in the liquid is linear between the enthalpys of a pure components corresponding to this 0, x equal to 0 and the enthalpy of the other components, more volatile components that x equal to 1, if you draw a linear curve between the two and the parallel H G and H L and we draw this lines starting from Q dash difference points and the several of this operating lines and take the ratios what we talking L 1 over G 2 equated to the enthalpy difference between Q dash and H G 2 and H G 1, one can show that this is also consistent with the geometries.

See all are we are trying to say here that the assumptions of equimolar flow rates, which we had you know L, all L(s) are same, all L bars are same, G(s) are same, G bars are same, they came all from thermodynamic considerations, where we neglected the heat of solutions we neglected the sensible heat in comparison to the latent heat of vaporizations, which is a very good you know assumptions here; based on that assumptions, we showed that the ratios are the same, then we make the molar balance around a plate to convince our say that even the individual molar rates of the liquid in one sections and the other sections will also be the same. Similarly, the vapour will also be the same.

Now we go back to this H x, y diagram, and we understand now the way we did it earlier classes two examples we had; in reality of course, these enthalpy curves they are not linear. So, there is a variation, the slant concave downward or concave upward, but more or less we can draw you know, a step line between the two ends. In that case, this line will be more or less parallel and linear you know to the second line. So, it is also a thermodynamic simplifications, but it is not very far from reality; we have gone through several examples given in the tray ball book or the Dutta book the second you know reference book, we will see that more or less these enthalpy curves they are linear or at least you can make that the assumptions with a reasonably good you know simplifications.

So, if you make that assumptions of linear, and then look at this ratios, it turns out to be that this is consistent with a thermodynamic criteria or the simplification which we obtained to say that L n over the G n plus 1 is same as Q dash minus H G n plus 1 and Q Dash minus L n etcetera. So, we must understand here that all the McCabe-Thiele method just simple method it sounds like a very rigorous assumptions, but it is not very far from reality, and most of the times of practical design calculations, how to calculate the number of plates, condenser load, heat boiler load, most of the people they prefer this McCabe-Thiele method, it is easy to construct, it is easy to work on unlike the previous methods, which is very, very rigorously based on your enthalpy balance.

So, we have the maths here, thermodynamically then we switch to this geometry on H x, y diagram, we showed that they are consistent with each other. And they also consistent with our physical assumptions that why the molar flow rates of the liquid and where the molar flow rates of the vapor constant; why because the number of moles of the vapor which condenses is same as the number of moles of the liquid which evaporates or vaporizes. So, L body is conserved L is conserved G is conserved and G bar is conserved. So, it is very important that before we take up an example or you know, we will see how we can construct this x - y diagram number of stages theoretical stages, we keep our concept clear about the physics, about the thermodynamics and about the geometry.

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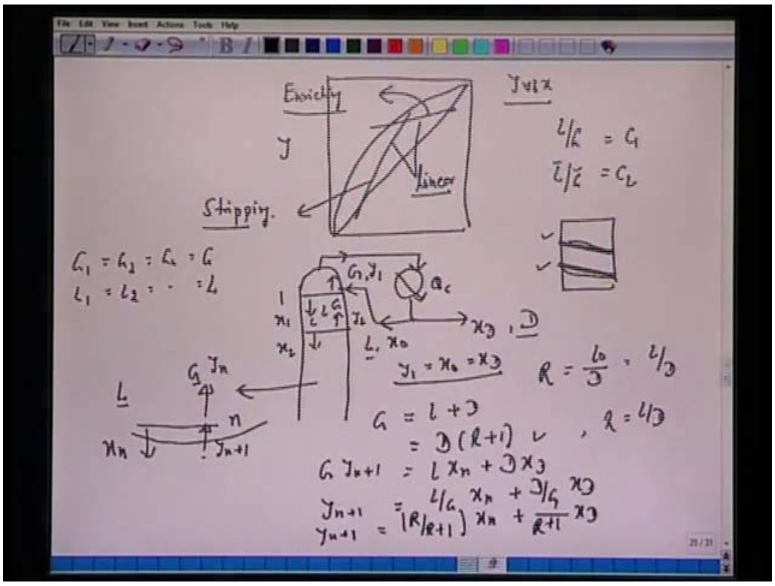

So, let us continue with this or we said that if we have this y x diagram., so now, from now onward you are going to work on this y verses x diagram. So, this is the 45 degree lines and we have this y x diagram like this; we are saying that because of that assumptions of L by G or L bar by G bar, constant C 1 C 2, if we have something called operating lines then these operating lines will be linear. We will see how to construct these operating lines.

So, whether we have the operating line for these enriching sections, enriching or rectifying sections, and we have this line for a stripping section or exhausting section, then these two lines will be linear. And this linear of course, just for the last time you say here that we are assuming that enthalpy balance and enthalpy of the vapour and enthalpy of the liquid is a linear and parallel.

In reality of course, it will be like this is there will some slant, but it is a very good assumptions. Now with that introductions, now let us see how we can construct the number of a stages, let us see how we can do it with the simple example or with this distillation column with a total condenser. So, we have these plates here the top plate one, the vapour, which arrives to this condenser as a molar flow rate G. So, note this here that we do not have any subscript, why because we said that all G 1(s) G 2(s) G n is the same. So, the molar flow rates of the vapour is G here without any suffix; however it has a mole fraction which is 1 y 1, because the mole fraction will change as the rectification goes on.

So, now we have this condenser Q c vapour condenses total condenser rate to give rise to this x D, and we have this reflux x 0 with the flow rate L. Now, again here we are not writing any any subscript for L L 0 L 1 L 2 L 3 etcetera, for the same reason that the molar flow rate of the liquid in this section is also constant as L. So, this reflux goes back to this distillation column with the plate.

So, we have y 1 and we have this x 1, but still this L and L, plate two we have now y 2 and we have x 2, but notice we just have L and we just have G, all L's are same here and all G's are same here. And it is a total condenser, so we have y 1 equal to x 0 equal to x D molar flow rate is D here. Now, we can you know do the same analysis which we did earlier, G 1 which arrives here is same as L plus D the reflux liquid and the top product D here; reflux ratio r is L 0 over D, but all we say this equals to L by D. So, no subscript here which means we can write this as D R plus 1.

Now, let us take an arbitrarily stage here say n. So, the vapor leaving has a mole fraction y n plus 1 liquid leaving this tray has mole fraction x n all we have just 1 G and all liquid which we have here is L, no no subscript here excuse me. So, we write in G equal to L plus D D R plus 1, if we now make a total species balance we have G y n plus 1 excuse me. So, we have this y n plus one here, and here we have just y. So, we are taking this envelope of this stream and this stream G y n plus 1 equal to L x n plus D x D, and then we have y n plus 1 equal to L over G x n plus D over G x D.

So, now with these definitions that G equal to D R plus 1, it is not difficult to show now we are obtaining y n plus 1 equal to R over R plus 1. So, we are writing in terms of reflux ratiosl L by G, we have G equal to L plus D, and G also equals to D R plus 1, and here we have r equal to L by t alright; that means, L by G is R by R plus 1 into x n plus x D and D by G will be one over R plus 1.

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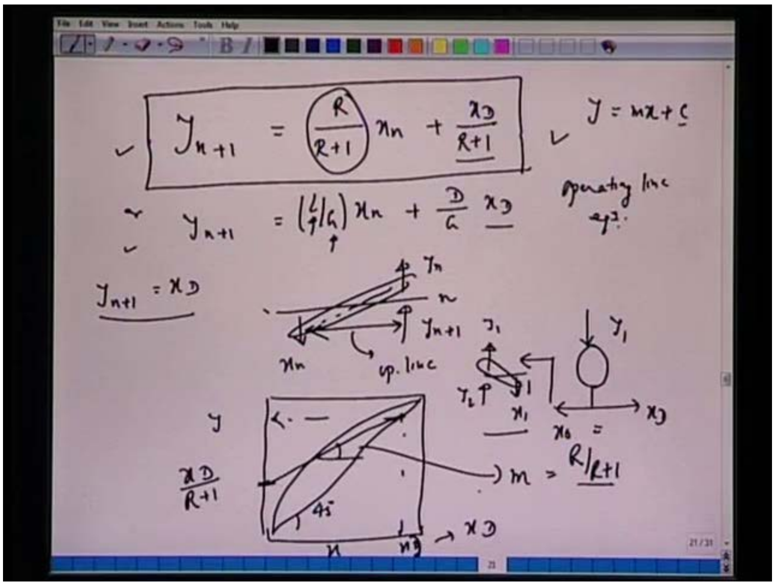

So, essentially we have got the operating line equations, which we can box y n plus 1 over R over R plus 1 x n plus x D over R plus 1. So, that the question of the operating line terms of reflux ratios, which is you can say or we obtain same as L over G, where if the ratios internal reflux ratio we call it L by G, no suffix that is more important that some McCabe-Thiele methods, all L(s) are same, all G(s) are same, L by G constants x n plus you have D by G x D. So, either this or this we have the operating line equation operating line equation for rectifying sections. What does it do? Well we have n here, so this is y n, this is x n, two are in equilibriums at the tie line, and what we have here is y n plus 1 between y n plus 1 and x n from the operating line given by this equation or this equation that is operate line does it connects between the mole fractions of the two stream leaving the liquid leaving here and the vapor arriving that plate. So, we made that envelope to obtain these equations here. Now we can go back to this D x y diagram sorry x y diagram and we can mark coordinates.

So, we have here y and we have x, we call we have just total condenser, so we have x D here alright and the vapour which arrives here is y 1, this is x 0 same as x D same as y 1 this vapor goes, this liquid goes to the top column with 1, we have this y 1, liquid which leaves here is x 1, two are in equilibrium we have y 2 and so forth.

Now let us see, what we do we on this x y x diagram. So, if you start with x D we are suppose to draw this operating line. Now if we put x D in this equations, you can show that y will y n plus 1 will also be equal to x D alright; that means, this point of x D at 45 degree line same as this y here will satisfy this operating line. In other words if you take this point here and take the intercept it is a equation of a straight line y equal to m x plus c. So, c is the intercept if you take the intercept at x D over R plus 1 right and connect with this should be able to convince yourself that this slope m will be same as this R over R plus 1. In other words you can take a intercept, you can take a slope here, when it intersects that 45 degree line what coordinates, we will get will be same as x D.

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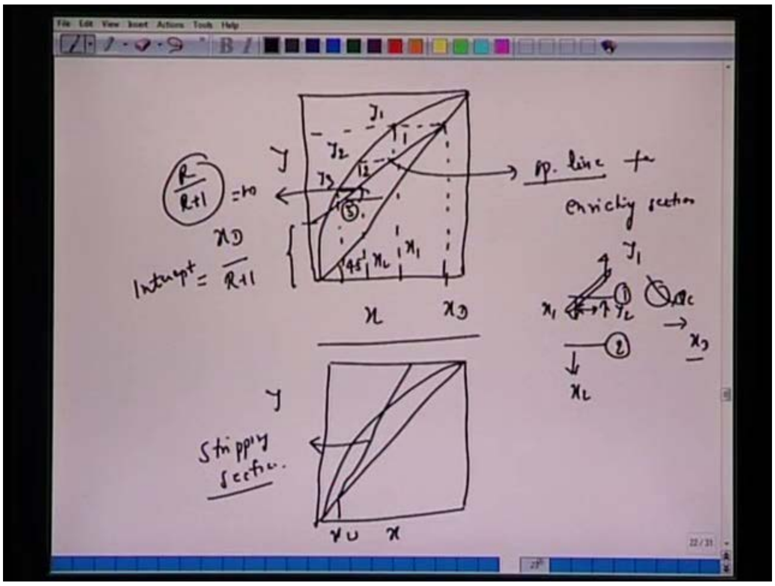

So, it is your convenience, hardly draw this. So, generally when you draw this y x diagram, the state of taking the slope here is more convenient to a start from x D, take this intercept at the 45 degree line here y equal to x mark the graph at R over R plus 1 same as L over G connect with this point, this will become the operating line equations, operating line for enriching sections, and we have this y equal to x equilibrium curve.

So, I can now look at this we have this y same as x D; first point which we locate here is your y 1. So, we are starting now stage one, stage two, the vapour which arrives here is which leaves this stage one and arrives to this condenser, we have this composition of the top product leaving x D y 1 same as x D. So, we have y 1 here, let us mark this point; this y 1 is in equilibrium with this liquid at x 1. So, we have this x 1; x one and now we have this y 2 connected by this operating line. So, we have this second point y 2.

So, we have the similar story tie lines gives us equilibrium, two streams leaving the stages are in equilibrium and the steam which arrives this stage one and the leaves they are in connected by the operating line. Second we start with x D same as y 1, so this line satisfies this operating line as we said earlier that if you take the slope of the operating line and take start from the intercept, we will also reach this point y equal to x, but it is more convenient to start from the intercept x D equal to y at 45 degree line; mark this intercept connect it, you measure the slope, you will get the same slope as before we have R over R plus 1. This is the intercept which is actually x D over R plus 1.

So, we have R over R plus 1 the slope and we have the intercept, which is x D over R plus 1. So, we have this y 2 again let us start; this y 2 is in equilibrium with this x 2. So, we have this x 2, again this x 2 and now we have this y 2 and so forth. So, starting from this, now we can construct the stages; the first stage is one, second stage is two, this stage one, this is stage two and so forth we have the stage three etcetera. This kind of constructions can also be done for the bottom sections. So, if we do x and if be a y here like this, we starting from x w and with the equation of the operating line one can also obtain this, we can also have the number of the stages here, this is stripping sections.

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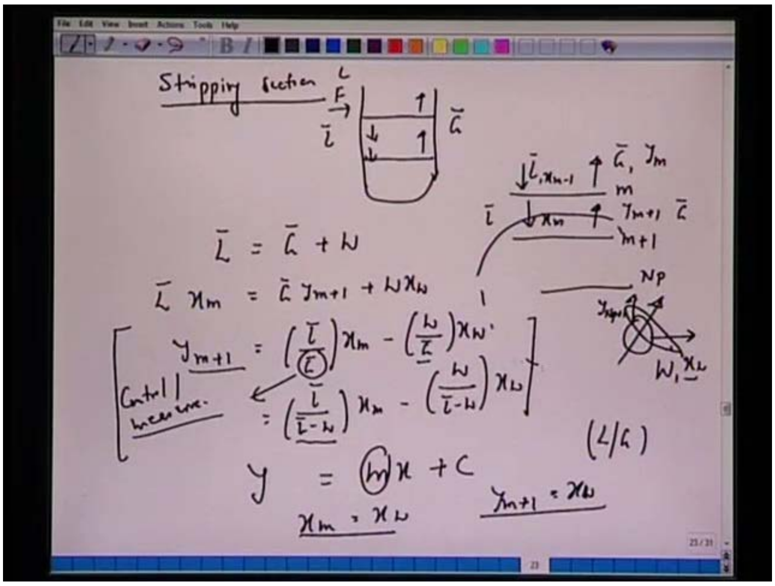

Stripping section

$$\bar L = \bar G + W$$

$$\bar L x_m = \bar G y_{m+1} + W x_W$$

$$y_{m+1} = \left(\frac{\bar L}{\boxed{\bar G}}\right) x_m - \left(\frac{W}{\bar G}\right) x_W$$

$$= \left(\frac{\bar L}{\bar L - W}\right) x_m - \left(\frac{W}{\bar L - W}\right) x_W$$

Control / measure.

$$y = \boxed{m} x + c$$

(L/G)

$x_m = x_W$

$y_{m+1} = x_W$

So, let us now focus on these stripping sections and here also we assume, now we have the similar assumptions that the bottom column or molar flow rates are the same. So, there is no suffix for our L bar of course, L bar is different from L, because there is introduction of every L. We will come back to this later when you make this molar balance for the feed feed tray. So, over L bar is same no suffix and all vapour leaving all vapour leaving the stage as G bar. So, we can do the same thing we can choose a plate m, so the vapour leaving is G bar liquid arriving here is L L bar this mole fraction will be y m this mole fraction will be x m minus 1, we have m plus 1 this vapor is y m plus 1, this liquid x m, but the same molar flow rate no suffix here G bar G. If you take this envelope with x m and y m plus 1 with re-boiler where the stream has this molar flow rate say w and we have x w. So, this goes to the last plate which is n p, now since we have this re-boiler this is y n p plus 1. So, these two streams are in equilibrium; x w and y n p minus 1.

Now if we make this balance here for the species balance and the total balance L bar equal to G bar plus w, so L bar equal to G bar plus w. So, this is the envelope which you have made (( )); and similarly we have L bar x m as G bar y m plus 1 plus w x w, and we have we can rearrange to write y m plus 1 over L bar L bar over G bar x m minus w over G bar x w, which is same as L bar over L bar minus w x m minus w L bar minus w x w. So, instead of G bar we are writing it L bar minus w, because very often it is very difficult to control the molar flow rates of G bar. So, it is very difficult to control as well as to measure.

So, generally most of the times in the distillation column we have the data for the liquid. So, we can write G bar is L bar minus w, two are the same that is our operating line. Notice this that this ratio is also a constant here this ratio is also constant here equation of this line is also y equal to m x plus c, we have the slope as L bar by G bar, compare this to the previous one, when we had the slope of L by G. And now we have the intercept which is negative here w over G bar or w over L minus L bar minus w. But again notice if we put x m as x w, you should be able to get y m plus 1 same as this x w, all it means this coordinate also satisfies you know, lies it lies on this operating line as well on this 45 degree line which means, now we can construct this operating line for this stripping sections without finding this negative intercept.

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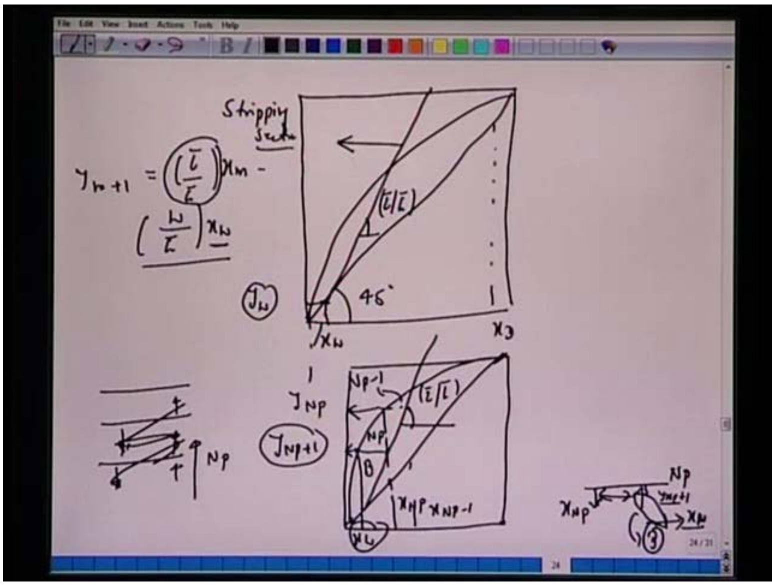

So, similar the previous case, where we said that either we take a slope and the intercept or it starts from this x D and connect with the intercept one can do the same thing here, this x W also satisfy equal to this y m plus 1 or y w also satisfy this operating line as well as this forty five degree line alright. So, the equation of operating line for this lets rewrite here y m plus 1 as L bar over G bar which is a constant x m minus w over G bar x W. So, with this slope of L bar by G bar and from y W x W, because y W x W this coordinate satisfy this operating line, we can draw this operating line. So, this is the operating line for a stripping section.

Again make a note here that instead of this, suppose some body starts with the slope and the intercept of this, which would be negative here right; one can one will obtain the same line, but there is no point it is really done like this, because we realize immediately that x W and y W, it lies on this forty five degree line as well. So, if you put here will get the same results here, its better to take the start from x W y W at forty five degree line and take the slope of L bar over G bar to obtain this line here alright.

Now, we can start construction of the plates in the stripping sections, the way we have done it for the top sections. So, we have this y verses x diagram, if we start from x w, so recall we have this bottom product x W and the vapor which arrives to the top bottom plate at NP through this re-boiler is in equilibrium. So, we extend this line first coordinate which will we get is y n p plus one. So, this y n p plus one two streams are in equilibrium y NP plus 1 and x W x W and y NP plus 1, they are in equilibrium. So, we have marked this point now y NP plus 1 and here we have liquid x NP, they are connected by the operating line. So, first if we have x W, and then we have drawn this operating line by the by the slope of L bar by G bar, this will give us x NP alright; again extend this, we have the second vapor compositions y NP alright extend this, we have now x NP minus one and so on.

First stage is of course, the boiler, which we have done in case of panchan-sevarit method as. So, we have boiler, then this stage is NP, then this stage is NP minus 1 etcetera. So, we have been able to construct we go from the bottom approach NP, all the time tie lines and we have this operating lines, tie lines, the operating lines, similarly we have done for this top sections will complete our construction here.

So, let us summarize today’s lecture. We started with a new method McCabe-Thiele method; this method you must have realized by you know its must simpler than panchan-sevarit method of course, there is assumptions we talked about that assumptions, thermodynamic assumptions that leads to this conclusion that all L(s) are same in the top sections, all L(s) are same in the bottom sections, all G(s) are same vapor flow rate at the top sections, all vapor flow rates are same in the bottom sections. With that assumptions, we made species balance, we made a total molar balance and we show that the equation of the line for the top as well as for the bottom section is a straight line y equal to m x plus c, the slope is given by L by G, and here its by L bar by G bar.

So, if you have the operating line, we have the equilibrium curve y verses x instead of h x y equilibrium curve for the panchan-sevarit methods, we can again restart the constructions for the both sections. So, we start at from x t, we realize that that vapor composition of the plate one it is same as the liquid composition, which leaves the top product total condenser. So, from here there y 1, then comes to the first plate y 1 is in equilibrium with x 1 we mark to the equilibrium curve x 1 with y 2, we go to the second stage or go to the operating line, then the second stage etcetera. Similarly we start from the bottom composition x W, we draw this operating line either from the slope and intercept or from the slope and starting from 45 degree lines, where we have marked x W and y W. Make a note that y W is a hypothetical point the composition of this vapor y W does not exist in the distillation column, it just lack mathematically that coordinate of y

W equal to x W satisfies this 45 degree line as well as your operating line. So, it is easier to start constructions from the 45 degree line.

Take a slope of L bar by G bar instead of looking for the intercept, which is negative in this case for the bottom column, and then take a slope. Similar thing we did for the top column we start from x t at the 45degree line and then the draw the line of L by G right. So, we can construct on the number of stages for the top column and for the bottom column.

Next lecture, when we meet, we will see that how we can locate the feed. If you remember in the previous class also we had spent a time on this location of the feed tray from the enthalpy energy balance here also will make some energy balance for this feed entry, and see that we can mark or we can come up with another operating line, we call it q line. So, we have one operating line for the top section, one operating line from the bottom section and one we have this q line, feed line; all three lines will intersect at one point (( )).