Mass Transfer II

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Lecture No. # 11

In today’s lecture we take an example, this time on hydrodynamics. So far, what we did was on mass transfer, and in hydrodynamics essentially we look at loading and flooding conditions. So, if you recall, earlier we said that if we have a packed column, and the liquid and the gas. They have countercurrent flows, they flow counter currently then there is a possibility of loading or flooding. In other words, if the liquid flow rate which you know liquid which trickles from the top has a high flow rate or large flow rate relative to the gas flow rate, it is possible that the liquid will prevent gas to flow through, through the column; in that case the column will be flooded with the liquid.

On this, at the same time if the gas flow rate is very large there is a possibility of channeling mall distribution inside the gas, which will prevent liquid to flow from the top. So, one has to optimize the liquid and gas flow rates or the ratios of liquid and gas flow rate to ensure that the pressure drop is reasonable and there is no loading and flooding. So, in this case we also said that we have a very well known quite popular graph propose earlier plotted by Pickford or Sherwood Eckerd, and so we are going to make use of that same plot to address this problem of loading and flooding through this example.

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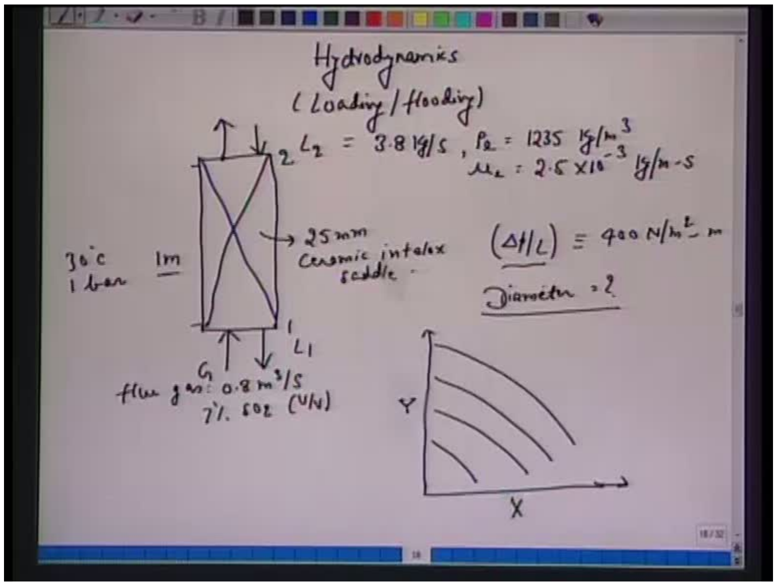

So, let us take this example on Hydrodynamics,. And we said we look at loading and flooding criteria. We have a packed column, absorption and we have gas flowing through this liquid trickles from the top and this is the packed column. So in this example, it says column is packed with 25 millimeter ceramic made intalox saddle packing. So, this is the name of this packing. And here it is given the flow rate of this flue gas is 0.8 cubic meter per second, temperature is given 30 degree centigrade, pressure is 1 bar and this flue gas contains 7 percent of sulphur dioxide and it is given volume by volume. So, essentially 0.07 is a mole fraction of this. And this is treated with the liquid or may know certain aqueous ammonical solutions the flow rate is 3.8 kg per second.

So, two means this level is 2 and this end is 1, physical properties of the liquid is given rho l 1235 kg per cubic meter, viscosity of the liquid is given as 2.5 10 to power minus 3 kg per meter per second and height of this column is given here 1 meter. So far, we have not said how to determine this height of the column that will be the next topic, right now all it says 1 meter height and the liquid flow rate here is L 1 pressure drop per unit length. This must not exceed or it is given as 400 Newton per meter square per meter. So, that is a pressure drop per unit length, and the question asks is what is the diameter or possible diameter of this column.

So, before we take up that graph, if you recall we had this Eckerd plot or the plot proposed by Eckerd or later modified by Pickford which you called flooding and loading graph. You had these curves and here we had certain quantities which we are suppose to evaluate on the X axis, on the Y axis. So, let us do this first.

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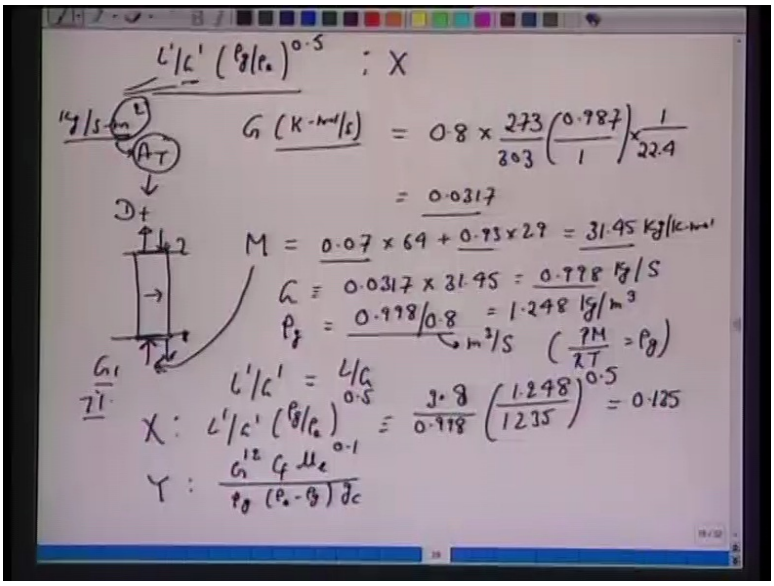

$L'/G' (\rho_g/\rho_L)^{0.5}$: X

G (k-mol/s) $= 0.8 \times \frac{273}{303}\left(\frac{0.987}{1}\right)\times\frac{1}{22.4}$

$= 0.0317$

$M = 0.07 \times 64 + 0.93 \times 29 = 31.45$ kg/k-mol

$G = 0.0317 \times 31.45 = 0.998$ kg/s

$\rho_g = 0.998/0.8 = 1.248$ kg/m^3 m^3/s $\left(\frac{PM}{RT} = \rho_g\right)$

$L'/G' = L/G$

X: $L'/G' (\rho_g/\rho_L)^{0.5} = \frac{3.8}{0.998}\left(\frac{1.248}{1235}\right)^{0.5} = 0.125$

Y: $\frac{G'^2 C_f \mu_L^{0.1}}{\rho_g (\rho_L - \rho_g) g_c}$

First we have to determine L prime by G prime rho g over rho l to the power 0.5. So, this is on your X coordinate. So, you are suppose to you are required to you require to visit that graph given in the Treybal, given in the Macabesmith a well known well popular graph which was generated in 1938 by Eckerd. Later modified, you might see several versions of this. Eckerd Pickford plots where you have this loading and flooding criterion. So, go to the plot and see what we have the X quantities, here you have L prime, L bar over G bar, rho g over rho l to the power 0.5. Similarly, you have certain quantities on Y which are suppose to calculate. So, that is what we are doing here.

So, L bar, G bar now this is your kg per second per meter square. Now, this meter square is your cross sectional area per unit cross section area, that is why we suppose to find out, so that we get A T, and from there we get the diameter of this column. And so, the flue gas composition is given here, let us calculate G as kg mole or kilo mole per second, and so let us make it kilo mole per second. So, this G would be 0.8 that is a cubic meter per second. So, let us make pressure temperature corrections 273 divided by 303 303 0.987 over 1, so that is a pressure correction 1 bar and then we know 22.4 liter per mole. So, we have 0.0317 kilo mole per second of this gas flow rate, so it is given to us. Molecular weight of this you know for this flue gas we can calculate at 0.07 into 64 plus

0.93 into 29. So, 7 percent is sulphur dioxide rest we assume, now it is air. So to average molecular weight of this flue gas at the entrance 31.45 kg per kilo mole.

Now, it is also important that we understand that in this column the mass flow rate of the gas will change as the sulphur dioxide is stripped was absorbed in water or on the solvent easily we have 1 here and we have 2 and molecular weight, we have calculated outstanding point out here that this is based on this G 1 conditions, why because at 1 end we have very high flow rates. So, liquid which also trickles from this you know it enrich sulfur dioxide. One does the calculation based on end 1 rather than based on 2. So, we have calculated this based on this 7 percent of sulphur dioxide which is in this gas phase. So, G, molecular weight is 31.45 kg per kilo mole which means now we can calculate G in terms of kg per second 0.0317 multiplied by 31.45 equals 0.998 kg per second. Rho g gas density 0.998 divides by 0.8 all right, to make it 1.248 kg per cubic meter.

So, this is the volumetric flow rate this was given to us cubic meter per second and just now we calculated kg per second. We can get the gas density alternatively, if we get the same result if you calculate P M over R T for rho g you will get the same result, you must check as an exercise. Now, we can calculate X L prime over G prime, now notice that this L prime and G prime that is given in kg per second per meter square, but the ratios of L prime over G prime will be same as L over G. So, we calculate this quantity rho g over rho l to the power 0.5 it is on the X axis. This is equal to 3.94 over 0.0137 1.248 over 1235 to the power 0.5. Liquid flow rate is given 3.94 3.8 kg per second so, will correct this 3.8 kg per second and this is what we just now calculated G 0.0137.

This is 31.45, 0.0317 that is a kilo mole per second so, we need to replace this 3.8 divided by 0.998. So, we have this correction L prime 3.8 and 0.998 and if you do this we have this quantity 0.125 and similarly, we have to determine the quantity on Y axis, but there we have this Y axis we have G square 1 C f mu l to the power 0.1 divided by rho g rho l minus rho g c. This is also your you have G prime, now we suppose to evaluate this quantity G prime, we know there is L prime over G prime from the X axis.

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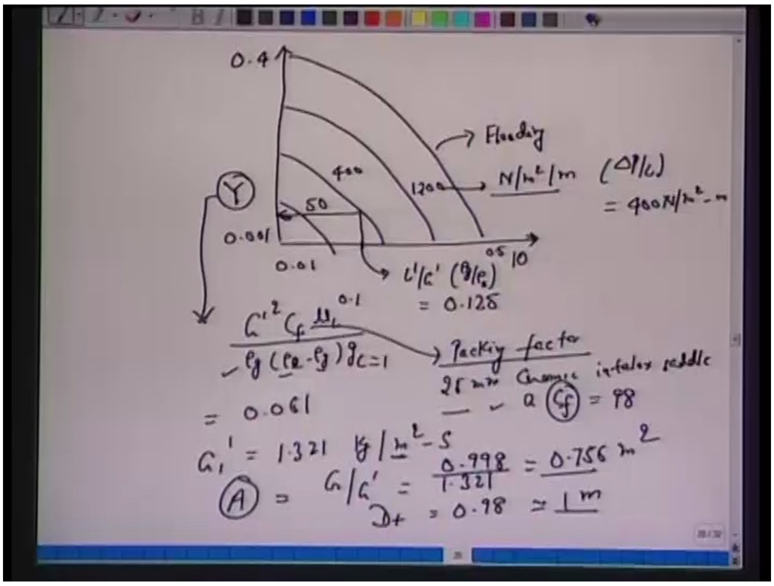

$(\Delta P/L) = 400\,N/m^2\text{-}m$

$L'/G'\,(\rho_g/\rho_L)^{0.5} = 0.125$

$\dfrac{G'^2 C_f \mu_L^{0.1}}{\rho_g(\rho_L-\rho_g)g_c} \quad g_c=1$

$= 0.061$

$a \quad C_f = 98$

$G_1' = 1.321\ kg/m^2\text{-}s$

$A = G/G' = \dfrac{0.998}{1.321} = 0.756\ m^2$

$D_t = 0.98 \simeq 1\,m$

So, now we can go back to that plot, take the plot from Treybal or from some text book. We have this plot 0.01 typically you have 10 here; you have 0.001, 0.4 and you have several of this curve. The top curve off course, this is flooding curve. And then you have several of other curves with the different pressure drop. So, this is 1200, for example 400 and all the units here are Newton per meter square per meter. So, that is your del P over L now we know this quantity which was L prime over G prime rho g over rho l to the power 0.5 this we calculated as 0.125. We mark this point pressure drop per unit length if you recall it was given in the example, it is 400 Newton per meter square per meter.

So, from here, we come to this curve and we calculate this quantity on the Y which is G square 1 C f mu L 0.1 over rho g rho l minus rho g g c, we should also recall that the C f is packing factor which means we are suppose to go back to the table we discuss couple of lectures ago for this packing of 25 millimeter ceramic intalox saddle. So, if you look at that one of the stables you will see type of the packing, size of the packing and you will see factor a interfacial area as well you will see a quantity C f which is a packing factor given by the manufacturer. So, take this factor calculate this, take this number which would be 98 from the table and calculate this quantity, if you look at this graph the quantity would be 0.061.

We know all of this rho l, rho g, g c 1 in these unit C f viscosity properties is given as an exercise, we just do the calculation you should get the answer G 1 equals 1.321 kg per meter square per second. So, this is your kg per meter square per second. So, which means the area of the column cross sectional area of the column A is G over G prime which is 0.998 over 1.321 to give 0.756 meter square. So, that is the cross sectional area and based on this you get the diameter of the column as 0.98 or approximately 1 meter, all right. So, what we obtain from this loading and flooding curve here, all right.

So, of course, now this example we had on hydrodynamics the idea is that we are not heating this flooding curve, . Some examples or some problems could be you know rephrase in different way, knowing L prime G prime or the ratios of the two flow rates on the X axis, you may be ask to calculate what is the maximum gas flow rate for which we have this flooding. So, all its suppose to do mark this point hit this flooding curve you get G prime and we obtain as we said if you recall we said that we operate on 70 percents most of the absorption columns operate on 70 percent of the flooding limit. So, once we know that flow rate take the 70 percent of this get your new del P by L pressure drop per unit length here.

In this case of course, this was given 400 Newton per meter square per meter. So, the problems can be rephrased, the idea here is that he must understand that the flow in the absorption column is countercurrent, we have the gas flowing from the bottom and we have the liquid flowing from the top. There is always a possibility of one flow rate relatively being very much larger than the other flow rate, trying to prevent the flow of one from the other getting into this column so, one has to watch out for this flooding limit. Of course, when you take a very larger flow rates of the gas you have the pumping cost if you have a very large flow rate of the liquid you also have a pumping cost. So, the cost of the pump cost of the blower for the gas also has to be taken into considerations while designing in absorption columns.

So, again we have this cost or the economic to play a role here, just like in the previous lectures we said that height of the column or the amount of the liquid flow rate or the amount of the gas, all of this they also play a major role in the economy which we cannot ignore. So, I am sure at some levels you have a design codes in which you are going to address this economy part of it. So, when you design a column one has to watch out for the hydrodynamics so that the columns not flooded it is a stable and given that conditions how much is your velocity, superficial velocity based on which you have a Reynolds number based on that you have a mass transfer coefficient based on that you have a flux and one can determine the height of the column.

How many, how much distance we require how much height we require to rectify this flue gas or treat this gas from one level to another level. So, that is the second topic we are going to take after this so far what we have done equilibrium curve operating curve which is linear for absorption or for stripping columns one above this equilibrium curve one below the curve and now we have this hydrodynamics. Now, next step would be that we should be able to determine how much what is the height of this column, right? This flow rate kg per second per meter square that decide by the diameter size of the tower size of the columns so size of the column comes from hydrodynamics pressured of considerations height will come from mass transfer that is more important in this course.

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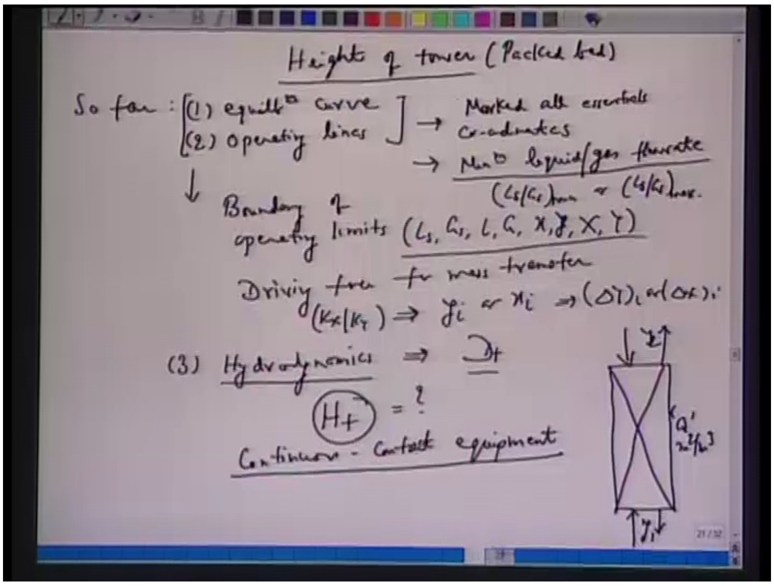

Height of tower (Packed bed)

So far: (1) equilibm curve
(2) operating lines → Marked all essentials co-ordinates
→ Minm liquid/gas flowrate $(L_s/G_s)_{min}$ or $(L_s/G_s)_{max}$.

↓ Boundary of operating limits $(L_s, G_s, L, G, x, y, X, Y)$

Driving force for mass transfer
$(K_x/K_y) \Rightarrow y_i$ or $x_i \Rightarrow (\Delta Y)_i$ or $(\Delta X)_i$

(3) Hydrodynamics ⇒ D_t

$\boxed{H_t} = ?$

Continuous - Contact equipment

So, let us start this second part of this chapter, how do we determine height of tower or absorption column? And we are saying that we have this packed bed so, essentially we have a packed bed column and we are looking at height of tower. So far, we discussed equilibrium curve, discussed operating curve, operating lines, two together we are marked all essential coordinates and based on this we also determine minimum liquid or gas flow rate. Recall, in the previous example we said that how to determine this L s by G s min or max depending upon we are talking of absorption column or we talking of stripping columns. So, this is what we have down here but this does not give us height of a column, right?

All it says, it gives us a boundary these two 1 and 2 is decides us a boundary of operating limits L s, G s, L, G, x, y, capital X, capital Y. These are the quantities you know we can determine, we can locate, and we can also obtain at any location what is a driving force for mass transfer? This can also be obtained from 1 and 2. So, again here you should recall that we have another line which has the slope of K X over K Y which if we take the ratios that will give us y i or x i and so far, we can have del Y i or we can have del X i. So, driving force from mass transfer that is what we do from 1 and 2 then we went ahead and we did hydrodynamics gives us stable operating condition from the operation point of view. This gives us the diameter, what we interested now is height H t. Height of a packed bed tower or absorption column which is nothing, but again recall this is nothing, but continuous contact equipment.

So, we not talking of trace a stages this is a continues contact equipment a column packed with certain packing having, certain meters square per cubic meter interfacial area certain type of packing two phase are allow to come in contact and we want to achieve concentration reduce from y 1 to y 2 in this treated gas. So, this is an objective how much height we require, how tall should be the column. Now, here before we get into mathematics, let us understand the approach. Traditionally what approaches we have, first we must recognize that it is a continuous contact. Of course, the contact could be as complex as possible it is not a very simple contact that the liquid flows from the top and gas flows from the bottom.

We have all types of packing right? Could be Berl saddle, could be Lessing rings, could be Raschig rings and could be Intalox you know saddle which we have seen here. So, there between two phases gas and liquid interfacial areas will vary from one location to another locations, there could be channeling, there could be mall distributions. For our purpose of course, we have ideal situations where there is no channeling and there is no mall distributions. So, whatever interfacial area we have the packing profiles you know between the phases it is available for mass transfer, but still hydrodynamics is quite complex here, right?

You require again you should be able to revisit the previous lectures, you require a very extensive c f d computation fluid dynamics to resolve the hydrodynamics insist packed column. We confined our self to this bulk properties or bulk variables like Reynolds number, Reynolds number is less than one based on the packing size we say that it is a laminar flow, Reynolds number is very large hundred, one thousand we say that if there is a trouble and then we define a mass transfer coefficients and we said that all complexities of hydrodynamics are embedded in mass transfer coefficients. So, as an engineer we make use of this K X knowing Reynolds number, knowing Schmidt number we can calculate Sherwood number we can calculate mass transfer.

So, mass transfer coefficients and then we can predict the rate etcetera so, that is the approach we are going to take it. More important here is that we are going to address continues contact it is not a trace like in distillation columns. So, that will address separately in absorption column most of the absorption columns we have this packed column, there are some industry trace also where you can have a trace and the two steams can be brought in contact we confine this course or the discussion in this course to this continues packed bed column, . And when we say continues packed bed column there is an approach, well you know well known it is called N T U and H T U approach. So, number of transfer units and height of a transfer unit.

Again before get into the mathematics we are trying to say here that this column which has five meter tall can be divided into several of small units 10 units, 20 units, 5 units each unit has a height. We have N T U number of transfer unit and height of a transfer unit so; N T U multiplied by H T U should give you the height. So, that is the approach we have, we will see how do we calculate these quantities N T U and H T U which will give us the height diameter mind you we have already fixed from hydrodynamics and there also it is also possible and that you know in one of the design courses you will see it is quite iterative process, in other words you start with certain diameter to ensure that there is no loading and flooding you calculate the height.

You may have to go back and change check the diameter why because maybe it was cost profit cost of this column consideration taking into consideration the cost of the packing, cost of this tower, cost of your liquid or the gas you know handling system power cost, blower cost etcetera becomes too much. It is always a very vigorous process eternity, you require to do a lot of programming that you will do separately in some course at third or fourth year level.

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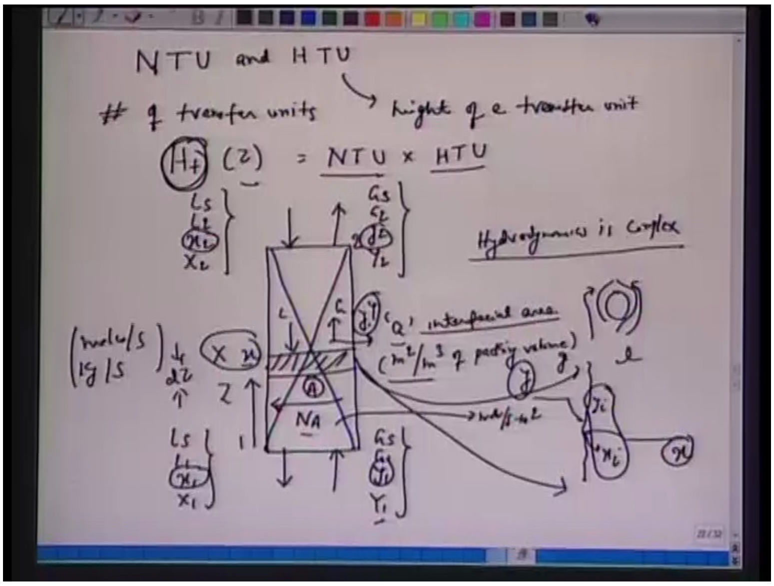

So, now, we confine our discussion to N T U and H T U to determine the height of a column. So, we have N T U and H T U, number of transfer units and we have height of a transfer unit. So, the height of this column if you denote by H t of z this is nothing, but N T U multiplied by H T U. So, we suppose to define and determine N T U and H T U to get height of this column. Let us draw the schematic again. We have this schematic of this absorption column it is packed with certain packing there is the first thing; it is pack with certain packing. So, we know the interfacial area of contact, the packing manufacturer he supplies this meter square of interfacial area per cubic meter of packing volume.

So, generally the manufacturer he supplies this quantity, a or from the hand out from the tables this quantity is known to us, also known to us is that we have a given gas which has a specification G s, G, G 1, y 1 and capital Y 1. So, you are familiar with this nomenclatures that G s does not have any subscript and 1 because this level is one here this gas which is treated now will have the specs G s notice there is no subscript again, but we have G 2, small y 2 and capital Y 2. We have discussed this in the previous class the meanings of this to treat this we have been given the liquid which has the flow rate L s notice there is no subscript we have L 2 we have x 2 we have X 2 and this liquid which trickles out which flows out from here has L s.

Notice there is no subscript with L s, we have L 1, x 1 and capital X 1 and all this flow rates we can have moles per second or one can also have kg per second if you multiply with this molecular weight. The two phases, they mix they come in contact through this packing’s so we say the hydrodynamics is very complex. Typically you have liquid film over this and the gas flows pass from this. It is quite complex here, for our purpose always that there is an interface very clean interface from mathematical calculations one side it is a gas and one side it is a liquid recall the concept of bulk film concentration drop interfacial concentrations and then we have this bulk concentrations.

So, we can have y x two bulk concentrations at any locations and we have y i and x i and they are related through this equilibrium or thermodynamics so, these are the four at any locations, location from here if it is height at z if we choose d z and you expand this you have the situations. Of course, we must not taken we are trying to repeat here that things are quite complex here, the flow hydrodynamics is quite complex. Hydrodynamics is complex, for our engineering calculations we have define this neat a clean interface between the two phases gas and liquid with the bulk phase comes mole fractions and bulk phase concentration or mole fraction in the other phase liquid.

So, here we will have L this will be G and we can say we have x general capital X or we can have y and we can have capital Y, here and here, there is a transfer of the species N A we call this is mole flux, moles per second per meter square so that is the mole flux. When the gas flows from this at any locations at Z we have taken a differential height d Z at which the concentration in the liquid phase is x in the gas phase it is y there is a transfer of this species A which is sulphur dioxide here for example, from gas phase to the liquid phase. So, the concentration changes from y 1 to y 2 and the liquid picks up the sulfur dioxide and concentration increases from x 2 to x 1 we have an ask to find out how long is a column, what is a height of this column H t or z to achieve this to reduce the concentration from Y 1 to Y 2.

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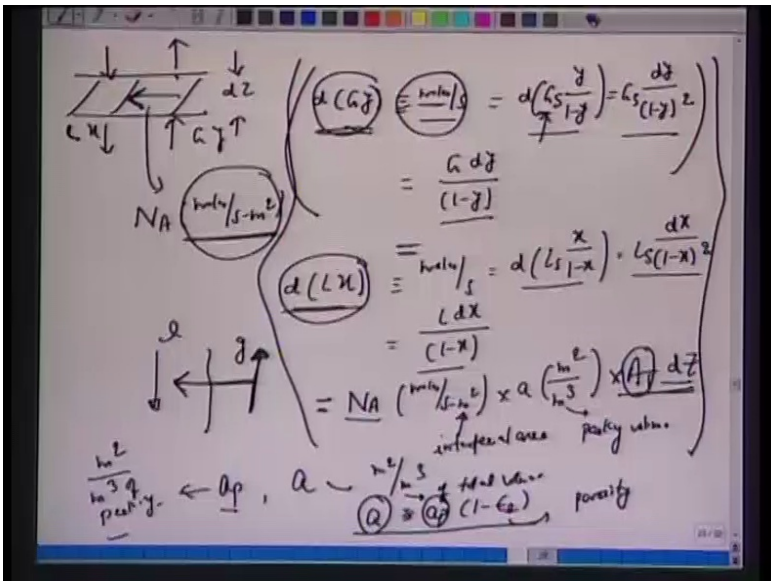

So, all we do we make a species balance across this dz and see what we obtain mathematically so, across this differential element of dz we have gas flowing like this and the liquid flowing like this l and g.

So, if you make a species balance over this differential you have d G y change in the gas phase concentrations d G y all right so this is nothing, but moles per second in one second how much moles of G changes across this all right, that is a d G y why does it change because there is a transfer of a species a from gas phase to liquid phase to so d G y this will equal to or can be written as d G s y over 1 minus y. So, we are trying to bring this G s here because we realize that G s does not change along the height, along the column it is a solvent flow rate solvent does not evaporate or the gas does not get dissolve. Berl gas or the carrier gas air nitrogen does not get absorb in liquid or the liquid does not evaporate in to gas phase. So, we write d G s then G s is constant we can take it out then this becomes G s one can show that this should be G s d y over 1 minus y square and then this can also be written as G d y going back to G s to G with 1 minus y.

So, several ways you can write what is a concentration let us go over one second, in one second so much moles of sulfur dioxide changes across from across this d Z y, it is getting absorb in water so, d G y equals d G s, y over 1 minus y G s is constant does not change along the column G s is outside the differential you have to or you can obtained d y by 1 minus y square again you can go back and write G s over 1 minus y as G so, we have G 1 minus y that is the concentration species balance we have made in the gas phase, but remember this will also be equal entire quantity which we have calculated and we have written a different forms can also be written in terms of liquid phase.

So, whatever happens in the gas phase in one second if five moles changes across this d Z the same five moles will appear in your liquid phase. That is the species balance under steady state there is no accumulations. So, if the gas flow rate is five here and two here what has happen to five to two three must have gone into the liquid phase. So, that is the balance we are trying to make the quantity which you have written in terms of gas phase and the mole fraction of y we should have a similar expressions for the liquid phase. This also equals by intuitions are otherwise this L x.

This is your L this is your x and this is your y d L x this is also moles per second this will also be equal to d L s takes over 1 minus x y, look at the analogy so, as the same L s d x over 1 minus x square this will also be equal to L d x over 1 minus x. All this quantity this equal, this equals this, equal this will also be equal to this will also be equal to this and so far, because that is your species balance under steady state and the most important there all of this quantities we will also be equal to N A. So, now we are talking of flux why this quantity has change from across this d z by d G y or by d L d L x because there is a transfer of sulphur dioxide from G phase to L phase in terms of moles per second per meter square.

So, now, we have getting this into business of this mass transfer diffusion flux equations, we have N A moles per second per meter square, let us write down moles per second per meter square that is the flux across the two phase from gas phase to liquid phase. So, whatever changes here this a decrease and there is an increase which is balance by this gas to liquid will equal to N A into a that is a packing specifications meter square per cubic meter, make a note this meter is square is not the cross sectional area this is interfacial area of mass transfer and we have this packing volume.

So, this multiplied by area cross sectional area of the column into d z that will be our moles per second so, moles per second per meter square meter square per cubic meter of the packing volume into A T cross sectional area into d z to it will you moles per second. Now, we have the entire species balance in the two phases gas to liquid change in the gas phase whatever is lost in the gas phase is gained in the liquid phase and that is also equals from the mass transfer point of view N A into a into A T into d z.

Here, it is very important that we also make a little bit careful the two types of nomenclatures use in most of the text book one is a p one is a, a p generally you have meter square per cubic meter of packing say Berl saddle or Raschig Rings this a is meter square per cubic meter of total packed volume, total volume, which means a can be related to a p into 1 minus epsilon b where b epsilon b would be the bed porosity. So, let us be careful in terms of using the certain nomenclatures whether it is a or a p or what is supplied by this manufactures. So, we all we have done we have made this a species balance between the two phases gas phase and the liquid phase where is the balance comes through this flux N A through this interfacial area a meter square per cubic meter of the packed volumes, .

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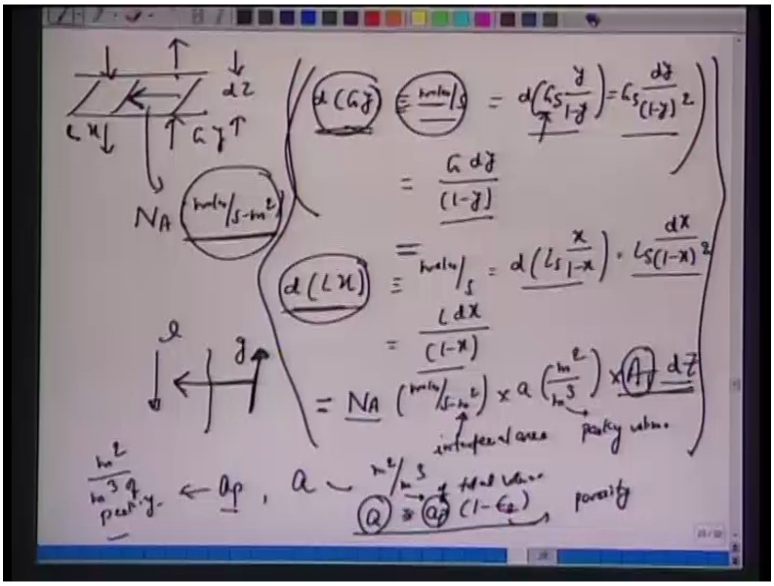

Now, we can go back and it is still we can bring the expression for N A diffusion flux. So, what is this N A now that is the missing quantity you recall this expression for N A it is still holds good this N A is nothing, but consist of two components one is bulk transport y A plus N A plus N B and one is your diffusion equations fixed law D A B d y A over d Z again be careful this Z is now your diffusion length it is not the packing length which we are trying to determine.

So, let us be careful with this diffusion length N A plus N B minus D A B d b Z in our case N B is 0 no transport of the bulk across the interface, so N B is 0 and again this similar exercise which we did in our first couple of lectures N A by N B plus will be 1 and this N A can be shown to be equal to D A B R T Z where the Z you know in it is a, it is a diffusion length not the height of the column total pressures l n 1 minus y 2 over 1 minus y 1 this we have done it after integrations you can go back and look at your class notes. Essentially, we have two planes of one and two mole fractions y 1 and y 2 and we have this diffusion length Z where we applied if you integrate N A writing down the equations at any location between this and integrate with N B equal to zero we should be able to obtain this.

So, most important here is that we recognize that again this quantity can be written as K y mass transfer coefficients into y 1 minus y 2 mole fraction difference. So, this is the place where we defined mass transfer coefficient in our earlier class where K y we write or equals D A B P T over R T, Z, l n, 1 minus y i over 1 minus y. This what we can write here K y is the mass transfer coefficients of course, this is the very general expressions, but in our case we have also said earlier that we calculate K y from Reynolds number and Schmidt number or from Sherwood number or from some correlations, all right.

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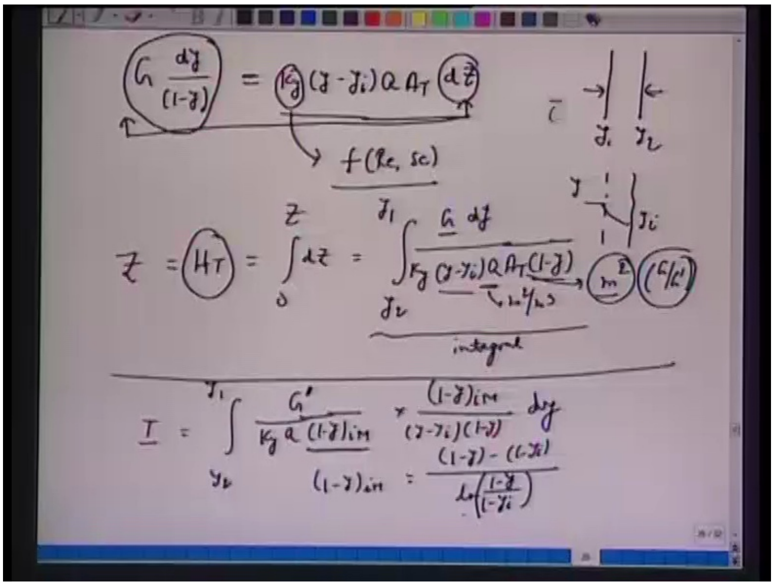

$$\boxed{G\frac{dy}{(1-y)}} = \boxed{K_y}(y-y_i)\,a\,A_T\,\boxed{dz}$$

$$K_y \rightarrow \underline{f(Re, Sc)}$$

$$Z = \boxed{H_T} = \int_0^Z dz = \underbrace{\int_{y_2}^{y_1} \frac{\underline{G}\,dy}{K_y\underline{(y-y_i)}aA_T(1-y)}}_{\text{integral}} \quad \boxed{m^2} \quad \boxed{(G/G')}$$

$$\underline{I} = \int_{y_2}^{y_1} \frac{G'}{K_y a \underline{(1-y)_{im}}} \times \frac{(1-y)_{im}}{(y-y_i)(1-y)}\,dy$$

$$(1-y)_{im} = \frac{(1-y)-(1-y_i)}{\ln\left(\frac{1-y}{1-y_i}\right)}$$

Which means now we go back and we write G d y over 1 minus y? So, this was the change in the gas phase and this equate to the flux which is K y y minus y i so, this what we have y 1 minus y 2 we have the diffusion length or diffusion length y 1 minus and y 2 in our packed columns now we have y at any locations and we have bulk concentration from bulk mole fractions y 2, y I, so we are writing y minus y i instead of y 1 minus y 2 into a cross sectional area and d Z.

So, this is the place is the equations should be able to recognize that now we are going to get the height of this column. Whatever we have to change in the gas phase this has been balance this has been made equal to this mole flux multiplied by cross sectional area or super facial area etcetera and we have written now K y, we have this introduce this K y not unknown quantity we have earlier said that this K y should be calculated from the operating conditions knowing Reynolds number Schmidt number. We can calculate this K y. So, now we have the height of this column Z or H t height of the tower equals zero to Z d Z, and then this will be equal to y 2 to y 1 remember y 1 is larger than y 2, G d y over K y minus y i a cross sectional area at A T into 1 minus y.

So, if we can calculate or we can evaluate this integral we should be able to get height of this column, area super facial or this interfacial area. Already, we have discussed this comes from the supplier the manufacture of the packing beta square per cubic meter is known to us A T cross sectional area meter square. This comes from hydrodynamics very often pressure drop considerations, which you have discussed earlier this nothing, but G over G prime that gives us cross sectional area from where we get this A T.

So, all quantities we have. we have to integrate this y or this quantity here recognize in the G changes along the height of the length of the column so, very often this is a quite complex, but one can simplify this. The same quantity, this integral i can be written as y 2 y 1. Now, we have G over A T, we can write as G prime so that is nomenclature we use kg per second per meter square cross sectional area or moles per second per cross sectional area G prime over K y a, we can rearrange to write this as 1 minus y i M. So, this is not the first time you have introduce this nomenclature here i M it is a logarithmic average, will write it. We are dividing and we are multiplying by this quantity 1 minus y i M over the previous quantities y minus y i from here and 1 minus y from here and we have this d y. 1 minus y i M it is a logarithmic average between 1 minus y or logarithmic average of 1 minus y and 1 minus y I, l n 1 minus y over 1 minus y i.

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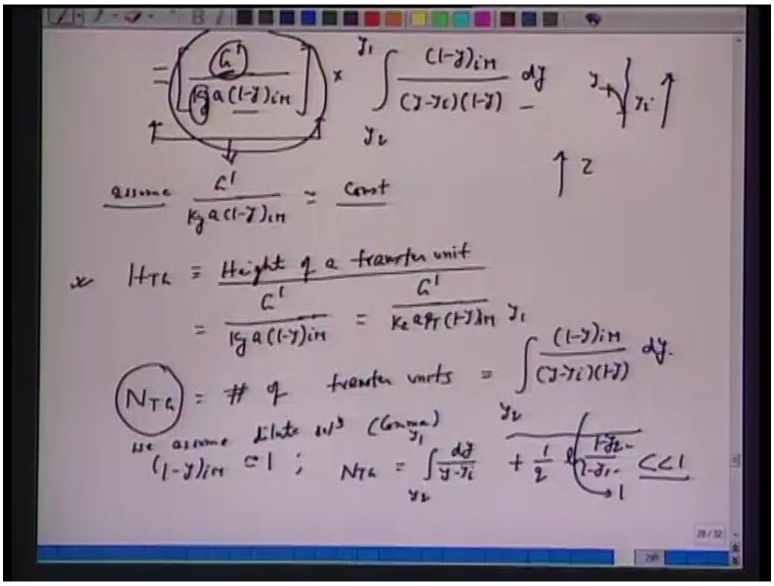

$$= \left[\frac{G'}{K_y a(1-y)_{iM}}\right] \times \int_{y_2}^{y_1} \frac{(1-y)_{iM}}{(y-y_i)(1-y)}\, dy$$

assume $\frac{G'}{K_y a(1-y)_{iM}} \simeq$ Const

H_{TG} = Height of a transfer unit

$$= \frac{G'}{K_y a(1-y)_{iM}} = \frac{G'}{K_G a P_T (1-y)_{iM}}$$

N_{TG} = # of transfer units $= \int_{y_2}^{y_1} \frac{(1-y)_{iM}}{(y-y_i)(1-y)}\, dy$

we assume dilute sol^s (Gamma)

$(1-y)_{iM} \simeq 1$; $N_{TG} = \int_{y_2}^{y_1} \frac{dy}{y-y_i} + \frac{1}{2}\ln\frac{1-y_2}{1-y_1}$ << 1

This integral can be rewritten as G prime K y i 1 minus y i M into y 2 y minus y 1, notice y 1 is greater than y 2 1 minus y i M over y minus y i 1 minus y d y, so we are talking of y and y i at any locations, there is a concentrations difference between y and y i as a driving force for mass transfer.

And most important that make a note here that we have taken out this quantity outside the integral. So, we assume that this quantity is more or less constant G prime K y a 1 minus y i M is more or less constant that is an experimental observations it remains constant along this column along the height of this column, although G changes right moles per second or kg per second of the gas phase changes, but the ratio of G prime K y 1 minus y , y also changes along this column and remember K y also depended upon y if we recall the previous expressions, but the ratios remains more or less constant here. This quantity has been taken out. Now we call this H T G height of a transfer unit so, height of a transfer unit is now G prime as K y a 1 minus y i M and this can also be written as G prime K G based on the partial pressure P T 1 minus y i M.

We are defining this quantity here height of a transfer unit we will see later what is the physical meaning of this for the time being the quantity which is outside the integral is H T G and the quantity which is inside the integral N T G is number of a transfer unit or number of transfer units. So, we have number of transfer units as N T G given as y 2 y 1 minus y i M over y minus y i, 1 minus y d y, all right. So, if you can evaluate this quantity we can calculate number of transfer units.

This integral is still quite complicated, right? So, if you look at this expression for height of a transfer unit and a number of transfer unit must recognize that y changes along this length, right? If we taken a differential elements y changes from y 1 to y 2. Mass transfer coefficient will also change go back and see that definition for K y be obtain 1 minus y i and logarithmic average of 1 minus y and 1 minus y i, interfacial concentration also changes. So, the logarithmic average also changes in other words one has to redraw the curve of y x equilibrium curve take a points x and y on the graph.

And then calculate y y i at if you locations by taking the slope of k g, k l, k y, k x, remember when we said earlier that on an equilibrium curve one can draw the operating curve as well as one can also obtain the driving force from mass transfer, if you take a point take a slope of k y k x minus k y by k x wherever it intersects that is y I, so from that we get y i one can take several such points and can evaluate this integral in principles it is very possible for this course and in fact in most of the industrial applications we have the dilute cases, right? One percent of sulphur dioxides we do not have 70 or 80 percent of sulphur dioxide or benzene in the coal gas etcetera.

Most of the time very often we have the dilute gases you know more or less maximum like one percents so if you have one percent two percent you know one can make this dilute gases assumption that simplifies all our calculations here. With that assumptions let us assume that to be assume let say that we assume dilute solutions which is of course, quite common in that case you can show that 1 minus y i M, its approximately 1. That is one thing and this integral N T G which you have written can be written as y 2, y 1, d y, y minus y i plus half l n, 1 minus y 2 over 1 minus y 1 and again this quantity is 1 because y 1, y 2 mole fractions is much less than 1 it is a dilute gases.

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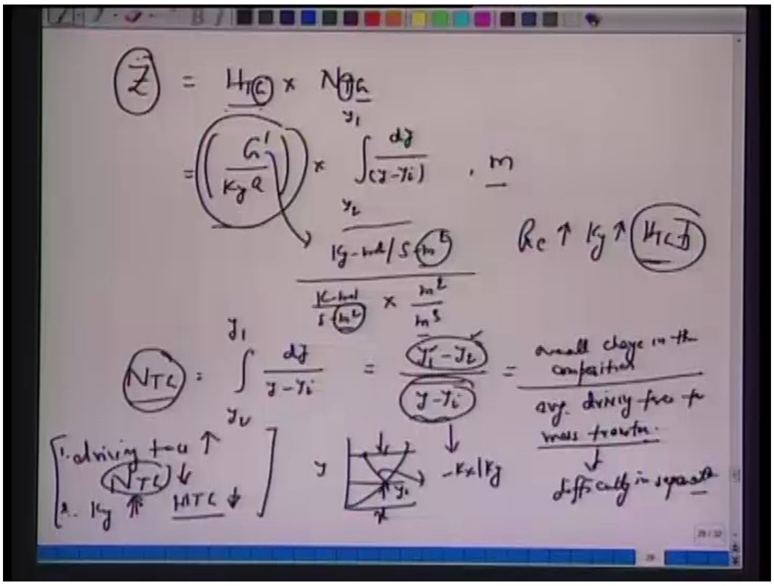

$$Z = H_{TG} \times N_{TG}$$

$$= \left(\frac{G'}{K_y a}\right) \times \int_{y_2}^{y_1} \frac{dy}{(y-y_i)} \quad , m$$

$$\frac{kg\text{-mol}/s\,m^2}{\frac{k\text{-mol}}{s\,m^2} \times \frac{m^2}{m^3}}$$

Re ↑ k_y ↑ H_{TG}

$$N_{TG} = \int_{y_2}^{y_1} \frac{dy}{y-y_i} = \frac{y_1 - y_2}{y - y_i} = \frac{\text{overall change in the composition}}{\text{avg. driving force for mass transfer}}$$

↓ difficulty in separation

$-k_x/k_y$

1. driving force ↑ N_{TG} ↓
2. k_y ↑ H_{TG} ↓

So, now we have very simple expressions for height of a column Z equals H T G multiplied by N T G, height of a transfer unit based on the gas phase number of the transfer units based on this gas phase this equals H T G multiplied by N T G where H T G is G prime K y a, so 1 minus y i is approximately 1 so, that is we have here and N T G is y 2, y 1, d y over minus y i. Now we have the quantity for N T G based on this we can get the height of this column in terms of meter, so this G prime is kilo mole or kg per second per meter square, cross sectional area and we have this K y which is also kilo mole per second per meter square of interfacial area. So, this interfacial area its meter square cross sectional area, but we have which is meter square per cubic meter of your packed bed columns. So, from here it is easy to show that you should be able to obtain the unit of meter which is height of the column and this is your dimensionless. So, we have the expressions for Z number transfer units in terms of number of transfer units and height of a transfer unit.

Let us conclude this section today’s lecture with the physical meaning of height of a transfer unit and a number of transfer units. Of course, the total height we understand we required it is a five meters to rectify or to purify our gas from y 1 to y 2, but what is number of transfer units and what is height of a transfer units look at the expression for number of transfer unit, this N T G is y 2, y 1, d y over y minus y i.

We can write as also as if we make this you know integrations we can also write as y 1 minus y 2 over y minus y i average. So, what we have? How much is the suppression here? Overall change in the compositions so, one end of the column to the other end of the column y 1 minus y 2 its overall change the compositions from 1 percent to say 0.001 percent, ten thousand p p m to one p p m that is y 1 minus y 2 across this column and what is y minus y i it is nothing, but the average bar for the average driving force for mass transfer and how do we obtain up y i recall on this equilibrium curve y and x, we draw this point take the slope of minus K x over K y.

So, at any location if this is x and y and take the slope this will hit to give you y i that is y minus y i is a driving force for mass transfer in the column. So, we have number of transfer unit as a ratio of overall change in the compositions per average driving force for mass transfer its quite natural that if driving force increases number of transfer unit will decrease. We require only four units five units instead of ten units if the driving force is much larger so, that is one thing for a given compositions so, y 1 y 2 generally are known now a number of transfer unit depends upon how large is the driving force.

If we have a small driving force we require larger number of transfer units and look at the expression for height of a transfer units, if mass transfer coefficient is very large, , then for a given Z height of a column will also be a small that also very consistent with the physics of this problem will come back to this again and again to take an example to see the meaning physical meaning of number of transfer units it is nothing, but difficulty in separations, larger number of units shows that there is a driving force for mass transfer is a small. Similarly, height of transfer units if the Reynolds number is very large mass transfer coefficient will be large and height of a column will be smaller.

For a given suppressions we require a very small column to transfer or to cause a change in the compositions across any differential elements or across a transfer units and two together multiplied by this gives you height of a column, thank you.