Fluid Mechanics

Prof. Viswanathan Shankar

Department of chemical Engineering

Indian Institute of Technology, Kanpur

Lecture No. # 09

Welcome to this lecture number 9 on this N P T E L course in fluid mechanics for chemical engineering undergraduate students. In the last lecture we started discussing a new topic and the topic relates to description of motion in fluids.

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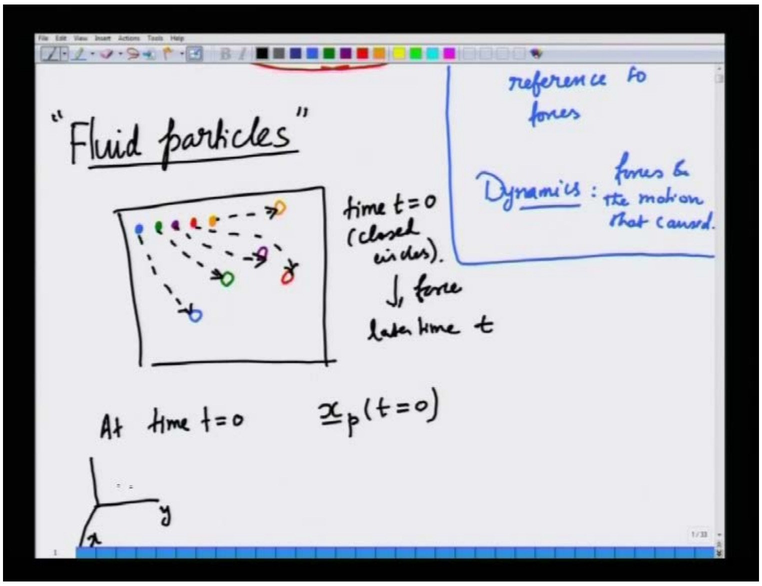

So, this topic is called fluid kinematics or flow kinematics, as I told you in the last lecture in any branch mechanics there are two aspects to it, one is dynamics and the other is kinematics. Kinematics refers to description of motion without reference to forces that cause the motion, these two forces and dynamics is the next branch which will come to little later. Where we worry about the forces and the motion, that are caused by applied forces that are caused by forces. So, the first job in any mechanical subject is to understand how to describe flow (( )). So, in this topic we are going to discuss kinematics and we are going to just describe flows how to describe motion in fluids.

So, in this context we introduced the notion of what is called a fluid particle? It is use full in the continuum hypothesis to identify what are called fluid particles, these are not real particles as in an as in case of molecule of a fluid, but they are high idealized hypothetical objects which are useful in describing flows. What are these fluid particles? Well imagine you have fluid the container of a fluid and a time t equal to zero there is no motion in this fluid the fluid is static. And let us say you can mark various points in the fluid using various color dye. So, I am just let me draw it slightly bigger so, that it is clearer.

So, you can mark various points in the fluid using various colored dye and this is a time t equal to 0 so, in principle you can do this for a continuously for all points in a fluid, because of fluid is a continuous medium within the continuum hypothesis. But for the sake of illustration, I am showing few points. So, what this points will do upon this is a time t equal to 0, I am using closed circles and at the later time upon application of force this some kind of force we need not worry at this point what causes the motion the some kind of force it causes a motion. At a later time t, all these points would move to some other positions in general. So, I am using open circles to denote these locations of these points at a later time.

So, this point for example, can move here this point could have moved here, this point could have moved here, this point could have moved here, and this point could have moved here. So, that we are assuming that the dye molecules are not diffusing or the diffusivity of the dye molecules are so small, that for our time scales of inter estimate material that the dye is diffusing. So, the dye faithfully represents a point in a fluid to the extent that you can resolve a point by the help of a dye molecule with the help of the dye molecule dye drop. And, this dye drop which you are using to identify a point will in general evolve in time up on application of forces, because of the fluid is moving.

So, this is roughly realization of this mathematical or abstract idea of fluid particle so, in principle you can identify. At time t equal to 0, the position of various particles so, with respect to our co ordinate system, as usual; whenever you analyze any problem in flow fluids, you have to put a coordinate system with respect to co ordinate system.

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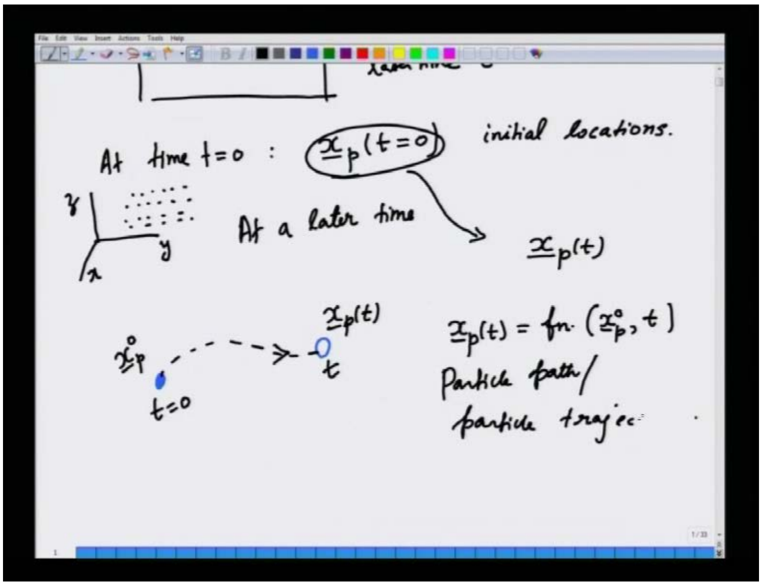

You can label various points at time t equal to 0. How do you label the points instead of using a dye molecule as I argued in this thought experiment, we can label each point by their initial locations, the idea is you are identifying each particle fluid particle on the basis of the initial locations with the respect to your co ordinate system. And, at a later time, what will happen is that, these initial locations the points that are identified by the initial locations will in general evolve to a current location. So, schematically, I will take one point and then at later time this will be like this. So, the trajectory of this particle at time t equal to 0 is here and then a time t is here.

So, this particle may move like this and at a later time all this so, this is the position at time t equal to 0 is denoted as x p 0 and this is the position of a fluid particle at time t. So, the position at time t will of course, of a particle will be a function of where it was time t equal to 0. This is called and of course, times because this is you have to follow this particle and at any time the location that this fluid particle occupies at time t will be a function of time itself as well as, where it was at time t equal to 0. This is called a particle path or a particle trajectory.

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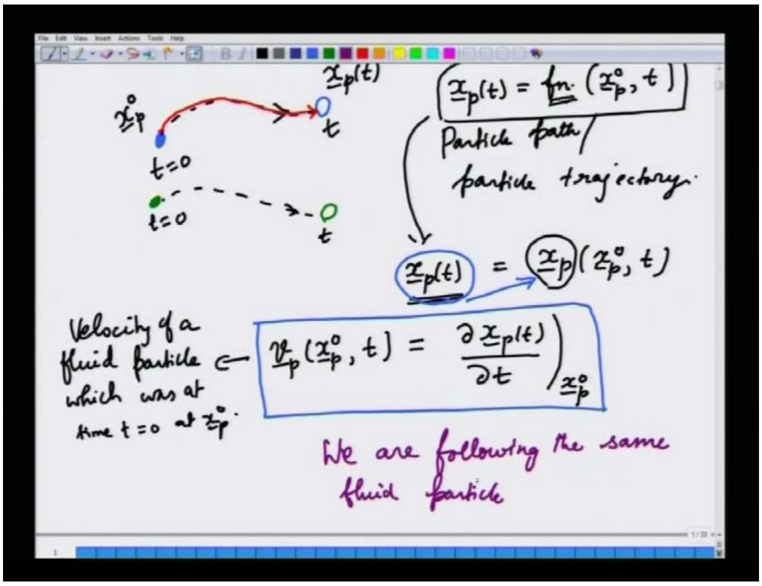

Now, what is the use of having such information? Suppose, you have to experimentally measure such information and you can do it for not just one point as I have shown here, but you can do it for many points. So, another point will generally in general move like this so it can so this point is at a slightly different initial position and the later time of course, it will occupy different position. But in general the idea is you can label all points based on the initial positions. You can follow the motion and represent it mathematically in this form. Usually, this function and form is written as x p at time t, is a function of x p 0 and time so, instead of writing a function like this you use the variable itself as a function so, this is a concise notation.

So, once I have this information and I can find what is the velocity of a particle, which was at time t equal to 0 at x p 0 at a later time t this is the velocity of a fluid particle, which was at time t equal to 0 at x p 0. Velocity from the fundamental definition in mechanics is that rate of change of position. So, we will have to simply take the position that the particle trajectory, which is given by this functional form, takes the particle trajectory and differentiate to time. By keeping the initial the particle label concept what you are keeping instant, that you are following the same particle and then you are measuring its trajectory as well as you can find quantities like velocity.

So, what is being kept constant in this time differentiation is very important, what is being kept constant is that the initial position of this particle or the particle label is kept constant in other words.

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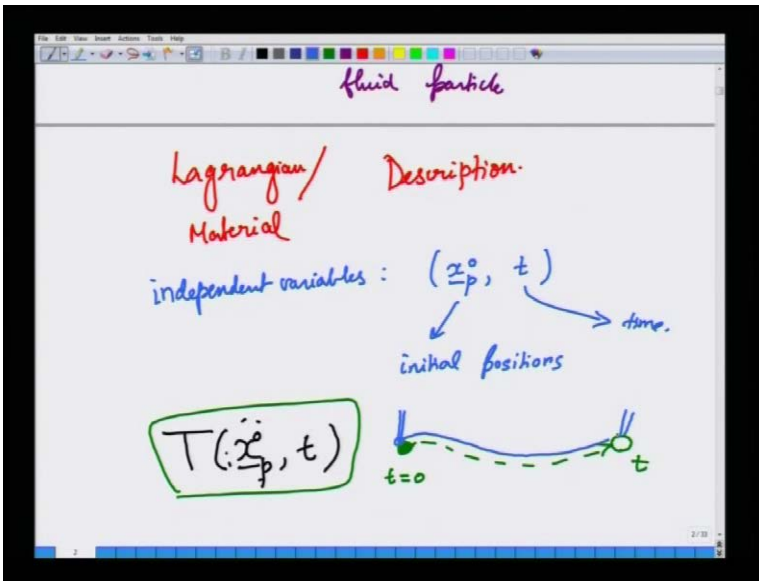

We are following the same fluid particles, that is the meaning of keeping x p naught constant. This description is called fluid motion is called the Lagrangian or some time it is called as material description. So, what this description means is that you are following properties of a fluid by following the position of each and every particle as fluid is moving. So, the independent variables in the Lagrangian description are the initial position of the particles and time t. So, initial positions these are the particle labeled remember that the particle labeled mathematically, but with the help of the initial position and this is time.

So, for example, I need not just worry about purely kinematic quantities like velocity or position, but I can also say things like temperature as the function in the Lagrangian description. The temperature of in a fluid flow will be depicted as the function of position in the initial position of fluid particle and time t so, what it means, physically is that suppose the particle is here at t equal to 0. And it is moving later on to time t, to some other location what this Lagrangian description of temperature means is that you are attaching a thermometer to the particle.

And you are moving along with the particle and you are recording the temperature in the thermometer as you follow the particle so, that is the meaning of keeping x p naught constant, you are attaching yourself, tagging yourself along with the fluid particle for example, we are interested in measuring temperature. So, we put a thermometer in our minds along with the fluid particle and measure the temperature of the fluid particle as you go along with the fluid particle. So, this is the Lagrangian description or Lagrangian temperature field.

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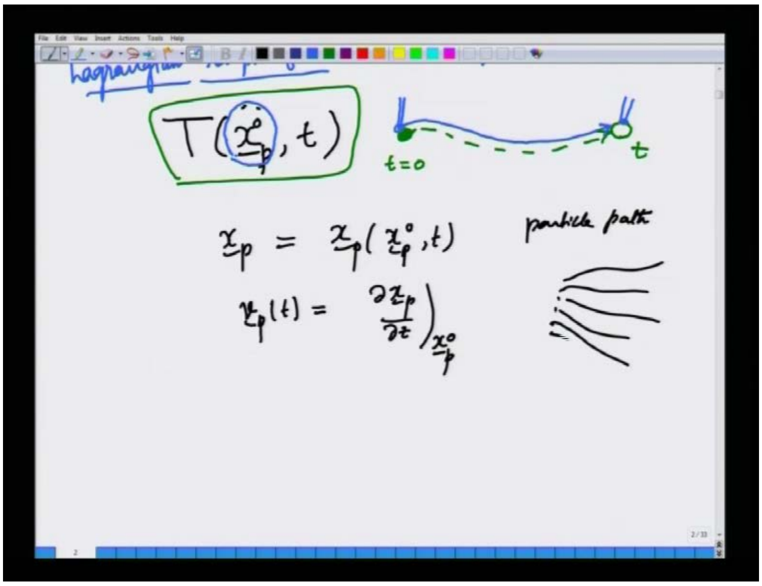

So, what is the advantage of Lagrangian description? We have the motion, the current position of particle as a function of it is initial position and time once, I have this is called the particle path or particle trajectory or simply the motion of the particle. Now, I can calculate the velocity of the particle at the later time by taking the partial derivative of the particle trajectory by keeping x p naught constant. Because x p is the function of x p naught and time so, various points in the fluid will move differently to various other locations. So, if I want the velocity of this particle I have to simply follow this particle at time t and then take its time derivative at time t. So, t is this is the rate of change of particle is kept constant the rate of change of position with respect to time at a time t. Now, this is the velocity of a fluid particle.

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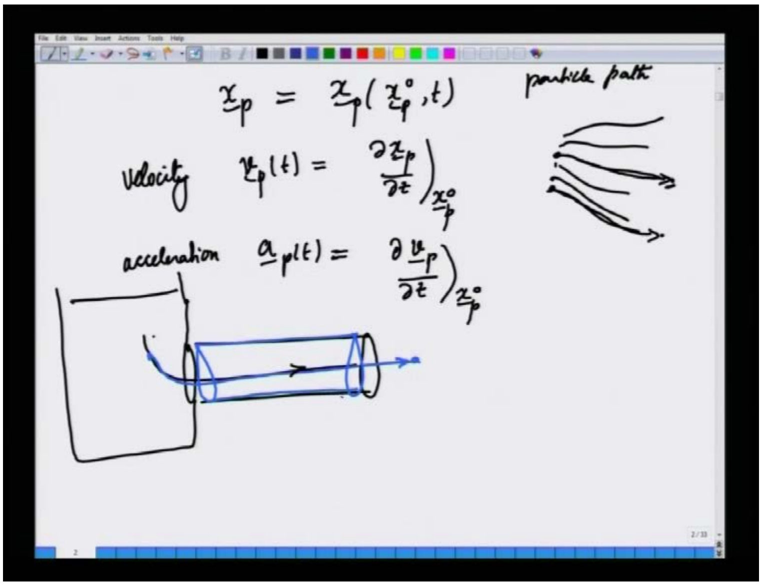

Now, acceleration in mechanics or kinematics is simply the rate of change of velocity that of a particle so, that is also very simple. Simply take the rate of change of velocity of particle, by keeping the particle identity to be the same, that is you are following the same particle and you are finding the rate of change of its velocity that will be the acceleration. Now, while this is a very similar to what is being what is normally done in Newtonian particle mechanics of point particles and objects, this is not really suited ideally for fluid flow problems, because in a fluid for example, if you are interested in flow in a pipe.

So, this pipe will be in general connected to some reservoir, it is figuratively shown like this so, if you are interested in pressure drop across the pipe that is required to pump a specific flow rate. Then you are not truly varied about various fluid particles, the identity of fluid particles that are entering and leaving. Because all you are interested in is what is the force that is being experienced by this pipe what is the drag force and consequently what is the pressure drop? So, the Lagrangian description where in we follow the same particle as a function of time is really not very useful especially, when you are considering fluid flow problems.

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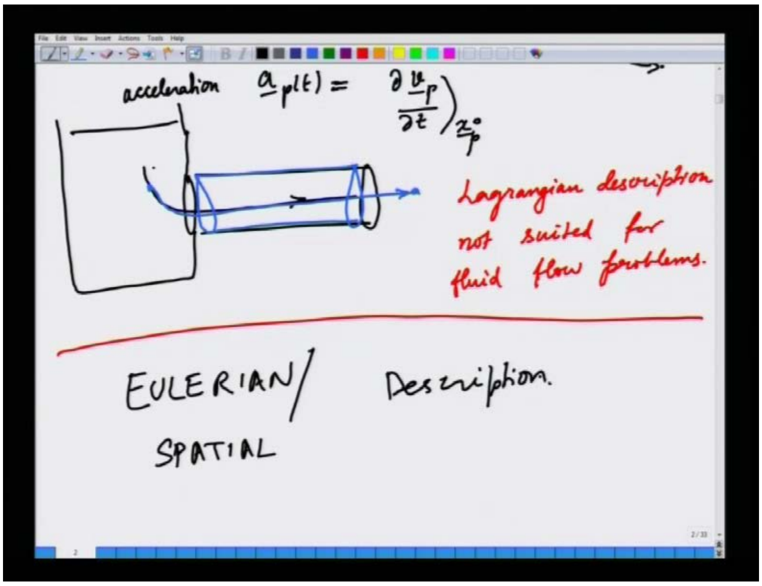

So, instead of Lagrangian description what is normally followed? So, the Lagrangian description so, let me just write this not suited for fluid flow problems. So, what is normally done in fluid mechanics is what is called the EULERIAN description or SPATIAL description of motion so, what this we mean is the following.

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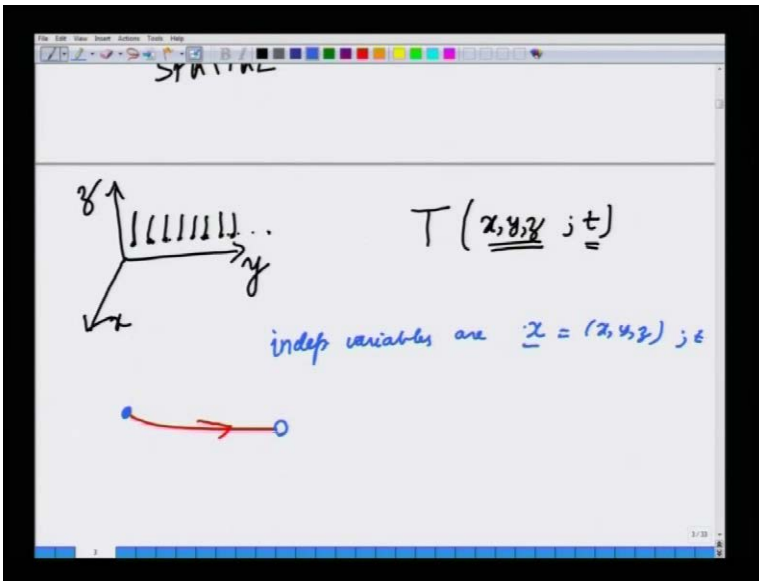

In the Eulerian description we again put the coordinate system x, y, z with respect to which we measure coordinate laboratory coordinates system let us say the coordinate system is fixed in the lab. Now, here quantities such as temperature are measure as a function of the three fixed coordinate x y z and time so, what we do here is that you take a thermometer and then keep the thermometer at like given point, then move the thermometer to various points and then keep measuring the temperature. So, if you want the temperature at a given time so, you have to if the temperature is changing as the function of both x y z as well as time, then what you have to do is you have to put many thermometers at various locations.

And each thermometer it wills each thermometer will locater will read will indicate the temperature at that location as the function of time. So, the in the Eulerian description the independent variables are the spatial position of a point x y z and time. So, what is the fundamental difference between Lagrangian and Eulerian descriptions in a fluid flow problem, in a fluid flow context? Suppose, fluid is flowing and in the Lagrangian description you will follow this material point or fluid particle and then you will measure its temperature as a function of time.

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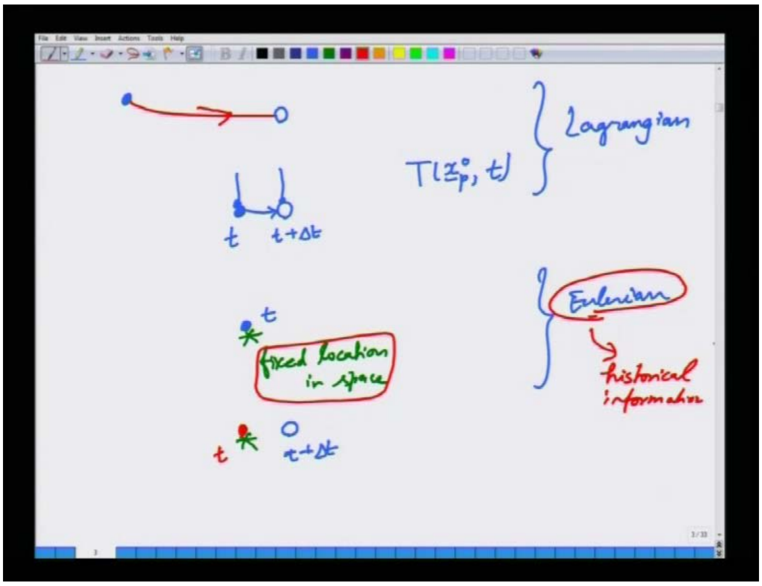

Suppose, you are just worrying about a small time interval t and little later to t plus delta t. In the Lagrangian description if you put a thermometer, this thermometer will measure the temperature of a fluid particle which was here at t time t and which was here at a time t plus delta t. So, you are keeping x p naught constant and you are measuring temperature as a function of time so, this is the Lagrangian. In the Eulerian description in contrast suppose, you have well let us fix the location suppose you fix the location so, this is the fixed location in space at time t. Let us say this location is occupied by a particle colored with blue that is this particle was residing at this point at time t, but at a later time what is going to happen? So, this let me just write that this is Eulerian.

At a later time what is going to happen is that a same location which is denoted by a star here; this point will not will locate the blue point will not be locating will not be residing at the same location. Because at a later time this blue point in general moves to, this is a time t at time t plus delta t, but this location will be occupied by some other point at time t so, the thermometer at a fixed location in space records. It does not record the history of a same fluid particle rather it records the temperature is same the temperatures of various fluid particles that happen to be at a given location at various times.

So, the sense of history of temperature history or velocity history or that this historical information of a given fluid particle is lost. Historical information is lost, because we are not following or tagging along with the same fluid particle instead, we are sitting at a same point and we are merely measuring the temperatures that various fluid particles are going to occupy, that point at various times.

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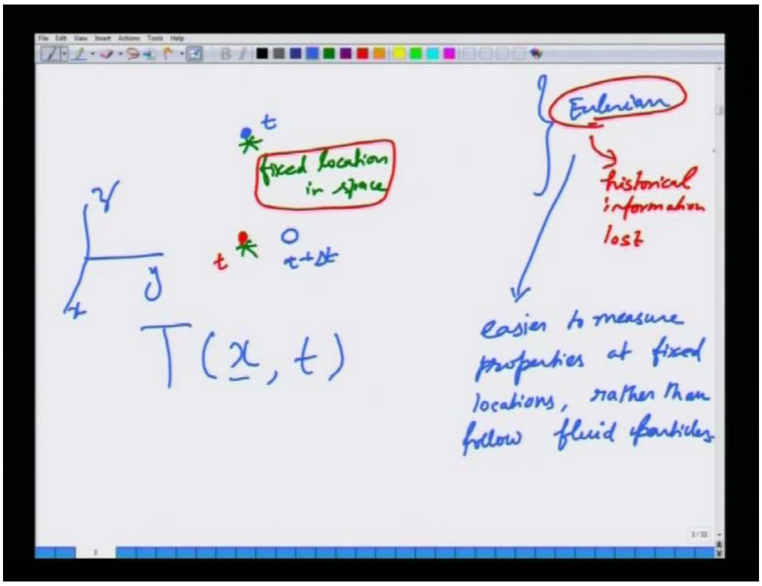

But, the Eulearian description even though it has lost by its construction it has lost the sense of history of information of a given fluid particle. But it is still very useful, because in laboratory it is easier to measure temperature, easier to measure properties at fixed locations, rather than follow fluid particles or material particles. So, in fluid flow problem there are two reasons why Eulerian description is preferred over Lagrangian description. Firstly even if it was feasible having the information about what are the various fluid particles that are occupying let us say a given section of pipe is not relevant to many practical questions, such as what is the pressure drop.

Because here we are not really worried about which fluid particle is coming and exerting a drag force. We are merely interested in the force that is experienced by involves of the pipe or shear that is moving or on so. So, the historical information is not practical importance in general, in fluid practical applications. And also even if you want to measure such historical information in the Lagrangian sense it is not easy to measure in lab, because you have to really follow the same fluid particle and it is not easy. Rather, it is easier to fix probes such as velocity probes or pressure probes, temperature probes at a given point, in special location or at various fix points in a spatial locations rather than moving along with a particle.

So, in the Eulerian description the independent variables are the fixed locations of various points in space and time so, the Eulerian description is very easy to measure in laboratory.

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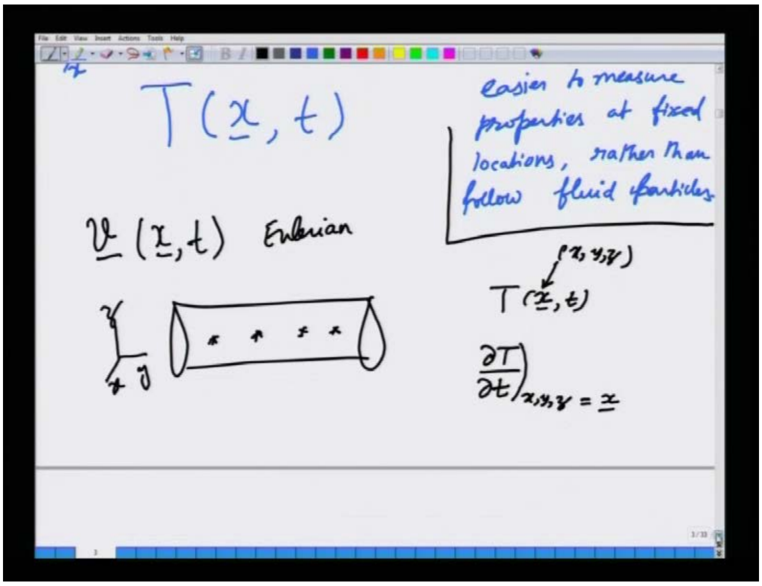

So, if you are interested in kinematic quantity such as velocity of a fluid flow is described in the Eulerian description as a function of various points in space and time suppose, you have a flow in a pipe and you put a coordinate system x y z. So, you can sit at this point and measure its velocity as a function of time, then change the location of observation and then or you can put multiple probes for velocity and then measure the velocity at various locations, fixed locations in space as well as time. That is the main crocks of Eulerian description. But the there is the problem with Eulerian description in the sense, that suppose; I have this information temperature as function spatial location and time. And suppose, I take this partial derivative temperature is a function of x vector is a combination of 3 variables so, x y z and time.

So, when I take when I say partial temperature by partial time, I am keeping the location x y z constant, I will figuratively denote this as vector x.

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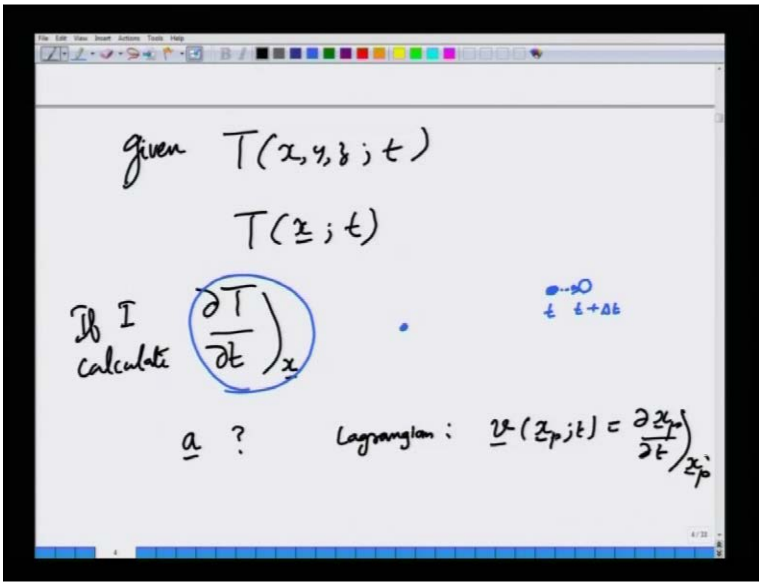

So, suppose, I have given this given temperature field let us say from an experiment so, the temperature field is given as T x y z time or in short form I will simply write this as T x t given this information. Suppose, if we calculate if I calculate at a spatial location, this is not telling me how the temperature is changing so, this is the partial derivative so, if this fluid particle is moving from time t to t plus delta t, its temperature can in general change as you follow the particle. But this is not giving that information what this derivative giving you is that if you sit at a point what is, that rate of change of temperature that you will feel at that point locally.

So, the sense of history is lost in the Euleruian description, while it is not critical in probably applications like temperature suppose, you are interested in kinematic quantities like acceleration. What is acceleration? In the Lagrangian description, if you recall, this is basically the rate of change of its position by keeping the label of the particle constant.

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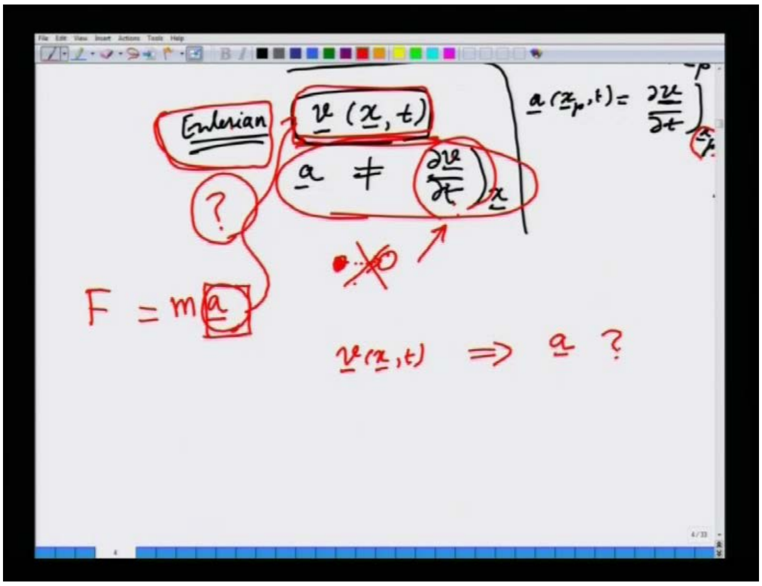

But in the Eulerian description v is written now not as a function of the initial positions, but rather the spatial locations in a coordinate system. So, first of all I do not have this particle path information. Even, if I have the velocity information in the Eulerian in the sense, I cannot calculate acceleration as partial v partial t, because now I am not keeping I am not following the same particle, I am merely sitting at a same particle point in space. Where as in the Lagrangian description, acceleration of the particle is the rate of change of it is velocity, because you are following the same particle. So, what is being kept constant is this so, in the Lagrangian description it is very clear; what is acceleration, because you are merely keeping the initial location of the particle constant you are following the same particles.

But in the Eulerian description, if you have the velocity field like this you cannot compute acceleration by simply taking the partial derivative. Because this does not have information as to how a given particle is moving, given particle is moving as function of time this information is not there in the Eulerian description. So, we cannot compute accelerations from the Eulerian velocity fields. So, you may ask why is this is an issue the reason why this is an issue is that when you want to eventually go to dynamics you are going to apply the Newton's second law of motion to continues fluid. The Newton's second law of motion says that the force on a particle an identifiable piece of matter this mass times acceleration.

So, we need the acceleration when you want to eventually write down equations of motion for the flow. But if you also want to work simply with the Eulerian frame work, because it is much simpler it is more useful. But fundamentally we need accelerations so, we need acceleration, but acceleration cannot be obtained from velocities, like this. So, how do I connect the acceleration to the velocity field Eulerien velocity field? That is the question is given the Eulerian velocity field v as a function of x t how do I compute accelerations? So, there is a very nice frame work for doing this and this is the reason why we introduce the Lagrangian description although we are not going to use the Lagrangian description.

We do need the motion of the Lagrangian description in order to compute acceleration of fluid particles. So, instead of doing this for acceleration I am going to illustrate this for a derivative like temperature.

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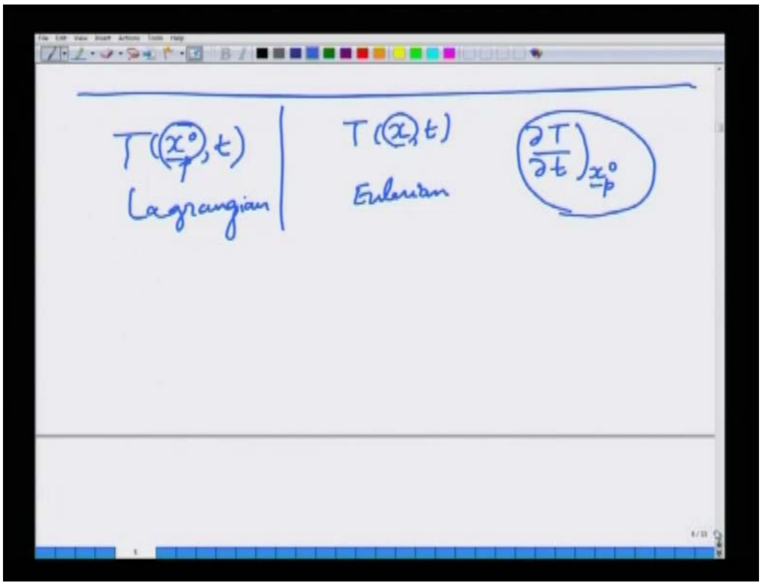

That is the question is given the spatial description or Eulerian description of temperature how do I compute partial T as I follow a particle, this is the Lagrangian time derivative. See notice, that the Lagrangian description and Eulerian description in differ merely by what are independent variables. This is the Lagrangian description, this is the Eulerian description so, they change not only by the independent variables that we chose to describe the problem with. So, I am going now, do this thing called Substantial or material derivative.

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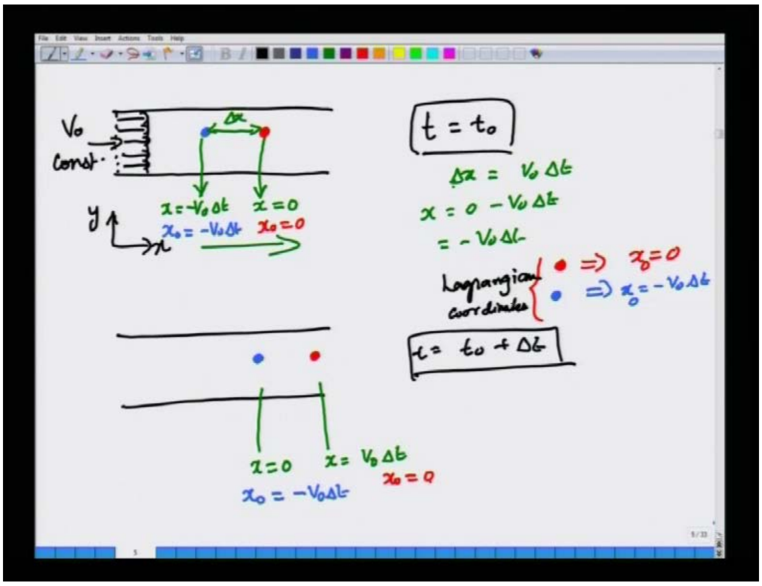

This will help us to calculate the time derivative as we follow particle from and Eulerian description. So, to motivate this it will take a very simple context so; imagine you have a channel in which fluid is flowing with constant uniform velocity. The velocity is constant in the sense that suppose, you call this x and y the velocity is constant in the y direction it is not realistic, but this is just for the sake of our illustration. So, you should do not vary about this part that y the velocity is uniform let us assume that the velocity is largely uniform. Now, let us imagine that there is a location so, let me introduce a location this is x equal to 0 and here a fluid particle, let us focus on fluid particles blue and red.

So, here there is a red fluid particle at time t equal to t 0 let us say we had a time t equal to t 0. And then we have another fluid particle which is blue in color at the location x is minus v naught delta t since; the fluid is flowing at a constant velocity. And so, we are looking at two fluid particles, which are separated by distance delta x and that is delta x is v naught delta t, where delta t is a time interval, that we are going to introduce at just shortly. So, imagine two fluid particles identified by their colors blue and red and we are following the motion of these two fluid particles as a function of time.

So, this is the time t equal to t 0 this is the situation the red particle is situated at x equal to 0 and the blue particle is situated slightly behind and since it is x is positive in this direction. So, this is at a distance minus delta x and since delta x is v 0 delta t x is 0 minus v 0 delta t so, this is minus v 0 delta t this is slightly behind the red particle. Now, at a later time, imagine after time t naught plus delta t what would happen? Let us try to draw this is x equal to 0 this is x equal to v 0 delta t. Since, the fluid is moving this particle will move eventually so, the red particle move from 0 to v 0 delta t and the blue particle will move from minus v 0 delta t to x equal to 0, this is a time t equal to t 0 plus delta t.

So, let us mark also at this point the Lagrangian labels of this particles is that initial position at time t equal to 0 so, the red particle is denoted by x equal to 0. That is the position of the particle at time t equal to t naught and the blue particle is denoted by x equal to minus v 0 delta t. So, the Lagrangian so, x naught I mean sense Lagrangian variables are denoted with this is the position at time t equal to t 0.

So, this particle while it is present it is also x naught is minus x naught is minus v 0 and x here the red particle x naught is 0. Now, even at a later time t 0 plus delta t this blue particle is still denoted by the same Lagrangian label, because the Lagrangian description uses the position of the particles at an initial time, let us say t naught, to label them. So, x naught is still minus v 0 delta t and x naught for the red particle is still 0. So, this is the initial position of the particle which is currently at x equal to 0 this is the initial motion of this red particle, which is currently at x equal to v 0 delta t. So, these are the Lagrangian coordinates these are the Lagrangian labels or coordinates.

Now, let us say we are having, if we are measuring temperature at this point we are measuring temperature at this point x, x equal to 0. So, we are measuring temperature by putting a thermometer at x equal to 0, and so here as well so, we are placing the thermometer at x equal to 0 this is the thermometer. What this thermometer is measuring as a function of time? At time t equal to t 0 it will measure the temperature of the red particle, while at time t equal to t 0 plus delta t it will measure the temperature of the blue particle this is the key to our derivation.

So, it is very simple, because the thermometer is fixed at the same spatial location x equal to 0, which I am denoting by this green color. But the points that are occupying the same spatial location at different the material particle that are occupying the fluid particles are different, because the fluid is continuously flowing.

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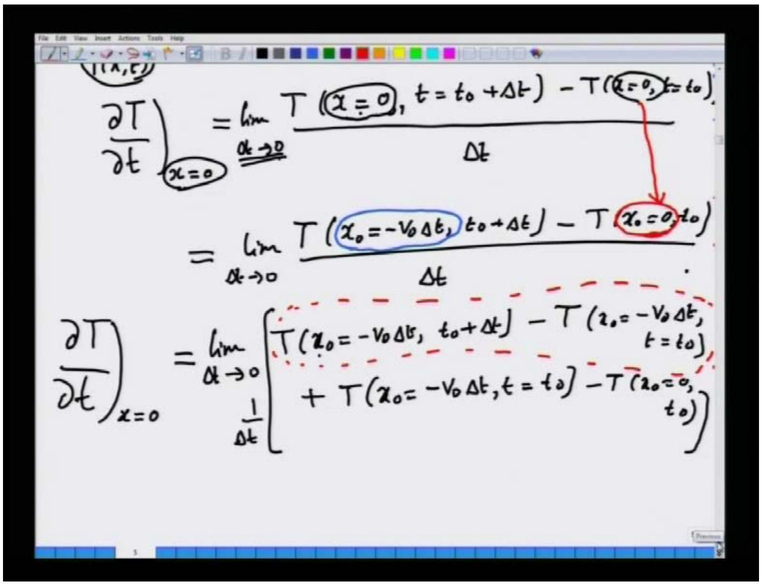

So, let us understand what is what we will do by calculating at the partial t of the temperature with respect to time at x equal to 0 at the same location. What is this is from fundamental definition of calculus this is t 0 plus delta t minus x equal to 0, t equal to t 0 divided by delta t in the limit as delta t goes to 0 this is the fundamental definition of partial derivative. Since, it is a constant we are keeping x is constant at 0 so, this label is the function of two variables t is a function of x and t is Eulerian description since, x is kept constant at 0, we see simply have to take the derivative with respect to time. Now, let us try to understand what this means, at x equal to 0 and t equal to t 0 plus delta t let us look picture, this is time t 0 plus delta t at x equal to 0 the particle that occupied is the blue particles.

So, the temperature that the thermometer will measure is nothing but limit delta t tends to 0 temperature of the blue particle which is identified by its Lagrangian variable. So, which is x naught is minus v naught delta t and the time is t 0 plus delta t minus here, the temperature at x equal to 0 time t equal to 0 is namely that of the red particle. Because at x equal to 0 at time t equal to 0 notice that it is the red particle that is occupying and the thermometer will measure at time t equal to delta t, t zero the temperature of the red particle. And the red particle is identified by its Lagrangian variable, which is nothing but, t x naught is 0 at time t 0 divided by delta t. So, essentially we are trying to measure the temperature at the given spatial location in this example, in this illustration.

So, we are putting thermometer at the position x equal to 0 the same positions same spatial location with respect to this coordinate system. And but at x equal to 0 at time t is equal to t 0 the red particle is occupying the location spatial location x equal to 0. At a later time, the same spatial location x equal to 0 is occupied by the blue particle. So, when you take this measurement and when you take the partial derivative of the temperature using these measurements. Partial derivative of temperature with respect to time is nothing but the partial derivative of temperature is limit time at that spatial location x equal to 0 is temperature at later time minus temperature is t 0 divided by delta t as delta t goes to 0.

But the key realization that we must have is that the temperature at x equal to 0 at later time corresponds to that particle which was there at a later time which is near to the blue particle. Now, the blue particle is denoted by it is Lagrangian labels x naught is minus v 0 delta t. Now, the temperature at x equal to 0 at time t 0 is due that of the red particles so, we can change from x equal to 0 x naught equal to 0, because the red particle is identified by it is Lagrangian variable which is nearly x naught equal to 0. So, this is the key realization when going from Eulerian to Lagrangian so, that we can change the labels from Eulerian to Lagrangian by knowing which particle was occupying the current position and the previous position and so on.

So, having done this, we will just do simple we are still having on the left side the spatial derivative the Eulearian time derivative of the temperature field. So, I am going to do a small mathematical simplification by adding and subtracting. So, let me write first two terms x 0 minus v 0 delta t time is t 0 plus delta t let me subtract time temperature at x o is minus v 0 delta t and t is t 0, and add the same thing again x 0 is minus v 0 delta t, t is t 0 minus T x 0 is 0 and then t 0 and then divided by delta t so, let we put 1 over delta t here that is of course stays. Now, here we are keeping x 0 the same.

So, what is this term? So, let us let me mark this with red so, that you can see what is this term? This term is nothing but the red term, that here is nothing but so let me write this from separately the red term is nothing but this write in red color so that it is clear.

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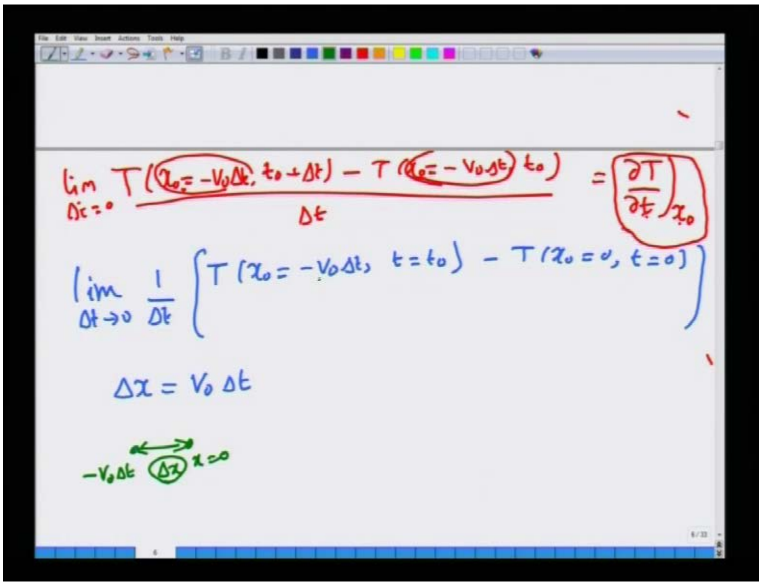

T at x naught is minus v 0 delta t at time t 0 plus delta t minus T at x naught is minus v 0 delta t at t 0 divided by delta t, that is of course there here common and limit delta t going to 0 is nothing but see here we are keeping x naught constant so, this is nothing but the time derivative as you keep x naught constant. So, this is the time derivative as we keep x naught constant, this is the Lagrangian time derivative as you follow the same particle. Here originally, we are measuring the time temperature at the same location, but here this part of this expression corresponds to the rate of change of temperature with time as you follow the same particle, because the particle is being fixed here.

So, this is what we are often we want to calculate the rate of change of temperature with time, as you follow the same particle. But, there is one more piece here which we will have to tackle so, what is that piece let me write it in blue color. So, we still have this additional piece limit delta t tending to 0 one over delta t T x naught is minus v naught delta t, t equal to t 0 minus T at x naught is 0 at the same time divided by delta t. Now suppose, you consider the change in position delta x, delta x is nothing but v 0 delta t recall that this point, the 2 points x the 2 points is separated by at distance delta x, x equal to 0 and this earlier distance which is nothing but minus v naught delta t, that so, delta x is separating distance is v 0 delta t that is so, delta x is separating distance is v 0 delta t. So, this is nothing but we can write this as so, one over delta t.

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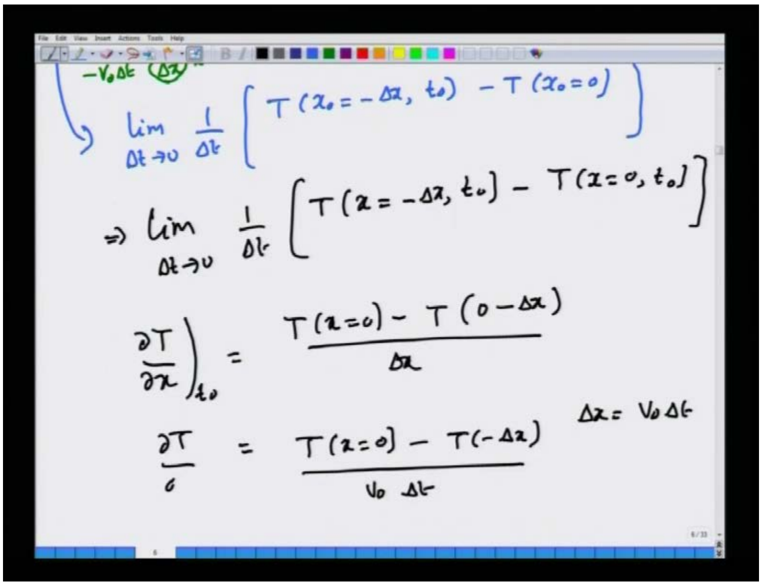

T x naught is minus delta x t 0 minus T x naught 0. Now, if you look at what is partial derivative of partial t partial x at x equal to 0 you can write this as T at x is 0, but we can also before I do that we can also change the labels now. When x naught is 0 x is also 0 this is nothing but so, let us go back to the figure in the previous slide. When x naught is 0 x is 0, when x naught is minus v 0 delta t x is also minus v 0 delta t. So, we can change this to the Eulerian description as limit delta t going to 0 1 over delta t. When x naught is minus v 0 delta T x is also minus v 0 delta t, t 0 minus T when x naught is also 0, x is also 0 at t 0 this is what we have.

But if you recall what is the fundamental definition of the rate of change of temperature with respect to position at a given time? Let us say this is T at x equal to 0 minus T at x equal to 0 minus delta x divided by delta x, but in our case delta x is nothing but v 0 delta t. So, I can write this as so, instead of delta x and write v 0 delta T here so, T at x equal to 0 minus T at minus delta x divided by v 0 delta t this is partial t, partial x at constant time.

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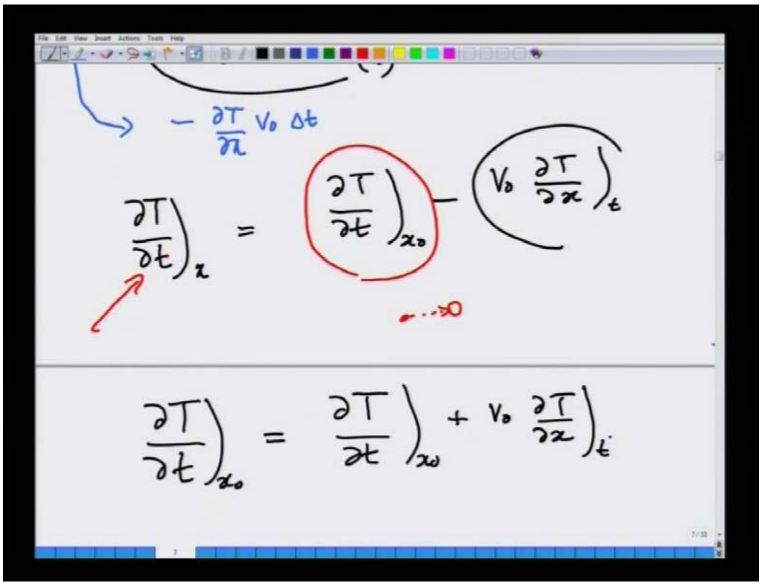

So, I can pull this v 0 up here and realize that what I have here in this expression is nothing but I have here T at x equal to minus delta x t 0 minus t at 0 this is nothing but minus partial T by partial x times v 0 delta t. So, if I go back to my original expression where I have two terms, if you remember let us go back to this expression I have two terms. So, let me just simplify here itself partial T, partial of temperature with respect to time at x equal to 0 so this term is simplified as so, we have still one over delta t, limit delta t going to 0 well we are taken the limiting process. So, let us remove the limits now this is nothing but you have the first term we already simplified it in to partial T, partial t at x equal to 0.

The second term is now simplifying to minus v 0 delta t partial T partial x. Now, what we are after is naught so, let me just write on this result once again partial T partial t at constant x is nothing but partial T partial t at constant x 0 minus v 0 partial T partial x at constant time. So, what we are after is this term, because this is something that is easier to measure experimentally where this is something is difficult, because this is the rate of change of temperature as you follow a point, where as this is a rate of change of temperature at a fixed point.

So, finally we have rate of change of temperature when you follow a particular fluid particle is equal to rate of change of temperature at a fixed point this term will go to the other side, if you take the negative sign to the other side, becomes positive at constant time. I am sorry this is x now, because this is in the other side this is x.

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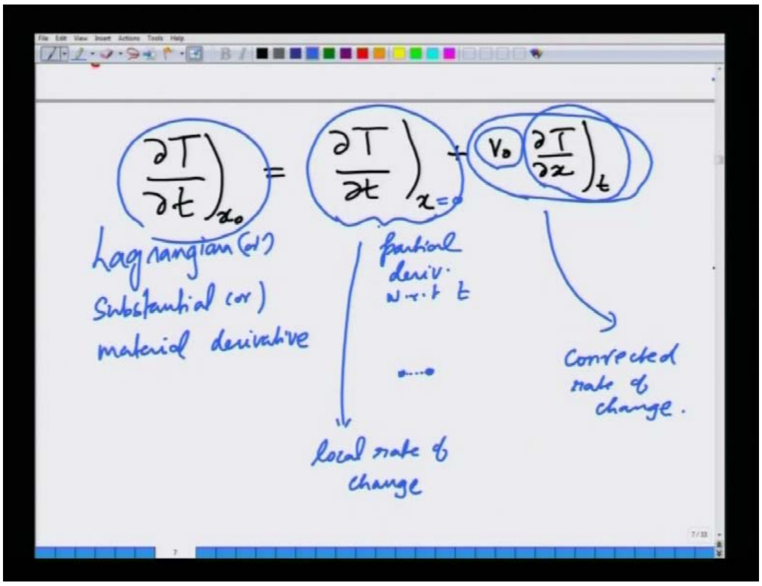

So, this is called the Lagrangian or substantial time derivative or in fact sometimes it is called the material derivative this is the usual partial derivative of time with respect to time. When I take partial derivative of temperature with respect to time, I have to keep the spatial location constant so, this is the spatial location at a given point space .We can call it x equal to 0 in this example but in general it can be x now, this one is what is called the convicted rate of change it is called the convicted rate of change of temperature. So, here what this is telling is that particle that was here will move from one location to another by virtue of flow that is where this velocity is coming in. And the temperature difference is going to field is given by the Eulerian spatial derivative of temperature.

So, this is essentially that temperature difference between these two points as given by the Eulerian description. And, if you multiply by the velocity of the particle by the velocity at which the particle is moving, then that is going to give us the convicted rate of change. So, this is sometimes called the local rate of change, this is called the convicted rate of change. So, in general therefore, we can write so, this is the Eulerian derivative this is the substantial derivative this is the normal partial derivative, sorry this is the Lagrangian derivative, this is the normal partial derivative and this is the connected rate of change.

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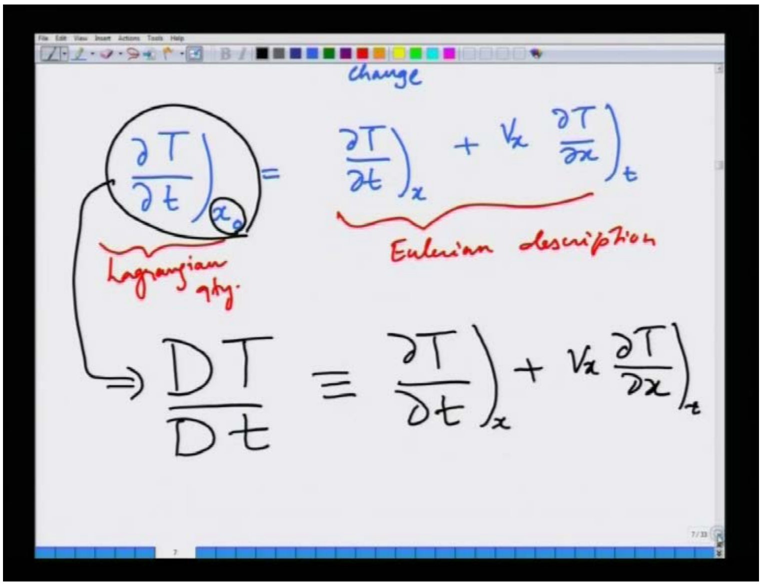

So, in general, the rate of change of any property like temperature as a function of time for fixed Lagrangian particle is given by the rate of change of temperature with respect to fixed point in space. Let us still keep x as a single variable, I will generalize it to more three dimensions shortly plus the velocity in the x direction the times partial t partial x at a given time. So, this right side can be computed completely from Eulerian description whereas, this is inherently Lagrangian quantity, because you are following the same particle. Now, Some times in text books you will find that instead of having this same symbol with respect to different independent variable being kept constant this is normally denoted by D T D t.

So, capital D is reserved for substantial derivative for a single, if the temperature functions of only single coordinate x and time so, it is the velocity in the x direction time’s partial T partial x constant time.

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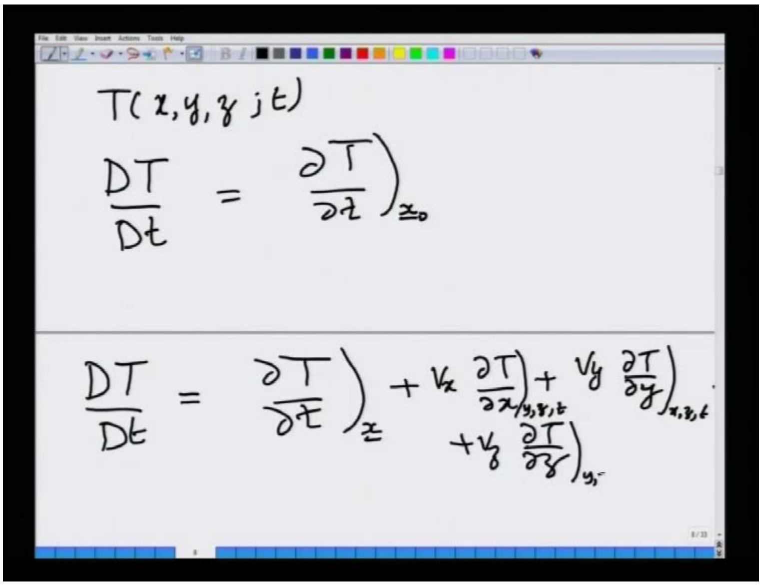

Now, I can generalize this two more dimensions instead of just having the one dimension. Suppose, the temperature is a function of not just x, y, z and time, then what is the substantial derivative? Substantial derivative is the time derivative of temperature as you follow a particle. So, what is it? So, we can generalize very straight forward in a straight forward way is partial T partial t at constant spatial location plus v x partial T partial x plus v y partial T partial y plus v z partial T partial z. So, again so, here partial T partial x is calculated by keeping y, z, t constant this is calculated by keeping x, z, t constant it is calculated by keeping y, x and t constant.

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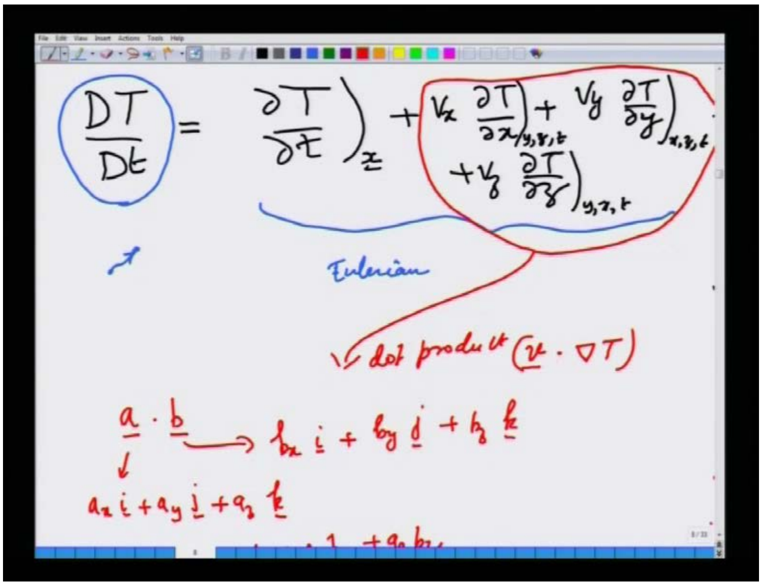

So, that is the basic idea of a substantial derivative where in by purely having completely Eulerian information, we are able to calculate the rate of change of a quantity like temperature, as you follow particle infinitely at a later time delta t. Now, if you try to see whether we can write this in a slightly better form now, this is let us look at this part of the equation and see whether we can write this slightly in a more compact form. So, this is like, if I have two vectors a and b and each vector is given by a x in terms of its Cartesian coordinates b is similarly, given as b x i plus b y j plus b z k, then a dot b is nothing but a x b x plus a y b y plus a z b z. Likewise, if you look at this expression it look a dot product of two vectors.

One vector is the velocity; the other vector is the gradient of the temperature let me just explain how this comes about.

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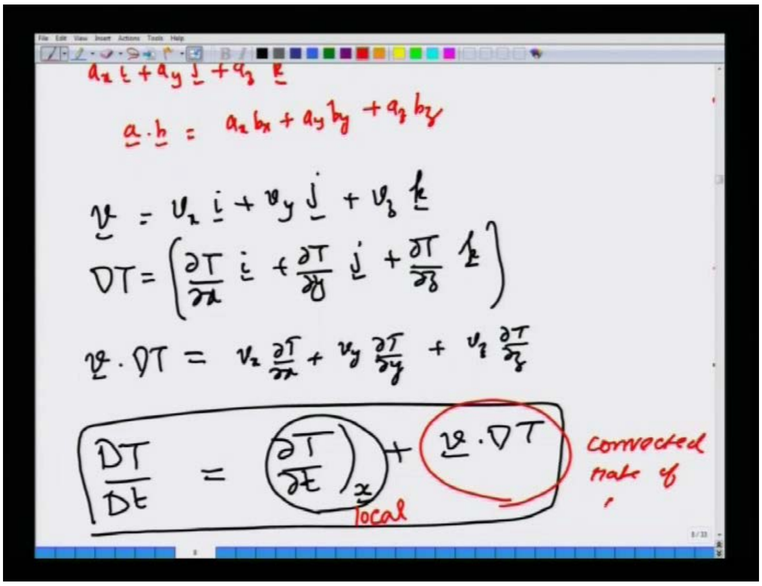

So, if you look at velocity is v x i plus v y j plus v z k, look at gradient of temperature it is nothing but partial T partial x i plus partial T partial y j plus partial T partial z k. If I take the two dot products, where the two dot product of this two vectors v dot del t, then you get v x partial T partial x plus v y partial T partial y plus v z partial T partial z. So, we can write the substantial derivative of temperature in a more compact form is as the local time derivative of temperature at a fixed spatial location plus v dot grad T, this is for three dimensions. This is the local rate of change sometimes this is called the local rate of change and this is called the convicted rate of change.

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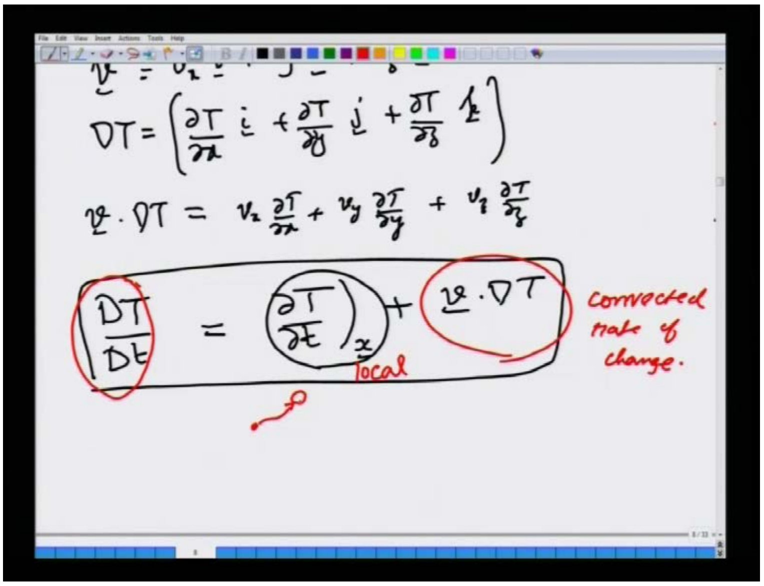

So, this is the very important concept in fluid mechanics, because this is a vehicle that allows us to calculate the substantial derivative idea. It is a vehicle that allows to calculate the rate of change of many quantities, as you follow a fluid particle from a given time to a later time purely based on Eulerian description quantities based on Eularian description. So, we will stop here and we will see you in the next lecture where we will continue future in fluid kinematics. Thank you.