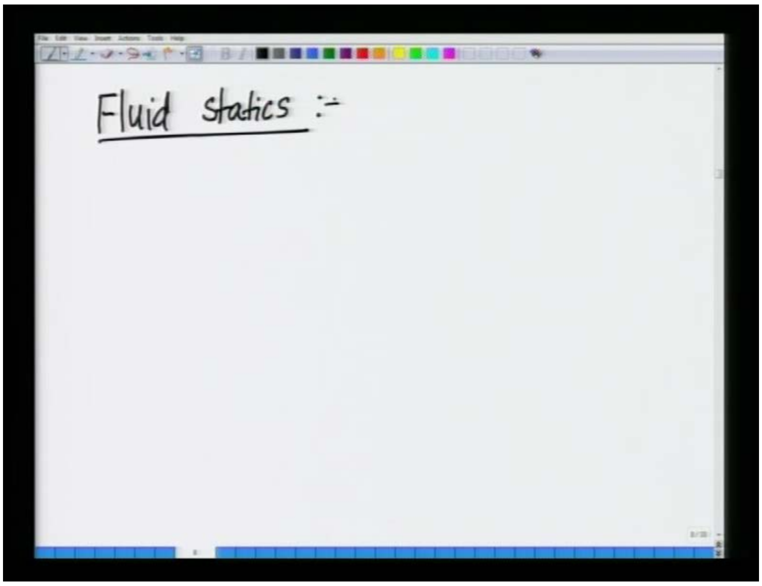
Fluid Mechanics

Indian Institute of Technology, Kanpur
Prof. Viswanathan Shankar
Department of chemical Engineering

Lecture No. # 06

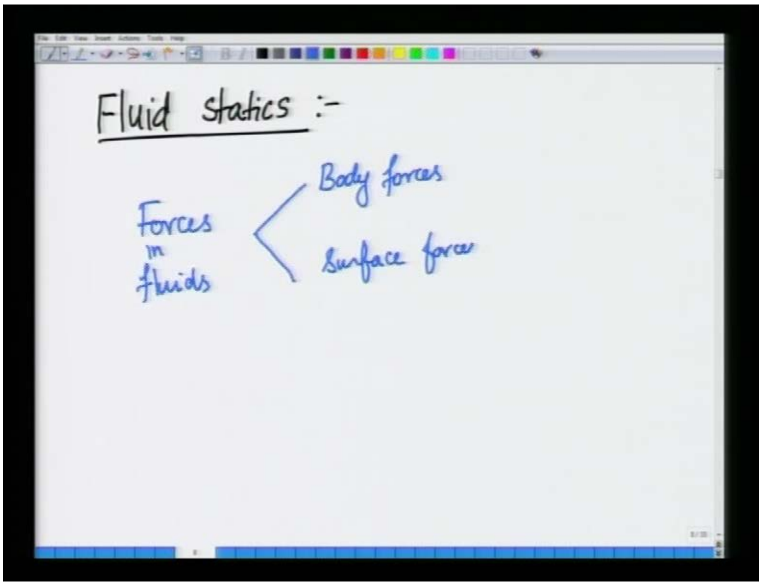
Welcome to this sixth lecture, in this course on fluid mechanics for chemical engineering undergraduate students.

(Refer Slide Time: 00:25)



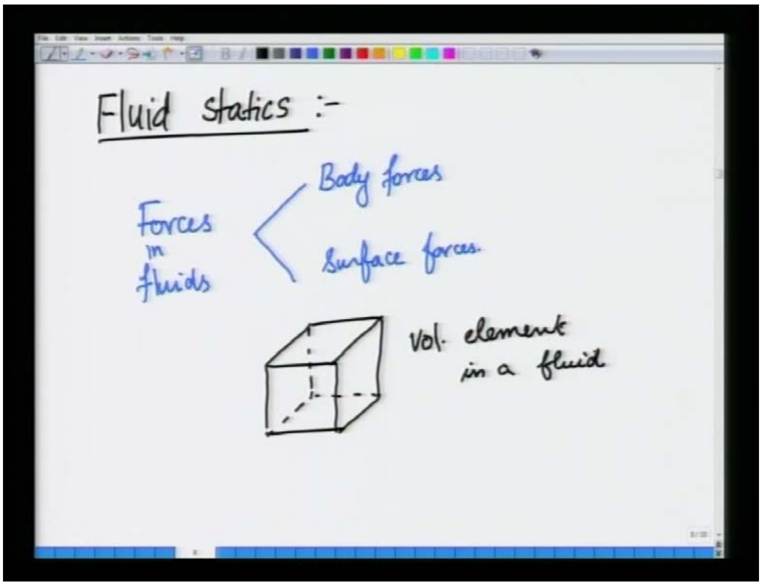
In the last lecture we were discussing f luid statics that is the forces that are present in a fluid that is not under any kind of motion that is the fluid static. And we will see that there are some interesting features that come about even in a static fluid and that will be the subject of our discussion for this lecture. If you recall, forces and fluids can be classified into two types.

(Refer Slide Time: 00:50)



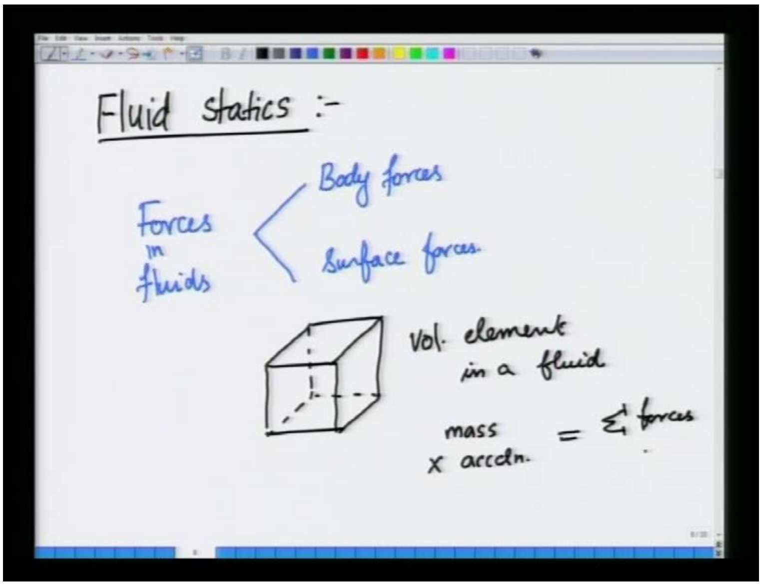
We divided them into broadly as body forces and surface forces. So, remember we are carrying on all our analysis within the continuum frame work, where the fluid is assumed to be a continuous medium in which various quantities such as pressure, density, temperature, velocity. They are all assumed to be smoothly varying functions of both spatial and time co-ordinates.

(Refer Slide Time: 01:33)



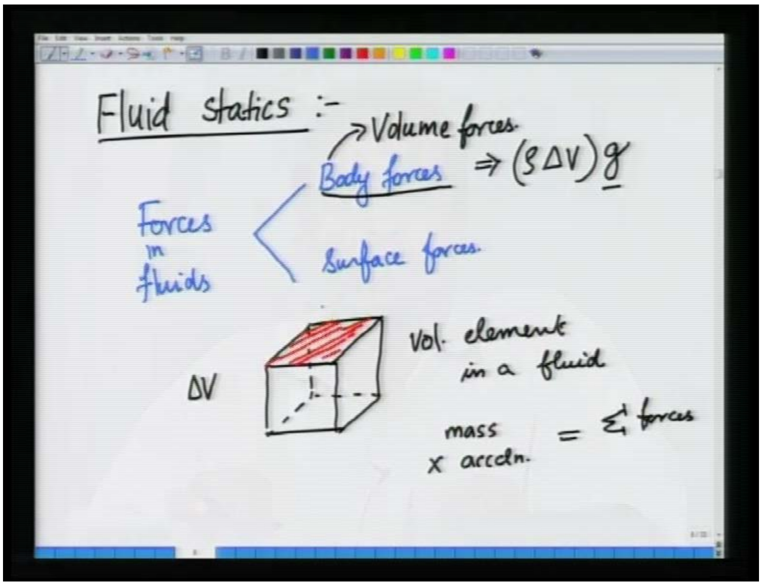
So, within the continuum frame work, if you consider an element of fluid for simplicity, I take a volume element, which is cubic in shape this is a volume element in a fluid. We can ask the question why is this volume element is in general going to move, if the fluid is not under static conditions. Well, it will move, because of the fact that if you apply Newton's laws of motion. So, mass time's acceleration of this volume element is equal to sum of all the forces in the volume element.

(Refer Slide Time: 02:02)



If there is an imbalance of all the kinds of forces that are present in the volume element then the fluid element will accelerate. So, it is important to know, what are the forces that are acting so, one force that is known to us is called the body force. The body force acts entirely through the entire volume of the fluid.

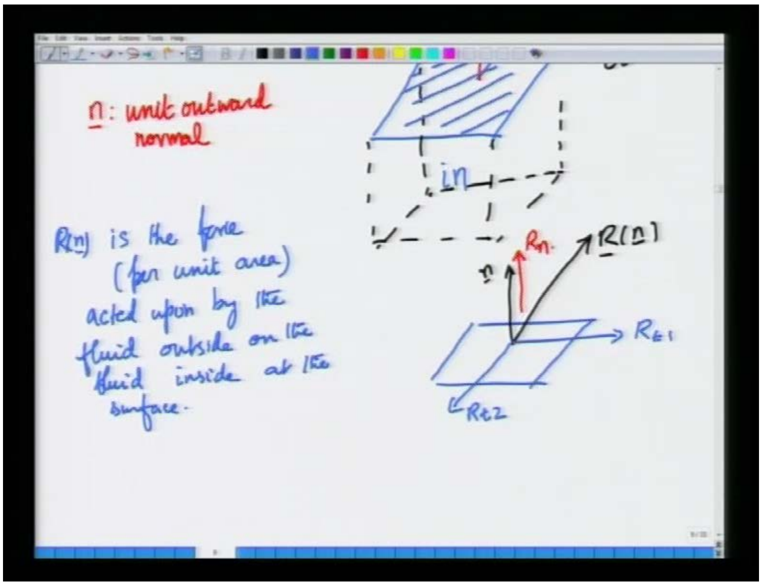
(Refer Slide Time: 02:26)



Well known example is that of gravitational force. Suppose, you have this volume element the volume to be delta V then, the mass of this volume element is rho times delta V times the acceleration due to gravity g, will give you acceleration as a vector. So, will give you a body force that acts through the entire element of the fluid, entire volume of the fluid. So, these are called body forces or sometimes these are also referred to us volume forces. In contrast, surface forces are forces that are acted upon only on the surfaces for example, if you consider the top surface of this cubic volume element.

The fluid that is present just immediately outside will exert a force on this surface. And this is called a surface force; the nature of the surface force is ultimately due to molecular interactions. For example, if you consider a very dilute gas, if you consider a tiny volume element within the gas, then the gas molecules present outside will be colliding with these molecules present inside and that will result in this force. And that force will just be present only on the surface it would not extend in to into the interior of the volume element. So, since these forces act only on the surface of the given volume element they are called surface forces and the surface forces in general. So, let us discuss a little bit more about surface forces before going to static fluids.

(Refer Slide Time: 04:07)

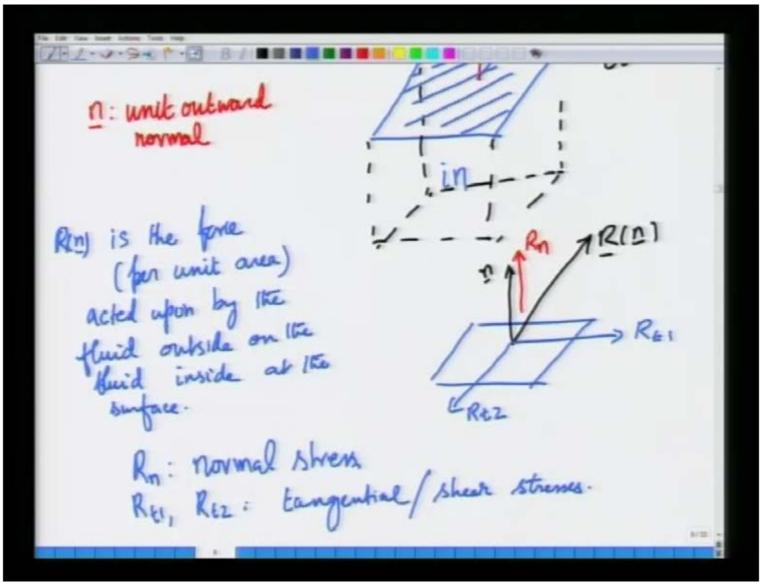


If I take a very simple surface element, this surface element there is fluid present outside. So, the surface element forms part of a volume element, which is the cubic volume element that we have chosen for simplicity. But let us, consider one of the top one of the surfaces namely the top surface. There is fluid present outside and there is fluid present inside and the every surfaces marked by a normal the unit outward normal let us call it n. So, n is the unit outward normal to the surface. It is a unit normal perpendicular to the surface and it is pointing from inside to outside therefore, it is called unit outward normal. Now, if you consider the forces in general that are exerted by the fluid that is present outside on the fluid that is present inside on this surface.

This force, which we will call R can point out in general in any direction. This R is a function in general; it is a function of the surface on which we are focusing that is a top surface. So, it is a function of the unit outward normal, but the direction of this force can need not be always in the direction of n. It can be in any arbitrary direction. So, R is the force, but per unit area, R is a function of n. So, let us write this as R of n, R of n is the force per unit area acted upon by the fluid outside on the fluid inside at the surface. The inside and outside are demarcated by a surface. So, R is the force that is x per unit area exerted by the fluids as present outside on the fluid that is present inside.

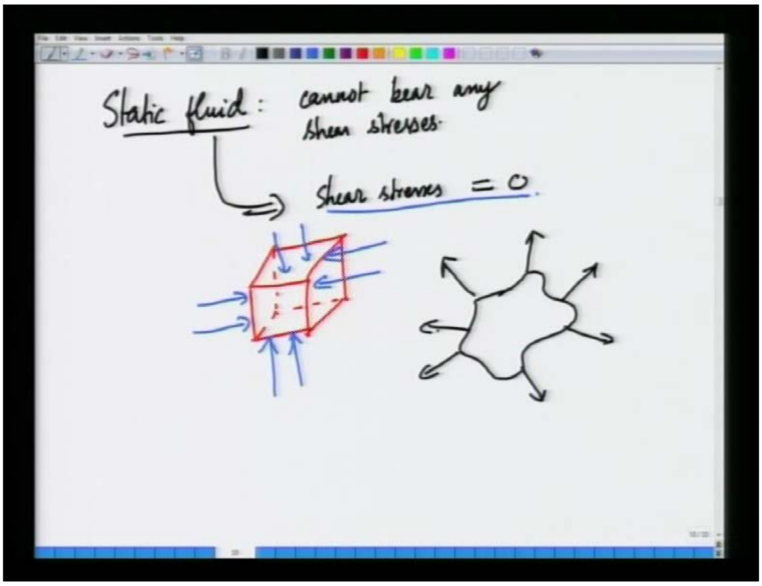
So, if you look at this surface of interest and let us say this is the unit normal. This R will can be resolved into a component that is parallel to the normal. Let us, call it R n and then it can also be resolved in the direction perpendicular to the normal. Let us, call it R t 1, R t 2.

(Refer Slide Time: 07:29)



Now, R n is called the normal stress, it is a normal component of the force that is being exerted by the fluid outside on the fluid inside R n is called the normal stress while, R t 1 and R t 2 are called tangential or shear stresses. So, in general the fluid that is present outside will exert a force on the fluid that is present inside only on the surface and this is because of viscous action. And this force can have a component along the normal as well as a component perpendicular to the normal.

(Refer Slide Time: 08:14)

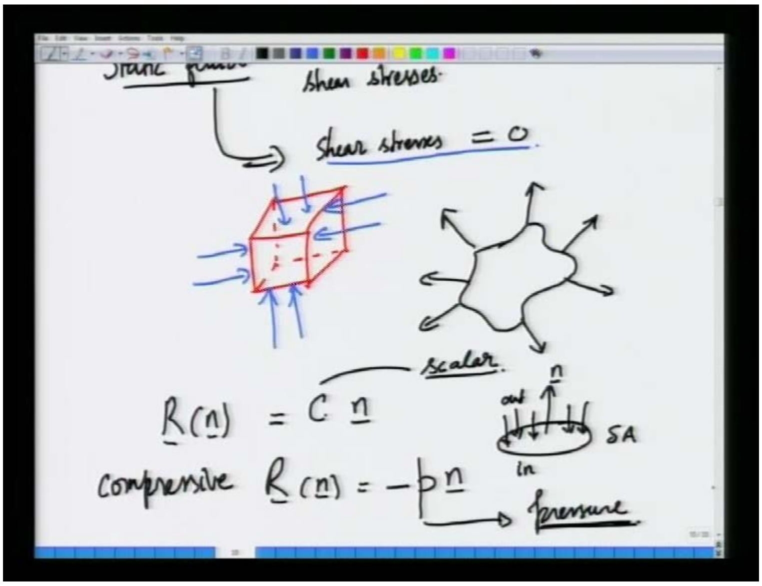


But when, we look at a static fluid, a static fluid by definition cannot bear any shear stress because if there is any finite non zero faces the fluid will start moving. So, the shear stress is in a static fluid must be 0. So, the static fluid is a fluid where, the shear stresses or the tangential component to the stresses are 0 so, the only, because otherwise, a fluid will start moving here. Now, considering a fluid that is completely stationary or static. So, if a fluid is stationary or static, if you take any volume element and enquire what is the force is on the surface. They cannot be any component that is parallel to the surface because if there were to be a force that is parallel to the surface then that will constitute to shear force or shear stress and the fluid will start moving.

So, we since, we are considering only static fluids where there is no motion, there cannot be any shear stresses. So, if you take a static fluid and if you take a volume element in a static fluid on each and every surface the force can exert only purely normally. If you take a static fluid and you take a volume element that is big then, regardless of the nature of the volume element. The force will act purely normally to the surface of the volume element. There cannot be any force that is tangential to the volume element. So, shear stresses are 0 in a static fluid.

Now, this need not the result that shear stresses are 0 and the force is acting purely normal to the surfaces holds for any arbitrary volume. Suppose, I take an very arbitrary volume, this is a three dimensional volume, but I can draw only a 2 d sketch of it. The force is always in the direction of the normal for any arbitrary volume element in a static fluid.

(Refer Slide Time: 10:31)

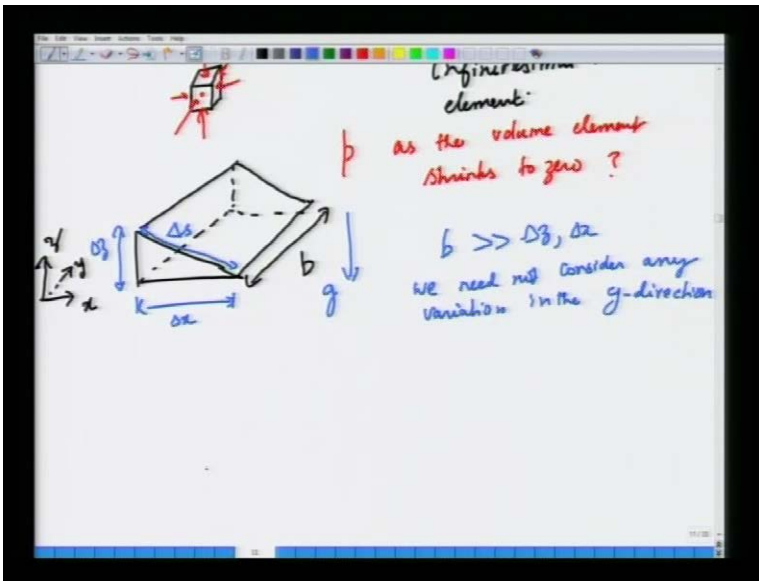


So, R of n in a static fluid must be; since, R is a vector and it is purely in the direction of the normal must be some constant times n, because r is purely in the direction of n and so, it must be some constant times n. So, R is a vector, n is a vector this constant is a scalar constant. So, in general quantities of physical interest can be classified as scalar or vectors. Scalars or quantities, which are described by a single value numerical value at a given point in space. For example, if you see temperature at a point, it has only when one value and it has no sense of direction at a given point in space in a fluid likewise, if you consider density. You can ascribe a value of density to a point in a fluid with in continuum hypothesis, but there is no sense of direction associated with density.

So, where as quantities as such as forces and velocities or vectors, which have a magnitude, that is a numerical value as well as a sense of direction at a given point in fluid so, this constant C is a scalar. So, if you take a surface area delta a in a static fluid and you ask what is R the force exerted by a fluid that is present outside on the fluid that is present inside it has to be in the direction of the n, it has to be purely normal to the area element surface element. Usually, it turns out that this normal force in a static fluid is compressive. That is the force is acting inwards while, the unit normal n is pointing outwards the force is pointing in the direction of minus n. So, R of n is normally, written has minus constant a scalar p times n where this p is called the pressure.

So, pressure is the magnitude of the compressive force that is exerted by the fluid that is outside on the fluid that is inside. And since, R of n is proportional to n and n is pointing outside and p has is compressive this negative sign accounts for this compressive nature of this force or stress in a static fluid. So, this is called the pressure.

(Refer Slide Time: 13:10)

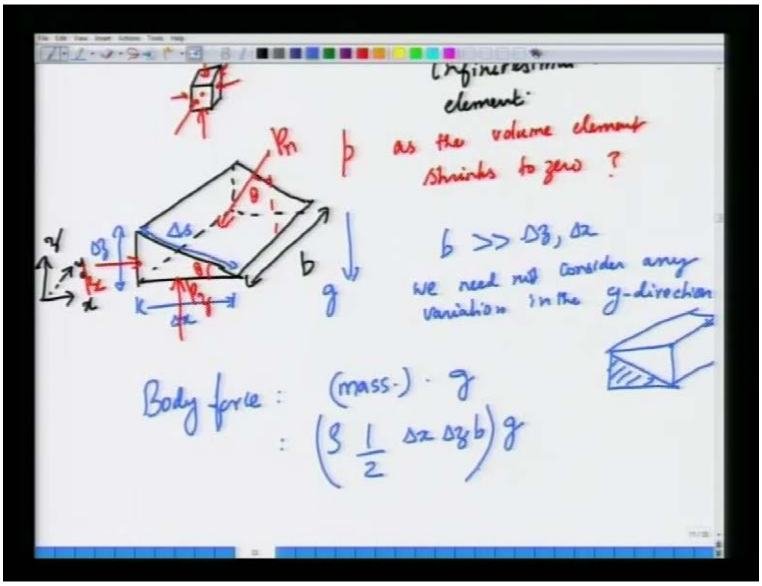


So, the pressure, pressure is denoted by the letter p is the normal force, normal compressive force per unit area acting in a static fluid. We can also; well, before I precede further the units of pressure is Newton's per meter square is one Pascal S I. In S I system units is p is one Newton per meter squared is one Pascal it is that is the standard units. We also showed in the last lecture that at a suppose, I will take a point in a fluid and I construct a tiny volume element about that point infinitesimal. Suppose, I take an infinitesimal volume element about a point and then, I asked the question, what is the value of pressure? So, this for simplicity this volume element is again, a cubic volume element.

So, there are eight faces one two or there are six faces to this cube. There are four side faces on top and bottom there are six faces to this cube. So, you can ask the question the pressure will act purely normally in all this six faces, but what is the value of the pressure p as the volume element shrinks to zero. So, the question is we know that pressure will act purely normally at each and every face in a static fluid. But what is the value of p is it same or it is different along the different faces we are going to show that it is in fact the same. So, in order to this we take a wedge like volume element and let us put a co-ordinate system, this is x and y is going into the board and z is so, y is the direction that is in the plane perpendicular to the board.

So, since and let us call the dimension perpendicular to the board as b and let us call this dimension as delta z and let us call this dimension as delta x. And let us say there is a body force of gravity, acceleration due to gravity g acting and let us call this top dimension as delta s. So, since this b is very very large compared to the thicknesses delta z and delta x. We need not consider any variation in the z direction, in the y direction in the plane perpendicular to the board. So, now first we will ask, what are the various forces in order to prove that pressure so, let us give some notation.

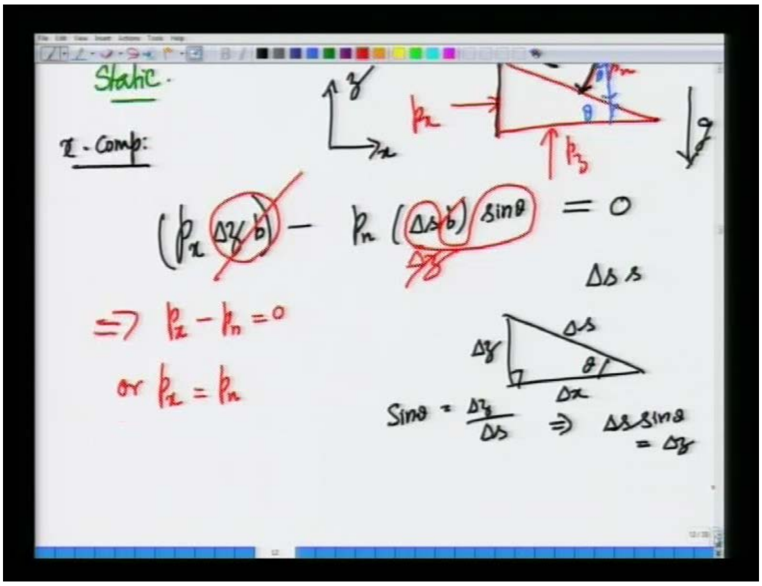
(Refer Slide Time: 16:55)



Let us call this p z; this is the pressure that is acting on a surface which is; whose perpendicular is in the z direction let us, call this p x this is the pressure acting on a surface whose, perpendicular unit normal is in the x direction. And let us call this on the pressure on the incline surface as p n. So, let us call this as theta, angle theta this is also angle theta by geometry. So, what are the various forces? Well, firstly what is the body force acting on this wedge like volume element? The body force is the mass times acceleration due to gravity mass is nothing but the density times volume. Volume of this wedge suppose, you consider a rectangle a cubic volume and then you divide it into half through the diagonal and then it is a volume element in the third direction like this.

So, the volume of this is half the volume of these rectangles so, I will put half delta x delta z time's b times g. This is the mass times acceleration due to gravity g. This is the body force that is acting on this wedge like volume element. What are the surface forces?

(Refer Slide Time: 18:17)

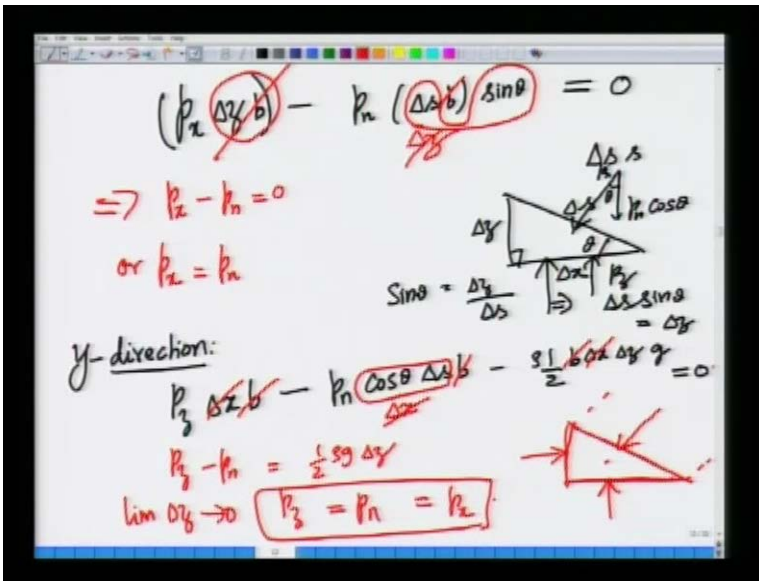


So, in order to this I will just consider a cross section of this wedge like volume element. I will take a cut in the y direction and this is p x, this is p z, this is p n and this angle is theta so, is this angle. So, this p n will have component in the horizontal and vertical direction. So, what are the various forces acting on this static fluid element, this fluid element is static. So, what are the various forces? Let us, take the x component of the force. So, just to remind you we had x like this and z like this. The x component of the force is the acceleration due to gravity is acting purely in the minus z directions.

So, the x component of the force balance will tell us that on this face the force is the pressure times the area of the face, which is nothing but delta x sorry delta z times b and minus this acting in the plus x direction. But there is also a force due to this force, which is acting normally to this incline surface. It will have a component in the horizontal direction or x direction, which is nothing but p n. So, the area of this face is delta s time's b and the force has a component p n sin theta, if this surface is completely vertical that is theta is phi by 2. The normal force will be purely p n, but this is surface is incline. So, there will be a component of this surface of this force on this in the x direction, which is p n sin theta.

So, the sum of forces must be 0, because by Newton’s second law the fluid is static. So, sum of all the forces must be 0. So, likewise, the z component but before I do that I can simplify this further. So, just from geometry delta s sin theta so, we have this, this is delta s, this is theta, this is delta z, this is delta x. Sin theta is nothing but delta z by delta s. So, which implies delta s sin theta, is delta z. So, instead of this delta s sin theta, I will put delta z. Then this equation will tell me that delta z b will cancel with delta z and b. So, this will implies that p x minus p n is 0 or p x is equal to p n. So, likewise, we can do a similar thing for the y direction.

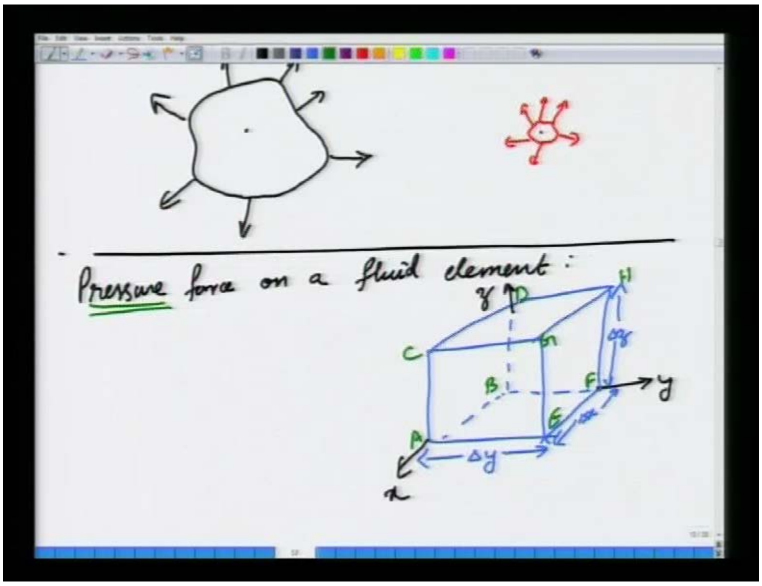
(Refer Slide Time: 21:15)



In the y direction again, we look at forces so, the forces will be; so, the force on this face is p z p z times delta x times b that is the force in the plus y direction. And there is a force the component of the force on the incline surface that will be p n cosine theta that will be acting in the minus z direction. So, will be minus p n cos theta time’s delta s time’s b also we have the acceleration due to gravity the boy force acting in this direction, which is minus rho half b delta x delta z time’s g and sum of all these must be 0. Now, just by geometry delta s cos theta is nothing but delta x so, I can cancel delta x right through the equation. So, I get and then I can cancel b right through the equation.

So, I get p z minus p n is equal to half rho g delta z and in the limit as delta z shrinks to 0, we get p z is p n, but we also showed p x is p n so, p z is p n is p x. So, if I take a surface like this, if I take a point and then I construct a volume element like this, which is extending in the third direction which we not worry. The pressure is acting in a static fluid as this volume element shrinks to 0 the pressure acting on this faces with different orientations. They are this in the magnitude of this force per unit area is exactly the same. So, the pressure; so, what we find is that you take a point, if you take a large enough volume macroscopic volume in a static fluid the pressure will be purely normal to this arbitrary volume element.

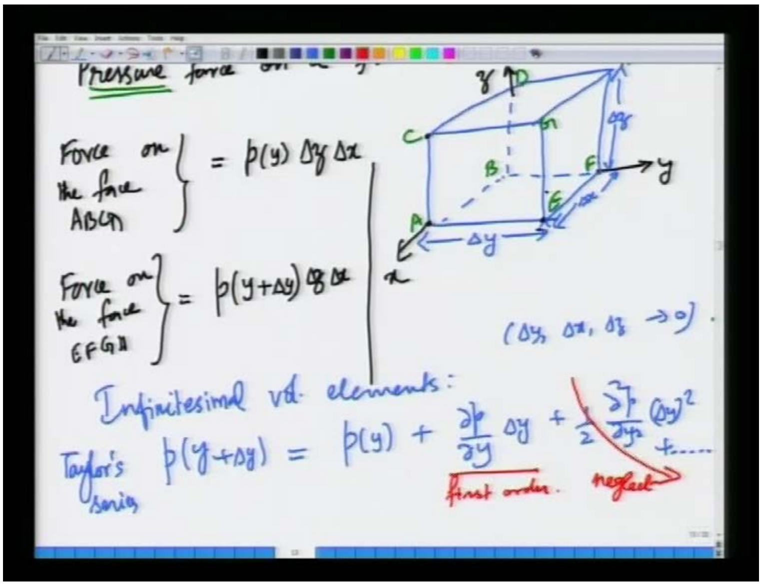
(Refer Slide Time: 23:33)



But as I shrink this volume element to a tiny tiny tiny poin then, the value of the pressure will be the same and purely normal to the surface of this tiny volume element. But as this volume element gets shrunk to a point even the magnitude of the pressure the numerical magnitude will be the same. And it is independent of the direction of the orientation of the surface. Now, the next task for us is to find, what is the pressure force on a fluid element? (No Audio Time: 24:16 to 24:27) In order to this, we will take a simple volume element again, take a cubic volume element. (No Audio Time: 24:34 to 24:43) And I will put co-ordinate system x y and z along the three directions and I will call this as delta y or and this as delta x and this width as delta z.

Now, let us also call put some labels for the faces let us, call this face A B C D; let us call this face E F G H. So, we want to find out, what is the pressure force, if this volume element is surrounded by a fluid and that is exerting a pressure under static conditions. What is the pressure force net pressure force acting on this fluid element?

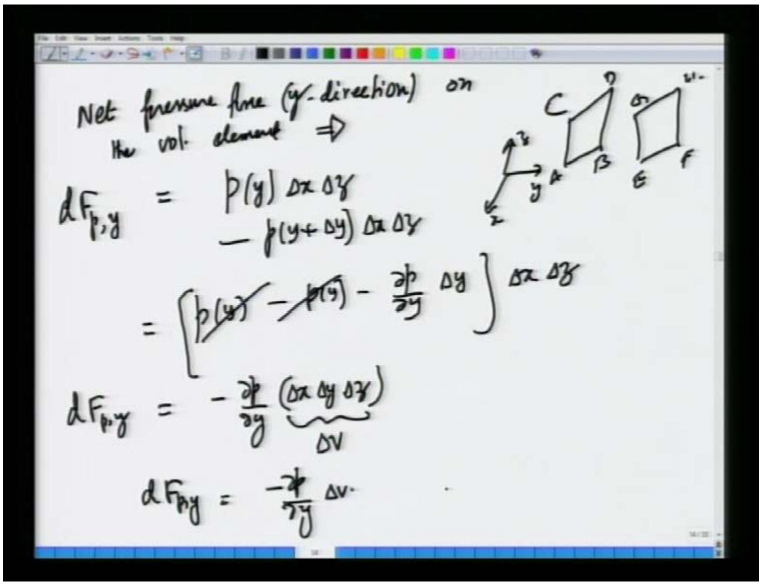
(Refer Slide Time: 25:45)



To answer this pressure on the face or force to be precised, force on the face A B C D. A B C D is this face is nothing but p of y. So, this pressure in general will be different from this point to this point. Since this is a tiny volume element the pressure at a given face is essentially the same, but it is value will change from one point to another times the area of this element. The area of this element is nothing but delta z times delta x. So, this is the force acting on the face A B C D, the force on the face E F G H is nothing but in general the pressure will vary from this point from to this point.

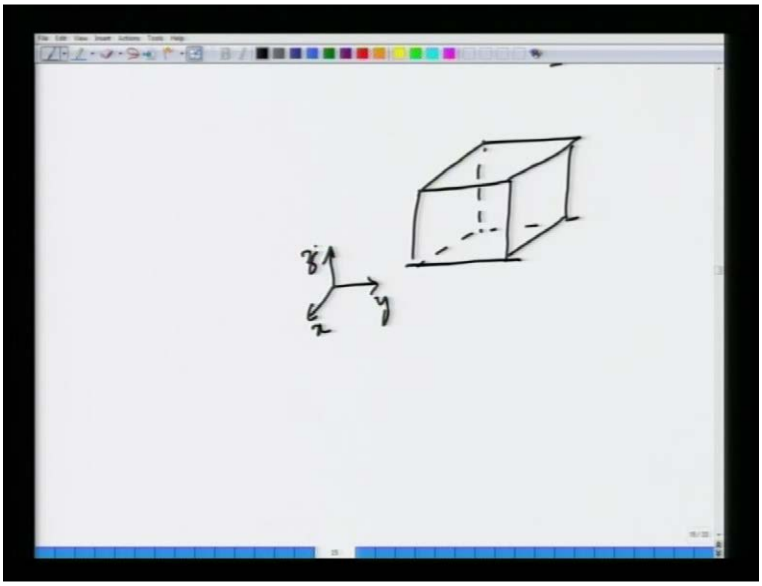
So, we write this as p of y plus delta y times delta z times delta x. Now, we are going to consider infinitesimal tiny volume elements, for infinitesimal volume elements that is as delta y delta x delta z tends to 0. We can always write by what is called a Taylor’s series. p of y plus delta y is nothing but p at y plus partial p by partial y times delta y plus half partial square p partial y square delta y square plus so on. Typically, this higher order sums are smaller so, we can neglect and we can keep our expansion up t only up to first order.

(Refer Slide Time: 28:08)



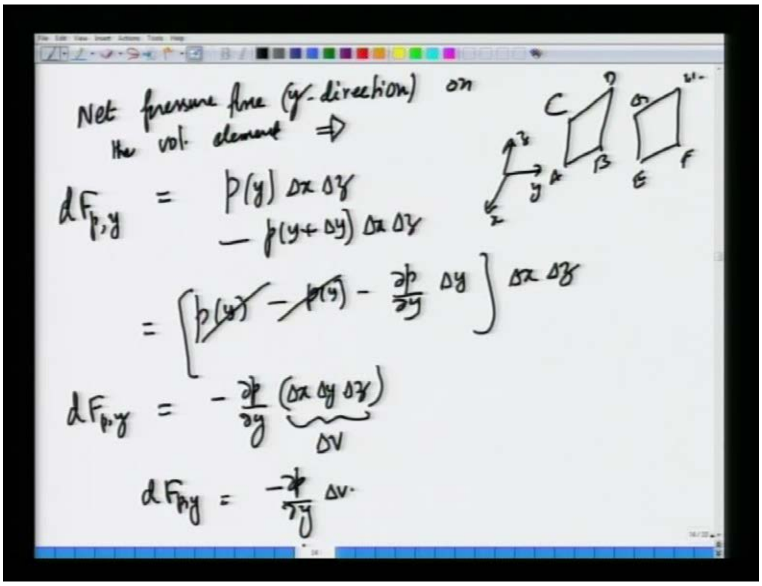
So, if such is the case, then the force the net pressure force in the x direction on the volume element is given by nothing but we have to worry about the force on the face A B C da nd the force on the face E F G H. So, this is p; so, the net pressure force acting in the y direction, because remember this is x, this is y, this is z, dF p y this is the net pressure force the y component of the net pressure force. So, this is the y direction is nothing but p at y times delta x delta z minus p at y plus delta y times delta x delta z. But I can use a Taylor series expansion to write this as p at y minus p at y minus partial p partial y time's delta y time's delta x delta z. So, these will cancelled to give dF p y is nothing but minus partial p partial y times delta x delta y delta z. This is nothing but the volume of the cubic element. So, I can write is minus partial p partial y times delta v.

(Refer Slide Time: 29:50)



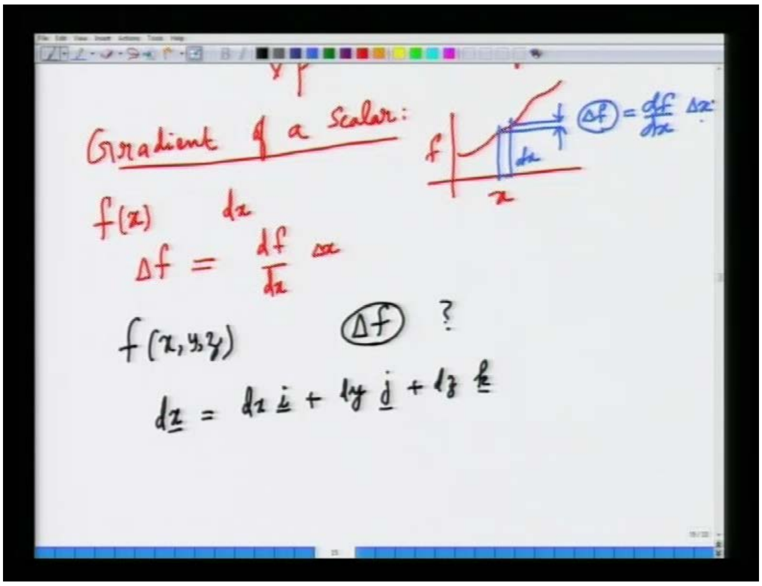
So, similarly, now what we have done is just to recapitulate, if you take a volume element a cubic volume element with x y and z like this. The net y component so, let me go back and correct any problems with notation.

(Refer Slide Time: 30:21)



So, we are this still doing it in so that is perfectly fine.

(Refer Slide Time: 30:34)



So, the net force in the y direction on this fluid element is due to the pressure acting acted upon by the fluid that is present outside and the fluid inside is this. This is the y component, but just by analogy similarly, you can do it as an exercise at home. You can show that d F p x, the net x component of the force due to pressure alone is minus partial p partial x delta v and dF p z is minus partial p partial z time's delta v. So, we can construct the vector force. So, if you have three components of a vector; suppose, I have a vector f and I have three components f x f y f z. I can simply reconstruct this is in the cartesian coordinate system as f x times i is f y times j is f z times k, where i j k are the unit vectors in the x y and z direction.

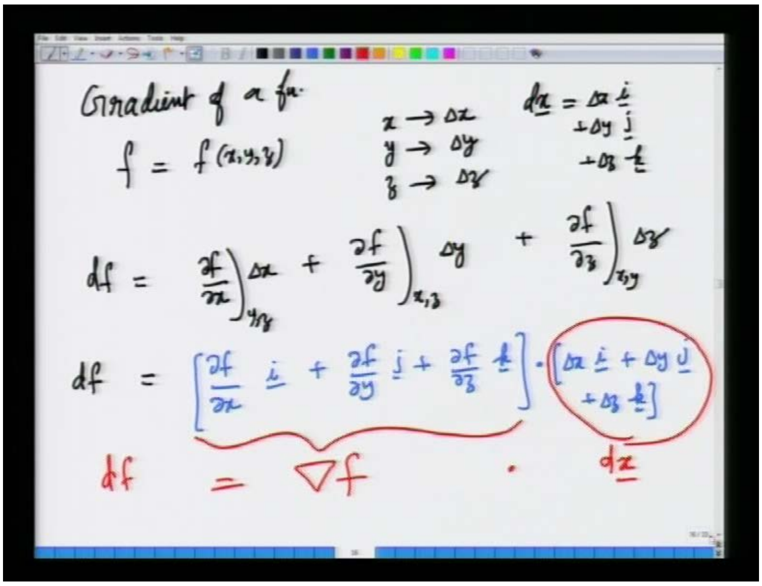
So, d F p can be simply written as dF p x i plus d F p y j plus dF p z times k, where i j k are the three unit vectors along the x y and z co-ordinates of the cartesian co-ordinate system. So, I can substitute for d F p x minus partial p partial x i minus partial p partial y j minus partial p partial z k times delta v. So, for those of you have taken elementary courses in mathematics, engineering mathematics, where you would be dealing with vector calculus. You would recognize the this is nothing but what is called the gradient of p. I will just take couple of minutes for those who are not familiar with these concepts.

So, gradient of a scalar quantity, what does it mean physically? So, let me just take two minutes to explain this before I discuss the physical significance of that result that we are just derived. So, let me just rewrite this d F p is minus del p times del v. So, what do you mean by gradient of a scalar? Suppose, I was single a function that is, where function of only one variable f of x. Now, if I want to know, if I change the value of x by an amount dx. Then the function f will change the delta f is the change that is obtain by the function that is incurred in the function as you change x by delta x is from definitions of calculus dF dx times delta x as the value of delta x becomes small, small and small.

The change in the value of the function suppose, I have function f verses x and I take a tiny element x of length dx. This change that is incurred by the function delta f this change is nothing but the slope at that point times the value of the displacement we are doing. So, this is true for normal functions, but suppose, you have a function that is not a function of single variable. It is a function of three variables not just one variable three, where the three variables are the three co-ordinates directions in cartesian co-ordinate system.

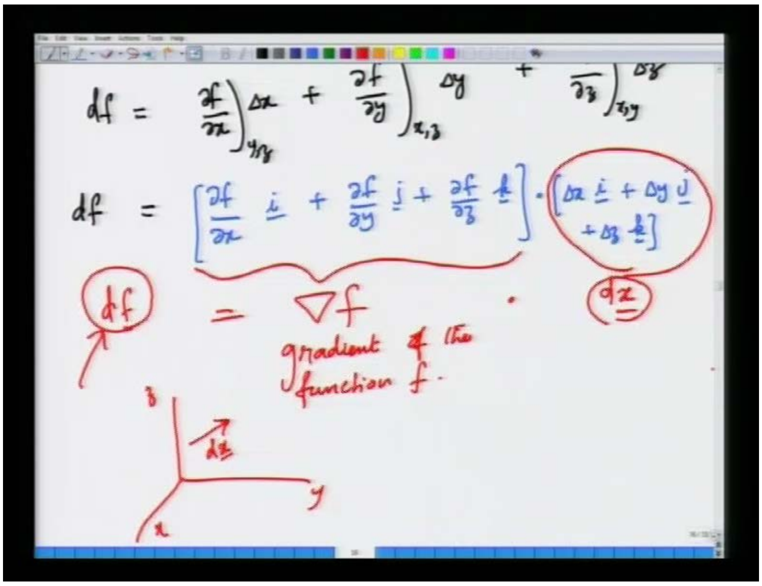
Then how do you think of delta f, what is the change in the function f? So, if you consider a function not just of one variable, but three variables. And you ask the question, what is the change incurred in the function as you proceed by a small distance d x, but x is no longer a scalar it is a vector, because you can undergo a displacement in the three co-ordinate directions dx times i times d y times j plus dz times k. Then what is the value, the change in the value of the function f?

(Refer Slide Time: 35:14)



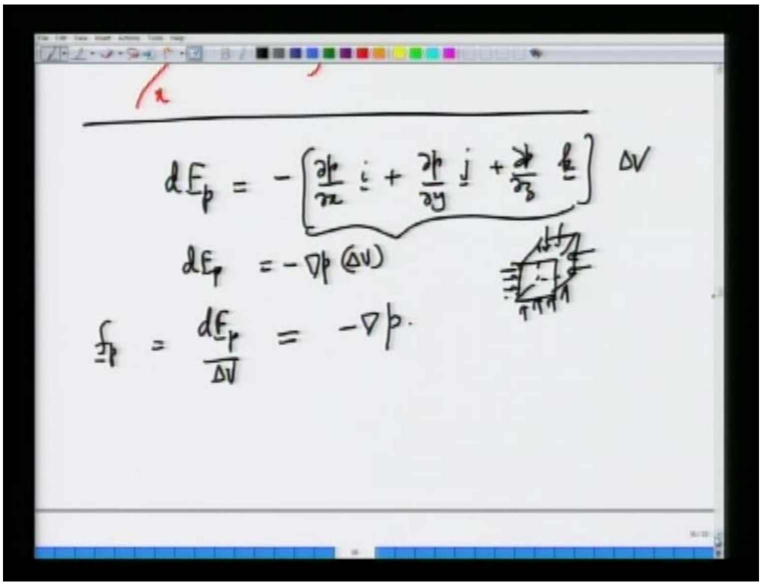
It turns out that gradient of the function has that information so, gradient of a function. So, if f is a function of x y z then, if x changes by a small amount delta x, y changes by a small amount delta y, z changes by a small amount delta z. Then d f is nothing but partial f by partial x time's delta x keeping y z constant, partial f by partial y constant x z times d y there by plus partial f by partial z x y delta z. So, this can also be thought of as or d f the change in the value of the function. This is the displacement vector that we are under gone is delta x i plus delta y j plus delta z k. So, I can also re write this as a quantity, which is a vector, which is nothing but partial f partial x i plus partial f partial y j plus partial f partial z k dotted, this is the dot product in vectors with delta x i plus delta y j plus delta z k. This is nothing but the displacement vector that we have undergone dx. This quantity is identified as grad f dotted with dx, this is df.

(Refer Slide Time: 37:05)



So, this quantity is called the gradient of the function f. So, the gradient of the function gives you information about the changes a function encounters as you suppose, you have x y z co-ordinate system and you are traversing any arbitrary distance delta dx. The gradient of the function has the information as to how much this function f will change that is d f as undergo a vectorial displacement d x about a point.

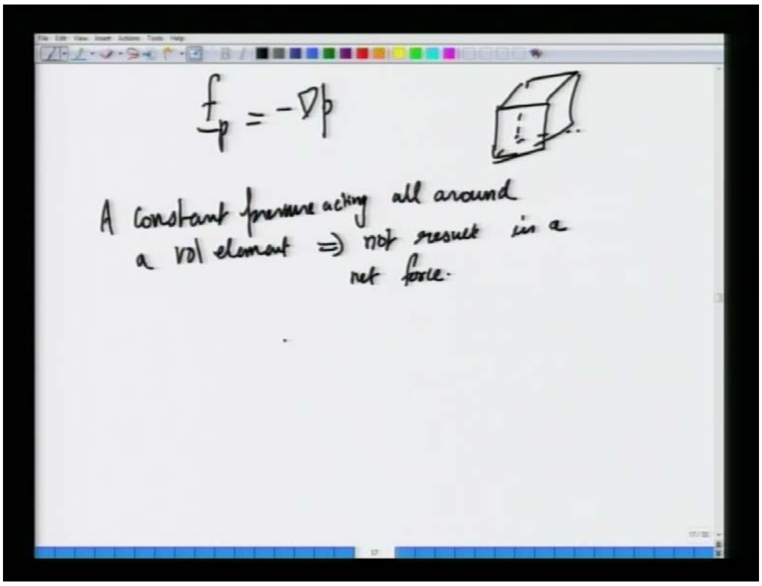
(Refer Slide Time: 37:48)



So, the quantity the expression that we derive. Now, going back to static fluids, the expression that we derived is that the force acting on a fluid element is minus partial p partial x i plus partial p partial y j plus partial p partial z k times delta v. This is nothing but minus grad p times delta v d F p. So, this force that is acting on a tiny volume element recall that our volume element is cubic in shape is proportional to that volume of the volume element. And therefore, I can define a force density that is small f p that is force differential force acting on this volume element divided by the volume is nothing but grad p.

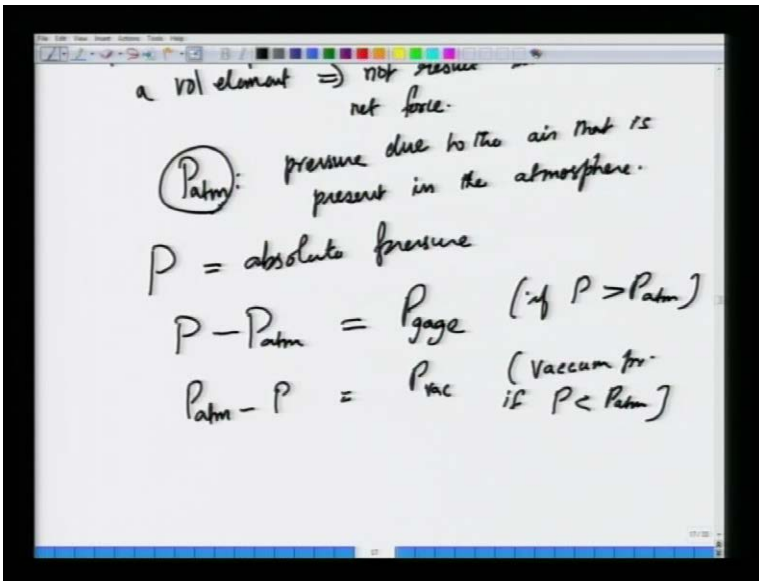
So, there will be a net force on this volume element only, if there is a change or variation in pressure. So, if each face has the same pressure can obviously all these forces will cancel out. There would not be a net force due to pressure variation, because there is no variation in pressure. Therefore, the forces will cancel out giving raise to no net force. But if for some reason there is a pressure variation in a volume element, then that will give raise to a net force on this volume element. So, this is called this is the net force acting on a volume element due to pressure.

(Refer Slide Time: 39:27)



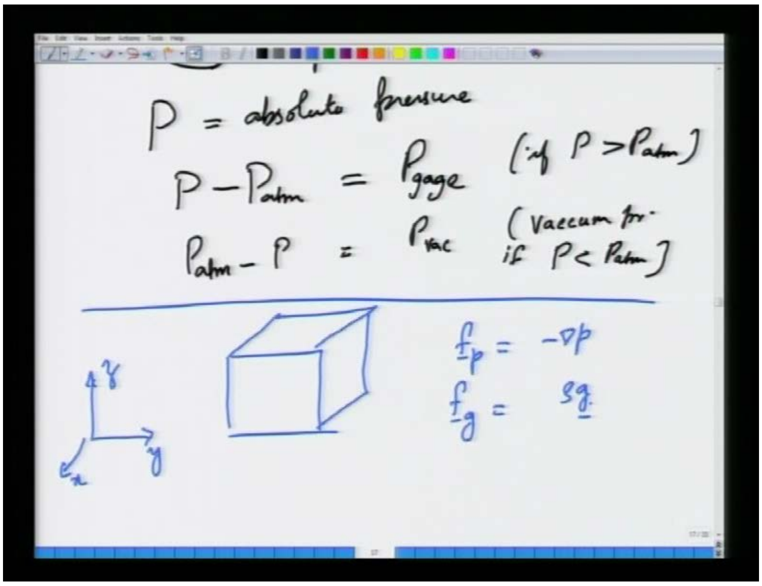
So, one can make a very simple corollary so, if the net force acting on a volume element is proportional to grad of p or the gradient of pressure. That means that if there is constant pressure acting everywhere on a volume element that will not give raise to any net force. A constant pressure acting all around a volume element will not result in a net force. Why is this important? This is important, because in many practical applications in terrestrial applications there is a constant atmospheric pressure that is acted upon by the air that is present. And this constant atmospheric pressure will not cause a net force on many problems of interest in fluid statics and fluid flow.

(Refer Slide Time: 40:31)



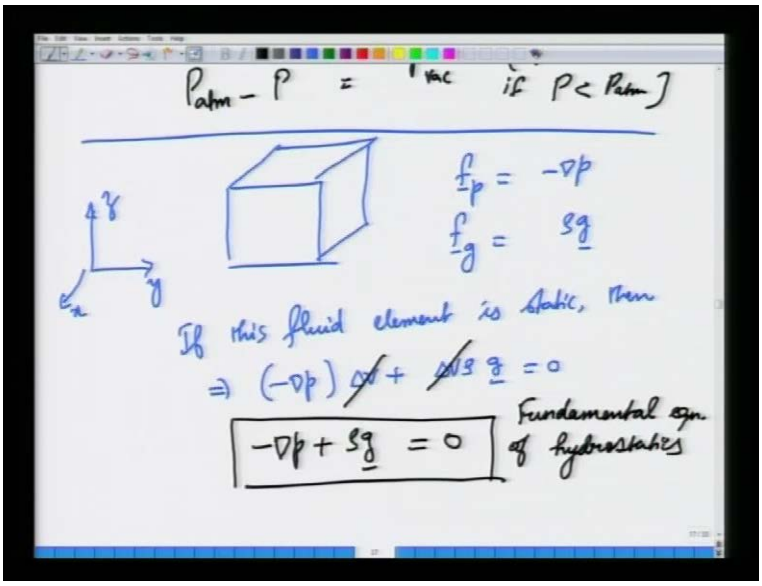
So, this force this pressure is called the atmospheric pressure. This is the pressure due to the air that is present in the atmosphere. (No Audio Time: 40:43 to 40:57) So, this is called the atmospheric pressure commonly denoted as p atmosphere. So, if you measure the pressure p of a fluid then that is called the absolute pressure. This is the actual pressure that is present in the fluid, but since, if you in many problems you have atmospheric pressure acting all around. It is only the difference between the atmospheric absolute pressure and atmospheric pressure that matters, this is called the gage pressure. So, this is if the pressure in the fluid is greater than atmospheric pressure. If it is less than the reverse of it is called the vacuum pressure. If p is less than p, this definition just ensures that vacuum pressure is a positive quantity.

(Refer Slide Time: 41:55)



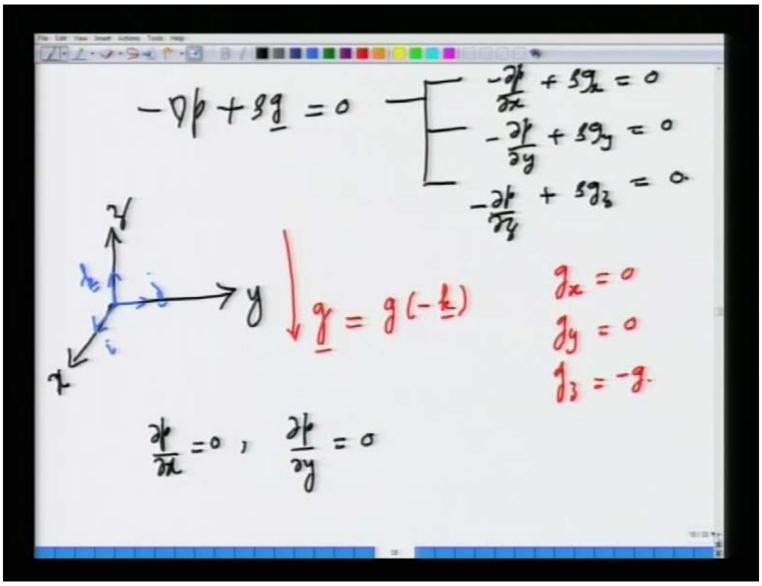
So, an uniform pressure acting all through all around our body will not result in a net force. Now, let us worry about; if you consider a volume element like this cubic volume element. We know that so, let us put co-ordinate system x y and z. We know that the force is due to pressure, the force per unit volume due to a pressure is a net force by the unit volume due to pressure is minus delta p and the net force per unit volume due to gravity is simply pro g.

(Refer Slide Time: 42:33)



So, if this fluid element is static is static. (No Audio Time: 42:35 to 42:41) Then the sum of all forces on this volume element must act to 0, which implies that minus delta p time’s rho delta v plus delta v rho g must also be equal to 0. So, if I cancel delta v right through we get minus delta p plus pro g equal to 0. This is the fundamental equation of hydro statics minus delta p plus rho g is 0.

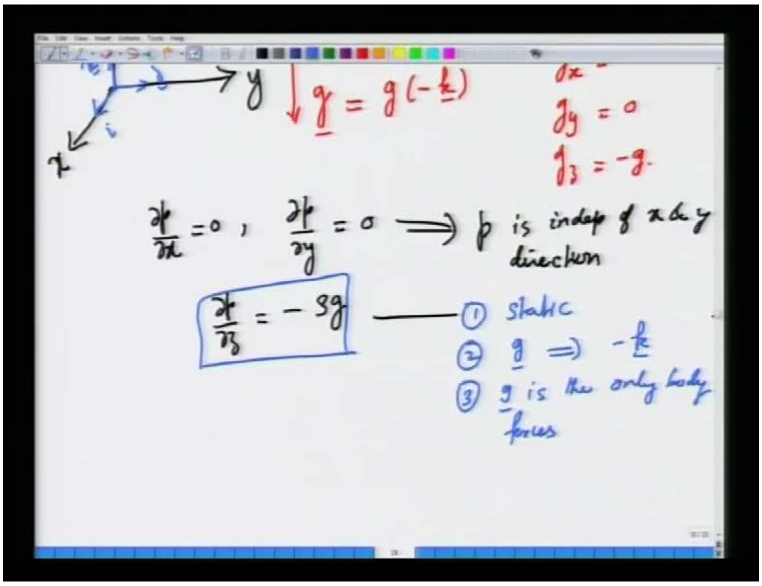
(Refer Slide Time: 43:25)



This is a vectorial equation any vectorial equation can be written in a three component form. So, this will have three forms, a three components minus dp dx plus rho gx is the component of the acceleration due to gravity vector in the x direction minus dp d y plus rho g y is 0 minus dp dz plus rho g z is 0. So, it is traditional or conventional in fluid flow to align the gravity vector vertically along with one of the co-ordinate axis. So, suppose, I call this x y and z and the three unit vectors are i j and k. And gravity is acting downwards so, g the vector g is written as minus or g times minus k. So, that means that g x is 0, g y is 0 g z is minus g.

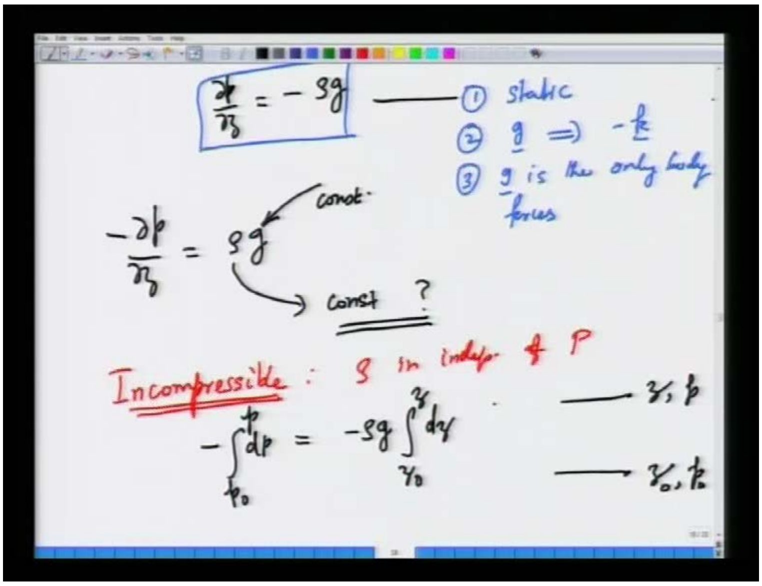
So, one can simplify this equations to write as partial p partial x is 0. All these are partial derivatives, because we are looking at variations in all the three directions. (No Audio Time: 44:56 to 45:07) So, you have partial derivatives partial p partial y is 0. There is no variation in the pressure in the x direction, no variation in the pressure in the y direction, but in the z direction partial p partial z will go as minus rho g, because g is z is minus g. So, if I take this rather side I will get minus rho g.

(Refer Slide Time: 45:14)



This is a fundamental equation; this is a fundamental result in hydrostatics. If you place the gravity vector in the minus k direction, then pressure is independent of x and y direction. And it is a function only of the z direction and the vary in which is going to vary is this. So, what are the assumptions of this, when this equation is valid? This is a very important equation. This is valid for static fluid and the gravity is aligned along the minus k direction and the gravity is the only body force. There can be other body forces in general, but just in this introductory discussion we keep think simple so, we take gravity to be the only body force.

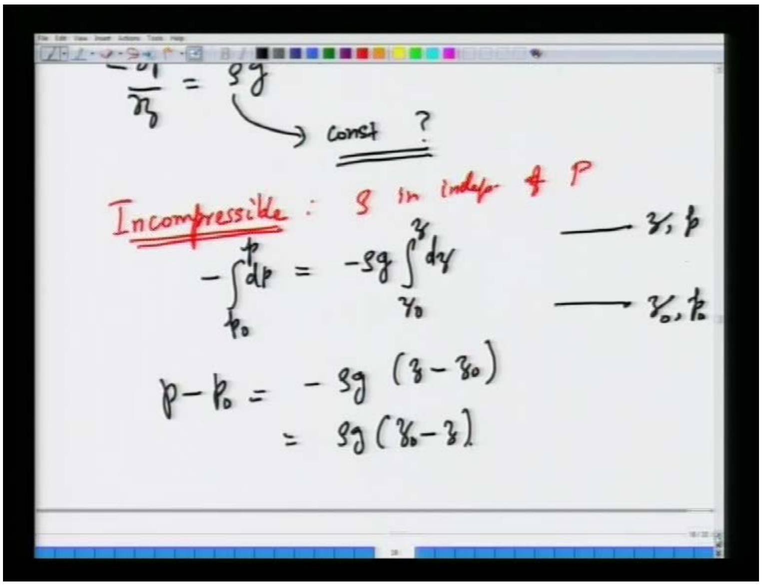
(Refer Slide Time: 46:24)



Now, can we solve this or can we simplify this further? So, you have minus partial p partial z is rho g in order to integrate this. We need to know whether rho and g are functions of z or not, g is practically a constant in terrestrial applications and most engineering and situations so, g is a constant. So, next thing is this rho a constant or not? Is it a constant? Well, it depends on context to context in many fluid flow problems, the fluid can be consider to be in compressible that is rho is independent of the pressure. There is no change in the density, because of the fact that the pressure in the fluid is changing. So, rho is considered to be independent of pressure then the fluid is in compressible and rho is independent of and rho is a constant.

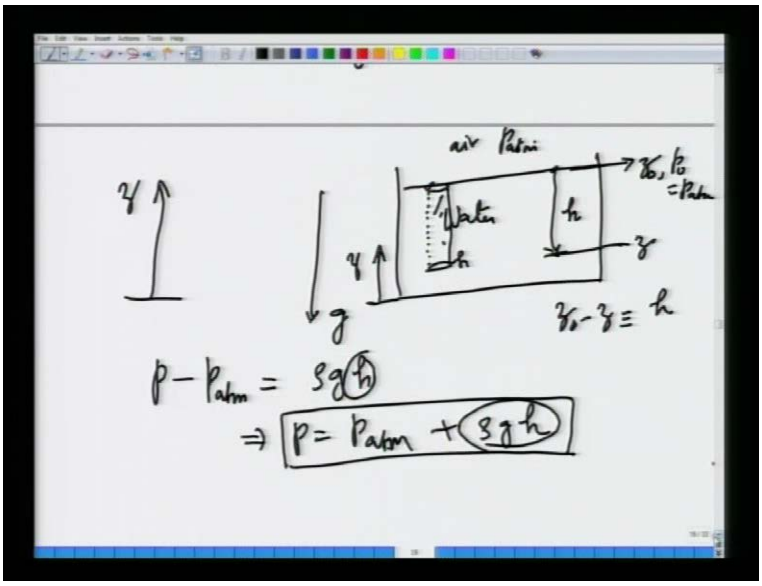
So, if you consider the density to be constant then this equation can be integrated in a straight forward way. So, you write minus integral z 1 or rather if I integrate, if I take two points in the fluid where the elevation is or the height is z naught and z. And where z is z naught p is p naught and where the vertical co-ordinate z pressure is p. Then I write minus integral p naught p dp is minus rho g. Since, rho is considered to be a constant for an in compressible fluid I will integrate this from z naught to z dz.

(Refer Slide Time: 48:08)



So, p minus p naught is minus rho g times z minus z naught or is equal to rho g times z naught minus z.

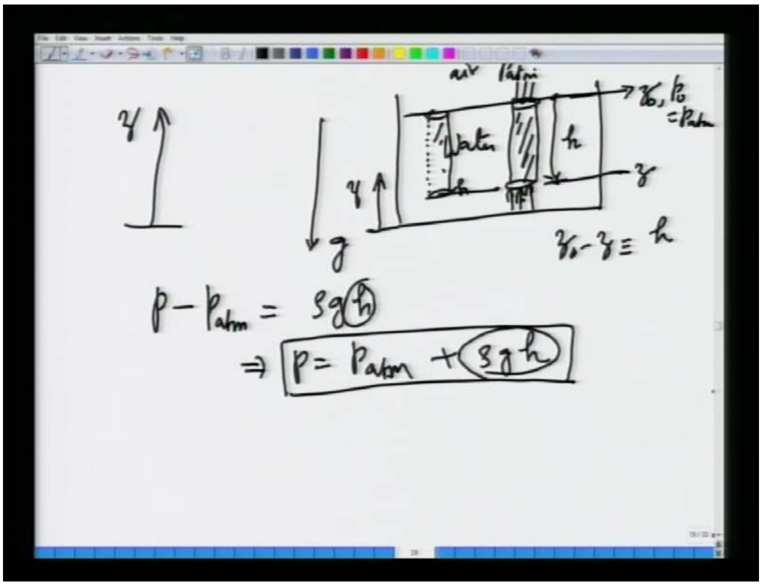
(Refer Slide Time: 48:24)



Now usually, what happens is we can; so, the way it is we have done the problem is z is pointing like this and gravity is pointing in the opposite direction. Suppose, you have trough of water and there is a free surface. This is let us say water and here it is air the pressure is atmospheric. Suppose, you take z naught z equals z naught as the interface, where the pressure is atmospheric. So, p naught is p atmosphere and let us take z naught minus z as h. So, h is the depth from the free surface, where the pressure is atmospheric. So, we can write p minus p atmosphere is essentially rho g h, where z naught is the location, where the pressure is atmospheric and z is any arbitrary location. So, this is z, z naught minus z is denoted by the symbol h.

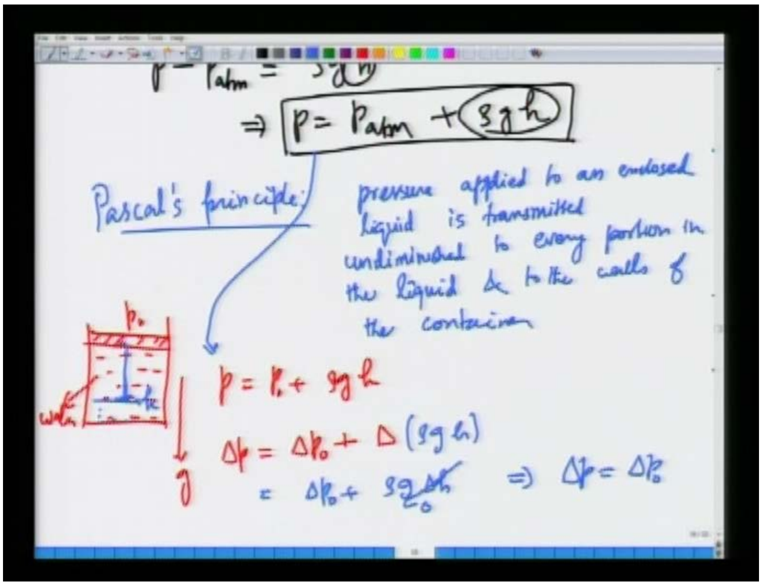
So, which implies p is p atmosphere plus rho g h. This is a fundamentally equation in hydrostatics. The pressure varies so, if you consider the pressure here in the air to be atmospheric and if you look at each and every point in the fluid as you go down in h. So, this is z, as z decreases then the depth of the water increases from the free surface. Then the pressure will increase compare to an atmospheric pressure due to the weight of the liquid column that is present above. This is so, we take any h the weight of this liquid column will act to increase the pressure.

(Refer Slide Time: 50:17)



So, this is the fundamental equation of hydrostatics in incompressible fluids. And the pressure increases in an incompressible fluid, because of the fact that the pressure at any point in the fluid has to; suppose, if you take a force, if you take a cylindrical volume element. If you do a force balance the pressure here is acting normally like this, the pressure here is the atmospheric. The pressure has to balance not just the atmospheric pressure but also the weight of the fluid element that is present in between the two levels. So, this is a fundamental result in hydrostatics for incompressible fluids.

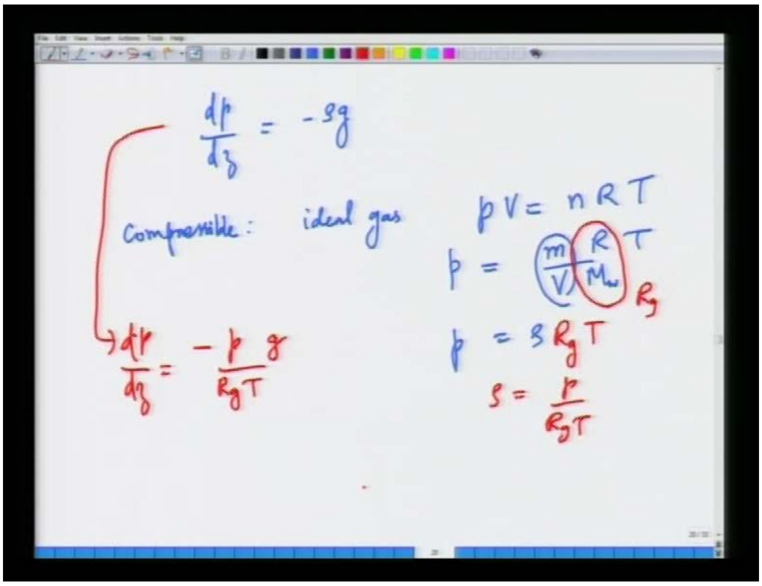
(Refer Slide Time: 50:58)



Now, you must have seen or learnt of what is called the Pascal's law or Pascal's principle for incompressible fluid, which says that pressure applied to an enclosed liquid is transmitted and diminished to every portion in the liquid and to the walls (No Audio Time: 51:33 to 51:42) and to the walls of the container. So, where does this principle come from? This principle merely is a consequence of this relation, this equation. Suppose, you consider a container in which you are imposing a pressure p naught and this is let us say water it is incompressible, there is liquid here. So, p at any; so, this is gravity. So, p at any location is p naught plus rho g h suppose, you change p naught to some other value.

The change in pressure is because of change in p naught plus change the change in pressure at any depth h from the surface is delta p. And there is change in p naught and then there is change in rho g h, but we are looking at the same h. So, this change in rho g h will amount to only change in h, because there is rho is constant for an incompressible fluid g is constant. So, rho g delta h, but we are looking at the same h so, delta h is 0. So, this implies that the pressure that you apply. The change in pressure that you make at the surface here will transmit undiminished to any in each and every point in a static fluid this is the Pascal's principle. Now, there are some other ways of integrating this equation.

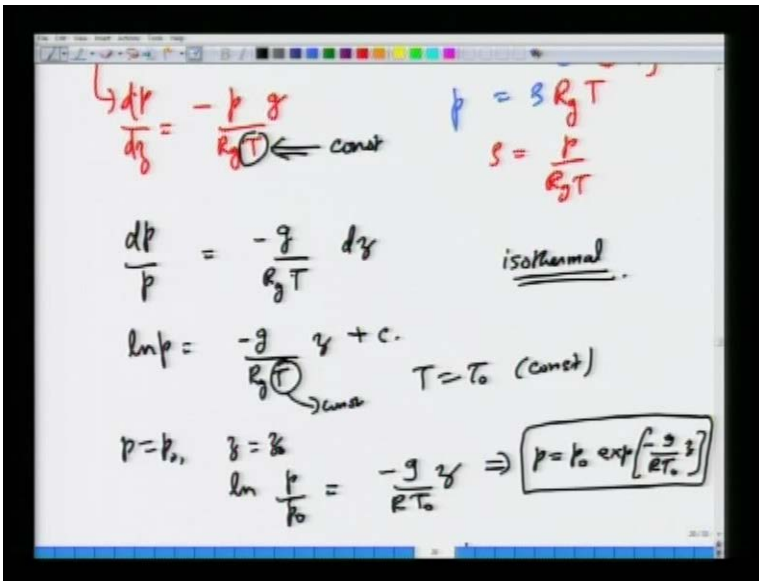
(Refer Slide Time: 53:14)



For example, you have dp dz is minus rho g. Now, we assumed initially the fluid to be incompressible. Now, let us say the fluid is not in compressible, but it is compressible and when the fluid is compressible the pressure and density are related through an equation of state. Let us assume the equation of state to be that of an ideal gas, where p v is given as number of mole times universal gas constant times t.

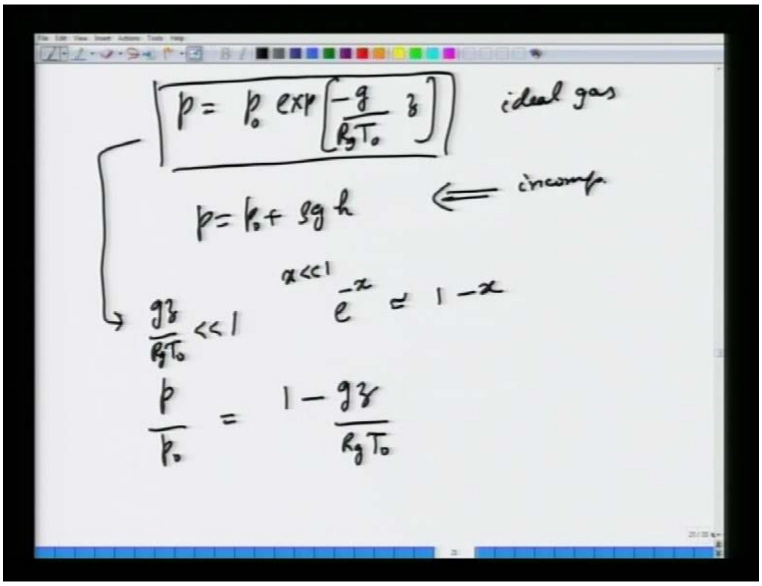
Usually, this is written has p is number of moles is mass divided by the molecular weight, the molecular weight of the gas and bringing V down R T. So, m by V is the density and R by molecular weight is called as specific gas constant so, let us call it R g. This not the universal gas constant it is specific gas constant. So, p is rho R g T; so, instead of writing rho g I am going to from this equation. I am going to write dp dz is instead of rho, I am going to write p by Rg T is minus p by Rg T times g so, this can be integrated.

(Refer Slide Time: 54:34)



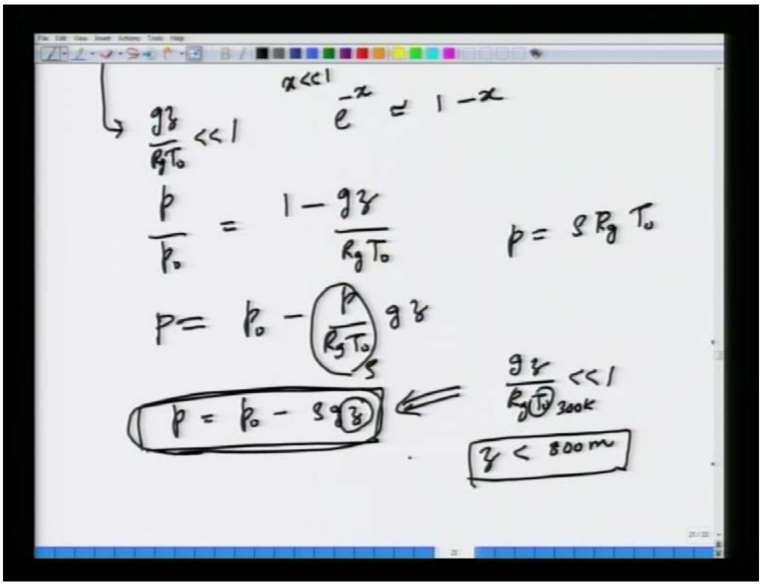
So, dp by p is nothing but minus g over Rg T dz. This is law of p is minus g by Rg T and the temperature is assume to be a constant. So, the here we are looking at an isothermal condition, but the temperature is fixed, the gas is a constant temperature so, t is constant. So, you can integrate this just to get t is a constant. So, you can integrate this trivially to give z plus c. So, if you assume p is p naught at z is z naught you can fix this constant. So, you get log p over p naught is minus g by R T naught so, t is t naught a constant it is isothermal. So, times z or p is p naught exponential minus g by R T naught times z. So, we can integrate this equation for static fluids not just with the incompressibility assumption.

(Refer Slide Time: 55:55)



So, even if you assume the flow the fluid to be compressible, but by supplying the ideal gas law. We got a simple relation for the variation of the pressure with the elavish. Now, this is for ideal gas and p is p naught plus rho g h, this is for incompressible. Now, this expression can be simplified, when g z by R T naught is much less than 1. If this is the case, when the elevations are not so high then I can; when n suppose, you have e to the minus x when x is small then I can write it as one minus x approximately so in the limit gz by R T 0 is small. So, this is the universal stress specific gas constant, which I denote by R g. So, p by p naught is written as 1 minus; so, since there is 1 minus gz by Rg T0.

(Refer Slide Time: 57:09)



Now, p by or p is p naught minus p by Rg T0 g is a from the ideal gas flow p is rho Rg T0 so, p by Rg T0 is nothing but rho so, p is p naught minus rho g z. So, this is the same equation that we obtained for incompressible fluids, but this is obtained as a special case, when gz by Rg T0 is very small compare to 1. So, even if you assume compressibility when the elevations are not so large. This essentially balls down to the same linear variation of pressure with respect to depth. So, when is this considered to be small? Well, when z is less than 800 meters roughly then, if plug in values of g and R g for air and T naught to be 300. You will find that this is small when z is less than 800 meters.

So, when you are looking at applications even, if you assume and have to be compressible when the elevations are not large. Then this linear variation in pressure with respect to the vertical distance is a very good one. So, we will stop here and we will continue in the next lecture. So, we will see you in the next lecture.