Fluid Mechanics

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Lecture No. # 05

Welcome to this fifth lecture on this n p-tel course on fluid mechanics for undergraduate students in chemical engineering in the last lecture, we discussed the basic difference between newtonian fluids viscous fluids and elastic solids and we wrote down we said that suppose.

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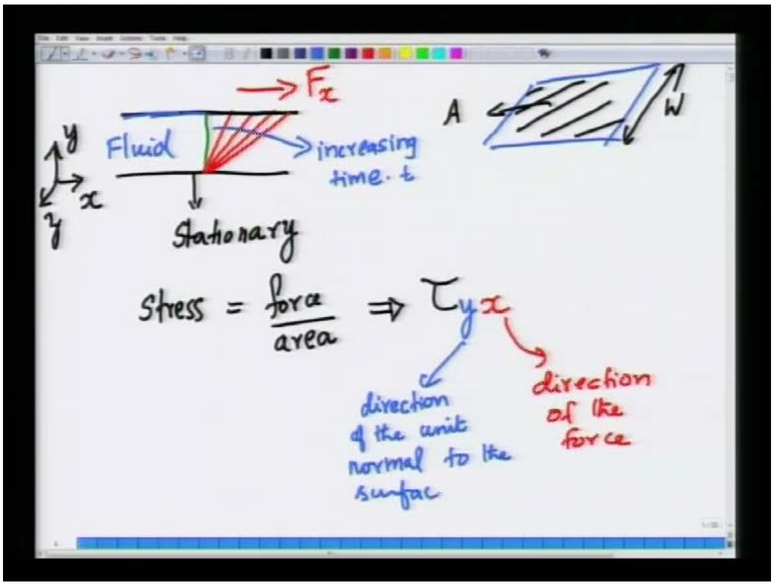

You take two plates and place a fluid layer in between them, a layer of fluid in between them, so we just remove so this is the layer this is fluid in between and suppose you exert a force on the top plate, f x so the top plate itself if you recall is a plate that is extending in the z direction, where the co-ordinates are as follows x y and z so this has an width w in the z direction.

And this is the area the top area a or which the force f is been applied and if the bottom plate is stationary, no force is being applied and its stationary then we said that if you follow line a line if you follow colored line in the fluid, which was a vertical before the application of force and if you follow the line after the application of force, this line will continue to tilt.

As a function of times so here time increases, increasing time, in this way. T as time t increases this line will keep on tilting. So, a fluid continues to deform on upon application of tangential or shear stress, shear force in contrast to a solid which deforms to some extent and then stops deforming upon application of force.

So, we said that the stress, stress is force divided by unit area force divided by the area upon which the force is acting. The stress is actually denoted by the symbol tau and we said that there are two suffixes for the stress, x is the direction of the force and y is the direction perpendicular to the surface upon which the force is acting.

So, it is a direction of the unit normal to the surface unit vector perpendicular to the surface is called unit normal. So, it is the direction of the unit normal to the surface, upon which the force is acting.

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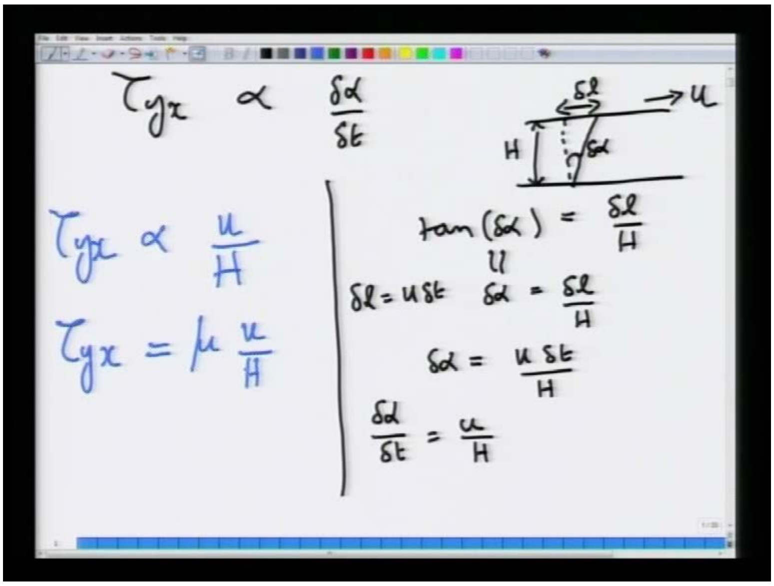

So, this is the stress this is the definition of a stress. So, tau y x we said must be proportional to the rate of deformation, which if you recall we call this angle as delta alpha since delta alpha continues to change with time, we said that the stress must be proportional to delta alpha by delta t and this was also found to be equal to based on geometry considerations, if you call this was the line before deformation this is lets say delta alpha and this is the small displacement delta l and this is the thickness of the fluid h and delta alpha tan of delta alpha is delta l by h but, for small enough angels this is approximately equal to delta alpha is delta l by h.

But as a fluid deforms continuously the top plate will move with a velocity v lets say all. Let us say u so delta l is u delta t so delta alpha is u delta t divided by h or delta alpha by delta t is u divided by h.

So, tau y x must be proportional to u which is the velocity at which the plate is moving and divided by the gap thickness in which the fluid is moving and the constant of the proportionality is the viscosity of the fluid.

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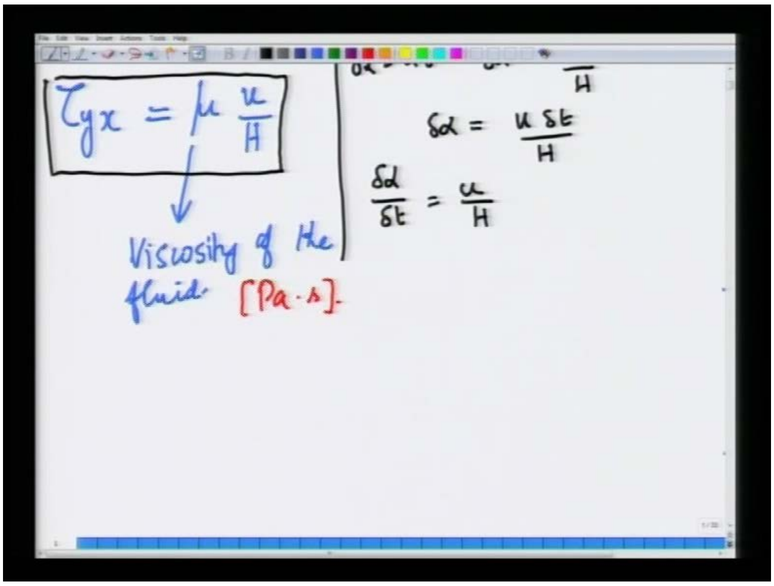

This is the viscosity of the fluid (no audio from 05:12 to 05:19) and we also gave you some examples of viscosity values, viscosity of the fluid is measured in pascal second in s I units and we also gave you some examples of the values of viscosities of various commonly encountered fluids like air water oils and so on.

Now what is the use of this formula well we will return to this formula in detail later, when we do differential balances but, at the outset this is an introduction to what how a viscous fluid behaves upon application of shear stress you can use this formula to find for example, suppose you have.

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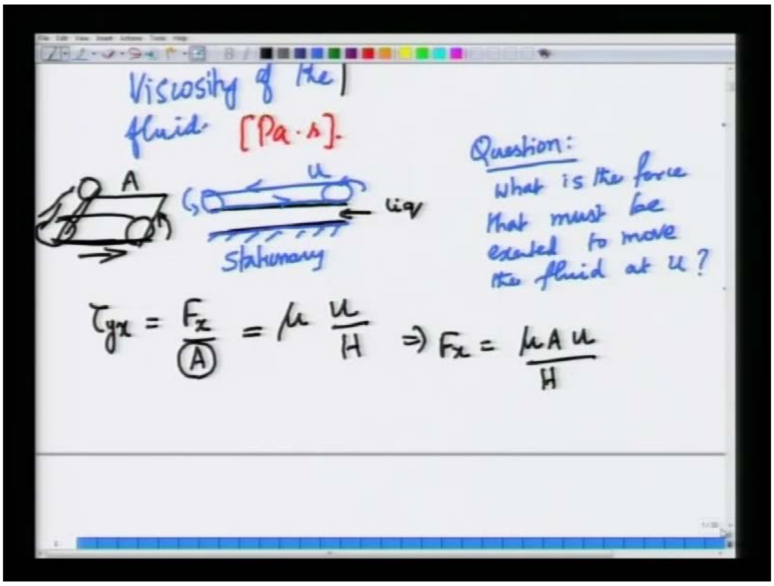

Fluid layer, a viscous liquid and let us say you have to push this viscous liquid and the bottom you have the bottom surface is stationary.

And let say you are moving the top surface with a belt with a velocity. So, this is rotating so this is a velocity u at which the belt is moving, so you are trying to drag the fluid at a constant velocity suppose you ask the question. What is the force that must be exerted (no audio from 06:43 to 06:51) to move the fluid at a velocity u?

Suppose you ask this question, well the answer is obtained in the following way we know that tau y x is the force f x divided by a, that area of the belt so remember this is a these are all the belt is a finite. So, this whole sheet is so this is a role cylindrical role in the third direction and this whole role is rotating a the constant velocity.

And the area of the top plate is or the belt surface is a so that is the area so this is the viscosity of the fluid times u divided by the gap thickness. So, force is therefore, mu a u divided by h suppose we know the values.

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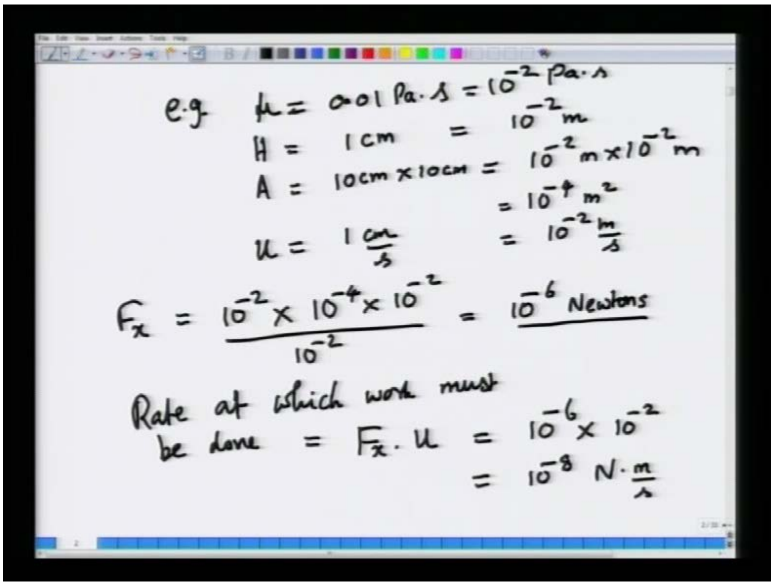

Suppose you are interested in dragging a viscous liquid using a belt, let say example viscosity is let us say 0.01 pascal seconds, let us say the gap thickness is 1 centimeter. And the area is so we should convert this in s I units which is 10 to the minus 2 meters, let say the area is 10 centimeters by 10 centimeters so this is 10 to the minus 2 meters times 10 to the minus 2 meters, which is 10 to the minus 4 meter square. And let us say the velocity is 1 centimeter per second which is 10 to the minus 2 meter per second so if you substitute everything in that expression f x is mu which is 10 to the minus 2 0.01 pascal second is 10 to the minus 2 times.

So, let me just write this is 10 to the minus 2 pascal second, so 10 to the minus 2 times area is 10 to the minus 4.Times velocity is 10 to the minus 2 divided by h is 10 to the minus 2 so this is about 10 to the minus 6 newtons, if the area is just the small amount then this is the force that must be exerted to move the belt at a constant velocity what is the rate at which work must be done. Suppose you want to move this conveyer belt using, some rotating machinery, so the rate at which work must be done by the machinery in order to keep moving this is work must be done by the external agency to move this belt is work done is force times distance rate at which works done is force times distance divided by time.

Distance by time is velocity, so it is the force times velocity its 10 to the minus 6 times velocity is 10 to the minus 2, so this is 10 to the minus 8 so this is newton then meter per second. So this is the rate at which work is done in order to move this fluid. So, this is 1 simple example of this equation that, we just wrote down for a simple fluid.

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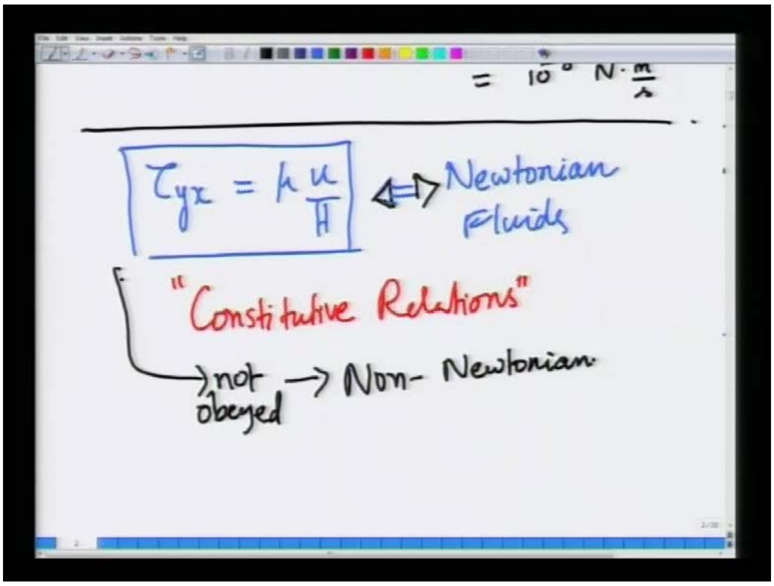

So we also said that this, this relation mu u by h, this describes what are called newtonian fluids, because this is only an expectation that the stress must be directly proportional to the rate of deformation. But, it turns out that commonly encountered liquids, respect this expectation. So, this is a material behavior and we will see that such relations are called constitutive relations.

Later these describes specific behavior of how a given fluid flows or responds to external applied stresses and there are fluids, which do not obey this for example, slurries and solutions of polymers, molten polymers, molten plastics and so on. They such fluids are called non newtonian fluids. Anything that does not obey if this is not obeyed they are called non newtonian fluids. So things that obey this they are called newtonian fluids. So, we also we will discuss much later in this series of lectures, as to how to understand with the behavior of non newtonian fluids a little later.

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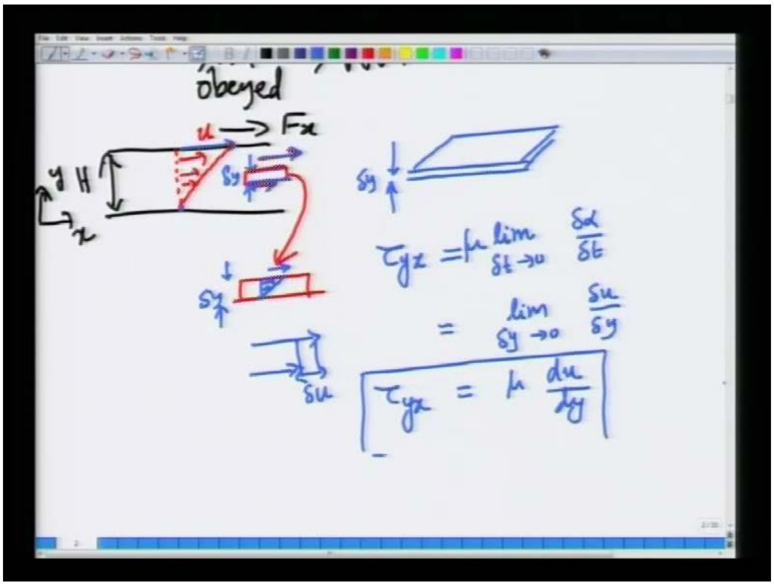

So, this expression for the, the shear stress as a function of velocity of the top plate divided by thickness was applied to a case of a finite fluid layer of thickness h, which is exerted upon which a force is exerted in the x direction and so on. But you can also think of a flow suppose you have, the same example a flow is here is the x y co-ordinate, this is fluid in between you can also think of considering a tiny slice of thickness delta y in the y direction.

So, essentially if you recall we are extending everything in the z direction so this is a tiny slice like this of thickness delta y in the y direction and you can try to apply so when a the fluid is stationary here fluid is moving with some velocity here. So, if you were to so if you were to plot the velocity, so the velocity will go from zero to some velocity u at the top plate in this example so fluid layers are sliding past each other in this simple example so if you take a tiny slice like, so the fluid that is flowing past here will have a greater velocity than the fluid that is flowing below this tiny slice, so if you are moving with the velocity of this lower fluid, the flow about this tiny slice will again appear. So, I am just expanding this tiny slice will appear light a linear flow like this.

So the thickness is delta y so because these two slices are fluid above the slices moving with the greater velocity then the fluid below the slice. So, if you move with the velocity of the lower side, then this will appear stationary the top will appear to move at a constant velocity. So the this expression can be generalized in the limit delta t going to zero delta alpha by delta t, this was the fundamental definition and delta alpha is by delta t is nothing but, limit delta t tending to 0 delta u by delta y, where delta u is the difference in velocity between these two points.

So we know the two if you take a slice, this magnitude of the velocity differs between these two slices in this case, so that magnitude of the velocity difference is delta u so as delta t goes to 0, this is the shear stress this is mu times this. So, tau y x is mu d u d y in the limit of delta t goes to 0, the velocity difference, so this translates to delta y goes to 0 becomes this. So, this equation is not merely applicable for thicknesses of macroscopic dimensions, Even if you take a tiny slice volume slice and then we try to look at how flow is going to happen how the shear stresses are related we will find that this is true even for a tiny slice.

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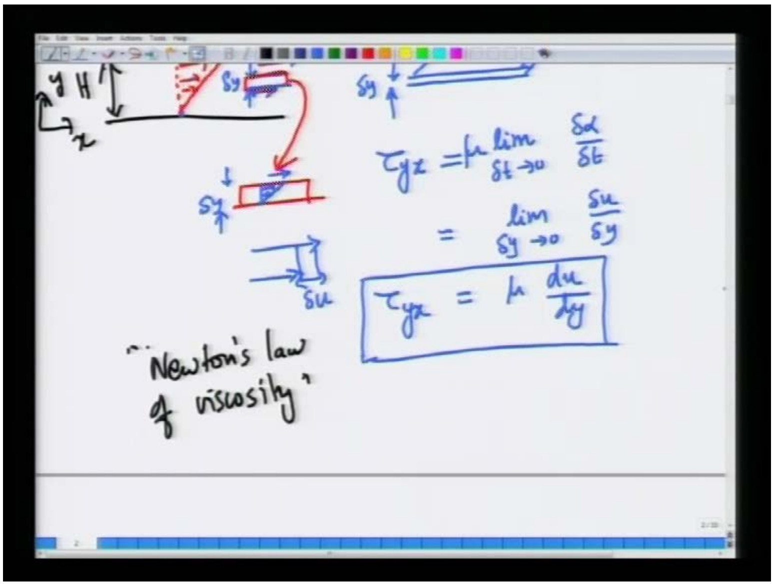

So this is called the newton's law of viscosity. (No audio from 16:03 to 16:11) now the next topic that we are going to discuss is fluid statics but, before I do that we will have to first understand what are the various forces that are exerted on a given fluid element.

So, let us continue along the same lines and then try to classify the types of forces, that can that can be experience by a given fluid element volume element of fluid and then we will proceed to fluid statics, so, let us imagine having continuing with the same example.

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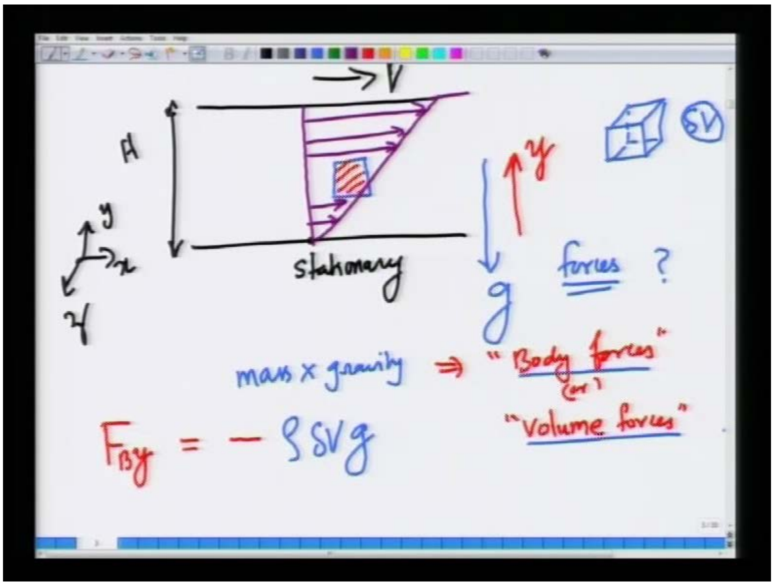

You have two plates, so the thickness is h this is x this is y this is z. And the top plate is moving with a velocity, let us say capital v bottom plate is stationary, so if you consider a tiny volume element remember that z is going outside into the board.

So essentially this volume element will look like this, into the z direction into the so that the z direction is going into the board. So, but this is the volume element this is the volume element but, if I want to write it in a so let me draw it separately, let me draw it separately here.

So, the volume element that we are going to analyze is somewhere something like this but, in a 2 d plane because this diagram has been drawn in a plane, that is cutting across any z location the volume element will appear like a slice. So, this volume element what are the forces that are acted upon on this volume element suppose you are doing this experiment in a lab and let us say gravity is acting downwards, now if you take this let us for simplicity assume this to be a cubic volume, that simplifies our discussion so take a cubic volume element instead of a rectangular volume element. So, this will appear like a square in the two d diagram.

What are the forces on this volume element well if the volume of this cubic volume element is delta v because there is gravity, there will be a force see whenever gravity acts on a mass there is a force acting in the direction of gravity m g. So, mass times acceleration due to gravity is a force this force acts through the entire body entire volume, such forces which act through the entire volume are called body forces or sometimes they are also called as volume forces because if you take a tiny volume this force due to acceleration due to gravity acts through the entire volume not just on a particular surface or a given location, so it acts on the entire volume.

So, this body force acting on this volume element and the body force is acting in the y direction only because, we know that the gravity is only in the we have align the gravity in the direction of negative y. So, this is negative because negative because gravity is acting so this is the direction of positive y gravity is pointing the direction on negative y so that must be negative mass is density of the fluid times the volume infantilism volume we are considering times, acceleration due to gravity the magnitude of acceleration due to gravity which is 9.8 meter per second square in s I units.

So, this is the force that is acting on this volume element due to acceleration due to gravity such forces are called body forces or volume forces, now apart from this because of the fact that the top plate is moving with the constant velocity. So, I told you that so, mething like this should happen.

So, there will be fluid flow layer by layer in this simple case and if you take a tiny volume element I am exaggerating this volume element for illustration sake but, this is a very tiny volume element if you take a tiny volume element, let us again draw this square separately here.

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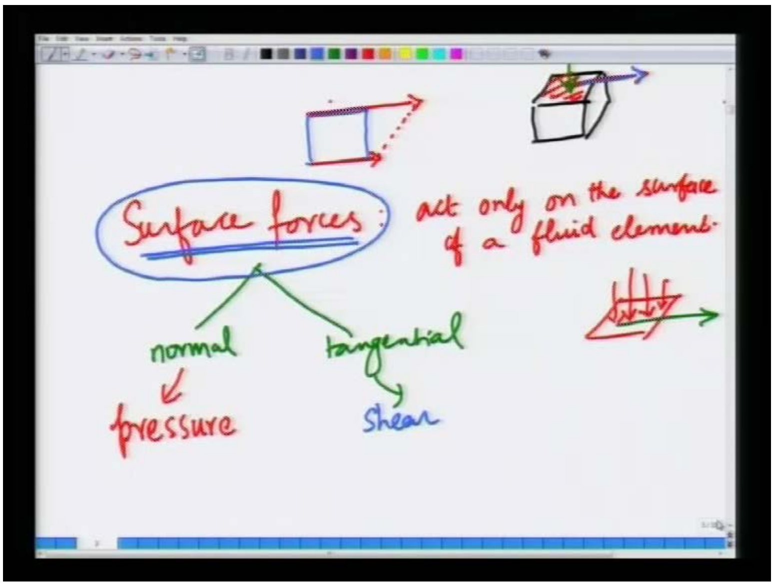

So, there is fluid flow above the square with some velocity and below the square with some velocity and this difference in velocity means at there is a shear stress exerted by the fluid on the top on this surface, such forces which act suppose you consider this, remember that it is a cubic volume it has a surface on the top I am drawing the 2 d version of it so there is a surface, so along this surface the top surface there is a force that is acting in the x direction due to the fact that fluid layers are flowing pass this surface and due to viscous action there is a stress or force per unit area or a stress, such forces are called surface forces.

They act only on the surface, of a volume element of a fluid the element when I say fluid element we mean a volume element in the fluid. So, if you take this volume element there is a tangential force on the top surface due to the fluid flow above in this direction and this will constitute a surface force this is essentially due to the tangential force is due to viscous action. But, there could also be force due to in the normal direction due to pressure those are also surface forces.

So, surface forces can be acting normal to a surface and tangential to the surface, the normal force to the surface is usually pressure dictated by a pressure also we will clarify a little later when we do differential balances this statement but, you can think of at this introductory level normal force on a element on a surface largely due to pressure this is acting perpendicular to the surface, so its normal therefore, it is a normal force where as the tangential force is acting parallel to the surface that is why is called tangential or shear force.

So pressure forces and shear forces can act on a surface and these are acting only on a surface and not to the entire volume. So, these are called surface forces, so this is in the context of a simple example like this like flow between two plates but, in general you could have a fluid that is flowing in a arbitrary way and you can construct any volume element.

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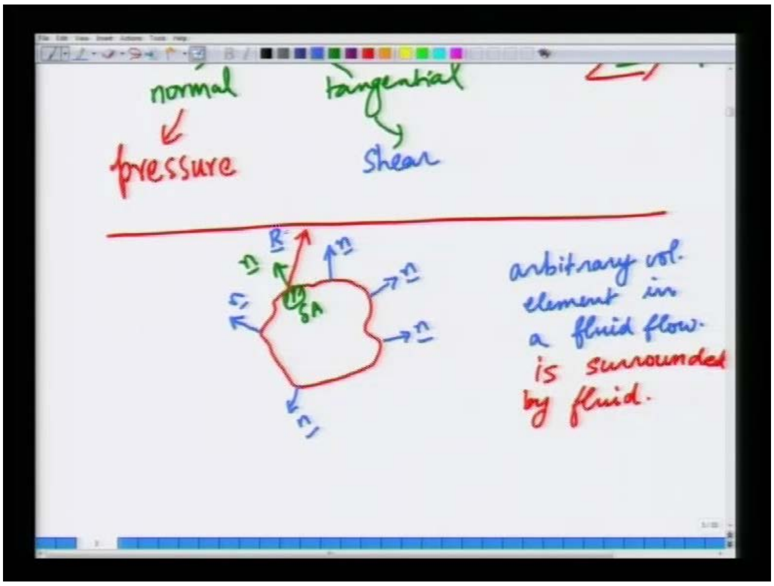

So, any arbitrary volume element and this volume element will have a surface and the surface is characterized. So, this is the volume element so this is a volume element in 3 d its arbitrary shape, but since I can draw only a 2 d version in this board, so I have drawn a 2 d version and the unit normal to the surface is n so the n suppose you consider this let me draw this slightly better. So, this is the arbitrary surface at each and every point this n will keep varying.

So, n will be like this here like this here like this here and like this here and so on. So, n keeps changing this is an arbitrary volume element in the fluid, in a fluid flow. So, this arbitrary volume element in a fluid flow is surrounded by fluid also, this is surrounded by fluid and if you look at a given point on this volume element and you worry about, let us say you take a tiny patch of area delta a, this has a unit normal n at this location and if you ask what is the force the force can in general be in any direction or the force per unit area which is denoted by the letter r stress vector it can point in any direction. So, let me just illustrate this again separately.

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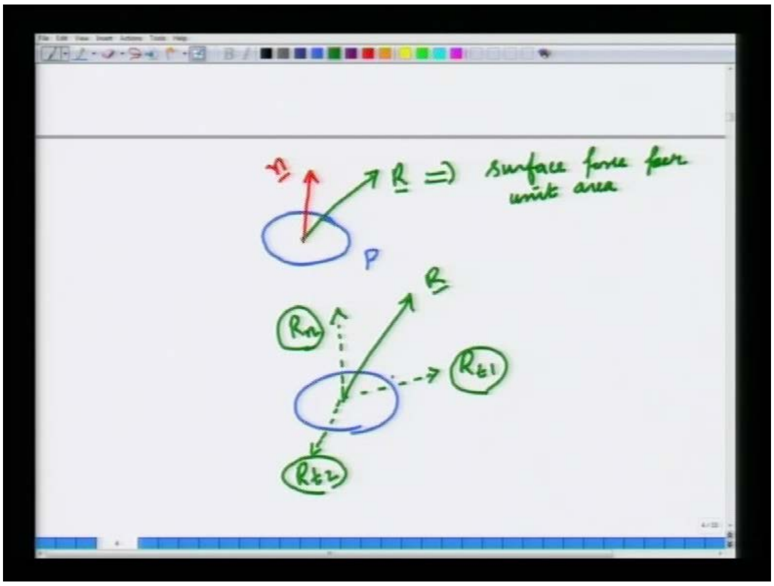

So you take any surface on a volume element, let the so take a point p on a volume element on the surface of a volume element and the surface has a unit output normal n and the force that is exerted by the fluid, that surrounding on this the surface force that exerted by the fluid is surrounding, this area on this surface can in general point in any direction so let us so that force is denoted by r.

This is the area now this r is of the force and that need not in general be in the direction of the normal but, it will have a component perpendicular in the direction of the normal. So, let us call that so I am going to draw this again here. So, this is r the direct the component of r and the direction of the normal is called the normal force per unit area so r is the surface force per unit area.

So, there is a normal component r n and along the surface there are two tangential components r t 1 and r t 2. So, these are the this is the normal stress these are the tangential stresses.

To any surface you can construct a normal unit outward normal and you can for this normal perpendicular to this normal there are two tangential vectors, this can be constructed in any way that you want I am just showing you for illustration in this manner, so this force that is exerted by the fluid on the outside on this surface can be resolved in the direction of the normal, as well as in the direction perpendicular to the normal that is parallel to the surface itself so these are. So, the surface force can have in general a normal component.

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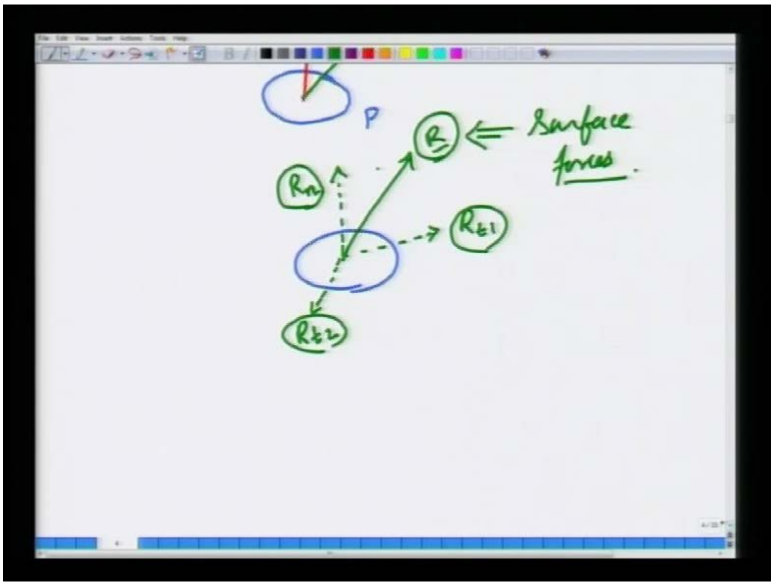

This is the surface force remember can have a normal component, as well as tangential component, now and I given you a simple example that the surface force is the surface forces normal as well as tangential components. So, the surface forces can, surface force is can have normal as well as tangential components, now I given you some simple illustration in which the tangential component of the surface force, why it occurs due to viscous action but, we will discuss more about this in detail a little later.

A clear understanding of what are the forces that can exert that that can be exerted on a volume element of fluid is necessary to understand, how to analyze a fluid a fluid mechanics because ultimately.

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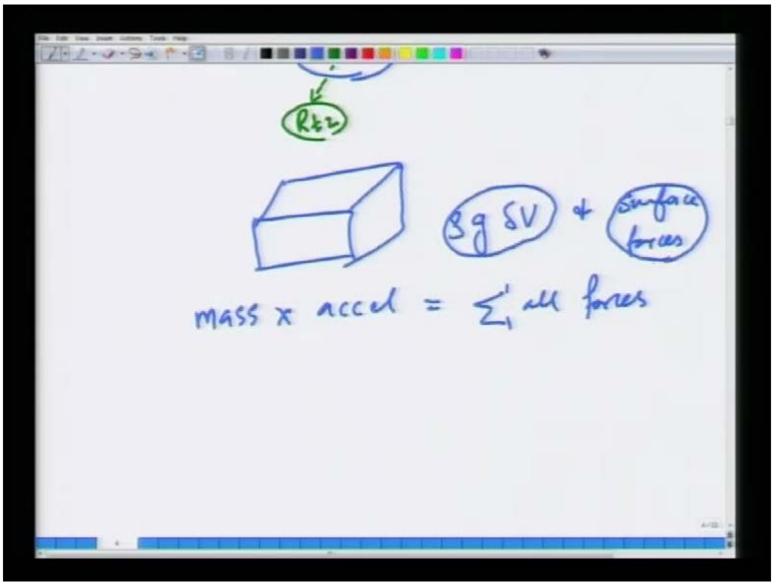

As I told you fluid mechanics is a branch of mechanics and if you take any tiny volume element and if you know, what are the forces that are acting the body force, which is rho g delta v and the surface forces, if you have a knowledge of body and surface forces then one can apply newton's second law of motion that is which says us mass times acceleration is sum of all forces.

Which can be used to compute the motion of the fluid so this is the program, rough program in any branch of mechanics but, in fluid mechanics the forces are not of various types one is the body force, which is acting entirely through the volume and the other is the surface force, which acts only on the surface the surface forces can be due to viscous action and as well as due to pressure.

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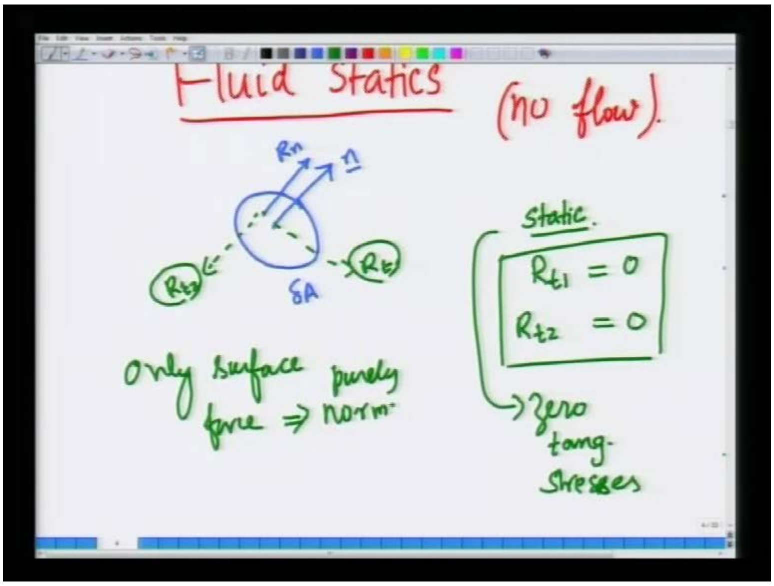

So, the first topic that we are going to consider is fluid statics. Now, we are going to consider fluid statics, deals with a fluid that has no motion, that is no flow. So, it is no flow this only a fluid is stationary.

Still there are some interesting aspects to the force distributions even in a fluid, that is even in a fluid that is not moving it is a stationary. So, let us understand what forces can act on at static fluid you take a fluid volume. Construct a surface at any point you take a point in the volume, construct a surface of area delta a this surface is denoted by unit normal n.

So, in general the surface forces, are in the direction of n which is what I called r n and in the direction perpendicular to n which is what I call r t 1 the tangential forces. These are the shear forces.

Now, we are now considering fluids that are not moving a fluid this static, the fluid is static, now what can we say about the forces the nature of surface forces, when the fluid is static by definition a fluid undergoes a flow, when there is a non zero shear stress, we just saw this in detail in the last lecture, where we constructed simple thought experiments, where we showed that a fluid continues to deform upon application of a shear or tangential stresses.

So, if you consider fluid that has no motion no velocity and no flow, then it implies that the fluid the tangential stresses in the static fluid must be 0. So, the tangential stresses must be 0 in a static fluid.

So, you cannot have tangential stresses, so the only type of surface forces that can act on a volume element, so the only surface force is the normal force because, if there is a tangential force on any point in a fluid that means it the fluid will start flowing.

Because, a fluid cannot resist any non zero tangential stress, even at a slightest application of tangential stress the fluid will start moving therefore, we can conclude safely that in a static fluid the tangential stresses are zero. So, zero tangential stresses (no audio from 33:30 to 37) and the only surface forces must be purely normal to any surface. Now, this is true, for suppose.

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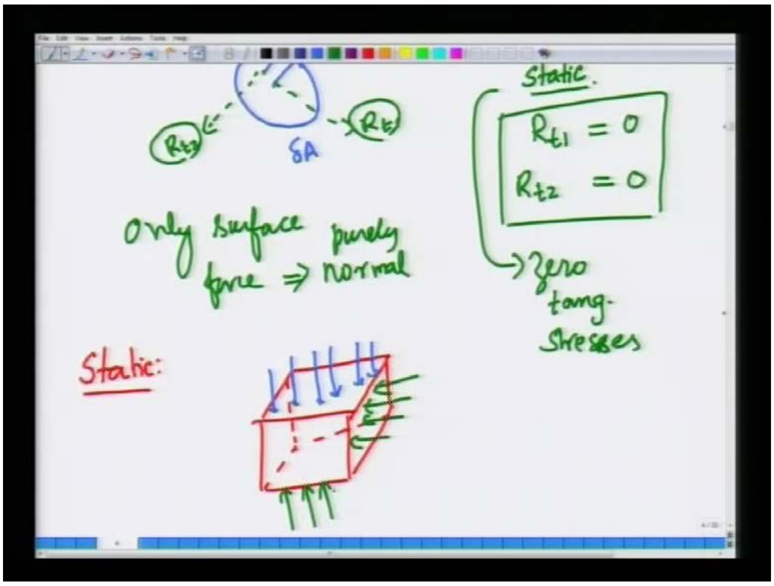

You take a static fluid and consider a volume, let us take a simple volume, like a cube so if you take any face on this volume, if you take the top face the forces will be purely normal, if the fluid is static because there is no other way no way the fluid can support a tangential stress.

If you take the side way, side face, here again the forces will be purely normal and same with bottom and so on. So, the forces will be purely normal in a static fluid but, this is true for any volume element because, you can construct a volume element the shape of the volume can be as arbitrary.

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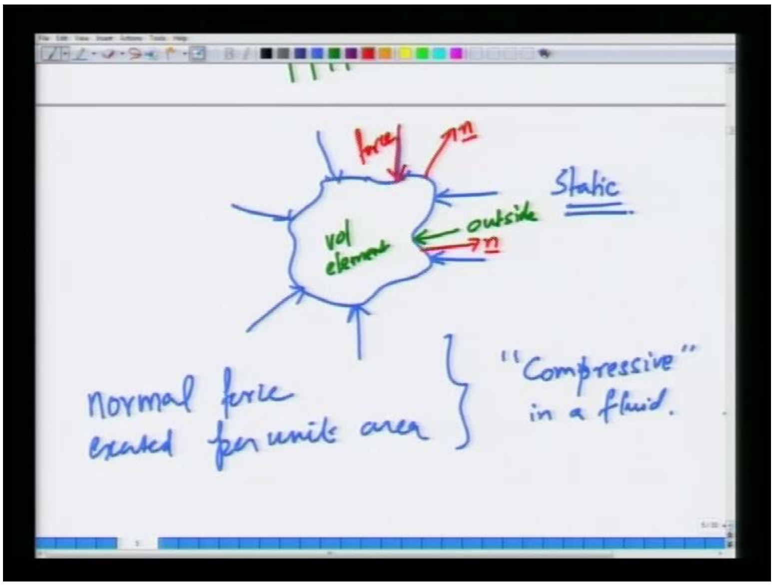

As you please, even in this complicated volume element the forces will be purely acting at each and every point on the surface of this volume element the forces has to be purely normal.

Because, by definition in a static fluid in a static fluid, if you construct a volume arbitrary volume element, the forces have to be purely normal because at each and every point in the fluid if there cannot be any tangential stress. So, the normal in a static fluid this so that can in general be normal force a static fluid can support normal forces.

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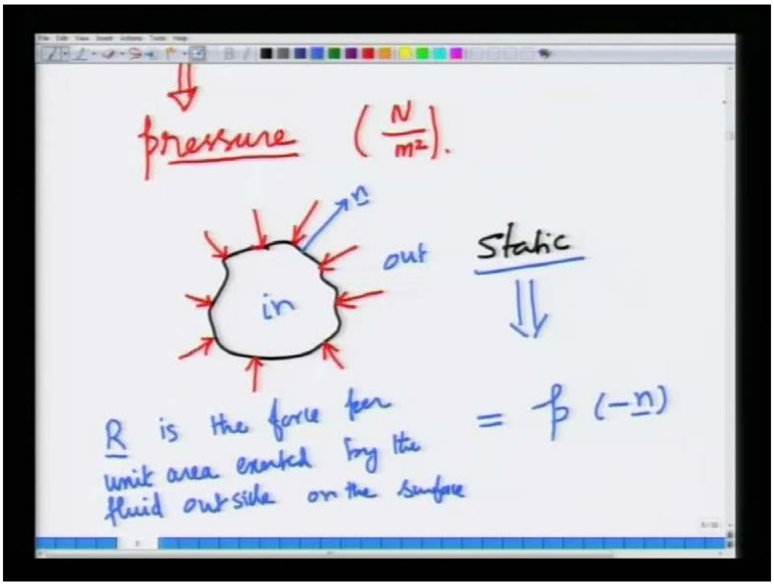

The normal force, exerted per unit area is usually compressive in nature in a fluid, that is if you consider this is the fluid inside the volume element, this is the fluid outside. So, this fluid outside exerts a force that is compressive the tenses compress the fluid inside the volume element.

And but, the unit outward normal is pointing like this, to the to this volume element at each and every point on the surface the unit outward normal will act like this but, the normal force will act in the direction, this is the direction of the force in the direction opposite to the unit outward normal.

This normal force exerted per unit area, this compressive normal force, exerted by unit area in a fluid is called the pressure. Since, its force per unit area it is dimensions of newton's per meter square.

The normal force that is experienced by a fluid, a static fluid element volume element the fluid is static. So, if you take a volume element I just told you that this volume element in a static fluid will experience only a surface force, that is purely normal to each and every point on the surface and this is compressive in nature that is called the pressure.

So, if you think if you say this is n and if you say r vector is the force per unit area, exerted by the fluid outside. (No audio from 36:17 to36:23) So, this is in, this is out on the surface, since the fluid is static this is, this has to be in the direction I mean since its purely normal it has to be either in the direction of n or opposite to n.

But the pressure is normally compressive so this is in the direction of minus n because n is pointing from in to out, where as the force is from out to in and the magnitude of the force compressive force is pressure, force per unit area is pressure p.

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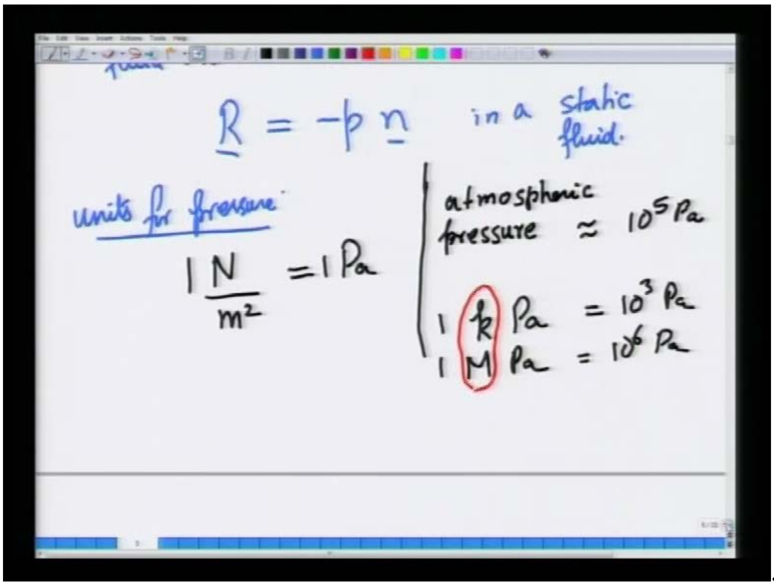

So R that is force per unit area is given by minus p n in a static fluid because, a static fluid cannot support any shear stress.

So, and a typically the units for pressure, is in SI units it is 1 newton a force per area newton per meter square, this is also equal to a pascal 1 pascal but, it turns out that the pressure that is experienced by us in the free atmosphere is called the atmospheric pressure. So, the atmospheric pressure is approximately 10 to the 5 pascals it is a large number, so pressure is normally expressed not just in pascals but, also in kilo pascals, mega pascals. Kilo pascal is 10 to the 3 pascal, 1 mega pascal is 10 to the 6 pascal and so on. So, it should be getting used to this prefixes to this units which tell you multiples of thousands.

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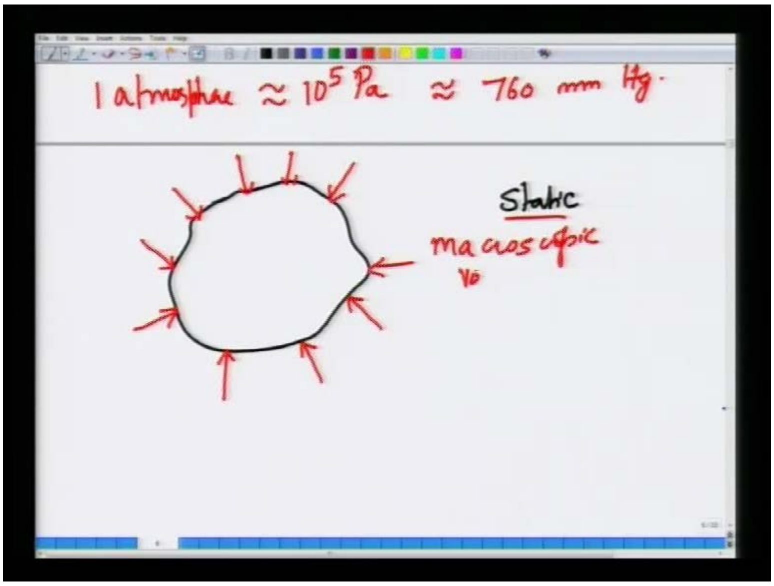

And sometimes pressure is expressed in terms of 1 atmosphere that is approximately 10 to the 5 pascals. Sometimes, it is also expressed, we will show little later in terms of columns of mercury in a mano meter its 760 m m Hg. We will see this a little later once, we finish the basics of a fluid statics, now let us first worry about what is the nature of pressure.

So if you take any macroscopic volume element in a static fluid, if you take a macroscopic volume element I have just told you, that the force will be purely compressive its force exerted by the surrounding fluid on the surface of this volume element purely normal to the surface and it is compressive.

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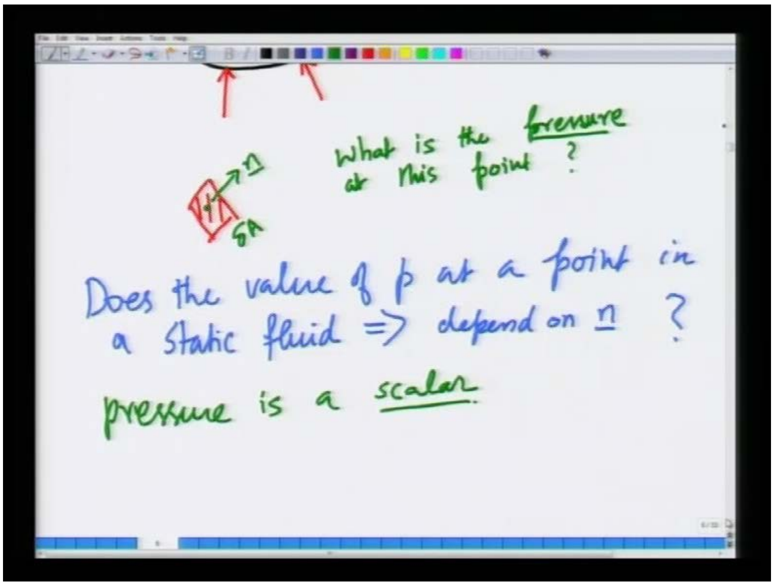

This is for a macroscopic volume in a static fluid. Suppose, I consider a point, in a volume and construct a surface, so construct a surface, this is a point and I can construct a surface with some orientation, whenever you construct an area there is orientation associated with it because I can orient this area in many ways. Now, what is the nature so this is really a point and about this point I construct a tiny area delta a and I ask the question, what is the pressure measured at this point. So, I can do this thought experiment I can orient this surface in one direction at a point. So, I take a point construct a very tiny small area and I oriented in a particular way I measured the pressure I change the orientation the fluid is static and this is really a very very small area. So, that we are shrinking it eventually to a point.

And as I change the orientation what is going to happen, what is the value of pressure, is if does the value of pressure. So, we will ask this question, if you do this experiment does the value of pressure at a point in a static fluid, (no audio from 40:48 to 40:55) does it depend on n the orientation of the surface essentially you are considering a point but, about a point you can construct a infantilism area.

So as I told you repeatedly in the continuum approximation, when you say at a point any property like density at a point you have to construct a tiny volume about that point, when you take consider a pressure you have to construct a tiny area but, you can construct this area with many infinitely many orientations of out of point, So, is there does the is the does the pressure that one obtains about a point as the area shrinks to zero in the continuum sense.

Does it still does it depend on n the orientation we will show that it will not, so the pressure is purely a scalar that is at a point pressure is force per unit area but, at a given point it.

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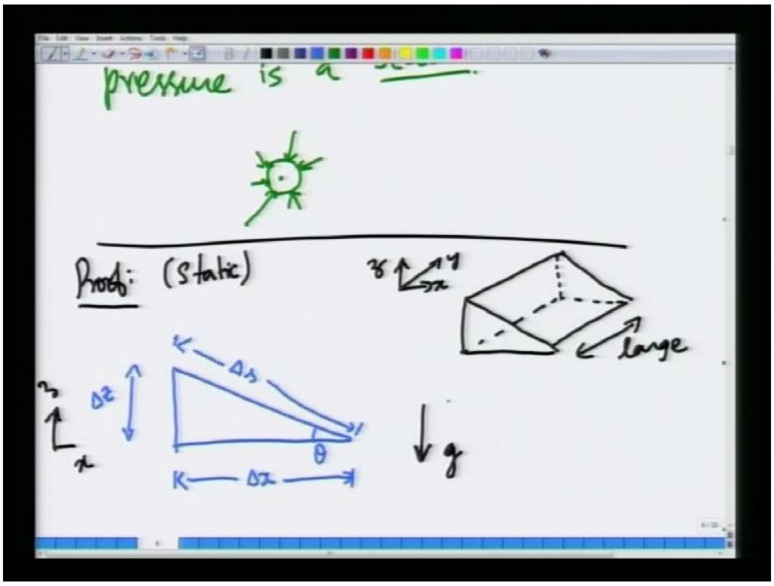

If you construct a tiny volume about a point at a of arbitrary shape orientation at the surface its always compressive its always in the direction of the in the direction opposite to the unit outward normal. So, it pressure does not have a sense of direction at a point it is the force per normal force per area.

At any surface, of a surface of any orientation at that you would construct about a point. So, how do we show this, we show this in a very simple way, so you construct consider so this proof, that the pressure is a scalar does not depend on the orientations about a many orientations, that you can have when you construct an area. So, let us take a volume that is wedge shape like this and this third direction is so large, that we will neglect variations in the direction. So, consider a only the cross section triangular cross section.

Now, this angle let it be theta, now let us put a co-ordinate system x z and y is going into the board that is not relevant. So, if you consider this x y into the board and z vertical, it is a gravity is acting with this. So, this is a static fluid the fluid is still static we are instead in fluid statics right now. So, let us not draw y here it is a purely a plane so this is x and z.

So, let us call this surface the let this distance be delta x, let this distance be delta z, let this distance be delta s, now there is also gravity that is acting down, the pressure at this surface and these are all infantilism links.

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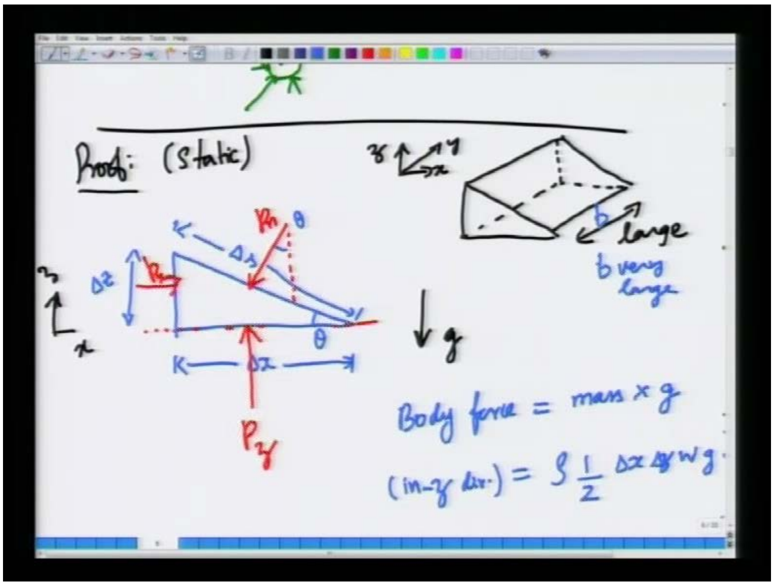

So, the pressure is approximately a constant the pressure on this bottom surface of this wedge like volume element, let us call it p z because this is p at this location z equal to zero and let us call the pressure here at this face p x.

And let us call the pressure, pressure is acting normally and its compressive, so it will act like this on this volume element, let us call this p n and by geometry you can verify that this angle is also theta. So, pressure will act on this incline plane incline surface that is the stop surface in a purely normal way.

So, what are various forces acting the body force due to gravity is the mass times acceleration due to gravity, the body force acts only in the minus z direction remember z is going up so body force in the minus z direction in minus negative z direction is equal to mass is rho times volumes, rho times volume of this wedge is basically half times delta x delta z times, let us call this width b, b is very very large.

So, W times g if this were a complete cube a cuboids you would say that the volume is delta x delta z times b but, you are cutting it into half by from across the diagonals. So, it is half delta x delta z w g that is the body force that is acting in the negative z direction.

Now, we will do a force balance, so and along the other shear a surface force is are acting the way we have shown here in the three faces, the surface force are acting along the three faces like this. So, now let us do a force balance, force is a vector, so you have to do the balance in component wise.

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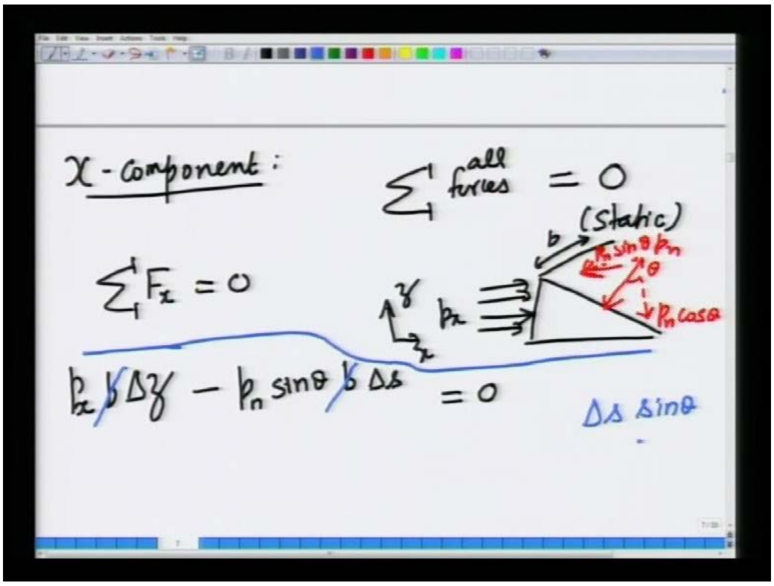

So, x component of the force, so first of all what is the force balance it is a static fluid, so by newton's second law sum of all forces acting on this volume element is 0 because the fluid is static this is a from mechanics.

So, sum of all the x forces must be 0, if it is, if you are worrying about the x component, so this is the wedge along this direction it is so this is plus x this is y, so along the plus x direction you have the force p x times the area, the area is b which is in this into the board times delta z I am sorry this is not y this is z that is way we drew times delta z this is the force in the plus x direction.

In this face and then you have this normal force p n, this normal force can be resolved into two and this angle is theta into two components, this is theta along the vertical, it is p n cosine of theta and along the horizontal p n sin theta, if such is the case then and it is acting in the minus, so this force is acting in a minus x direction, so I will put a minus sign times p n sign of theta, times b times delta s is 0. So, let us put 0.

This is the x component, now the b factor were cancelled right through, now from geometry delta s sin theta is nothing but, delta z.

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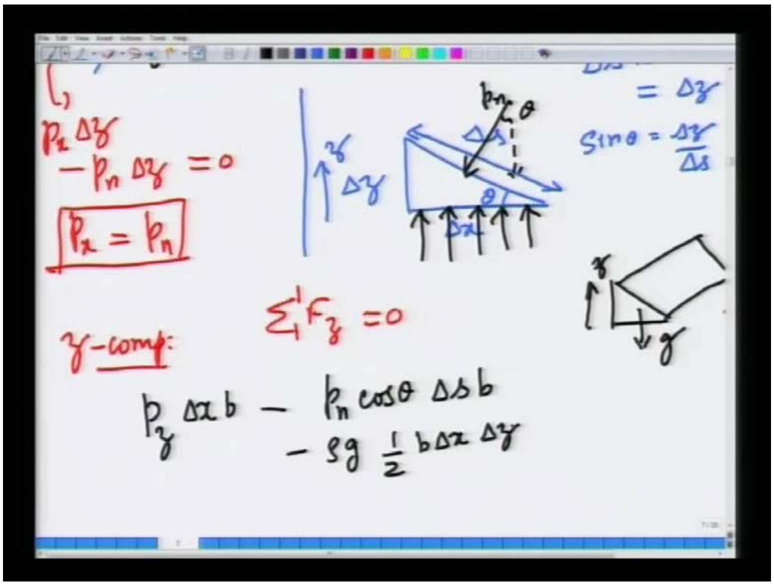

So, remember that this is theta, this is delta z, this is delta x, this is delta s. So, sin theta is delta z by delta s therefore, delta s sin theta is delta z, so in the above equation in this equation I write the first term as p x delta z minus p n sin theta delta s is nothing but, delta z is 0. So, we have shown that by using the x momentum balance.

That the magnitude of this force p x must be the same as the magnitude of the force per unit area here, so p x is p n, now let us do the vertical balance which is the z component, so here the force is the sum of all the force as in the z direction must be 0 along the z direction the force is so remember this is the z direction, so the forces are in this plane acting in the positive z direction p z times delta x times b and then you have in the negative z direction minus p n cos theta, that is the component of the force p n in the vertical direction.

But it is negative because it is acting in the minus z direction and then you have to multiply by the area of the triangular sorry the wedge area of this surface because, that is where this force is acting that is delta s times b.

And then in the vertical direction this is z there is gravity acting, so gravity is you have already found this is rho times g times half b delta x delta z is 0.

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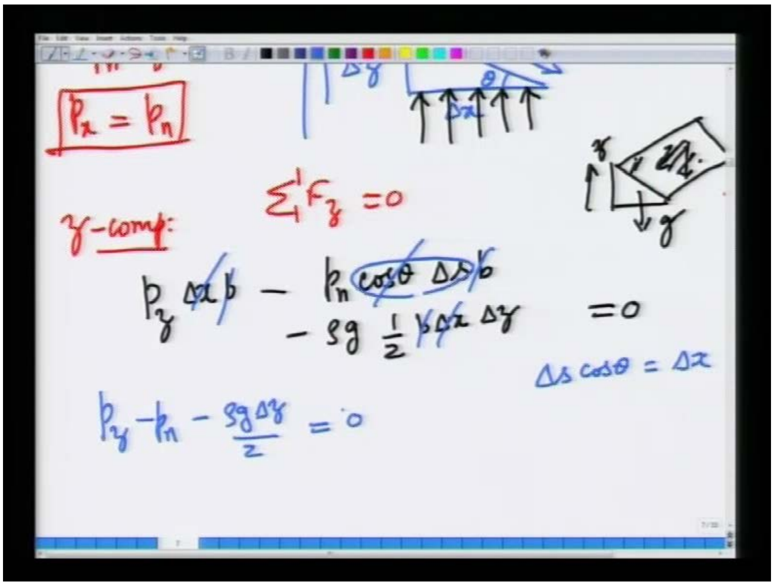

Sum of all force is gravity is acting in the negative z direction, so is this force which is the component of the force on this triangular surface on this face. So, we can sub cancel b right through and by geometry delta s cos theta is delta x, so we can cancel this, this and this to give p is a minus p n minus rho g delta z by 2 is equal to 0.

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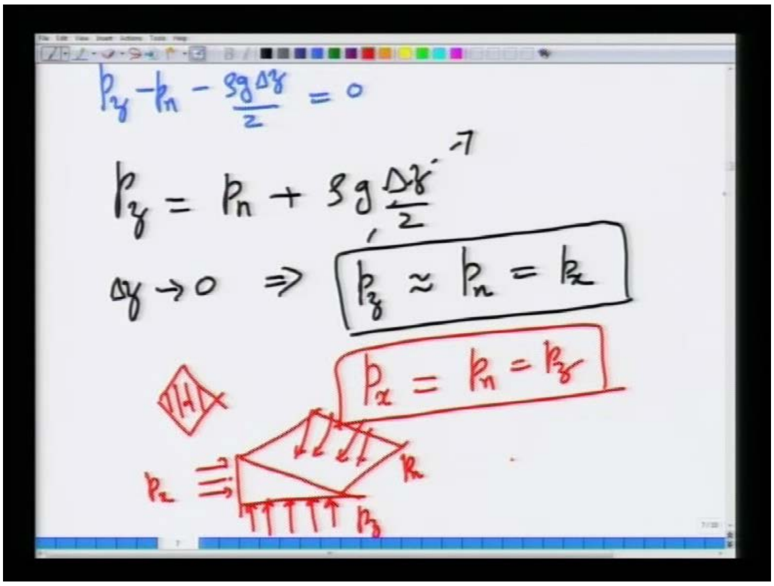

Or p z is p n plus rho g delta z by 2 but, in the limit as the volume shrinks to a point delta z will tends 0, so this term will be negligible so p z is approximately equal to p n in the limit and that is also equal to p x.

So, what we ended showing finally, is that if you take a point in a static fluid and if you want to measure the pressure, so you measure the pressure by taking a tiny area and the area can be oriented in any way.

What we have shown with this example is that p x is p n is p z is the same and it is independent of orientation of the surface, so we took this wedge, this is the wedge and we looked at the side face which is p x the bottom face which is p z, the pressure acting is p z and then incline surface the pressure acting is p n and we find that the pressure must be the same in a static fluid.

Regardless of the orientation of the surface at which it is measured. Now, we will stop here and then we will continue in the next lecture and we will worry about force distribution in a static fluid remember that in a static fluid there are pressure forces which are due to the surface forces and body forces balance between a pressure and body forces gives raise to force distributions of pressure distributions. So, we will continue with this in the next lecture and we will see you in the next lecture. Thank you.