Fluid Mechanics.

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Lecture No. # 39
Fluid Mechanics

Welcome to this lecture number 39 on this NPTEL on course and fluid mechanics for undergraduate chemical engineering students, the topic that we are discussing currently is turbulent flows.

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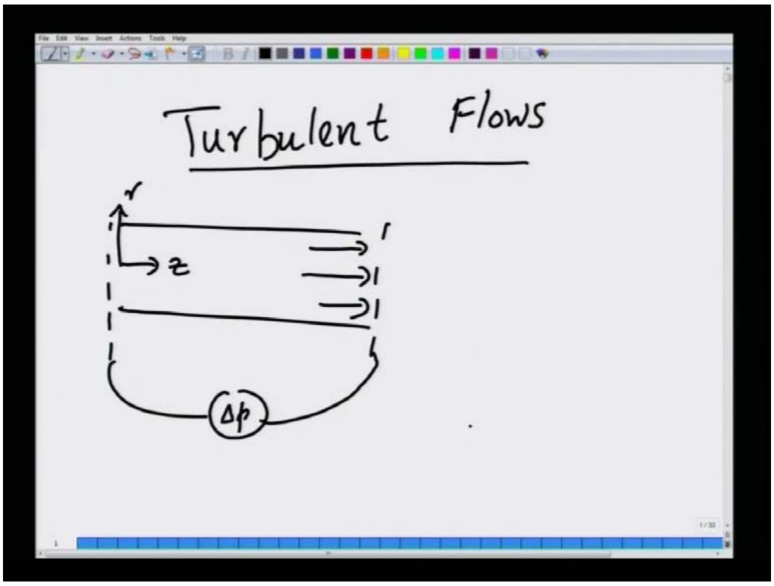

And we discussed in the last lecture lecture number 38 that Turbulent Flows are characterized by a large amount of fluctuations about the mean. Suppose, you consider a flow in a pipe and let say this is z direction of the flow and r is the direction normal to the flow the radial direction along the pipe. So while there will be mean flow if you apply a pressure drop between the ends of the pipe. While there will be a mean flow in the z direction there will also be a large amount of fluctuations about this mean flow.

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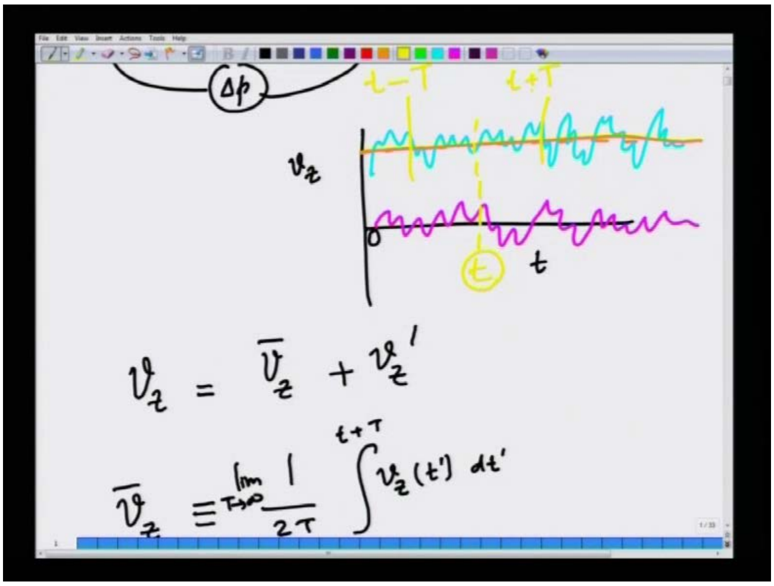

And if we you measure for example, the z velocity as a function of time using some probe at a given point in space in the pipe, you will find that there will be a large amount of fluctuations about the mean.

While if you measure, so this is the 0. So, this is the z component of the velocity, while on the other hand, you measure the r component there again fluctuations but, there will be fluctuating about the 0. Because, there is no net flow in the r direction the pressure gradient is only along the z direction. Therefore, there will be flow only along the z direction.

So, the fluctuations that are present in a turbulent flow play a very key role in data mining the stresses the represent in a turbulent flow. Now, so we decided that it is better to restrict ourselves to predicting, the mean quantity such as time average quantities velocities etcetera because, from an engineering prospective from the point of view of practical applications. What we want is actually average quantities such as friction factor or volumetric flow rate pressure drop and so on. So, these are all average quantities so it make sense for us to make restrict ourselves to predicting only the mean rather than the fluctuations.

But, as we saw in the last lecture and I as am going to retreat that point again the fluctuations too play an indirect role even in determining the mean, on the way that happens is like this.

You write the total velocity as a mean plus fluctuations, fluctuations are denoted by prime. The mean velocity is defined as 1 over t in the limit t tending to infinity 1 over 2 t times integral t minus T to t plus T v z of t prime d t prime.

What we are essentially saying is here, is that suppose you look at this velocity profile. This velocity data let us say this is actually velocity data now, we are going to take some time t. And we are going to integrate the velocity fluctuations in this region between minus t and plus t that is t minus tau to t plus tau and we are going to integrate this data and divided by 2 t.

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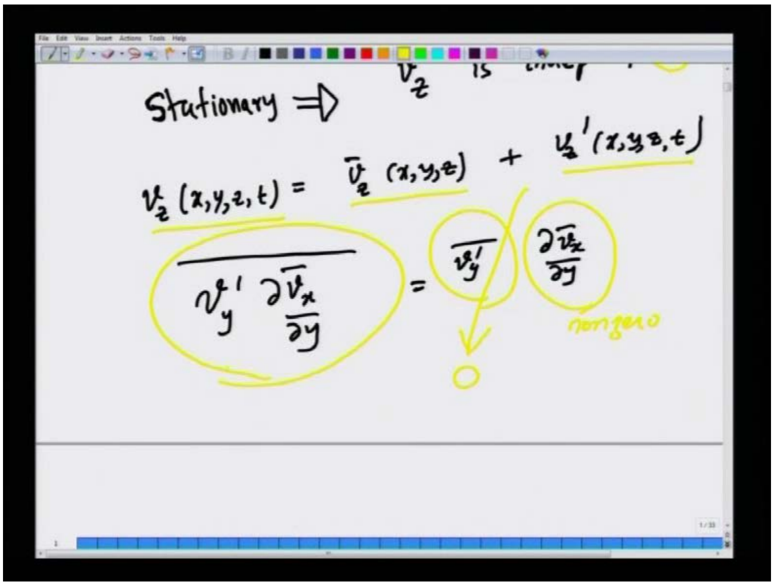

Now, if the flow is study in the mean, that is if the flow is stationary that would implied that v z itself is independent of time. What this means is that you could have imagine doing this averaging by shifting this t to some other value. Let say here you could also have t here and then do from t minus T to t plus T a small t minus capital T to t plus capital T. And the averages is thus computed will be indifferent of will be independent of where we choose t to T that is a meaning of stationary or study in the mean.

So, what we did was substitute this split v z is a function of x y z time plus v z bar, v z bar is already time average. So, it is a function only of x y z plus v z prime, which is a function of x y z and time back in the navier stokes equation. And we average the entire navier stokes equation while doing, so we neglected quantity such as like this v y prime d v x bar d y bar whole average. This would be essentially v y prime average times d v x bar by d y although this is non 0 but, the average of a fluctuating quantity is 0.

So, such quantities which occur as which occur linearly in fluctuating quantities in when substituting this expansion the total flow is the mean plus fluctuations back in to the navier stokes equation. You will encounter quantities like this and those are 0.

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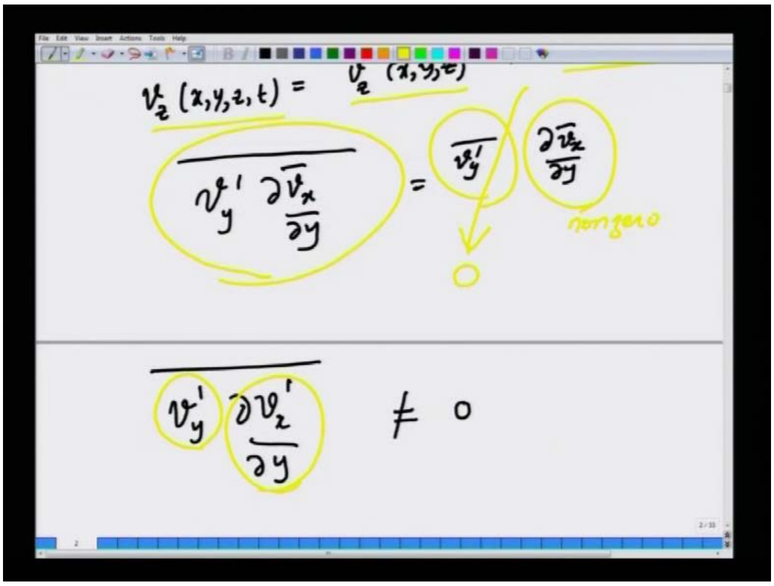

But, we will also encounter quantity such as this, the product of two fluctuations average and this is not 0, the product of two fluctuations is the average of two product of two fluctuations is not equal to the product of their averages. And this is because of the factor these two quantities could be correlated in time. So, when you integrate over particular instant particular extent of time they could in general be non 0.

So, the integral could be non 0 although the individual values if they integrate over time could be 0 because, if they positively correlated and then they could be non 0. So, that is the main reason why fluctuations play a role, even in determining the mean turbulent flow.

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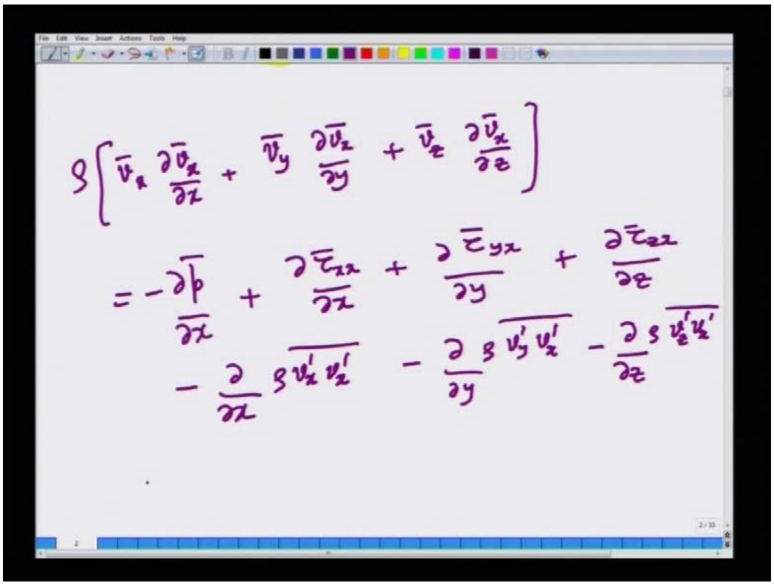

So, now after substituting all these and time averaging we finely ended up with this expression rho u x d x plus v y d v x d y this is the most generally expression we are of course, going to simplify this little later for specific cases x d z is equal to minus p partial x plus partial tau x x bar partial x plus partial tau y x bar partial y plus partial tau z x bar partial z plus. If you merely put bars in the navier stokes equation the study navier stokes equation you will just get this but, the fluctuations play a role.

Because, there are quantities such as this minus d d x of rho v x prime v x prime bar minus d d x of rho d d x of rho v y sorry d d y of rho v y prime v x prime bar minus d d. There is some minus sign so that is correct minus d d z of rho v z prime v x prime.

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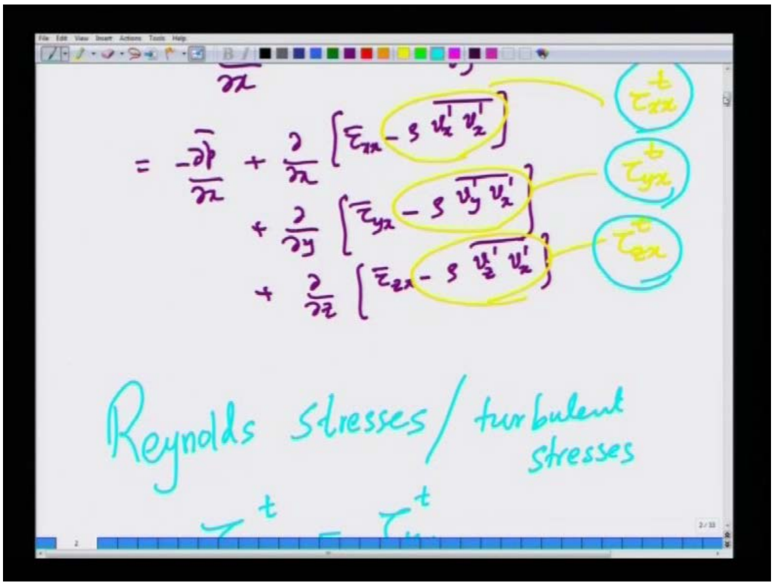

Now, these three so, can be again included back into this derivative minus partial p bar by partial x plus partial by partial x of tau x x bar minus rho v x prime v x prime plus partial by partial y tau y x bar minus rho v y prime v x prime plus partial by partial z tau z x bar minus rho v z prime v x prime put 1.

Now, these are termed as Reynolds stresses because, they appear as stresses and they appear in the same way as the viscous stresses, appear divergence of the viscous stress these are called turbulent stresses or Reynolds stresses. So, this is for example, denoted as tau x x turbulent this is denoted as tau y x turbulent and this is denoted as tau z x turbulent.

These are called Reynolds stresses (no audio from 09:22 to 09:28) or simply turbulent stresses. So, they are denoted as some tau x y turbulent and so on. There are also symmetric tensile therefore, so tau x y is tau y x a symmetric but, there origin lies purely in the fluctuating motion that is present often in a turbulent flow.

Turbulent flows are often characterized by rapid random fluctuations in both space and time. So, these are basically averages of quantities products of quantities and they are in general time dependent. Now, we have to solve this but, in order to solve this we have to tell something more about these turbulent stresses.

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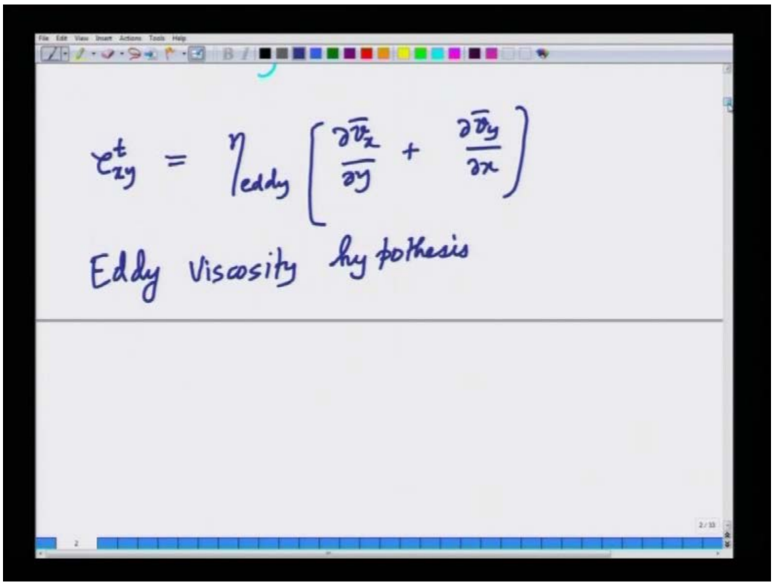

So, one option is to write turbulent stress in terms of a viscosity called eddy viscosity. Times the mean velocity gradients just as we wrote for the viscous stress this is called the eddy viscosity hypothesis, this is not a rigorous relation, this is merely hypothesis. So, this is called a eddy viscosity hypothesis.

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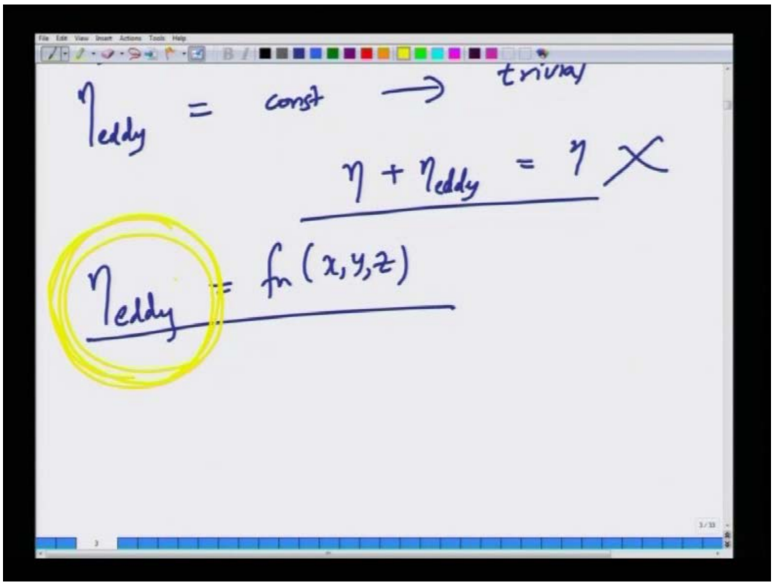

Now, if eddy viscosity which has the same dimensions as the normal viscosity if this is a constant. Then it is a trivial edition because, it changes nothing it is almost like your a increasing the effective viscosity by eta plus eta eddy. So, it will be another viscosity so, we are merely changing the character of the fluid by increasing it is viscosity by a tiny amount by some amount.

But, so this is not what is in what is happening in reality because this eddy viscosity is not a constant, it is a function of special quotients in general. Because, for the simple reason that if you consider turbulent flow fast a solid surface like turbulent flow in a pipe near the wall the fluc at the wall itself the velocities are 0 no slip condition and no penetration condition.

So, near the wall the velocity fluctuations will be small compare to the bulk of the flow. So, near the wall the magnet of fluctuations are small therefore, the turbulent stresses will be small. And hence since we are trying to model the turbulent stress in terms of the mean velocity gradient, somehow the eddy viscosity has to be small in order to keep turbulent stresses small close to the solid surface.

So, that physic that physical aspect has to be built in the model by saying that eddy viscosity is not a constant but, instead it is a function of special quotients. So, in some sense what we are trying to do in this type of hypothesis or in this kind of modeling is that our ignorance about the way in which turbulent transports momentum is sort of buried in the single parameter. So, one can often ask a question whether a single parameter, well it is not a single constant parameter, it is a function but, whether a single function can alone describe all the complexities of turbulent.

So, that is a fair criticism but, none the less for engineering applications, we will show that the eddy viscosity proves a very very reasonable tool to predict. For example, fiction factor verses Reynolds number relation in the turbulent region, with some suitable physically motivated approximations. And with some experimental input, we will show little later that will predict using this model. So, we will be content in this course by using the eddy viscosity like approach although it has it is own limitations, in terms of generalizing how this approach can be extended to other turbulent flows.

But, at least for turbulent flow through tools and rectangular channels and turbulent flow past of flat plate, all these kinds of turbulent shear flows it turns out that the eddy viscosity is a fairly reasonable model, especially if you are interested in engineering applications.

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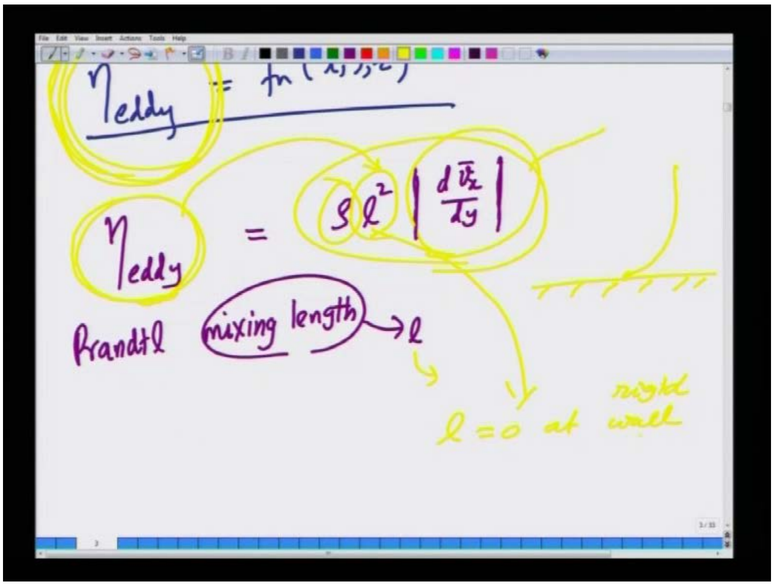

Now, so as I told you the eddy viscosity, has to be in some sense, we have to provide a model for eddy viscosity, otherwise it is not a constant that we have agreed. So, the model for eddy viscosity was first provided by Prandtl is called the Prandtl mixing length model. So, Prandtl wrote an expression for eddy viscosity like this, times the rho times l square, where l is called the mixing length rho is a density of the fluid. And this is the magneto the velocity gradient in the fluent. This is the Prandtl eddy mixing length hypothesis for eddy viscosity.

Now, the origin of this expression comes from kinetic theory of gases, where in if you consider the normal viscosity the protein shear viscosity of a liquid. The dynamic viscosity of a liquid, one can use for an if you consider the kinetic dynamic viscosity of a dilute gas. Such as air using kinetic theory you can write down the viscosity in terms of the density of molecules times. The mean free path square times the velocity gradient the magnet of the velocity gradient.

In a similar spirit Prandtl wrote down this. He imagine that instead of molecules you have this turtle and eddies, which are undergoing this random motion. And therefore, these eddies are able to transport momentum just as molecular collision transport momentum in a dilute gas. This is a hypothesis that the turbulent eddies almost act in a similar manner. So, he wrote an expression based on this mixing length.

So, the mixing length is physically the length over which a turbulent eddy loses it is identity. Just as what is the mean free path in the classical kinetic theory is a distance typical distance between two collisions of between two molecules. In some senses turbulent eddy moves and then ones it collies with some other eddy it sort of loses it is identity. So, turbulent eddy’s themselves are rough or loose concepts they are not rigorous concepts like velocity and verticality but, these are physical concepts, which are motivated by intuitive ideas rather than regress first principles.

So, key thing is that at the wall at if you have any turbulent, share flow at the wall there has to be some non 0 velocity gradient. So, the velocity gradient is not 0 at the wall but, you would expect the turbulent fluctuations to go to 0 at the wall. Therefore, eta eddy has to go to 0 at the wall, if this is not 0 and if this is not 0 rho is a density of the flow cannot be 0. So, l has to somehow go to 0 at the wall, at solid walls, at rigid surfaces.

So, this is something that we can say very in a very definitive way from straight forward physical considerations. That at a solid surface the eddy viscosity has to go to 0 because, otherwise if it does not go to 0 the turbulent fluctuations, the turbulent stresses will not be go to 0. But, if they eddy viscosity is 0 that means that we have to somehow say that this mixing length has to go to 0.

So, the mixing length is in fact so instead of saying in some sense, what we have done is to trade one unknown function to another unknown function because, everything else is known in the problem. But, this model does have some physical grounding in the sense that it is motivated by kinetic theory of gasses, where random motion of molecules and collisions transport momentum. Here, we are making an analogy by saying that the random motion of eddy’s in a turbulent flow transports momentum.

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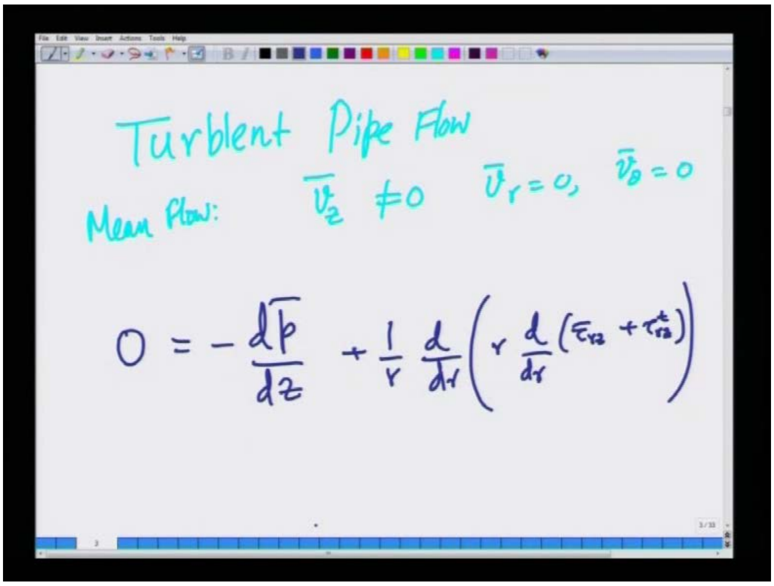

Now, we are going to go to a specific example Turbulent Pipe Flow, we have already seen that Turbulent Pipe Flow can be characterized using experiment in terms of the friction factor of the Reynolds number chart. And in the laminar regime, we know the exact analytical relation by solving the navier stokes equation and simplifying assumptions.

Now, can we make some progress in predicting the friction factor versus Reynolds number relation in the turbulent regime using the eddy viscosity kind of approach. That is the question we are going to answer.

Now, of course, Turbulent flow in a pipe must be addressed in cylindrical co ordinates. So, I will switch to cylindrical co ordinates. So, the mean flow will satisfy there is only one mean flow v z so v r is 0 and v theta is 0. So, many terms will drop out so, the mean flow is in the d r z momentum equation k is plus 1 over r d d r of r d tau r z by d r well but, we also have not just this, we also have the turbulent contribution.

So, let us write this as (no audio from 19:02 to 19:10) there are both the mean, viscous contribution plus the turbulent contribution.

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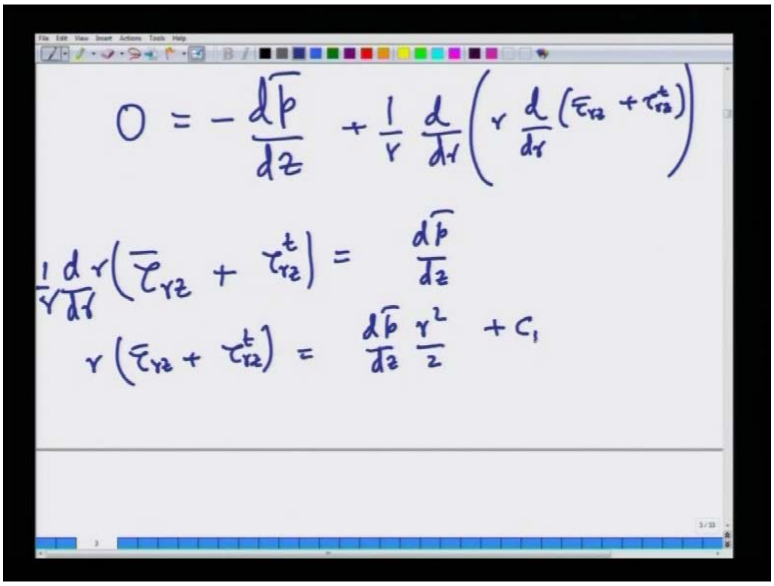

So, in some sense we can re write this very simply to yield t r z plus t r z t is nothing but. So, we can write is equal to d p d z and then we take r here and then integrate we will get tau r z plus tau r z t is d p d z times r square by 2. So, this is d d r of r so, you have an r of integration plus some constant.

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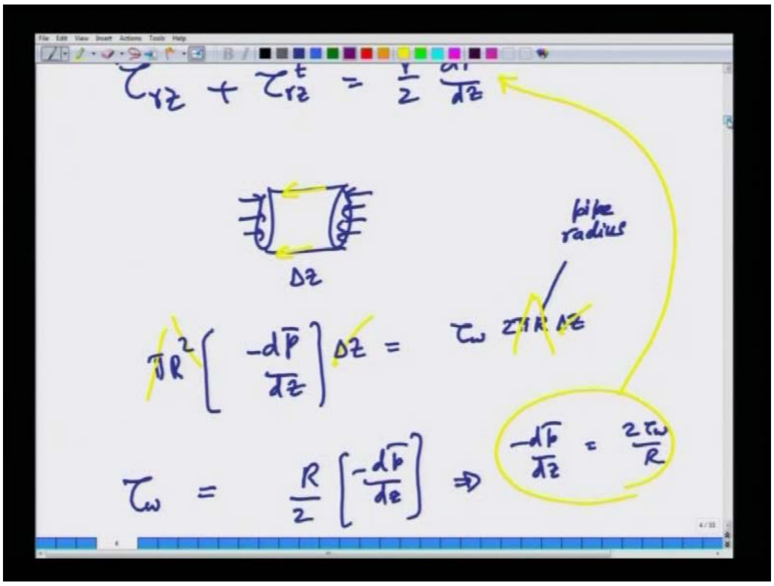

So, tau r z bar plus tau r z turbulent is d p d z R by 2 plus c 1 by r now, we can readily say that as r goes to 0 in the center of the pipe the stresses will be finite so c 1 has to go to 0. So, we can say very easily that tau r z plus tau r z turbulent is R by 2 d p d z. Now, if we do a macroscopic momentum balance so, it takes a tiny section of the pipe delta z do a macroscopic momentum balance. The forces here are the pressure forces on this side and this side, if you take this as the control volume now and on the surface, we will have viscous forces which will retire the fluid in the minus z direction.

So, you have d p d z times delta z is equal to the stress wall shears are excited by the wall on the fluid times 2 pie r delta z, where r is the radius of the pipe. Now, you can easily see that delta z delta z will cancel 1 pi will cancel and 1 r will cancel to give tau w is nothing but, r times minus d p d z by 2.

So, we substitute instead of d p d z instead so this implies minus d p d z is 2 tau w by r we substitute this term out here to give.

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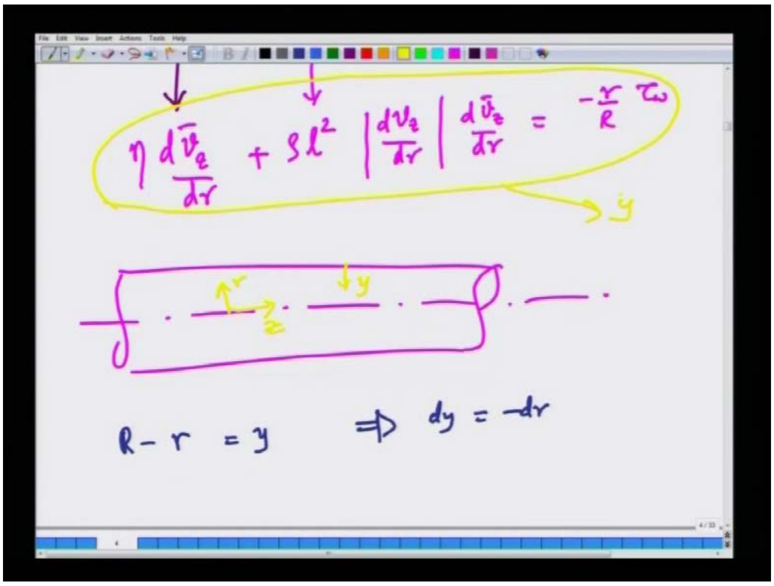

To get tau r z plus tau r z turbulent is minus r by R tau w. Because, that what it is you have plus d p d z here you have minus d p d z here. So, if you substitute in terms of tau r tau w the wall shade stress will get simply minus r the factors of 2 will go away.

Now, we know what is tau r z this is eta times d r and we know what is the turbulent stress in the r z direction that is from the mixing length model, it is rho l square times d z d r times d v z d r is equal to minus r by R tau w. The wall shear stress inserted by the wall on the fluid.

Now, so you have a pipe the access is along the center of the pipe. So, we said that r is along this direction z is along this direction now, I am going to define y from the wall towards the center. So, trivially r capital R minus small r is equal to y this implies d y is minus d r so when once, I use this and I can convert this entire equation in terms of and convert this in terms of y, which I will do very quickly.

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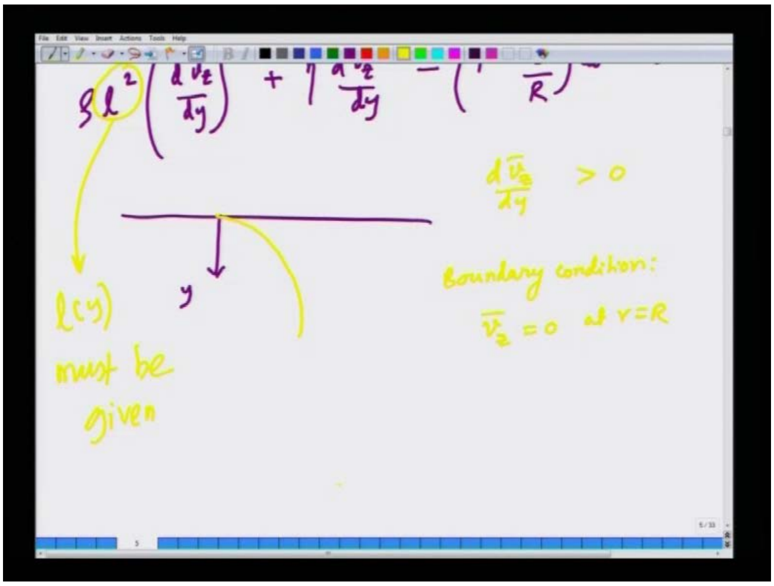

So, you have rho l square times d v z by d y whole square plus eta d v z by d y minus 1 minus y by r times tau w is 0. Now, notice that y is going like this and the velocity profile will be like this so d v z d y is a quantity that is positive. And that has been taken in account r so the mod is not required anymore because, the d v d d v z d y is positive. Now, the boundary condition is that (no audio from 25:58 to 25:05) v z bar is 0 at r equals capital R, therefore, no slip condition which is satisfied by both the mean and fluctuations.

So, far we have to we have are not specified what is l as a functions of y. So, we have to specify this that is another task that we have to do.

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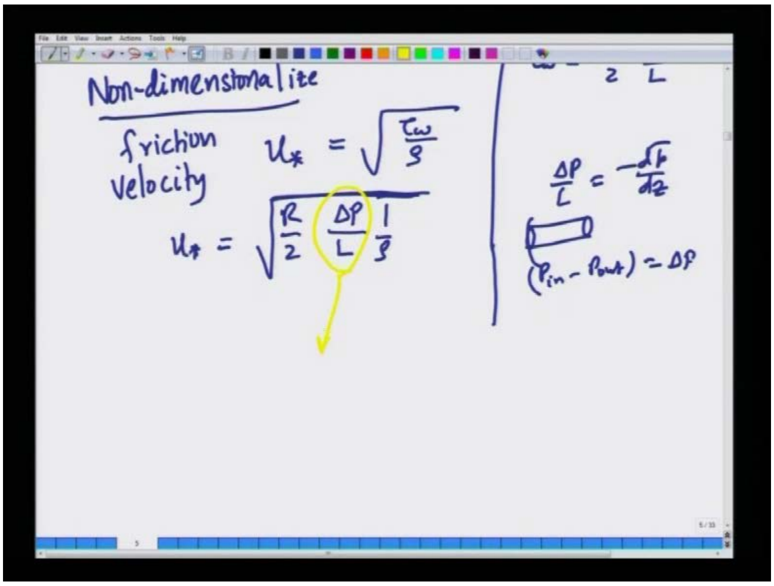

So, before i proceed further I am going to non-dimensionalize this, using turbulent scales. These are done conventionally the scales use for non dimensalising are not very obvious the velocities are scaled by the friction velocity, which is denoted as u star that is nothing but, square root of tau w by rho.

So, but, we know what is tau w in terms of the macroscopic momentum balance, tau w is R by 2 delta p by L so that something just that we derived using a microscopic momentum balance here. So, this expression so instead of d p d z I am going to write delta p minus d p d z I am going to write delta p by delta p by L minus L is minus that is pressure at the inlet delta p is nothing but, p inlet minus p outlet. So, that is the definition of delta p so u star therefore, becomes square root of R by 2 delta p by L 1 over rho with in the square root. Now, we know that delta p by L is related to friction factor for flow in a pipe.

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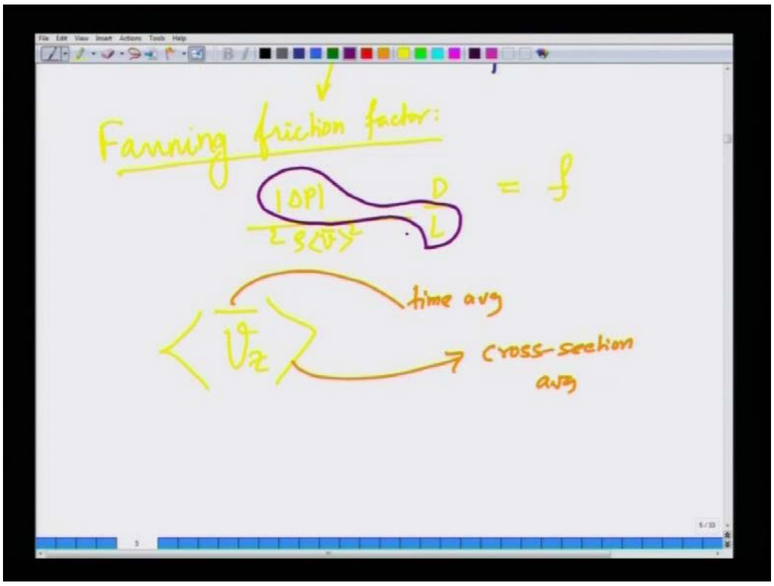

So, here I am going to use the Fanning friction factor which is differs by a factor of 4 from the Darcy friction factor which we have already pointed out the differences Fanning friction factor is defined as delta p divided by 2 rho v square times d by L is equal to f this is the Fanning friction factor. Now, there are 2 types of averages that are involved suppose I have a quantity, v z the over bar denotes a time average, the angular brackets denotes cross sectional average.

Remember that when we define friction factors, we have to define with respect to a cross sectional average velocity but, here we are considering turbulent flow that velocity is also time dependent in general. But, we are worried only about the time independent mean flow. So, there is an average with respect to space as well as the time period the extent of time. So, using this I can write what is delta p by L in terms of f and I can substitute it back here.

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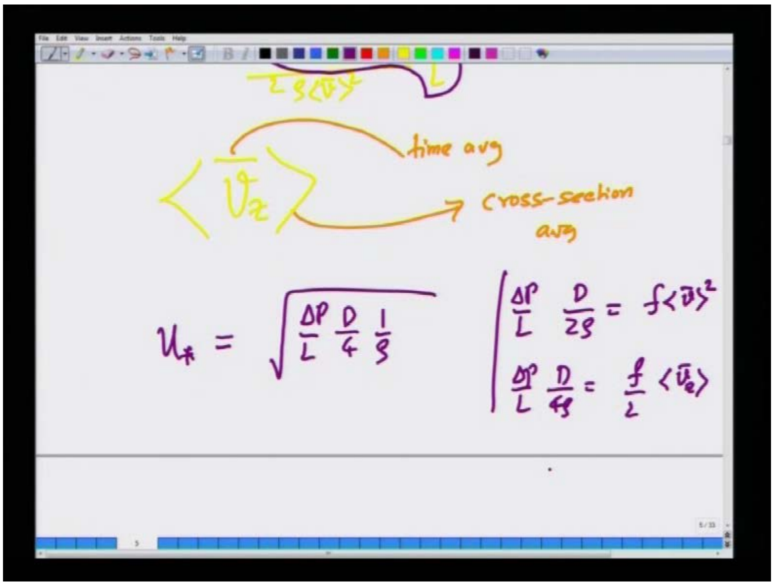

So, I will get u star the scale used for non-dimensionalization is nothing but, square root of delta p by L d by 4 1 over rho but, delta p by L times d by 2 rho is f times v square. So, delta p by L d by 4 rho is nothing but, f by 2 v z so, v z square.

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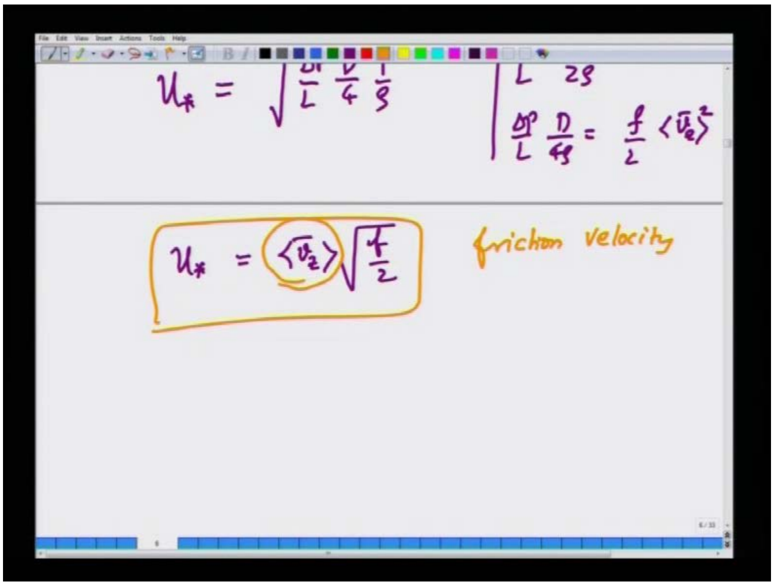

So, if you substitute this in here you will get u star is v z average square root of f by 2. So, this is a highly non-trivial velocity scale this is called the friction velocity because, normally we would non-dimensionalize things only by this. Here we are trying to multiplied by square root of f over 2. So, it is highly non-trivial in terms of non-dimensional using turbulent flow.

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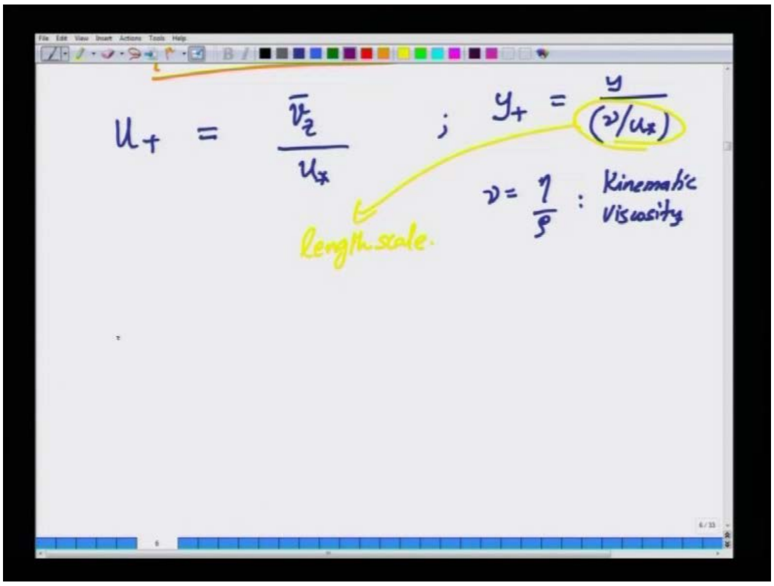

So, we will define the non-dimensional z velocity, as u by u star and we will define non-dimensional distance y plus as y by nu by u star where nu is eta by rho is the kinematic viscosity.

So, since we have used the length scale for non-dimensionalization is this not the radius of the q. And there is of very important reason why we are choosing a different length scale.

This length scale is not set by the macroscopic dimensions of the geometry such as pipe radius of channel width over length of a plate or something. It is inherent to the turbulent flow because, nu is the kinematic viscosity and u star is essentially friction velocity dictated by the turbulent flow.

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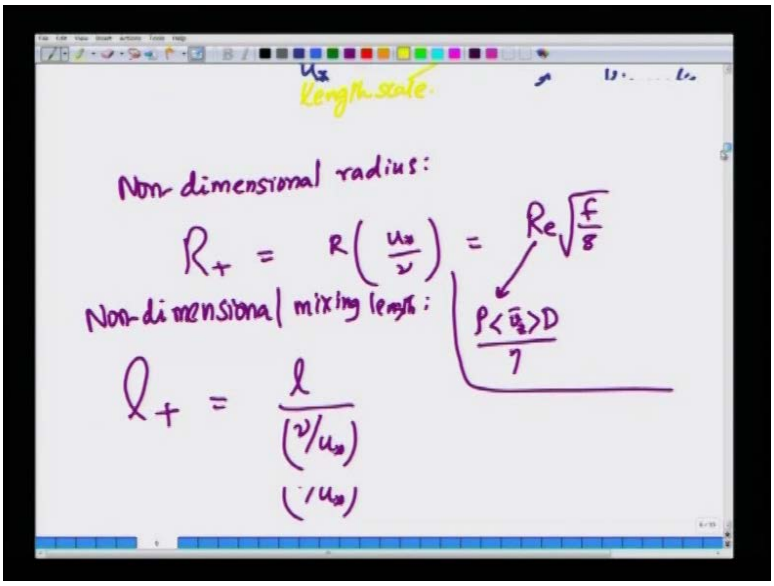

So, the non-dimensional radius will become (no audio from 30:35 to 35:43) will become R plus is R times u star by nu that becomes, R e times square root of f over 8, where R e is defined as (no audio from 31:00 to 31:06) it and you also have a non-dimensional mixing length.

Because, that also has to be known all the length scales has to be non-dimensionalize by nu divided by u star else l plus is l divided by nu by u star. So, after doing all these we can re-write this expression, we have this expression out here we can rewrite in terms of the non-dimensional variables we have just defined.

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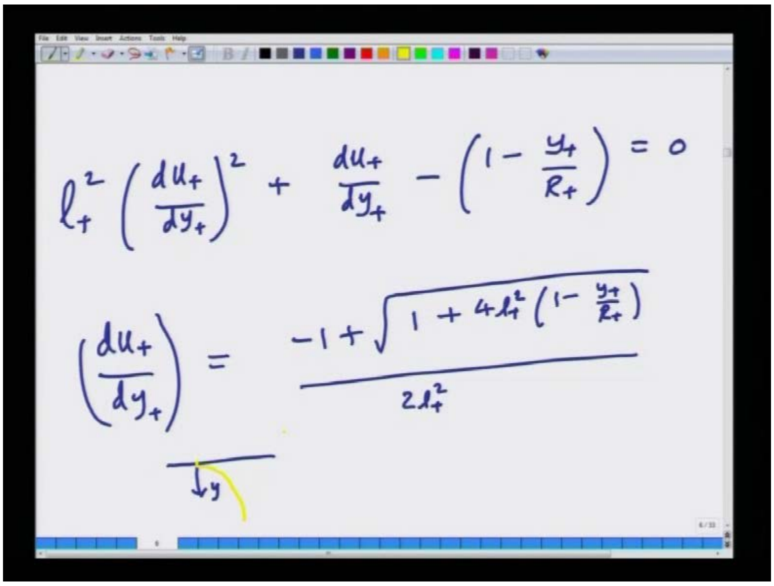

So, we will write this as l plus square d u plus by d y plus whole square plus d u plus by d y plus minus 1 minus y plus by R plus is equal to 0. Now, this is the quadratic in d u plus by d y plus. So, it is like a quadratic equation, so you can solve this quadratic for what is d u plus by d y plus sorry and so that becomes minus 1 plus square root of 1 plus 4 l plus square and so on minus y plus by R plus divided by 2 l plus square. The reason why I am we are not choosing the minus root is because, we know the d u plus by d y plus is a positive quantity.

Because, you know notice that y goes from the wall towards the centre, so the velocity profile will increase in some sense like this. So, d u by d u plus cannot be a negative quantity that is the case, then you cannot choose the negative root. Because, you already have a negative number in the first term so the second term better be positive in order to make d u plus by d y plus a positive quantity.

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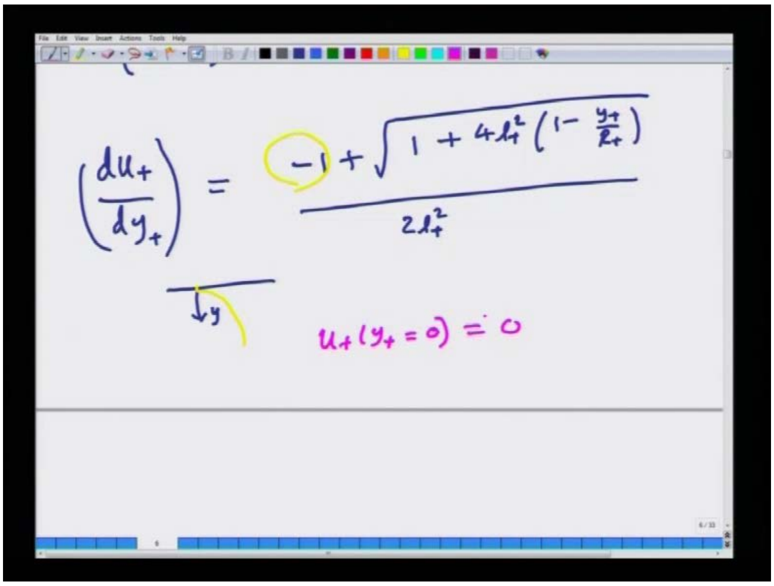

So and we can solve this equation by using the no slip condition at the wall u plus must be 0. So, we can integrate this equation.

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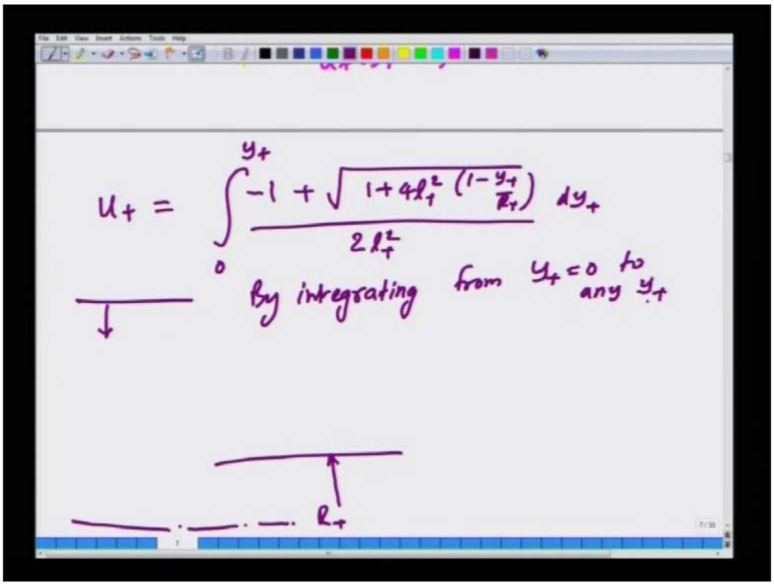

U plus is integral 0 to y plus minus 1 plus, plus square root of 1 plus 4 l plus square 1 minus y plus by R plus divided by 2 l plus square times d y plus. Now, this is by integrating from the wall towards the centre another way of integrating is which is also often useful. We use to integrate from the centre line that is from R plus towards the wall. So, this is by integrating from y plus equal to 0 to any y plus.

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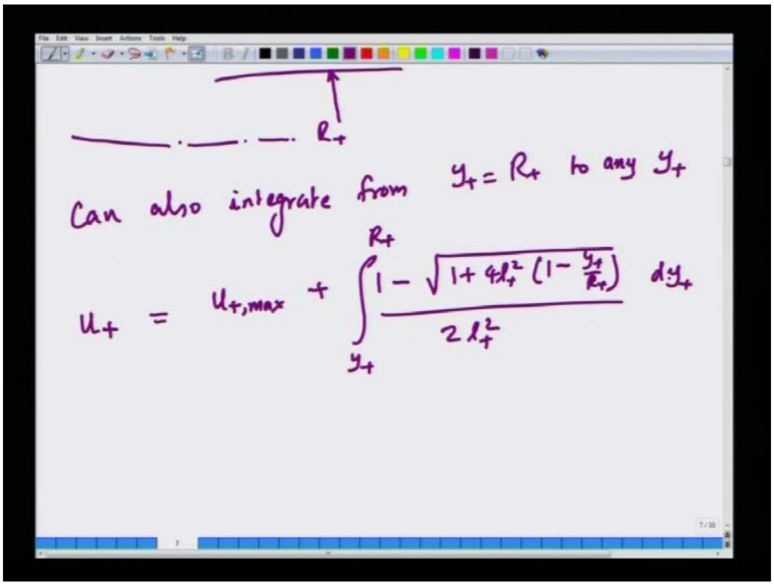

But, we can also integrate (no audio from 34:27 to 34:34) from y plus is R plus to any y plus. So, once we do that we will get u plus is u plus max plus integral y plus to R plus 1 minus square root of 1 plus 4 l plus square times 1 minus y plus by R plus divided by 2 l plus square d y plus. So, this is also possible so we will find use for both these expressions in the discussion to follow.

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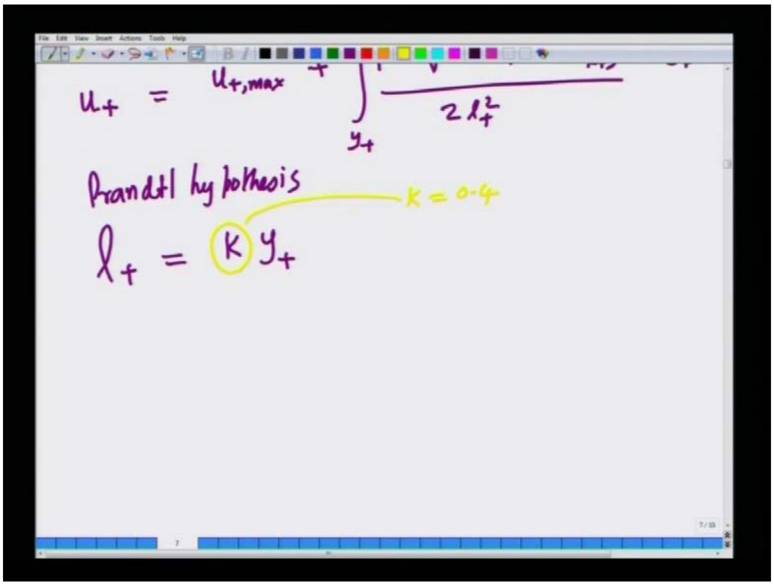

Now, prank dial propose for the mixing length a hypothesis for the mixing length. So, if the mixing length has to go to 0 at the wall so the simplest hypothesis is that it is some constant k and y plus. It is a linear function of y plus and experimentally it turns out that k works to be 0.4 to predict the friction factor.

We will come to that little later but, right now, we just treat k to be a constant which can be fitted by comparing with experiments. So, once we do that if you look at this expression. So, let us rewrite this expression again I am going to rewrite this expression again and make an important observation.

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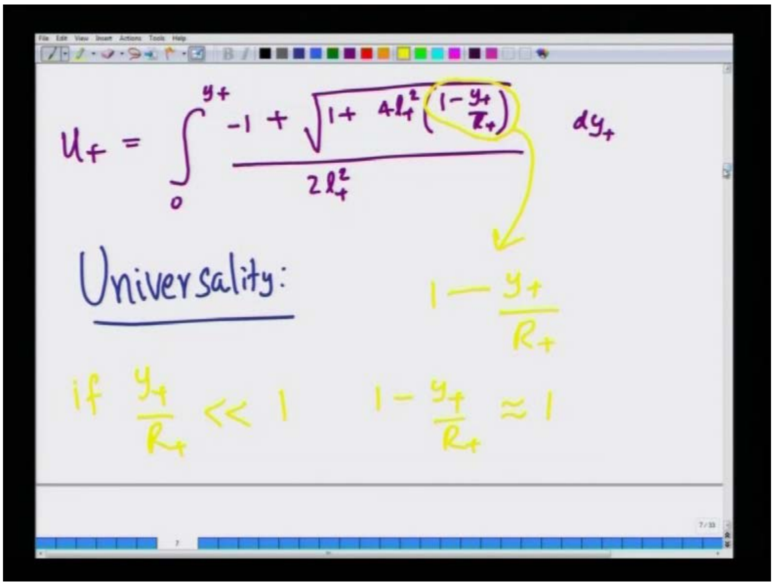

So, what we are going to do is to use this expression, where we had u plus as integral 0 to y plus times minus 1 plus square root under 1 plus 4 l plus square times 1 minus y plus by R plus divided by 2 l plus square times d y plus.

So, we are going to use this expression to analyze some (( )) and draw some very important conclusions about turbulent flow past solid surfaces the first thing is universality. Now, if you look at this expression the only way in which radius enters the problem is through this term. So, 1 minus y plus by R plus if y plus by R plus is small compared to 1 that is if you are fairly close to the wall. Then we can treat 1 minus y plus by R plus as approximately 1 and therefore, all the dependence on the radius of the pipe disappears from the problem.

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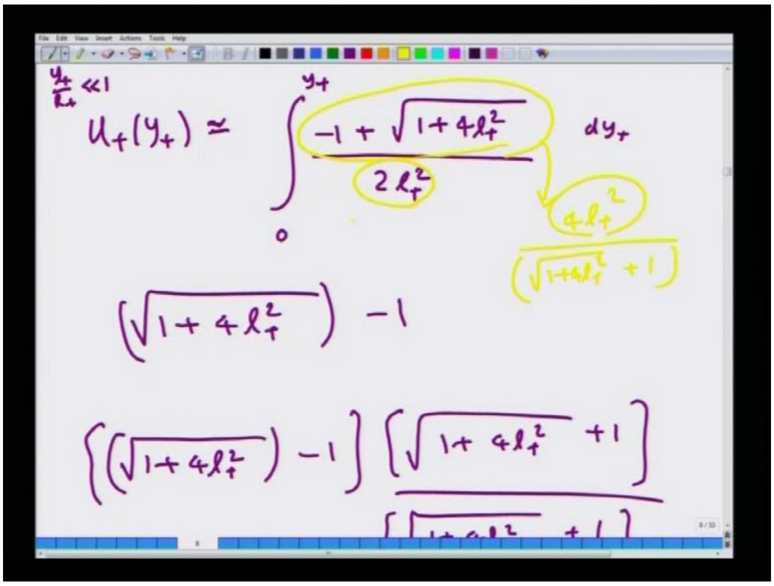

So, essentially you have u plus as a function of y plus as approximately, for under this condition that y plus by R plus small compared to 1. That this is approximately equal to minus 1 plus square root of 1 plus 4 l plus square divided by 2 l plus square times d y plus or you can simplify this further, we can write root of 1 plus 4 l plus square minus 1 as suppose, you consider root of 1 plus 4 l plus square minus 1 times root of 1 plus 4 l plus square plus 1 divided by root of 1 plus 4 l plus square plus 1.

I am multiplying and dividing by the same quantity this of the form. So, if I multiply it these 2 terms I get 1 plus 4 l plus square minus 1 divided by square root of 1 plus 4 l plus square plus 1 and the 2 minus once cancelled to give you 4 l plus square by square root of 1 plus 4 l plus square plus 1. So, essentially I can write therefore, that if I were to have this term I can replace this by this term here which we derived 4 l plus square, 4 l plus square by root of 1 plus 4 l plus square plus 1 this entire thing. This 4 l plus square will cancel this 2 l plus square to give you only a factor of 2.

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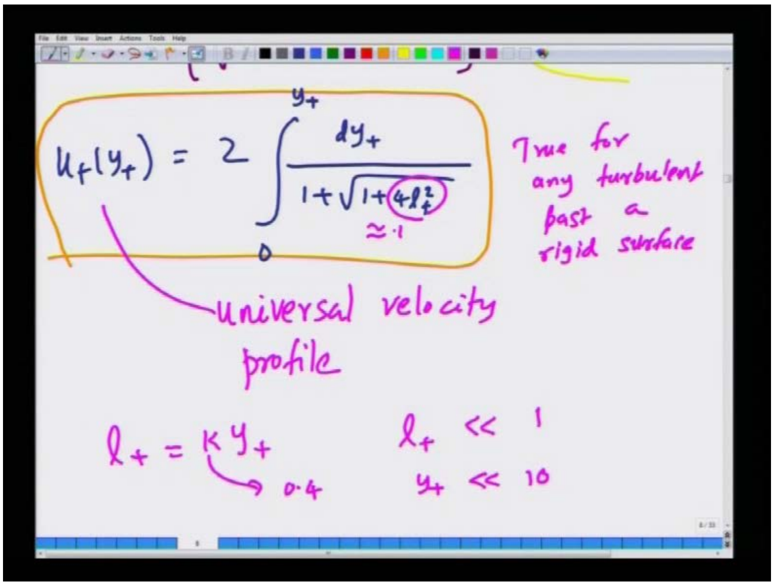

So, u plus as a function of y plus becomes twice 0 to y plus d y plus divided by 1 plus square root of 1 plus 4 l plus square. Now, if you look at this expression this expression has no radius of the pipe in it and there is no reference to the geometry of the pipe also it is fairly close to the wall.

So, this equation must be true for any turbulent flow past a rigid surface, turbulent shear flow past a rigid surface, be it flow through a pipe or flow through a rectangular channel of flow past a boundary layer for flow past a flat plate as long as a flow is turbulent. And the flow is in one direction and predominantly in one direction, these are called turbulent shear flows. Then this is in fact through so, this is a universal velocity profile close to the wall.

So, all turbulent shear flows close to the wall must exhibit this universal velocity profile. Now, if you are very close to the wall. So, we remember that prank dial's hypothesis is l plus is k y plus if you are very close to the wall. So, you will have l plus and k is somewhat 0.4. If l plus is much small compared to 1 or we can say that y plus is small compared to 10 it is 1 over 0. 4 it is about 10 then. So, k is 0.4 so it is about.

If k plus is small compared to 10 then you will have 1 plus 4 l plus square you can be treated approximately as 1. So, 1 plus 4 l plus square is because l plus is very very small compared to 1 you can do that.

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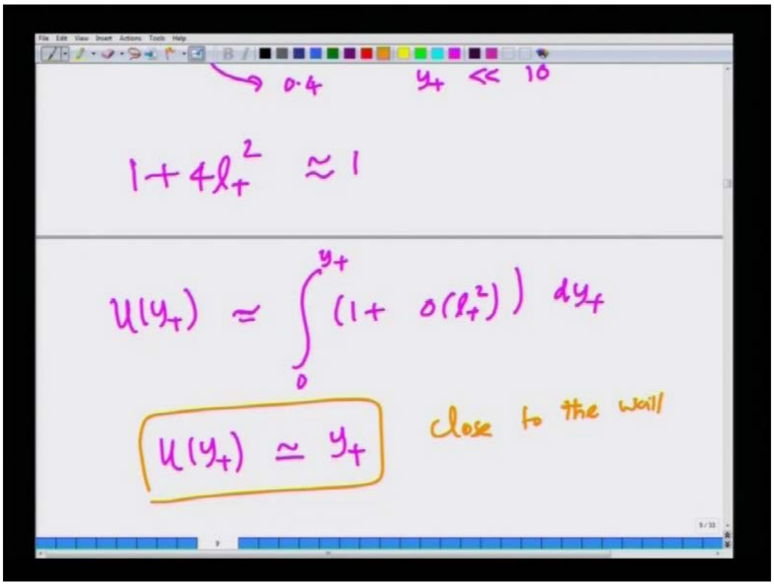

So, you will have u of y plus is approximately 0 to y plus 1 plus order l plus square times d y plus or you can say that u of y plus is approximately y plus close to the wall. So, his expression is in fact obtained to go back to our original differential equation. Go back to the original differential equation here.

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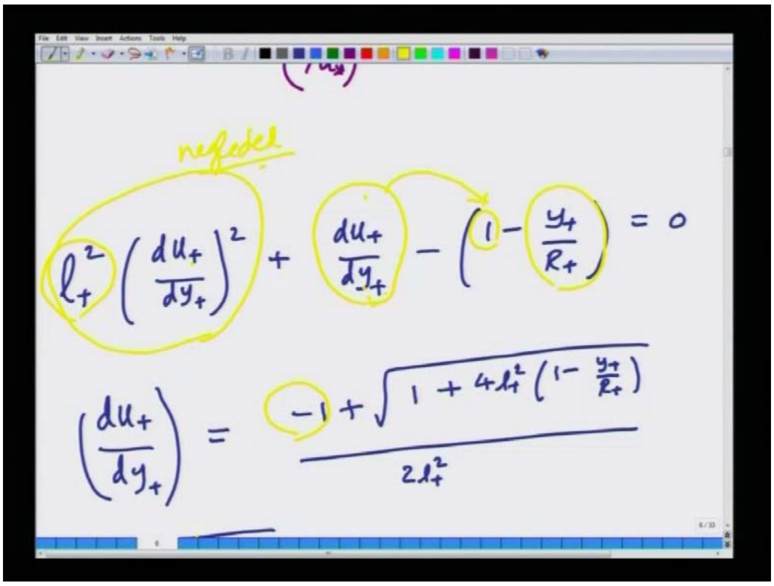

This expression is obtained by neglecting, this is the contribution to eddy viscosity this is the contribution due to the shear viscosity, normal viscosity. This is neglected close to the wall because, close to the wall the turbulent fluctuations are small turbulent stresses are small. That is what we mean by saying l plus is small compared to one. So, this term wins over this term and this term balances this term again this is small y plus is small compared to so, when d u plus by d y plus is 1 that means that velocity profile is linear.

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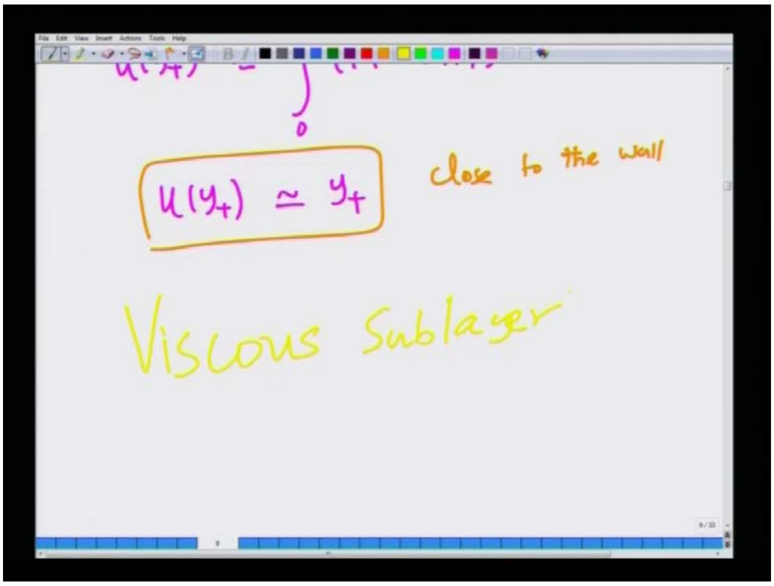

So, this region close to the wall is called the viscous sub layer. Sometimes the term laminar sub layer is used but, the flow is of course, is not laminar in this region. It is in fact turbulent but, the viscous effects dominate the turbulent stresses very close to the wall. And this is very universal feature of any turbulent shear flow past a solid surface this is not restricted to flow in a pipe, turbulent flow in a pipe.

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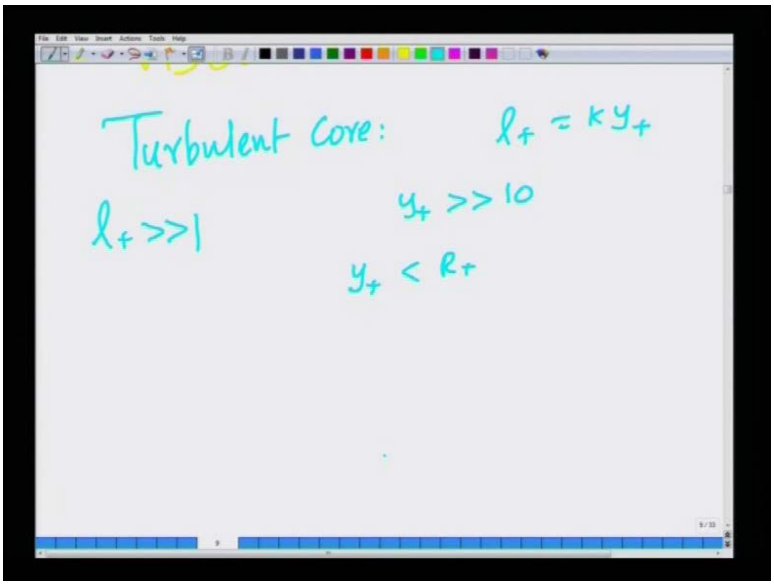

So, let us look at a Turbulent core that is in the centre of the pipe. Can we say anything more general so, we are far away from the wall. So, we have l plus large compared to 1 because l plus is k y remember k y plus and y plus is large compared to 10 but, still we are saying y plus is less than R plus.

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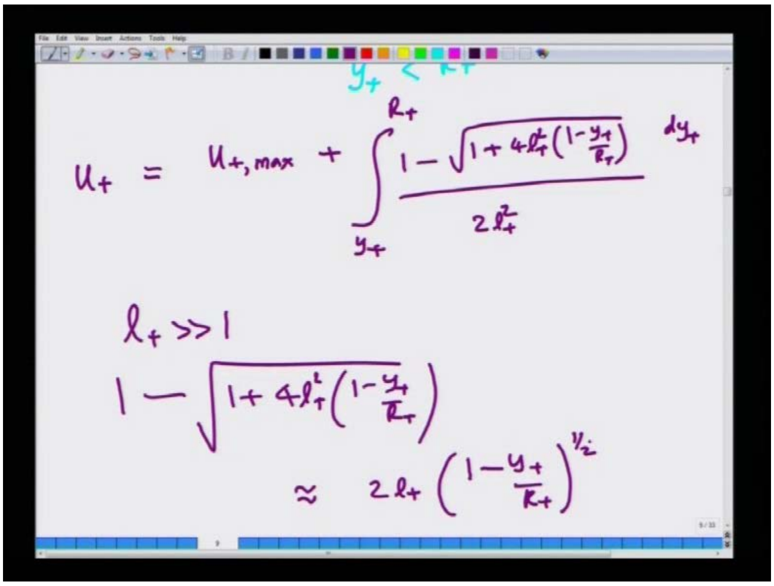

So, remember the other expression we had for u plus in terms of by integrating from the centre of the pipe u plus max the maximum velocity plus y plus to R plus 1 minus square root of 1 plus 4 l plus square times 1 minus y plus by R plus d y plus divided by 2 l plus square.

Now, if l plus is large compared to 1 so, you can say 1 minus square root of 1 plus 4 l plus square times 1 minus y plus by R plus is approximately is equal to 2 l plus times 1 minus y plus by R plus whole to the half if l plus is large compared to 1. Then we can say that this is approximately the just equal to this.

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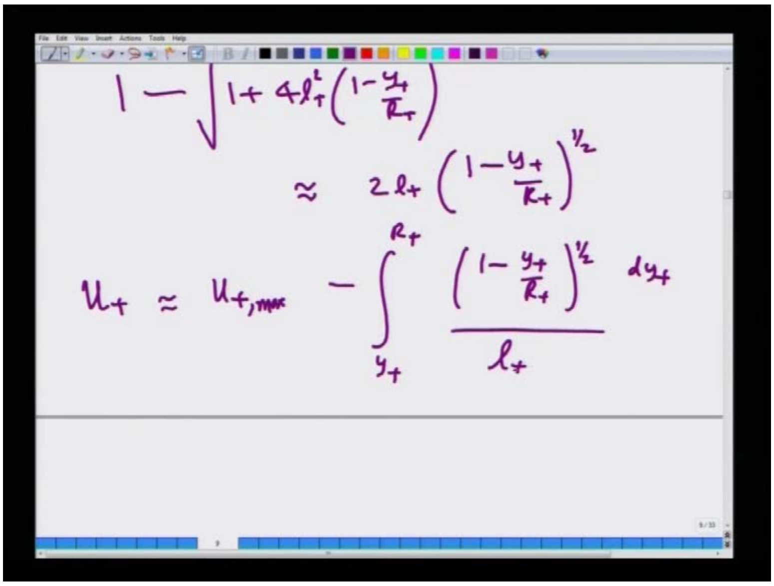

Now, so we can further. Therefore, write u plus as u plus max minus integral y plus to R plus 1 minus y plus by R plus to the power half d y plus divided by l plus.

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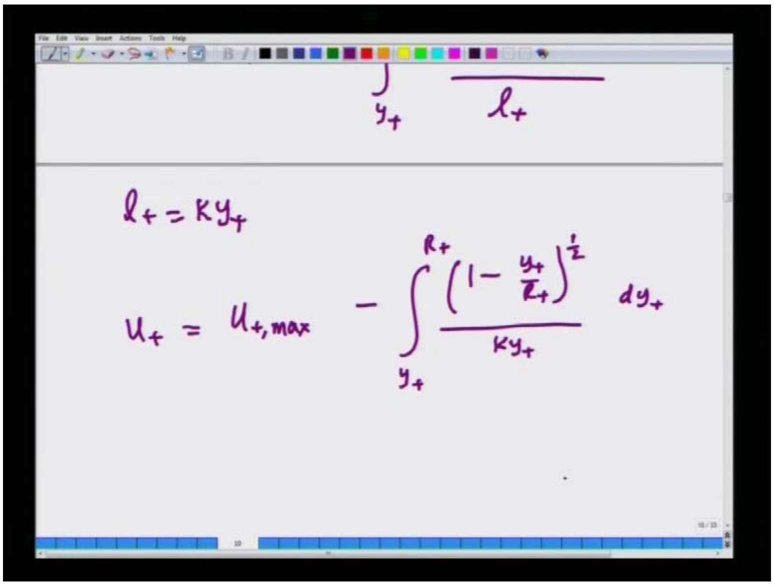

Now, l plus is k y plus this is the prankdial's hypothesis for the mixing length being a function of the distance from the wall. So, u plus is nothing but, u plus maximum minus integral y plus R plus times 1 minus y plus by R plus to the power half by k y plus d y plus. So, all we are saying is that when l plus is large compared to 1, we are expanding this term and that is the only simplification that we are making, when we do this whole exercise that is the only simplification we are making.

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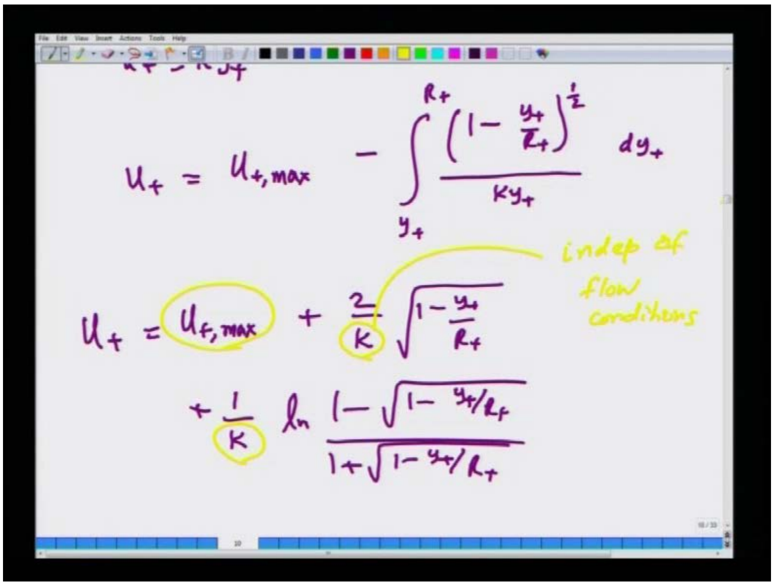

Now, if you look at this expression you can integrate this u plus is u plus maximum plus 2 by that constant k times root of 1 minus y plus by R plus, plus 1 over k logarithm of 1 minus square root of 1 minus y plus by R plus by 1 plus square root of 1 minus y plus by R plus. Now, here there are only two constants that are floating around u plus maximum and k. Now, k should be independent of flow conditions because, we are shown that it is the mixing length.

So, it should not be a function of the flow could be a function of geometry for example, this is pipe flow. But, generally it should not be a function of flow conditions but, u plus maximum is a function of flow conditions because, that is the maximum flow velocity of the pipe in a non-dimensional sense.

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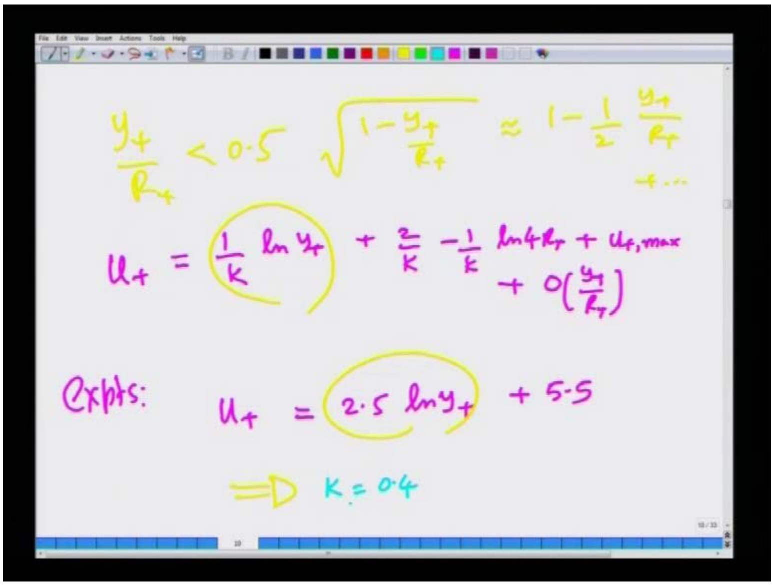

Suppose, you have y plus by R plus less than 0.5 you can expand the square roots, so root of 1 minus y plus by R plus is approximately 1 minus half y plus by R plus, plus so on.

So, when I substitute this back out here you get u plus is 1 over k lon y plus, plus 2 over k minus 1 over k lon 4 R plus, plus u plus max plus order of y plus by R plus. So, we are not close to the centre end but, at the same time, we are far away from the wall. This is called the turbulent core region and experiments tell you that u plus is 2.5 l n y plus, plus 5.5.

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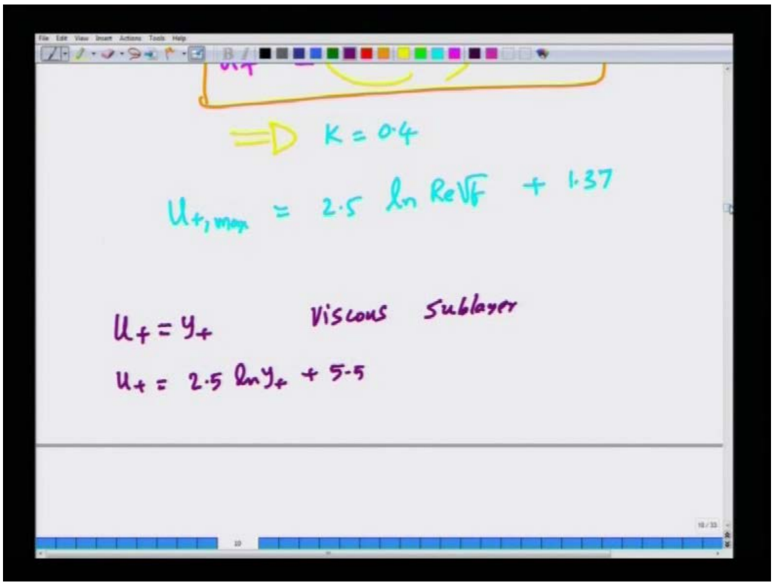

So, by comparing these two expressions, we can infer this implies that k is 0.4 and u plus maximum is 2.5 logarithm of R e root f plus 1.3 7. So, this is the description that we have for the flow that is far away from the wall. But, still little away from not close to the centre but, little away from the centre is called the turbulent core region. So, we have a description for very close to the wall where the viscous sub layer.

Where the velocity profile is just proportional to y plus and then we have a logarithmic velocity profile far away from the wall. So, we can patch these two relations so, we have these two relations u plus is y plus the viscous sub layer and we have u plus is 2.5 log y plus, plus 5.5.

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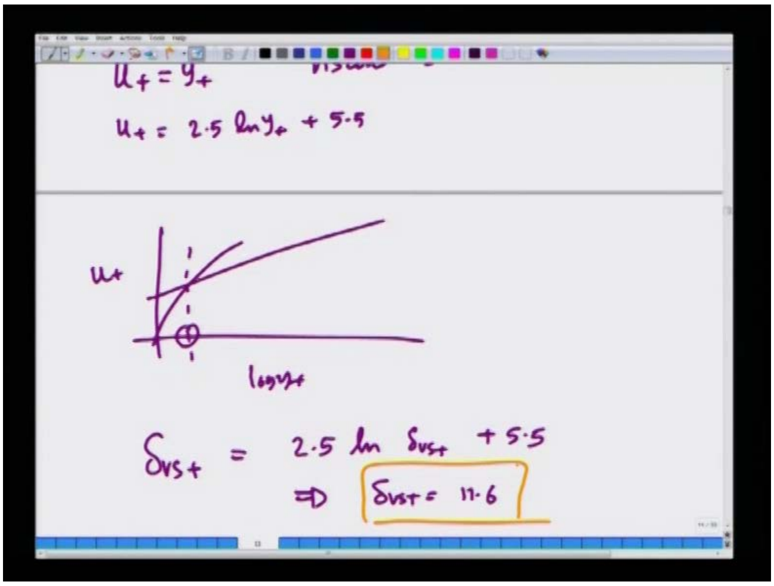

Now, if you plot these two curves and then say that the region where these two curves meet is the thickness of the viscous sub layer. Then so, you will have u plus as a function of log y plus, we have one (( )) total like this, another (( )) total like this.

So, if you equate these two and say this is the viscous sub layer thickness. Then you will find that delta v s plus, the thickness of the viscous sub layer. The non-dimensional thickness of the viscous sub layer is to 2.5 logarithm of delta v s plus, plus 5.5 if you solve this you will get delta v s plus it is 11.6. So, the non-dimensional viscous sub layer thickness is 11.6 in general.

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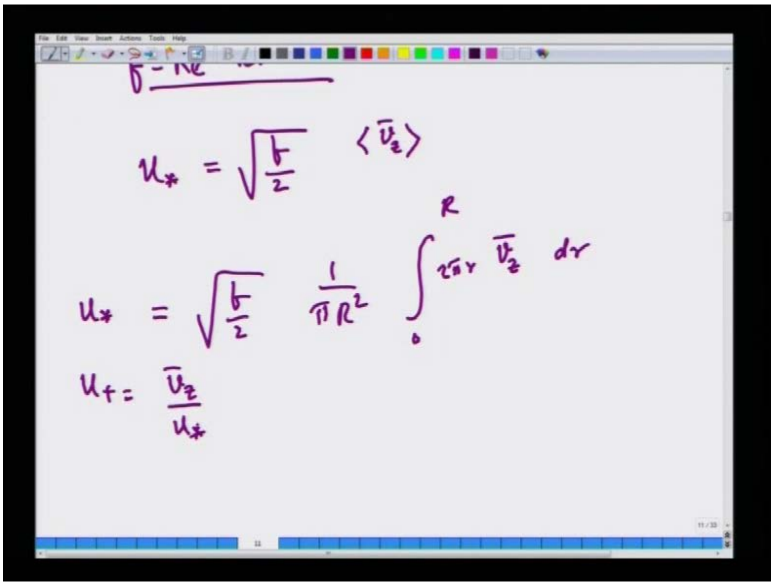

Now, we have all the required quantities to find the friction factor versus Reynolds's number relation. Now, you know that u star is square root of f by 2 v z, so this is nothing but, square root of f by 2 the definition of v z is 1 over pi R square 0 to R 2 pi r the time average velocity v z bar d r. And this is u star we know that u plus is v z by u star this implies.

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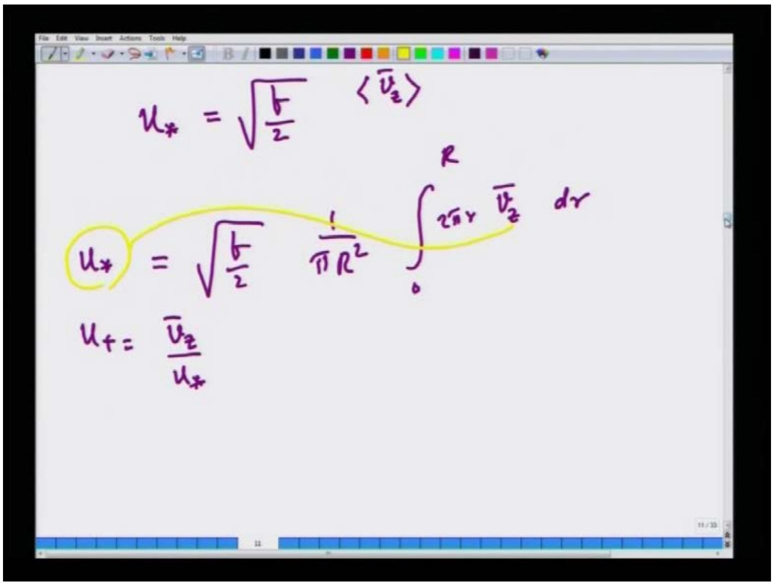

So, I can bring since u star is constant I can bring this u star to the denominator here and then call it u plus.

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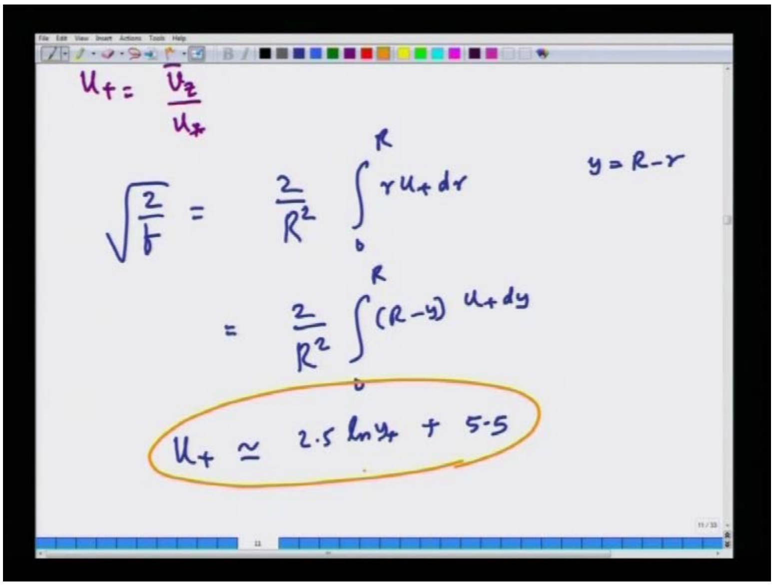

And then I can get a relation for refraction factor 2 by root f is 2 by r square 0 to r, r u plus d r. Now, I am going to convert from r to y so this becomes 2 by r square 0 to r capital R minus y u plus d y. Now, we know that u plus has a composite description, close to the solid surface here. The viscous sub layer little away you have the logarithmic region but, as a first approximation. If you use u plus 2.5 log y plus, plus 5.5 in the entire pipe, we are saying that, we are not going to account for the viscous sub layer. And we are not going to worry about the velocity of profile close to the centre of the pipe.

Because, this logarithmic velocity profile is valid only in the intermediate regime far away from the wall but, again little away from the centre of the pipe, we are not going as far as the centre of the pipe. But, as a first approximation if you just use this out here. Let us see what happens.

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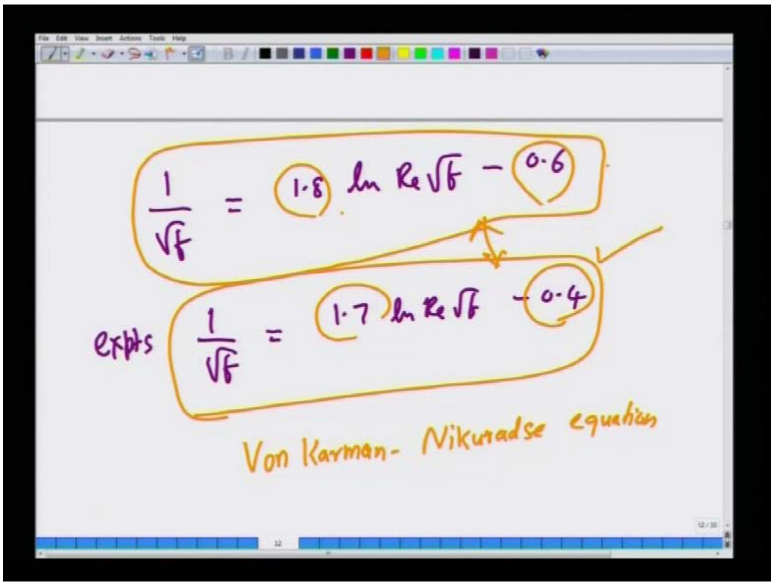

Then you will get 1 over root f is 1.8 log R e root f minus 0.6 but, this the expectation from the theory. Experiments show that 1 over root f is 1.7 log R e root f minus 0.4. So, this is a very approximate model after, we made so many approximations yielded this while experiments show this. So, that shows this gives some confidence in the efforts that we have made in understanding the turbulent flow in a pipe.

Because, we do get constant that are fairly closely to experiments of course, we have to correct them but, that is understandable because, we have made a whole lot of assumptions. And we have errors in actually extending the velocity profile all the way from the wall to the centre of the logarithmic velocity of profile, when in reality it is not valid.

So, we have made several approximations but, having done all these approximations. It is quite satisfied the result from this approximate analysis of turbulent flow yields very close results very close to the experimental result. So, this equation is called the von Karman-Nikuradse equation and this predicts reasonably the f versus R e data, that we already saw in when we discuss pipe flows and losses. When we discussed friction factor Reynolds's relation, the turbulent region flow for smooth pipes is reasonably captured by this relation. And we also have obtained the reasonable understanding of what is going to happen in terms of in detail.

what is the velocity profile and why it is changing using a simple machine length hypothesis due to prank dial’s, we will stop here. And at this point and in the next lecture, lecture number 40, we will try to summarize everything what we have done in this course. And also list out what is ahead in terms of what are the newer things that we could not have time to go through. But, which are necessarily, which are essentially very very important but, we could not cover them for lack of time and lack of scope of this course. So, we will stop here and we will meet again in the next lecture.