Fluid Mechanics

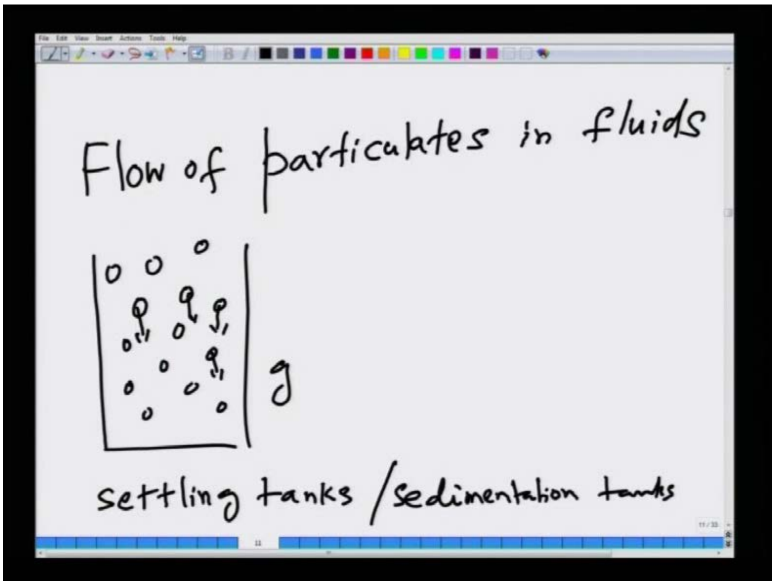
Prof. Vishwanathan Shankar
Department of Chemical Engineering
Indian Institute of Technology, Kanpur

Lecture No. # 36

Welcome to lecture number 36 on this NPTEL course on fluid mechanics for undergraduate students chemical of engineering.

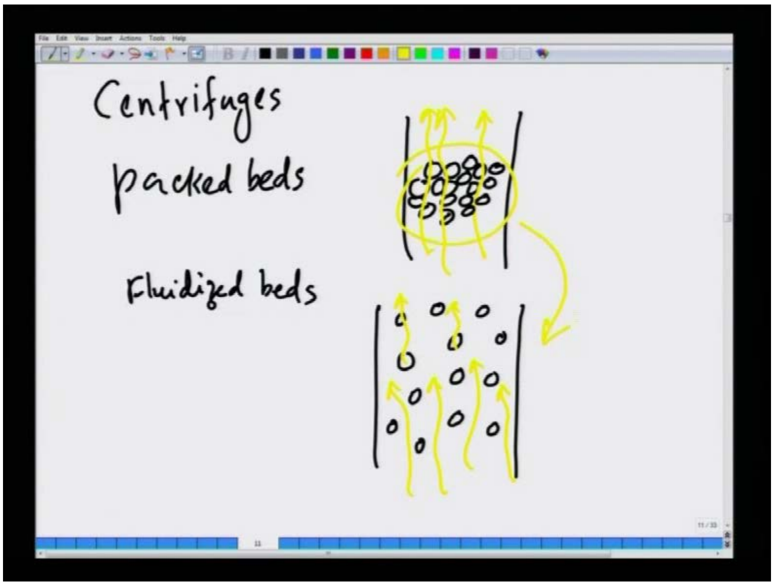
Today we are going discuss fluid mechanics, application of fluid mechanics to various chemical engineering processes. And one of the major topics in chemical engineering an application is the flow of particulates in fluids.

(Refer Slide Time: 00:33)



In many chemical engineering applications for example, you have a system like channel in which you have many particles that are settling in a liquid. These are called settling tanks. So, here particles are moving and settling in a fluid due to gravity and this is used to separate particles of different sizes because of their difference in settling velocities as we will see. So, this one example there is settling tanks or sedimentation tanks as they are called in many chemical engineering applications. You will have sedimentation tanks that are used to remove particulates from one liquid phase and then use the liquid phase for further processing in other unit operations.

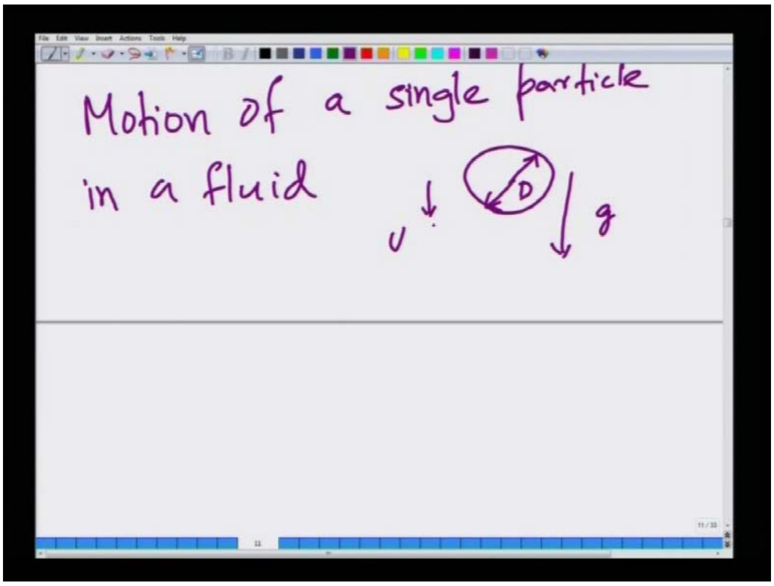
(Refer Slide Time: 01:44)



And you also have centrifuges to separate particles of different densities, particles or droplets of different densities. So, here the driving force here again it is also settling, but, under the influence of centrifugal driving forces. And you can you have other unit operations such as packed beds wherein you have a bed of particles which are in close contact and there is lot of gap in between the particles and fluid is flowing.

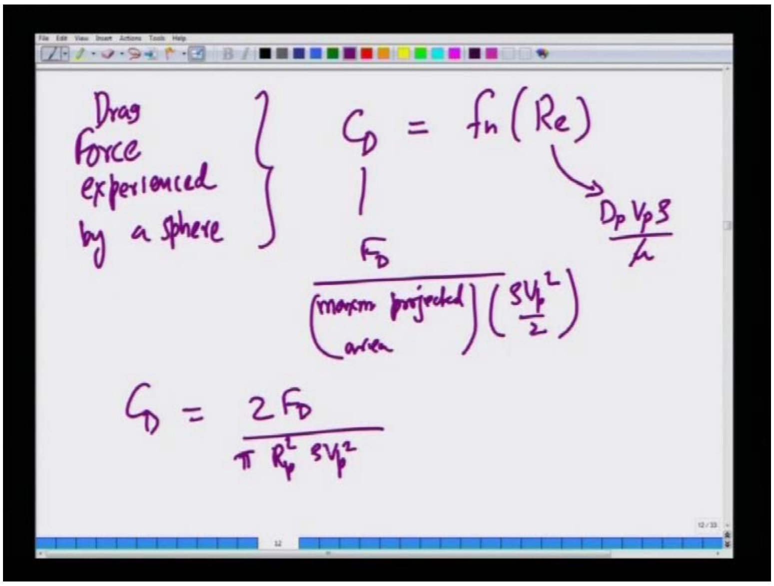
Fluid is flowing in the interstitial gap between these particles. And there are also other applications such as fluidized beds wherein you have the motion of fluids through a bed of particles. But, the particles themselves are no longer static, but, they are also in some sense fluid is suspended. So, they are also in a state of animated motion due to the fact that the drag force exerted by the fluid on the particles exceeds the weight of the particles. So, the bed of particles starts fluidizing after some velocity. So, all these are examples of motion of particles in a fluid. And in order to design these operations well in many chemical engineering operations unit operations you have to understand the fundamental mechanics of flow of particulates through a liquid.

(Refer Slide Time: 03:24)



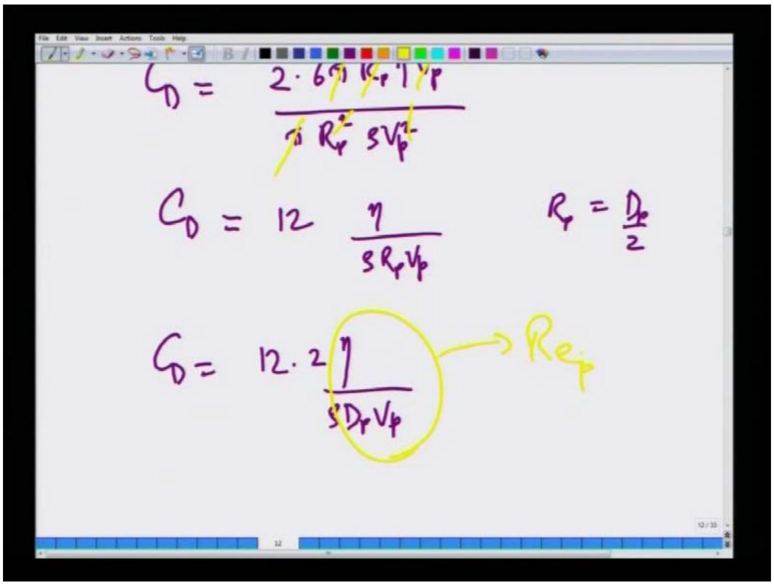
So, one of the simplest problem that we will first start with is flow or motion of a single particle in a fluid. (No audio from 03:24 to 03:34). So, essentially we will imagine that you have a spherical particle, rigid particle to begin of some radius or diameter D. And that is moving with a constant velocity under the influence of gravity. Let us say and we want to be able to predict what is the settling velocity of this particle; as a function of various parameters in the problem.

(Refer Slide Time: 04:04)



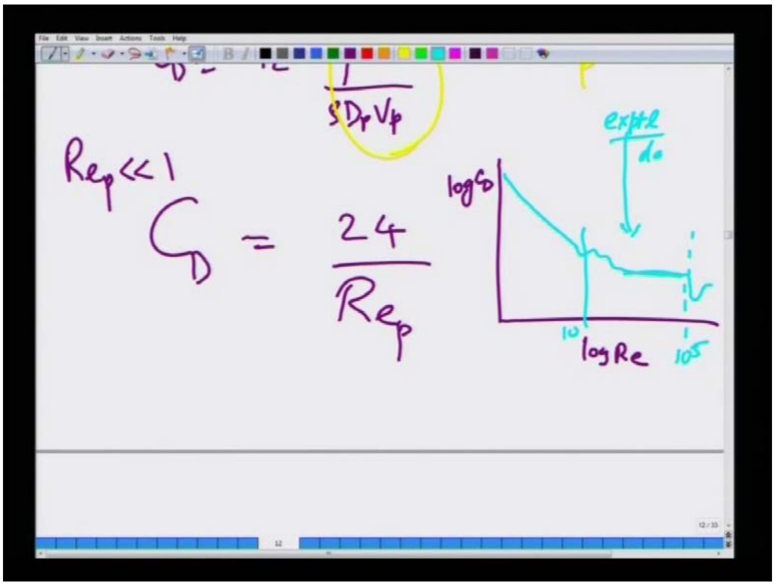
Now, we already know that the drag force experienced by a sphere is correlated in terms of non-dimensional groups C D as a function of the Reynolds number, where C D is nothing, but, 2 f D the force divided by maximum projected area times rho V p square by 2. And then Reynolds number is of course defined as D p V p rho by mu where D V and V p are the diameter and velocity of the particles. F D is the drag forced experienced by the particle. So, C D is nothing, but, 2 f D by pi or p square rho V p square.

(Refer Slide Time: 05:10)



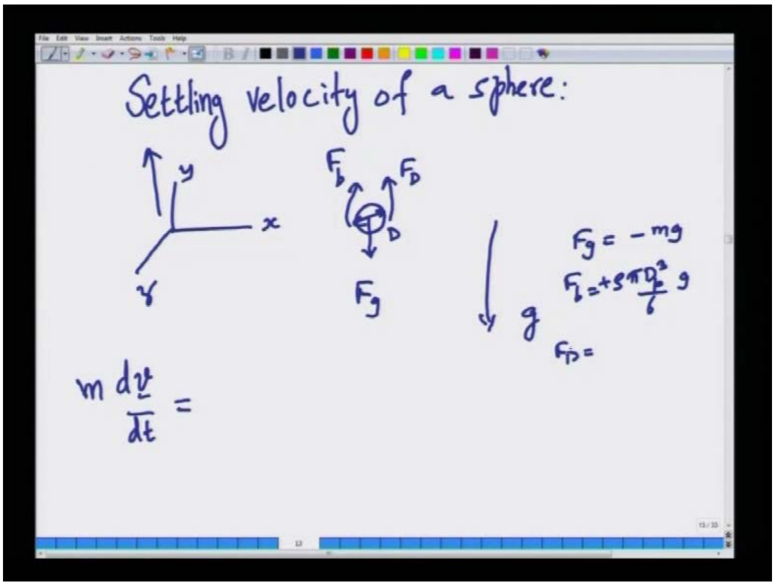
Now, if you substitute for Reynolds number small compared to one the drag force is given by the strokes drag law 6 pi R p eta V p. So, C D is given by 2 times 6 by R p eta V p divided by pi R p square rho V p square. So, C D for a spherical particle, at low Reynolds numbers given by, so, 2 times 6 is 12 and then the pi cancels off. One V p will cancel with one V p and R p will cancel with one R p will give you 12 by 12 times eta by rho R p V p; R p is D p by 2. So, C D is 12 times eta 2 by rho D p V p and this quantity is the Reynolds number based on the particle diameter and velocity.

(Refer Slide Time: 06:17)



So, the drag coefficient is given by 24 by divided by R e based on the particle for Reynolds number of the based on the particle much small compared to 1. Of course, at high Reynolds numbers we showed that we have log C D log R e plot and it will appear like this. For smaller Reynolds numbers is this constant. It is a straight line with slope minus 1, but, at higher Reynolds numbers of course, things are different and you will find that there is a reason of constant relatively constant drag coefficient. And then for Reynolds number of 10 to the 5 is about 10. You have transition from laminar flow to turbulence. So, we have this data. This is an experimental observation correlated in terms of drag coefficient and Reynolds number experimental data.

(Refer Slide Time: 07:17)

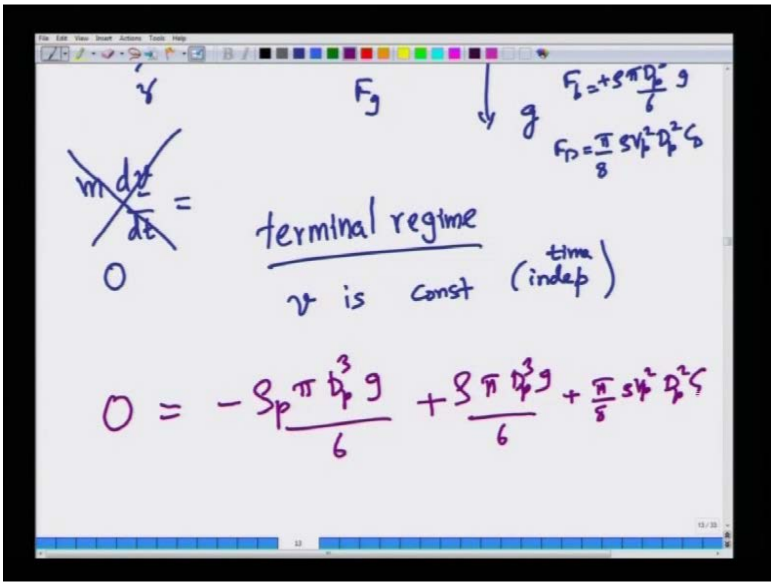


So, now we want to worry about settling velocity of a sphere. Imagine you have an infinite expanse of fluid and then you take a sphere and you drop it in the fluid in the liquid and there is gravity. After some time let us say you drop it with 0 velocity initially the velocity of the sphere is 0. After some time, initially, the sphere will accelerate because there is acceleration due to gravity acting on it, but, it is also being retracted by the fluid drag force. This is gravity force let us call it F g. The gravity force that acts and then there is drag force that acts in the opposite direction. There is also buoyancy force that acts in the opposite direction.

So, these are the three forces. Now, initially, there will be in imbalance if you write force balance for the particle. So, the mass times rate of change of velocity acceleration of the particle is sum of all the forces. So, let us assume a coordinate frame x y and z. So, the velocity mass times D V by D t sum of all forces and the gravity forces is in let us simply write the algebraic quantities of all the algebraic. So, let us use the sign convention the gravities f g is acting its minus m g f buoyancy is plus the volume of the sphere let us assume the diameter of the sphere to be some D.

So, the buoyancy will act in the plus y direction because it is buoyancy acts in this direction the direction opposite to the motion of the particle. So, buoyancy will act in the plus direction which is rho pi D p 2 by 6 time’s g. This is plus then there is drag force which is given by it is also in the plus y direction. So, the drag coefficient is defined in the following way. Let us look at the definition of the drag coefficient here. From here we can drag force which is nothing, but, pi R p square rho V p square by 2. Now if you write R p as V p divided by 2 then f D will be simply pi by 8 rho V p square D p square times C D of course, times C d. So, I will write if f D f D the drag force in terms of the drag coefficient.

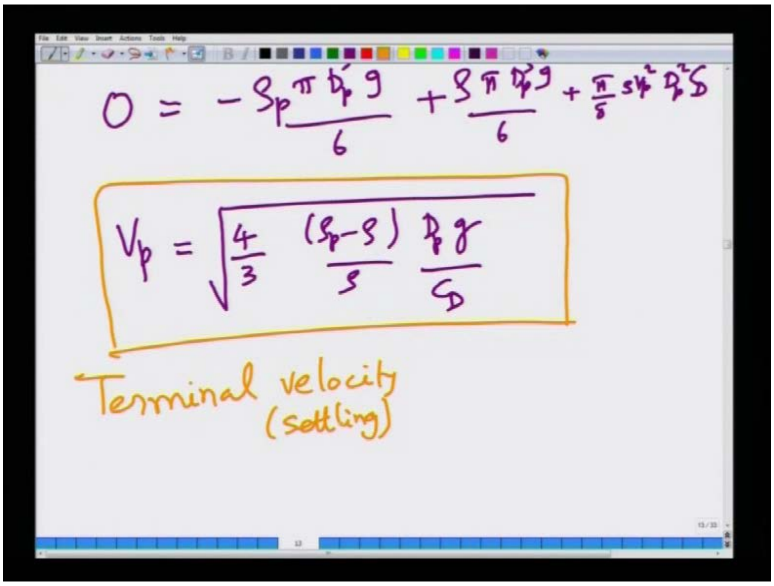
(Refer Slide Time: 10:16)



So, the drag force is pi by 8 times rho V p square D p square times drag coefficient C d. So, at steady state you will have no when the sphere has reached the terminal sphere has sphere has reached a velocity such as all the force balances its acceleration is zero. So, this is called the terminal regime, wherein the sphere velocity does not change with time the velocity of the sphere is constant that is independent of time.

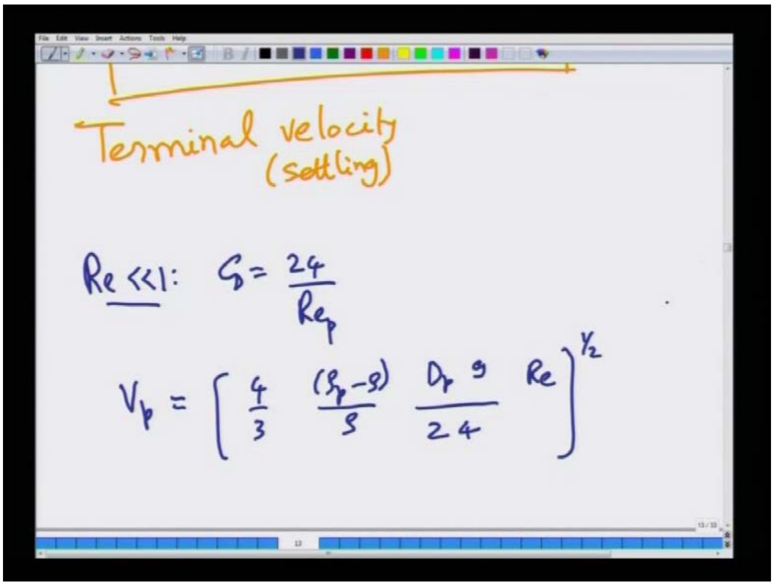
So, you have to simply balance all the three forces and the three forces are simply zero is minus rho p by D p Q g by 6. This is the weight, rho p is the density of the particle times volume of the particle times g is the weight of the particle; it acts in the minus y direction times rho times pi D p cube g divided by 6 plus pi by 8 rho V p square D p square times C D.

(Refer Slide Time: 11:30)



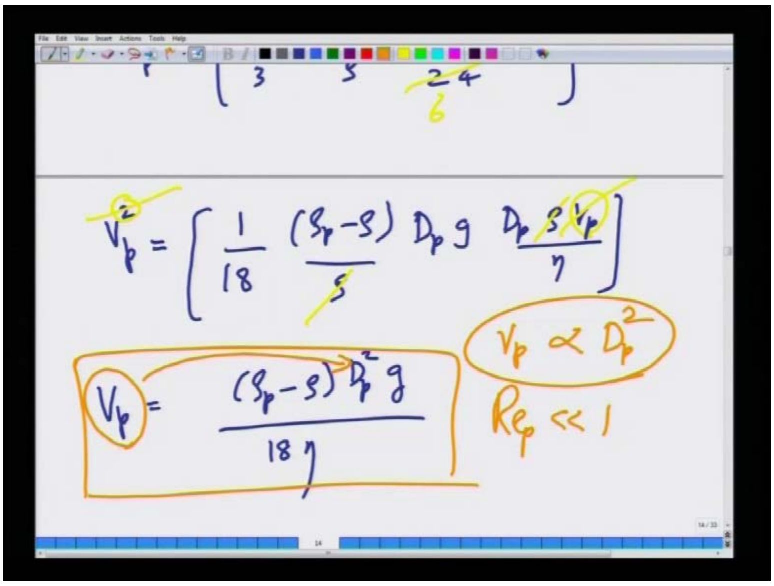
So, if I eliminate for V p we will get V p is equal to square root if I solve for this equation for V p you will get 4 divided by 3 times rho p minus rho divided by rho D p g divided the drag coefficient. So, this is the terminal steady terminal velocity of a sphere. Expression for terminal settling velocity of settling of a sphere of diameter D p which is moving on density rho p which is moving in a fluid of density rho.

(Refer Slide Time: 12:11)



Now, of course, we have to look at different regimes when Reynolds numbers is small compared to one we know that C D is 24 divided by Reynolds numbers based on particle diameter. So, V p becomes 4 by 3 times rho p minus rho divided by rho time's D p g by 24 Reynolds number to the power half.

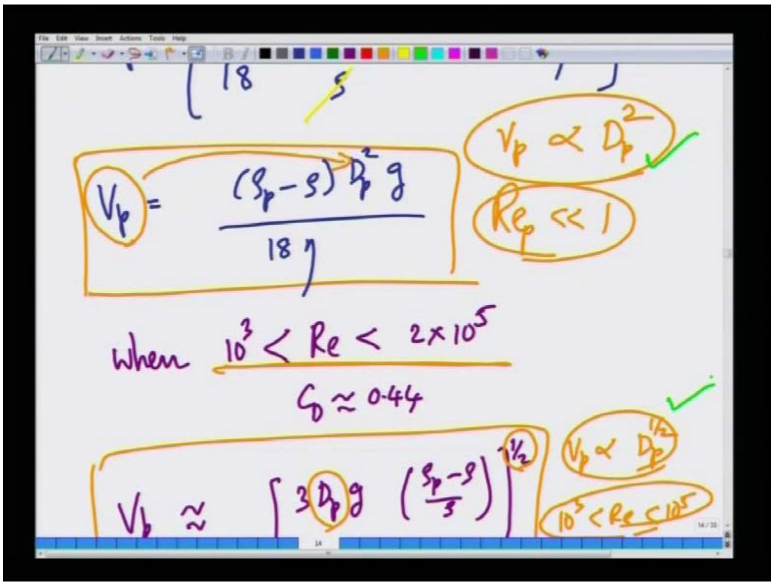
(Refer Slide Time: 12:39)



So, V p is nothing, but, or we can square the entire expression V p square is nothing, but, we will get. So, you have 4 6. So, you get 18 here you have 1 over 18 rho p minus rho divided by rho times D p g times. We have substitute for Reynolds numbers is D p fluid density rho V p divided by eta.

Now, this fluid density will cancel with this density one factor of V p will cancel with one factor of V p here to give you V p is nothing, but, rho p minus rho D p square times g divided by 18 eta. So, for low Reynolds numbers the settling velocity is given by this expression. So, notice that the settling velocity for low Reynolds numbers is proportional to square of D p particle diameter. So, that is the key if everything remains constant. If you just worry about the variation of the settling velocity on the particle diameter it goes as the square of the particle diameter. Now, let us look at high Reynolds numbers.

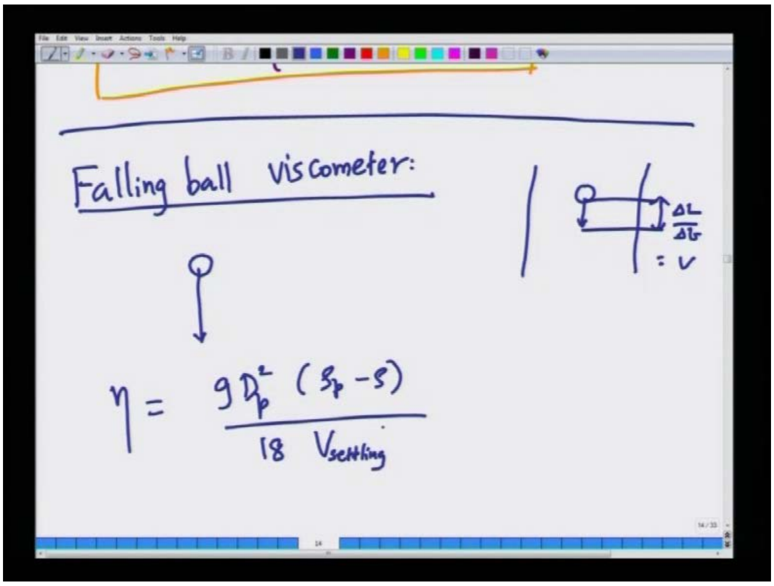
(Refer Slide Time: 14:12)



Now, I told you that when Reynolds numbers is in between 1000 and twice 10 to the 5 there is a region of constant drag coefficient. If you look at this data here C D is approximately constant. So, we are looking at this regime where C D is approximately constant. It is a number and the number is approximately 0.44. So, then we have to merely substitute 0.44 in this expression for C D and then we will get an expression for the settling velocity. And the settling velocity for this regime is approximately 3 D p g rho p minus rho divided by rho to the power half. Now, notice here that this is the settling velocity at the relatively high Reynolds numbers regime between 10 to the 3 and twice 10 to the 5, 10 to the 3 and 10 to the 5.

Notice that the settling velocity is proportional to diameter of the particle to the power half. So, there is a square root here also. So, the way in which the settling velocity depends on the diameter of the sphere is very different whether the Reynolds numbers is small or large. It goes as settling velocity goes as diameter of the particle squared at low Reynolds numbers while it goes diameters of the particle at half for high Reynolds numbers.

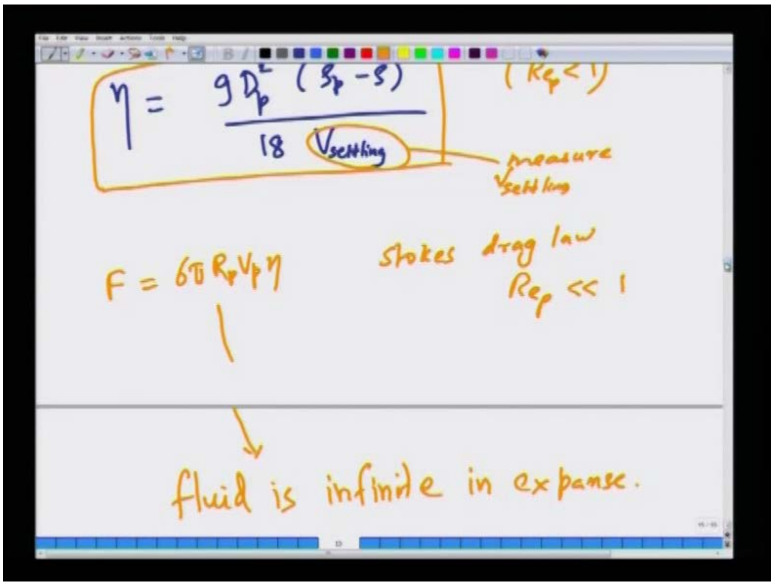
(Refer Slide Time: 15:52)



Now, this expression can be used to measure the viscosity of a fluid and such a device is called the falling ball viscometer. Essentially the idea is you drop a sphere in a fluid and wait for it. You take a sphere and replace it in a liquid whose viscosity you want to find and wait for it to attain terminal velocity. Calculate the velocity by noting how much distance it travels over a particular time t and that will give you the terminal velocity settling velocity. So, the settling terminal velocity is measured in the experiment by simply measuring the time particles takes to move a certain distance and from there we can calculate the viscosity of the liquid in which it is settling through other parameters.

So, divided by so, merely inverting this expression. So, if the Reynolds number is low. So, we can pull theta here and V p here and that is all we have done in this expression. And to get an expression for viscosity provided you know what is the diameter density of particle density of fluid and you measure the settling velocity; velocity of settling of the particle.

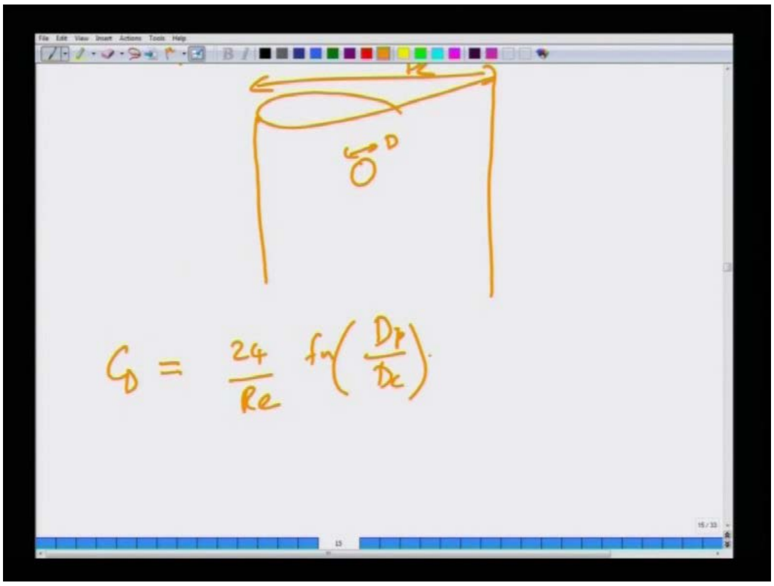
(Refer Slide Time: 17:11)



So, this is often used to measure the viscosity in many industrial settings because it is easy first of all to do such an experiment and this is assuming that Reynolds number is small. Because, you choose your particle dimensions such that the Reynolds number based on the particle is very small. Now, in the above when we used the Stokes drag law.

So, this is the Stokes drag law of valid at small values of particle Reynolds number. When we use the stroke drag law we are using, we are assuming that the fluid is infinite in expanse. That is the there are no surrounding walls. But in reality there will be surrounding walls.

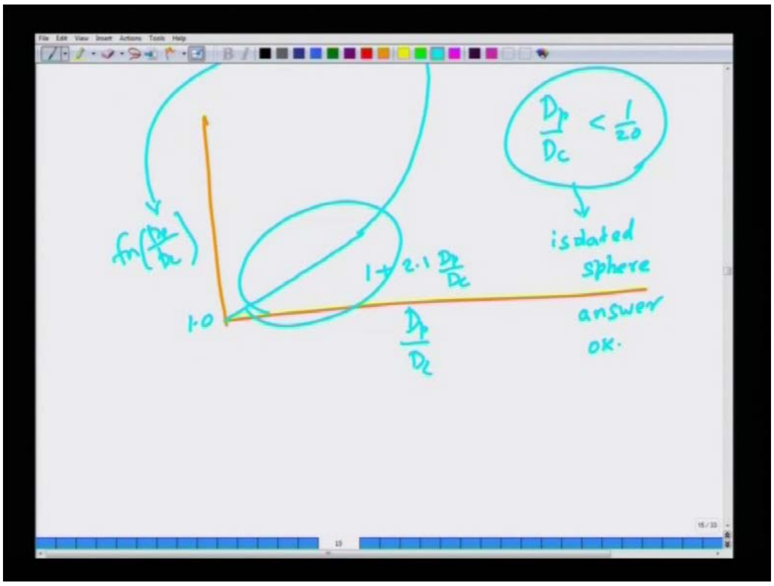
(Refer Slide Time: 18:21)



Suppose you may do an experiment in a container and therefore, you have diameter of the particle D and then the diameter of the container let us say D C. In principle you may expect that when the diameter of the particle is very small compared to the diameter of the cylinder in which it is flowing, it is moving. Then you would assume that the wall effect on the drag law drag law is negligible.

Because this drag law is valid only if the walls are infinitely away from the particle, but, in reality that is not the case because we do experiments often in laboratory where there are confining walls of cylinder or whichever container we are doing experiments. But, you can also correct for that. So, the drag coefficient is 24 by r e times some function of D p by D C, where people have found using experiments what this functional form is.

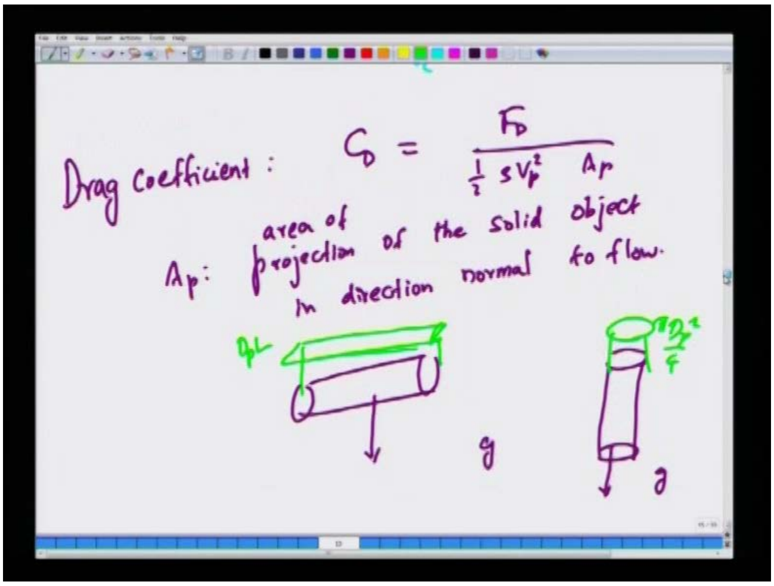
(Refer Slide Time: 19:17)



So, initially it goes as 1 plus. It goes 1 plus this function is what I am plotting as a function of D p over D C. Initially it goes 1 plus 2 point one times D p by D c. So, when D p equals D C of course, we expect the function to go to 1 and then it starts deviating. So, the confining walls tend to increase the drag because there is additional dissipation in the problem compared to when you have no confining walls. So, typically when D p by D C is less than 1 over 20 then the isolated sphere results is pretty good; answer is pretty good. But, when the confining walls or when D p by D C is such that it is not as small then you will start seeing the effects of containing walls confining walls and this is given by this correction.

So, we have seen what is meant by settling of a sphere and we will we are going to now first understand that it is not just that drag coefficients are defined only for spheres. You can have in many applications particles of other shapes also.

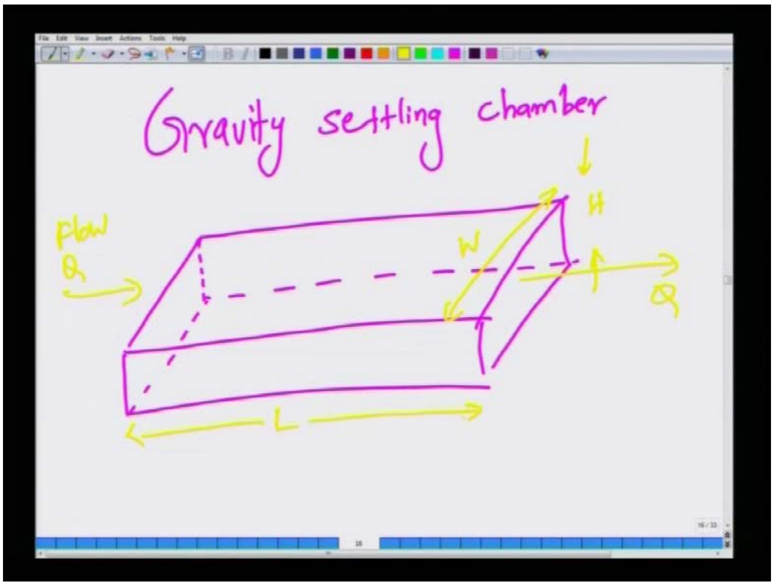
(Refer Slide Time: 20:53)



So, a general definition of drag coefficient is simply C D is the drag force divided by half rho V p square times A p where is A p is the projection of the solid object on a plane normal to the direction of flow; Area of projection of the solid object in the direction normal to flow. So, for example, if you have a cylinder that is moving settling suppose, you have cylinder settling like this and you have cylinder settling like this and the under influence of gravity the projected area here is very different. Here the projected area is simply it is going to appear like a rectangle. Here the projected area is going to appear like a circle of radius pi D p square by 4 or whereas, over here the projection area is pi D p times l.

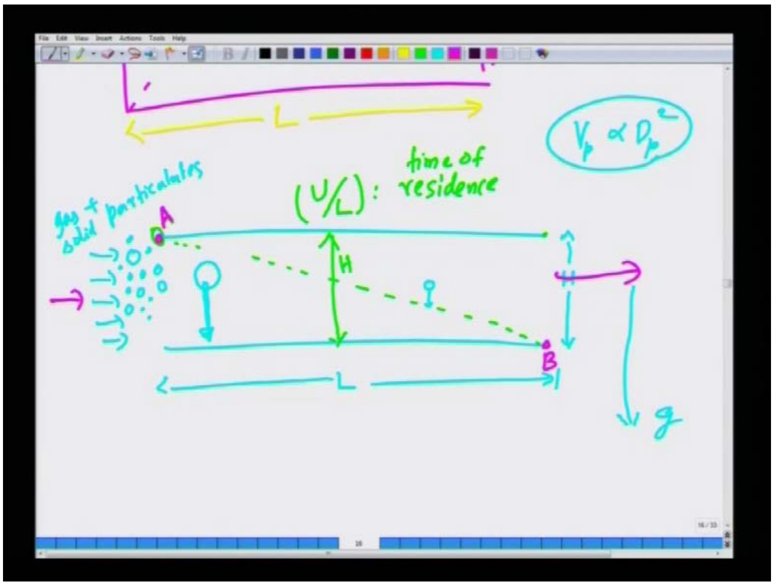
So, it is very different depending on sorry not there is no pi here it is D p times L because it is going to appear like rectangle of dimensions D p times l. So, the drag coefficient is defined differently because the projected area is very different and similar drag loss. I mean that this how the drag coefficient depends on Reynolds number is empirically found using experiments for other geometry such as cylinders as well. But, those are matters of experimental detail which can be found in text books or those are just matter just empirical data which can found in text books. But, the idea is that you do have similar drag coefficient versus Reynolds number relations for other geometries as well.

(Refer Slide Time: 22:55)



Now, I am going to do an application and the application is as follows. You want to have separation of particulates by virtue of the difference in their size for example. Suppose, you have how it is done is you see what is called a gravity settling chamber? (No audio from 23:14 to 23:23). Essentially, you have a box channel through which you have flow of a particulate. This is a suspension of it is this is like a suspension of some fly particulates like solid particles that are suspended in a fluid and the flow rate is Q. And width of the channel is W in this direction and height of the channel is H the gap in which particulate is flowing its flowing out in this direction and the length of the channel is l.

(Refer Slide Time: 24:20)



So, let us assume W is very large. So, we can treat the problem as a 2 dimensional problem. So, you imagine you have a channel height is H and the length is L and you have a mixture of gas solid mixture gas plus tiny solid particulates that flow in. And we want to make sure when the gas when this mixture flows out we want to separate out the solid from the gas. And that happens by the virtue of the gravity which acts in this direction and settling. But, typically in industrial applications this in incoming mixture will have a size distribution of particles not all particles will have the same size. It will have particles of varying sizes.

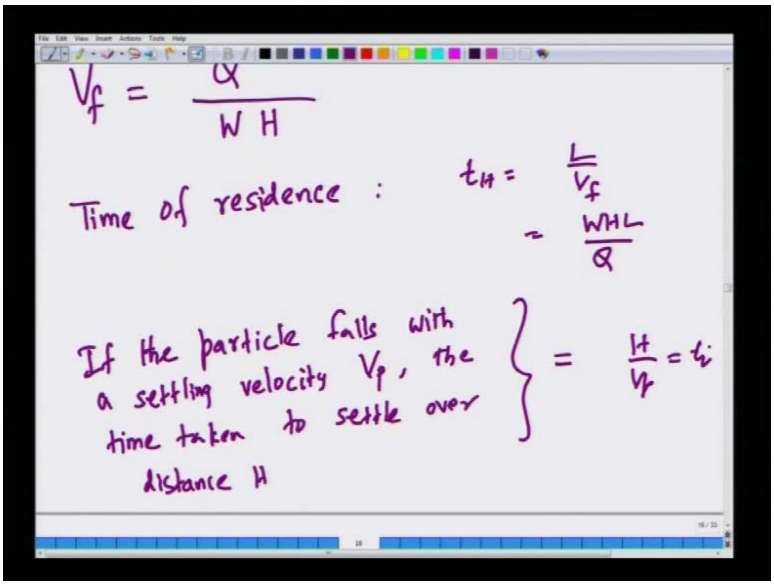
So, but, we want to know. So, from settling from the basic idea of settling you will imagine that bigger particles will settle sooner compared to smaller particles. Because, the drag force sorry the let us let us look at this settling velocity the settling velocity at low Reynolds numbers is proportional to D p square. So, the settling velocity is proportional to D p square which is what we derived just now.

So, it is higher by particles with higher diameters are going to settle more quickly compared to particles with smaller diameter. So, you can ask the question suppose I start from here. What is the smallest particle that I can capture in a length L? Suppose, you have a settling chamber of some length L and the gap with H what is the smallest particle that you can capture typically in a settling chamber. So, let us start with some position a here.

Now, if this particle has to be trapped over a distance L then it must settle or it must travel a distance of H within a time where wherein it resides in this settling chamber. That residence time is typically the average velocity divided by the length of the, this is typically the residence time; time of residence. A particle will approximately stay over a time of order u by L because it has to travel with an average velocity city u over a distance l. So, the time the particle spends in this chamber is u by l.

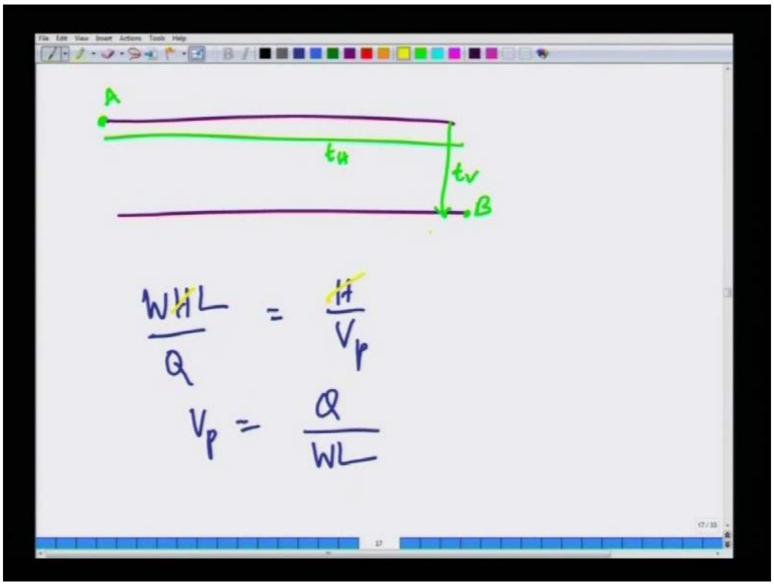
Now, within this time this particle which started out here must reach here as it moves. So, the particle will do this the trajectory of the particle will be something like this. So, if you consider a particle that is bigger than this particle that is anyway going to settle quickly. If you consider a particle that is smaller it is not going to settle. It is not going to settle within the chamber it is going to fly away with the flow. So, there is flow in this direction. So, which is that particle which will exactly start here at A and end here at B and that particle will be the cut off diameter which can be separated by this settling chamber.

(Refer Slide Time: 27:40)



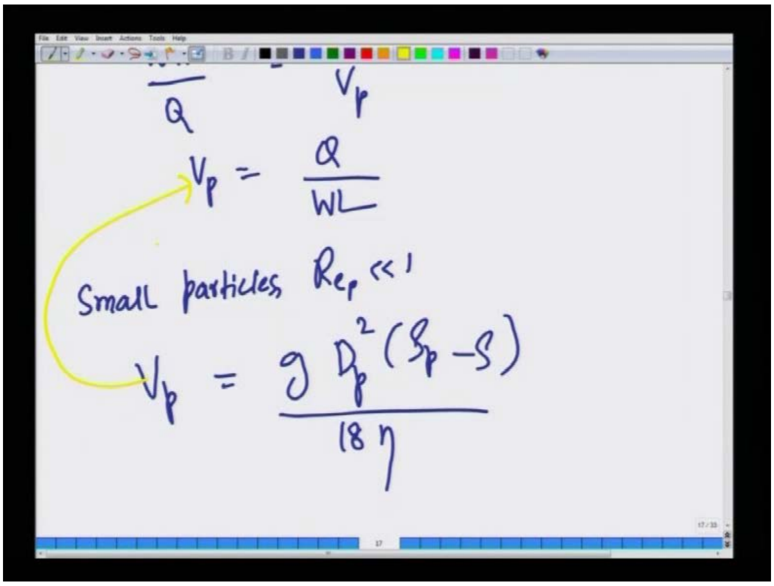
So, let us look at this analysis. The horizontal velocity of the fluid is simply average velocity of the flow is simply the volumetric flow rate divided by the area. Area is the cross section area of flow, which is width, which is into the board times the gap thickness h. A residence time of the particle the time, the particle typically spends in the chamber is simply L by V f which is nothing, but, W H L by Q. Now, if the particle falls with the settling velocity V p the time it takes taken to settle over a distance H is simply H divided by V p. This is let us call it t v.

(Refer Slide Time: 28:57)



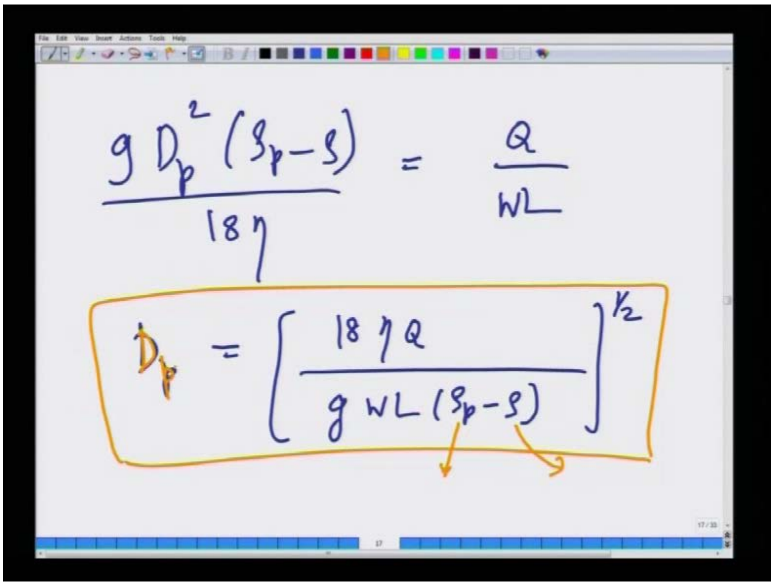
Now, if you imagine that you wanted a particle which started at A and ends exactly at B. For such a particle the time of residence which is the time it spends over a length L must be exactly equal to the time of settling. So, t H is the residence time velocity time. So, it must be equal to settling time. So, equate the two times we will have W H L divided by Q is nothing, but, H divided by V p or V p is Q divided by W L as H cancels from both sides.

(Refer Slide Time: 29:38)



Now, we have let us say, we have small particles and let us assume that Reynolds number of based on particle is small compared to 1. Then we will use the settling velocity that we just derived for such particles which is g D p square rho p minus rho divided by 18 eta. We are going to substitute this out here.

(Refer Slide Time: 30:06)

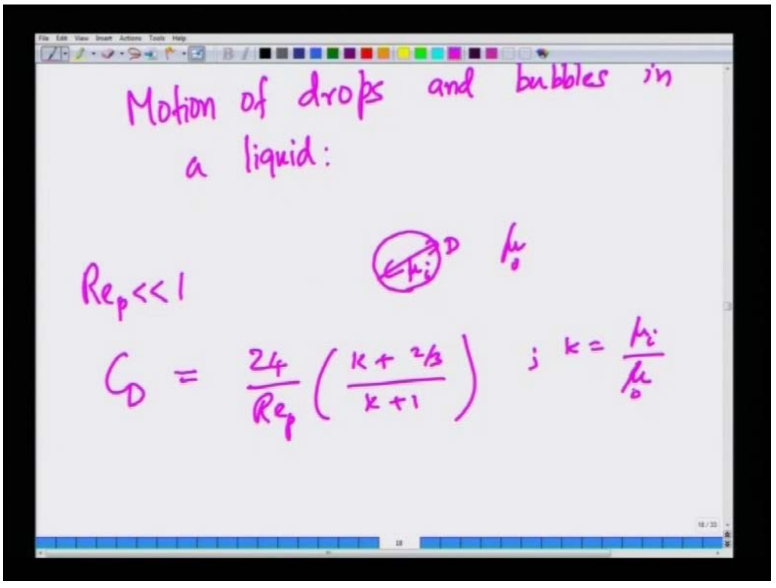


So, we will get g D p square rho p minus rho by 18 eta is Q divided W L or D p is nothing, but, 18 eta Q divided by g W L times rho p minus rho the whole thing to the power half. Now, this is the smallest diameter typically that can be captured by in the settling tank of this dimension. If Q is the volumetric flow rate of the mixture it has the viscosity of the liquid, g is the acceleration due to gravity, W is the width into the paper, L is the length of the channel, rho p is the density of the particles and rho is the density of the fluid in which it is travelled it is moving. This is the diameter of the smallest particle that can that typically the diameter of the smallest particle.

Now, if a smaller particle it is in principle possible that a smaller particle than this D p enters the chamber somewhat below A and it can be captured before B itself. But, that is something that is something that we can predict with certainty. So, this is typically because the incoming particles will come with distribution and each particle will enter the chamber at different vertical locations.

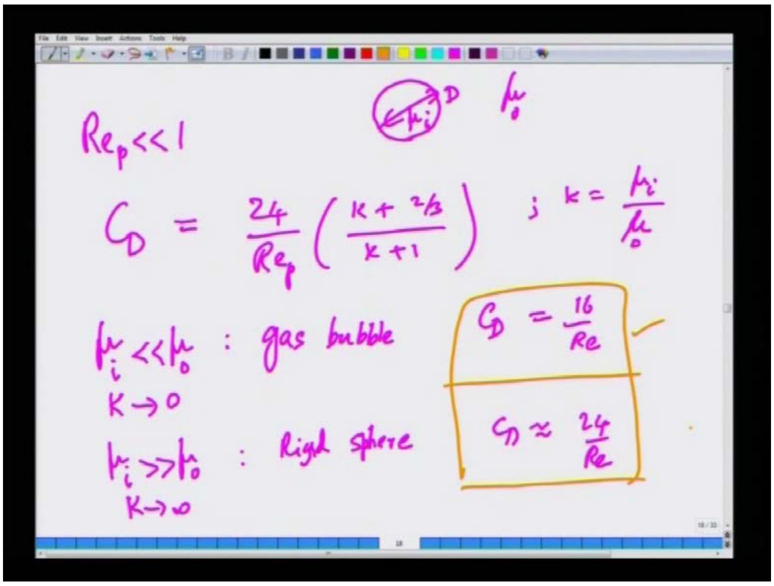
So, obviously, that can be smaller particles that can that can settle in the chamber, but, we can make with the following statement with certainly that no particle larger than D p will settle within the chamber. Now, so far we have been discussing the settling and motion of rigid particles. There is also case in many chemical engineering applications where you have to motion of drops and bubbles in a liquid. (No audio from 31:59 to 32:09).

(Refer Slide Time: 31:55)



Now, if you have a drop of some diameter D and let us say the drop viscosity is mu I and the outside viscosity is mu o. Then at low Reynolds numbers based on the drop small compared to 1 the drag coefficient is given by 24 divided by r e time’s k plus 2 divided by 3 divided by k plus 1 where k is nothing, but, mu I divided by mu o. So, the ratio of inner fluid viscosity drops viscosity to the outer viscosity.

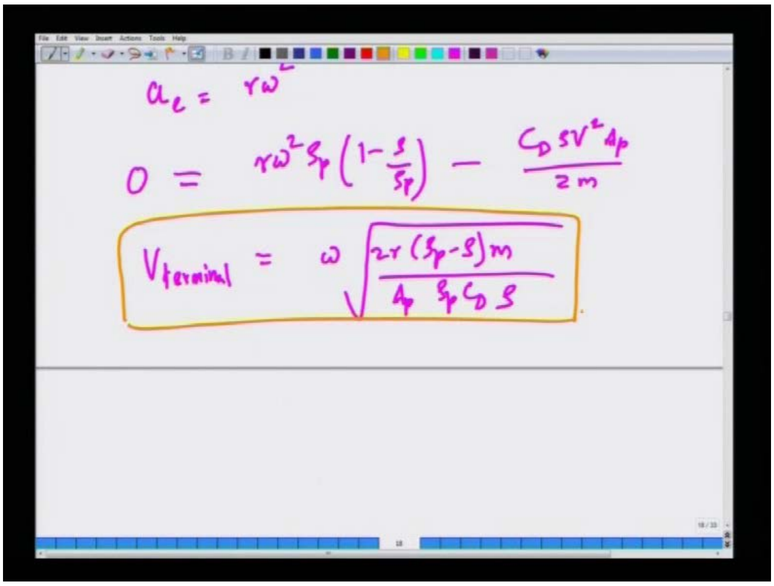
(Refer Slide Time: 32:57)



So, we have two limiting cases where mu is very small compared to mu o you get the motion of an air bubble or a bubble, gas bubble. And you will get C D in that limit. So, mu I is less than mu naught k tends to 0, if, I put k tends to 0 in this case I will get 16 by R e. Now, if mu I is very large compared to mu o that is k tending to infinity you get a rigid object rigid sphere. So, C D becomes 24 by l. So, it does reduce to the well-known cases of the; it does reduce correctly to the result of 24 by r e when the viscosity of the inner fluid is very large compared to the viscosity of the outer fluid.

And we also have a new result that for the motion of the gas bubble suppose you have a gas bubble in water air bubble in water it is going to rise. What is the terminal rise velocity at low Reynolds numbers? Well we can use this relation and then find out the terminal rise velocity in a very similar way that we did for terminal settling velocity.

(Refer Slide Time: 34:23)



Now, it does not just because of the driving force for settling is typically gravity in many applications, but, in many of in some application you do find the use of centrifugal driving forces that drive the motion of the particle in a fluid. So, the centrifugal happen forces happen whenever there is a rotation of fluid flow. So, whenever the direct of part of. So, when you have a suspension of a liquid in of solid in liquid and then if you want to separate these particles based on their sizes or densities. You can use gravity, but, gravity you cannot manipulate gravity because it is fixed on the surface earth.

So, if you were to use, if you want to speed up the process, if you want to have more control over the process, you could use centrifugal forces because these forces also appear as body forces. Because, this is like motion in an accelerating frame of reference it is a rotating frame of reference. So, you have fictitious forces which will act like body forces and that force proportional to r omega the acceleration centrifugal acceleration proportion to r time's omega square.

So, if you balance the centrifugal acceleration with the drag force you will get O is r omega square rho p times 1 minus rho divided by rho p. This is the net driving, net force along the direction of the body force because you have to account for buoyancy minus the drag force which is opposing this. So, C rho V square A p by 2 m. So, the terminal velocity in a centrifugal field is written as omega times square root of 2 r rho p minus rho times m divided by A p rho p C D rho.

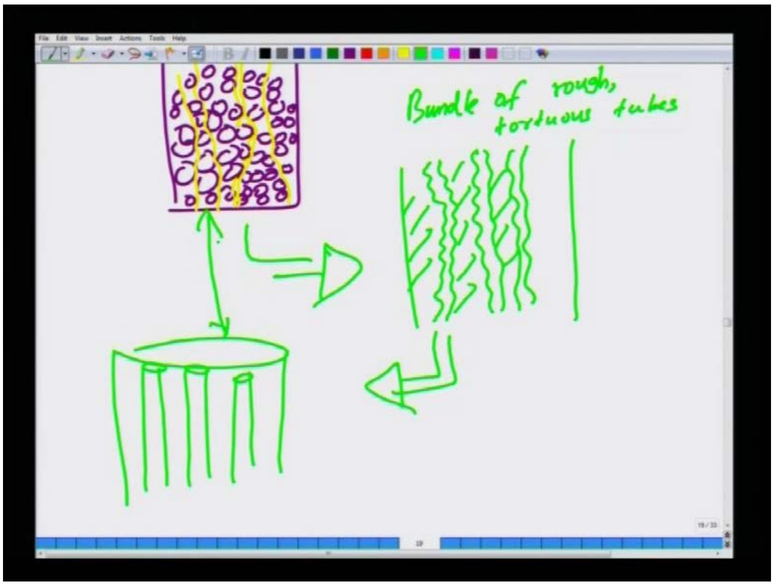
So, once you know what C D is you can actually predict the terminal velocity in a settling of settling even in a centrifugal field. This is often used in you know you know if you have a dairy product milk is basically emulsion of droplets of one liquid in another. Suppose, you want to remove fat from the milk and since fat has different density compared to the other constituents of milk you can centrifuge the milk to separate the fat from the rest of the milk.

So, this is how the process skimming is done in a milk using centrifuges and there if you want to design those centrifuges you have to have clear cut idea of what is the terminal velocity. In order to design the centrifuge, what is the size of the centrifuge you need and so on? So, such calculations are very useful in the design of these unit operations.

So, the next topic that we are going to discuss is related to flow of particulates in a fluid or we are going to look at the flow of fluid through a bed of particles which is slightly opposite different from what we just did. We so far, we considered the motion of particles in a fluid now we are going to consider the motion of fluid in a bed of particles. Such beds or such operations are called packed beds in chemical engineering applications. The reason why packed beds are used in many unit of operations such as adsorption or even reaction is that the packing of a higher surface area per unit volume for gas liquid contacting.

So, typically you may want to conduct an absorption process in a packed bed because you want to contact essentially a gas phase with a liquid phase and transfer species across the interface between gas and liquid. And clearly the amount of I mean the extent the rate at which the transfer can happen will depend on how much area is available for the transfer. And if you provide a packing that essentially increases the contact area of mass transfer per unit of volume of the bed. So, because of this high surface areas high interfacial area of mass transfer that is available these are often used in chemical engineering applications.

(Refer Slide Time: 38:29)



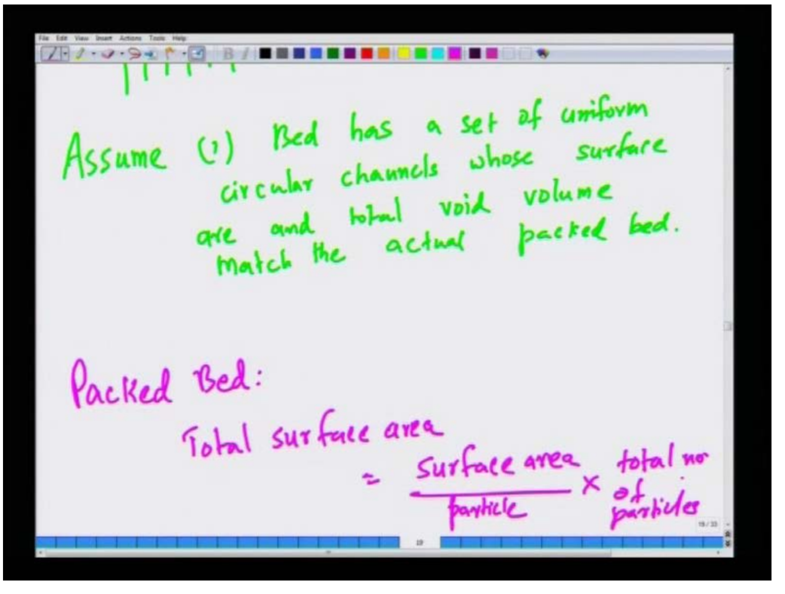
So, the next topic that we are going to do is packed beds. So, it is going to schematically appear like this you have a bed of particle and fluid is going to flow in this interstitial voids or interstitial spaces between these various particles. If you want to design a packed bed for a chemical engineering application, you want to know what is the pressure drop to make a fluid flow with some flow rate Q. So, this is the typical design question that comes up in designing packed beds. So, how are we going to do that? Now, if we were to draw a cartoon of the interstitial spaces, it may look like this.

So, and you can draw several such contours of interstitial spaces through which fluid is going to flow. So, like this and so on. So, there are going to be gaps through which fluid is going to flow and essentially the flow through a packed bed can be thought of as a flow through a bundle of corrugated tubes. So, you can imagine each path can be like a tube and there are several such paths. So, there is empty space within the tube and the rest is all solid filling. So, this is the model that we are going to use. A packed bed can be thought of as a bundle of rough and highly tortuous tubes of some complex curve cross section; rough tortuous tubes.

Now, we are going to assume further simplification since this problem is way too complex. We are going to assume this can be simplified as a bed of uniform circular tubes such that the surface area of this problem and the total void volume in this problem will match with that of the real packed bed.

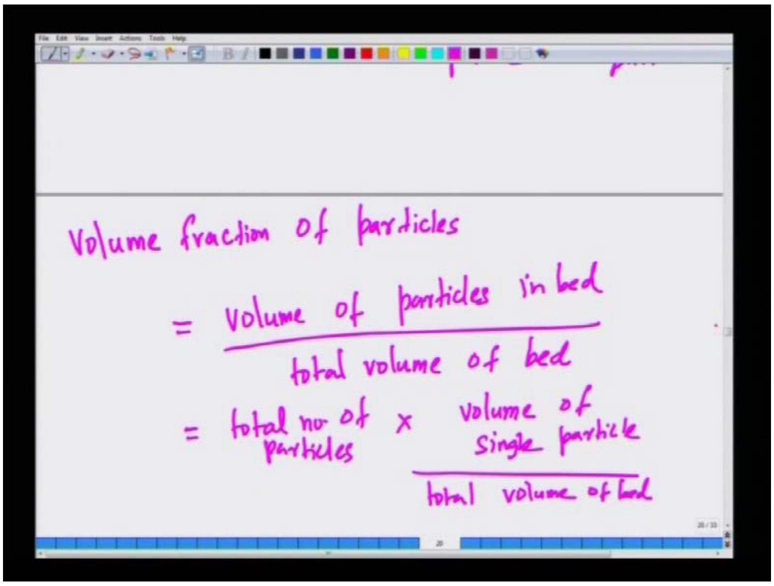
So, that is the only connection that we are going to essentially think of flow through a packed bed as a flow through a bundle of tubes. Although, in reality these tubes are extremely complex in their cross section and they are very corrugated. But, we are going to assume this is going to look like a straight tube, but, we are going to make the connection between the model and reality by saying the following that.

(Refer Slide Time: 41:36)



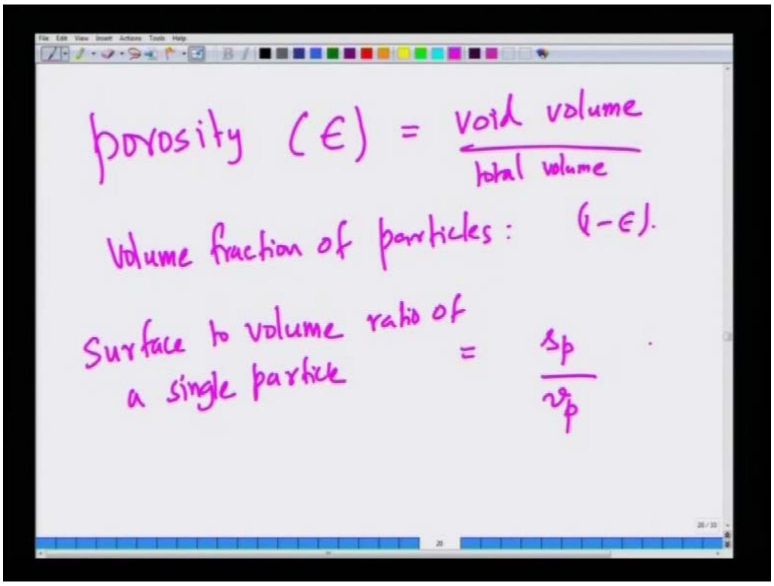
Assume that the bed has a set of uniform circular channels whose surface area and total void volume match the actual packed bed. So, this is the only connection we are going to make with the real packed bed. So, let us first look at the real packed bed. The real packed bed the total surface area is nothing, but, of particles is equal to the surface area per particle times the number of particles total number of particles. So, that is the total surface area available for the in the real packed bed.

(Refer Slide Time: 43:03)



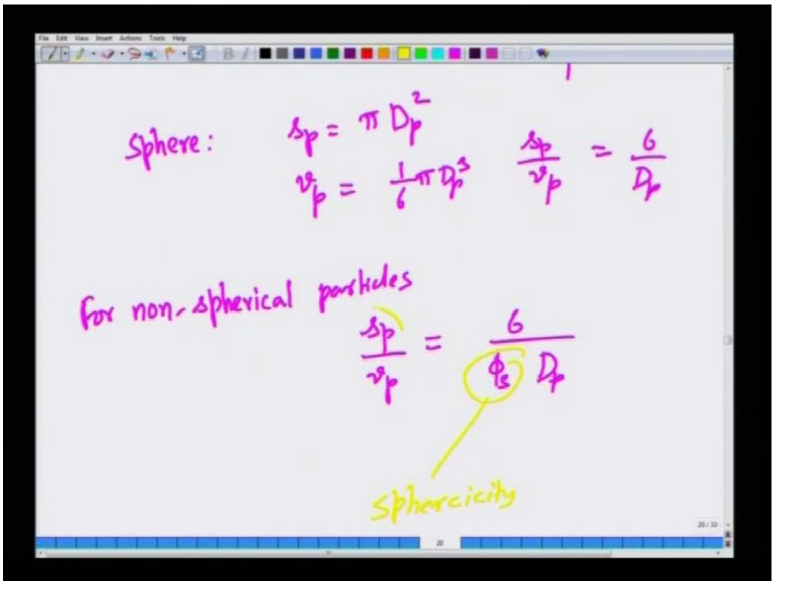
Now, the volume fraction of particles is nothing, but, volume of particles in the bed divided by the total volume of the bed. This is the volume fraction of particles it is essentially the amount of volume that is occupied by the particles divided by the total volume. This is nothing, but, the total number of particles times volume of one particle single particle divided by the total bed volume.

(Refer Slide Time: 44:09)



Now, the bed porosity is defined by the symbol epsilon is essentially the void volume divided by total volume. So, the volume fraction of particles becomes 1 over 1 minus epsilon. The volume fraction of particles becomes 1 minus epsilon because there only particles and void only 2. It is like of like two components system. If voids are occupying volume of epsilon then this can to occupy particles have to volume occupy the remaining volume. So, the fraction of that space is 1 minus epsilon.

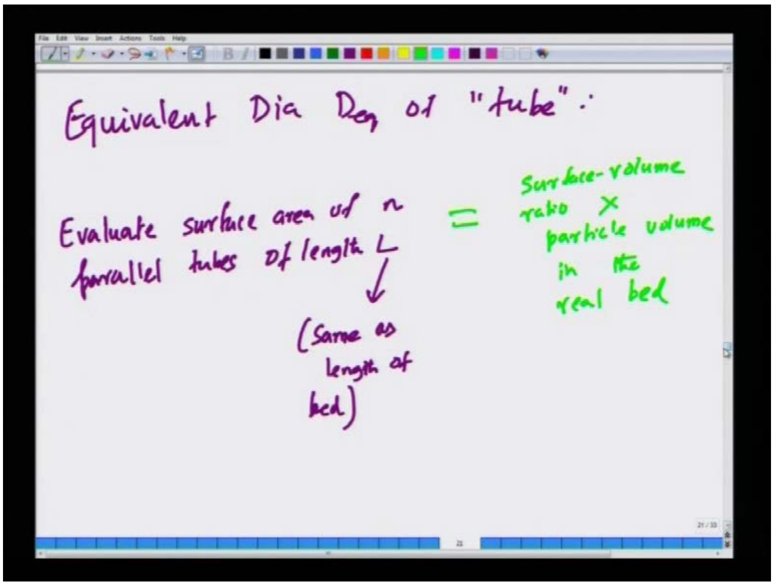
(Refer Slide Time: 45:16)



So, the surface to volume ratio of a single particle is surface area of a particle divided by volume of a single particle. For a sphere, if the particle is a sphere s p is nothing, but, pi D p square and V p is nothing, but, 1 over 6 pi D p q. So, s p divided by V p is 6 by D p for a sphere. For non-spherical particles because typically we will definitely have only non-spherical particles in practical applications s p by V p is written as 6 by phi s D p phi s is called a Sphercicity. So, it tells you about deviation from a spherical shape and phi s for some spherical shape cause s p by D p is simply 6 by D p for non-spherical particles phi s greater than 1. So, so this shows s p by D p is denoted by sphercicity parameter.

So, you write the, so, you calculate how sphercicity is calculated? You determine s p by V p for a non-spherical particle and then use this to determine sphercicity. It tells you the deviation from the spherical shape.

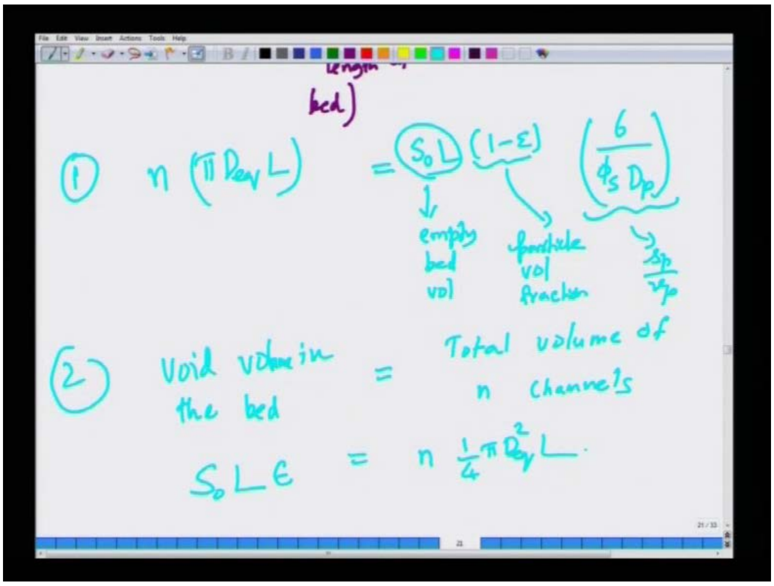
(Refer Slide Time: 46:38)



So, now we want to now find what is the equivalent diameter of the tube? (No audio from 46:38 to 46:46) through which we imagine fluid is flowing inside a packed bed in this bundle of tubes model. Now, first thing we will do is to evaluate the surface area of the tubes surface area of n parallel tubes of length L. So, here we are assuming that the length of the tube is same as the length of the bed same as the length of the bed which is a severe assumption. Because in reality the actual length of the flow of the particle sorry actual length of the fluid since the fluid is since, the tube is highly corrugated; it should certainly be more than l. But, we are now assuming because we do not know how what is the length of the actual flow path of a fluid inside this corrugated space.

So, we are going to assume the same as the length of the bed itself. So, this is an assumption. This is nothing, but, so, now first we evaluate this and then we equate this to this must be equal to the surface volume ratio times the particle volume in the actual bed in the real packed bed.

(Refer Slide Time: 48:29)

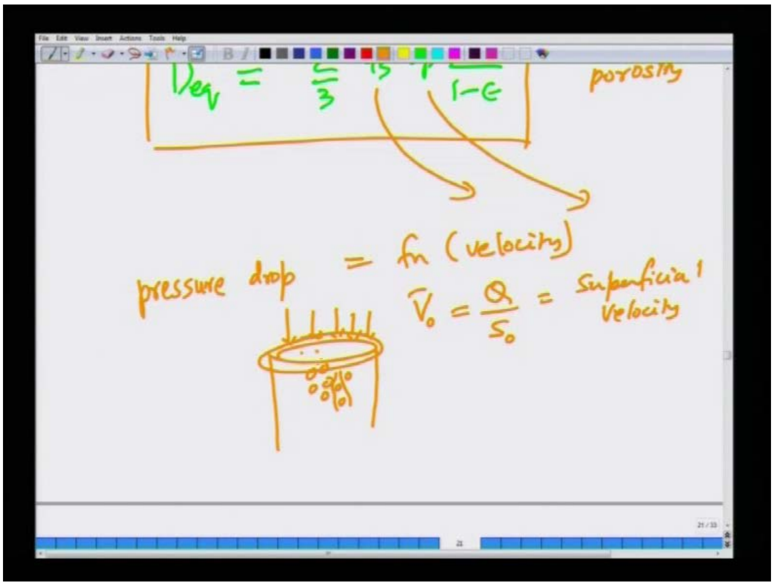


So, this is simple to calculate. So, number of tubes times pi D e Q time's l. This is surface area of a single tube through which fluid is moving and times n number of tubes is equal to. Therefore, the surface to volume ratio of a particle which is non-spherical is 6 by phi s D p times the particle volume in the bed particle volume in the bed is nothing, but, the cross section of the bed times the length of the bed times 1 minus epsilon.

This is the empty volume of the bed, but, 1 minus epsilon of this fraction 1 minus epsilon of this volume is occupied by the particles. So, we have to do that. So, this is the empty volume of the bed volume. This is particle volume fraction and this is the surface to volume ratio of a non-spherical particle if it is a sphere phi s is 1. This is one constraint that we have. The second constraint is that we have is that the void volume in the bed. The void volume in the bed is the same as the total volume of the n channels.

So, the void volume is simply s naught times L times epsilon. Because it is the empty bed volume and once you put in particles this is the void volume because epsilon is the void volume divided by the total volume. This is equal to n times pi times l. This is the cross section need of a single tube times its length times the number of such cha tube.

(Refer Slide Time: 50:30)



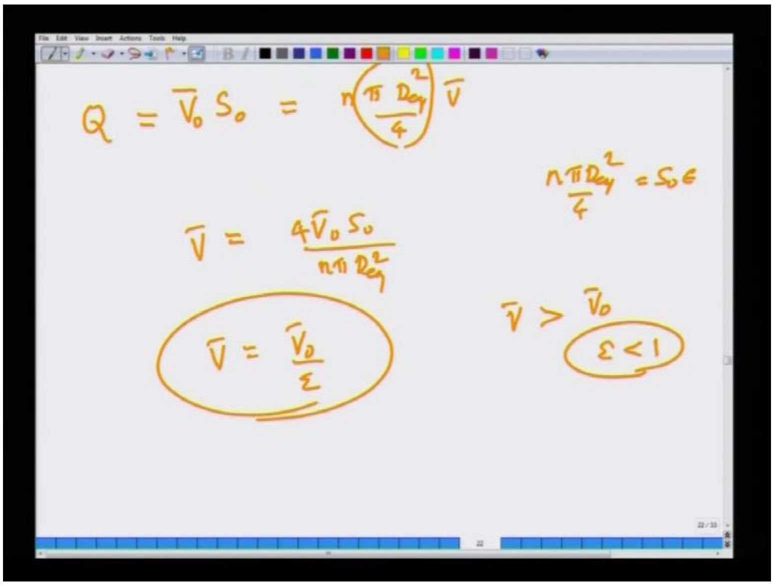
So, by equating these two. So, D equilibrium sorry b equivalent becomes therefore, 2 by 3 phi s. So, if you look at this term you can get L will cancel out; L cancels out. So, you can get D equivalent in terms of other properties, other quantities. So, D equivalent becomes 2 by 3 times phi s and also we have to eliminate for n by using this expression. So, essentially you can write instead of s naught L time's epsilon. You can write this expression out here instead of s naught L you substitute n times 1 by 4 pi D square L out here and then eliminate for D equivalent for D equivalent. So, we will get 2 by 3 phi s D p times epsilon by 1 minus epsilon.

So, this is the equivalent diameter of the various tubes based on the nature of the geometry of the particle the particle dimension and the porosity of the bed. So, we are trying to we have already gotten an expression for the equivalent diameter for these tubes through which we imagine fluid is flowing.

Now, the pressure drop depends on the average velocity in the channel; will be a function of the velocity in the channels in this various channels. The question is which velocity is to be used cause if you consider the bed. So, you have the cross section of the bed as s naught and volumetric flow rate that flows is q. So, this will be what is called the superficial velocity.

This is the superficial velocity of the fluid within the bed because once if you look at the actual velocity it is likely to be more than the superficial velocity. Because, the cross section flow is not actually the entire thing there is only some part that is available for the flow. So, it has to be more. So, how do we do that? How to find the relation between the superficial velocity and the actual velocity? It is also called the empty tower velocity. If the tower was empty without any particles then what is the velocity that you would expect in the bed.

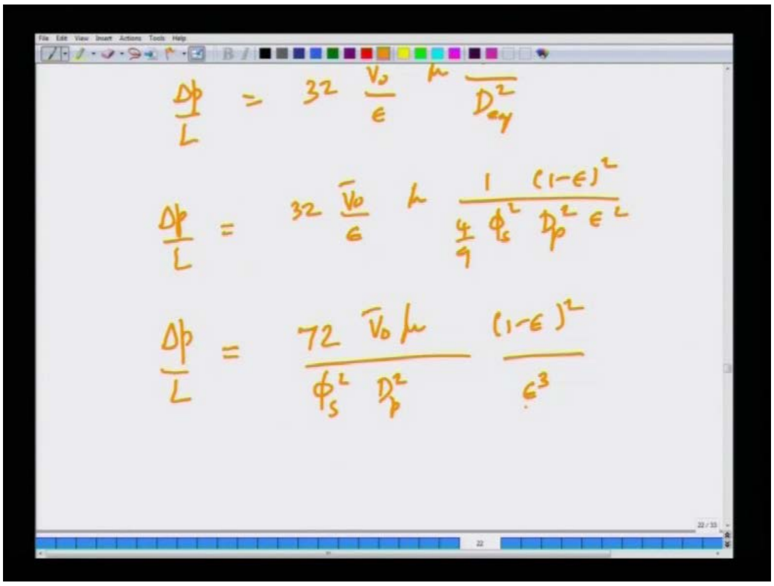
(Refer Slide Time: 53:14)



So, how do you relate this to? So, Q naught is V naught times s naught. Now, that is simply equal to n times pi D Equivalent Square by 4 times v. Because this is the volumetric flow rate in these n tubes each tube is of dimension of area these cross and there are n such tubes. This is the volumetric flow rate across the n tubes that we have modeled. So, V becomes therefore, V naught 4 V naught s naught divided by n pi D equivalent square.

Now, there is there is also this equation. You have this expression that relates n and s naught L e epsilon. So, once you substitute that, but, n times pi D equivalent squared by 4 is nothing, but, s naught times epsilon. So, V bar will become therefore, V naught by epsilon. So, we can see that V bar is greater than V naught because epsilon is always less than 1; epsilon is the porosity. So, this always is less than 1 and in the limit when epsilon is equal to 1 that is the whole bed is empty then we will get V equals V naught.

(Refer Slide Time: 54:37)



So, now we know what is V in terms of the superficial velocity of the bed divided by porosity and we know what is D equivalent. So, we know what is the velocity and the diameter of the pipes. So, we can use suppose, the Reynolds number is low, Then we can use the laminar flow relation. So, delta p by L is 32 V bar mu by D square it is 32 V bar is nothing, but, V naught by epsilon times mu by 1 by instead of D we will put D equivalent square equivalent diameter and then substitute the expression for D equivalent.

So, you get delta p by L is nothing, but, 32 V naught by epsilon mu 1 over 1 minus epsilon squared by epsilon square 4 by 9 phi s square times D p square just after substituting the expression for D equivalent that we just derived few minutes back. So, delta p by L becomes now after doing algebra the algebra here you will find that this is nothing, but, 72 times V naught mu divided by phi s square D p square times 1 minus epsilon whole square by epsilon cube.

Now, we will stop here and we will continue with this derivation in the next lecture where we will complete the derivation for pressure drop in packed bed both in the laminar regime as well as in the turbulent regime.