Fluid Mechanics

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Module No. # 01

Lecture No. # 33

Welcome to this lecture number 33 on this NPTEL course on Fluid Mechanics for under graduate chemical engineering students. The topic that, we are currently discussing in the last two lectures is higher Reynolds number flows, when fluid flow, typically fluid flows happen at very high Reynolds numbers of the order of few thousands in many engineering applications. Therefore, it is very useful to understand the basics of the fundamentals of fluid flow at high Reynolds numbers. At high Reynolds numbers, the inertial forces are dominant compared to viscous forces in the in the flow.

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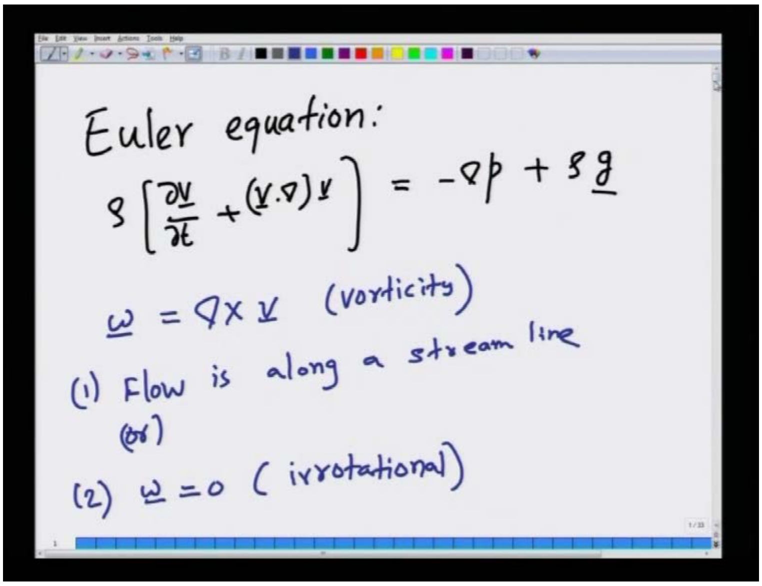

Therefore, as a first approximation, we can hope to neglect the viscous forces or viscous effects all together, there thereby ending up with the Euler equation, which is simply the Navier stokes equation without the viscous terms, this is the inertial part of the Euler equation and the pressure force is plus the gravity forces. Now, upon using a vector identity and by assuming that either, suppose if omega is del cross v is the vorticity of the flow then, either by assuming that flow is along a stream line line or another independent assumption, by assuming that vorticity is 0, that is the flow is irrotational.

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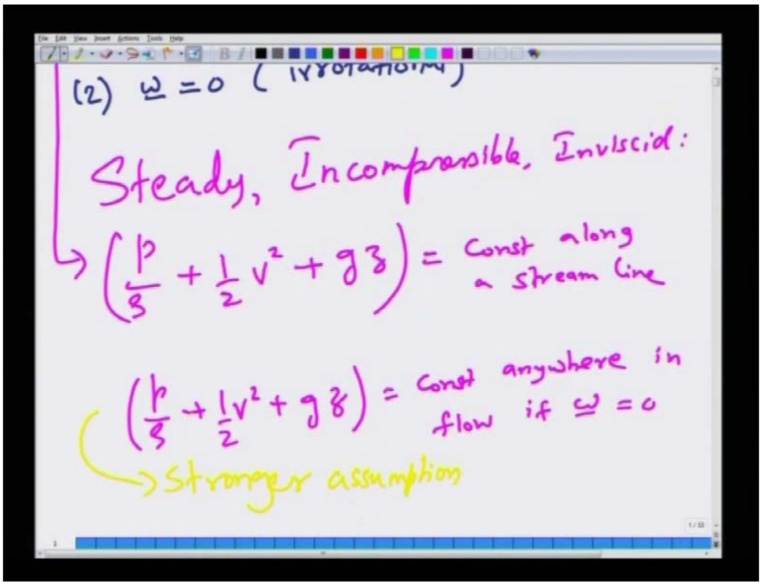

We were able to show, that the Euler equation simplifies to the Bernoulli equation for a steady, incompressible and inviscid flow, that is, flow without viscous effects. So, you have p by rho plus half v square plus g times z is a constant along a stream line, this is if use assumption 1, or it is a constant anywhere in the flow, if you use assumption 2. That is, if you assume that the flow is irrotational, anywhere in the flow, if omega is 0. So, these are two independent assumptions, as I told you in the last lecture, this is the more stronger assumption, because we are assuming that the flow is irrotational, but it leads to simplification, that this is a constant anywhere in the flow.

Another, so we will also pointed out we also pointed out in the last lecture, that irrotationality, although being the kinematic assumption is more likely to be seen at higher Reynolds numbers, when you are far away from solid surfaces.

So, we can assume the flow to be reasonably irrotational when the fluid flow is happen happening at higher Reynolds numbers, but close to the solid surfaces, rotational effects become important. So, we can imagine the solid surfaces to be sources of vorticity and they the vorticity will be confined close to the solid surfaces, but far away from solid surfaces at high Reynolds numbers, the flow is largely irrotational.

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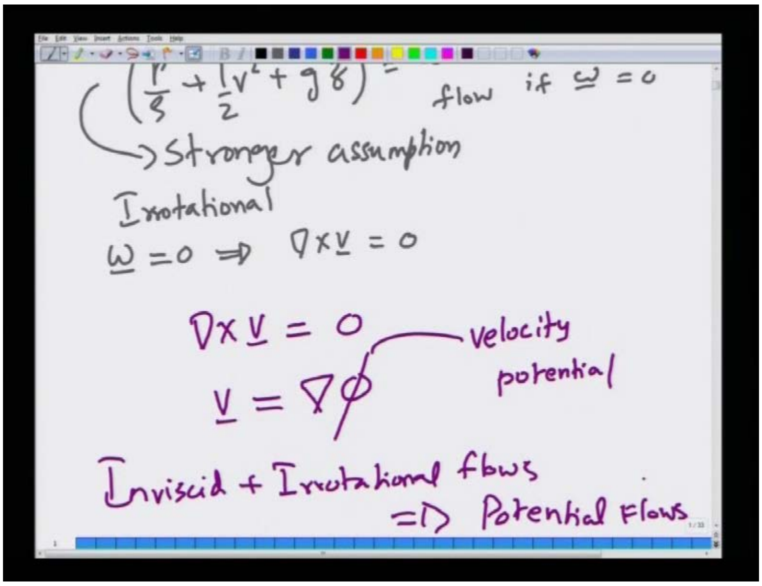

So, when you assume the flow is irrotational implies irrotational implies del cross v is 0, or whenever you write curl of a velocity vector is 0, then we can write v as a gradient of a scalar potential, this scalar potential is called the velocity potential. So, inviscid and irrotational flows are also called as, so inviscid plus irrotational flows are also called as potential flows, are also known as potential flows, because the velocity is written as a gradient of a potential.

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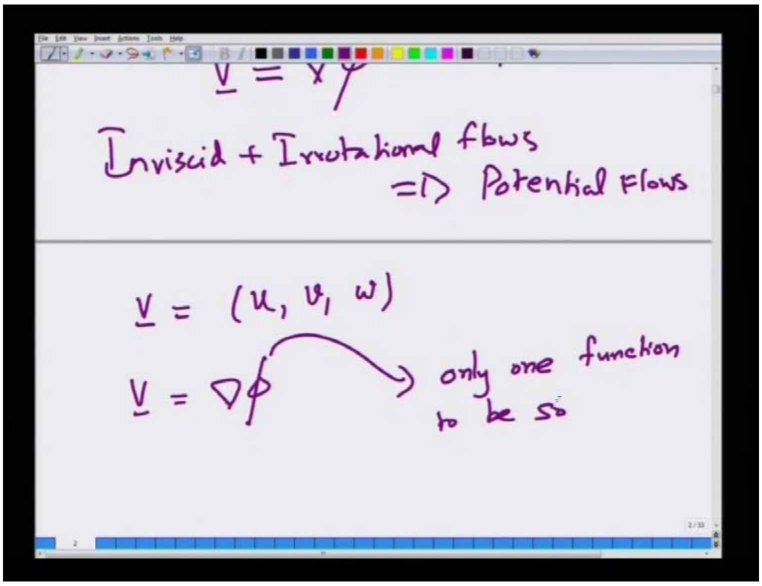

What is the simplification? The velocity is a three dimensional quantity in general, it has three components, each being the function of all the three coordinates directions. But if you use velocity as gradient of potential for irrotational flows, then you need to resolve only one unknown function, instead of three unknown functions. So, only one functions to be solved, so it leads to lot of simplifications.

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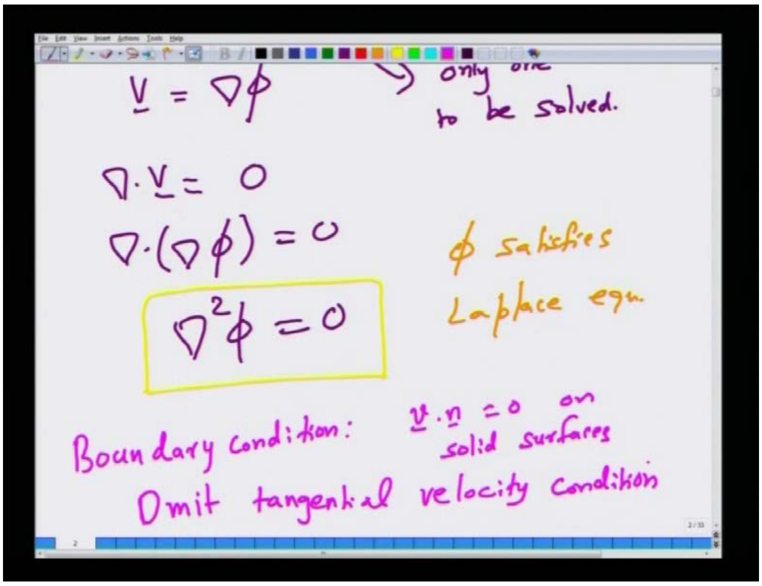

Another simplification that comes is that, when you couple this with the fact that the mass conservation equation for incompressible flows becomes del dot v is 0, this means that del dot del psi del phi is 0 or del squared phi is 0.

So, this is a Laplace equation, so this is a well-known Laplace equation, but importantly, it is a linear equation, so the potential satisfies the Laplace equation for the potential flows, we also pointed out that, the only boundary condition that can be applied is the normal velocity condition, because we are forgone v dot n is 0 on solid surfaces, because we have already sacrificed, we have already dropped the highest order derivative in the Navier Stokes equation namely, the viscous terms, so we cannot satisfy all the boundary conditions that are present in the problem that will lead to over specification of the Euler equation problem.

So, we have to necessarily sacrifice or forgo one of the boundary conditions. So, we we actually omit the tangential velocity condition in the inviscid flow regime. Now, whether this is really a good assumption or not, has to be seen, but already this, because the tangential velocity condition is satisfied, no matter what, how higher the Reynolds number is.

So, clearly the no-slip condition is being violated by not being by not satisfying the no-slip condition, we are neglecting some physics, how important and that is we will see a little later, and that will be done in the context of what is called the boundary layer theorem. But right now, let us stick to potential flows and in the last lecture, we mentioned that, it is much easier to solve the potential flow, because you have to simply solve del squared phi is 0 subject to v dot n is 0 on the solid boundaries. Now, that is merely del squared phi is equal to 0, is merely a statement of the fact that, the flow is irrotational and incompressible.

So, the momentum equation, the Euler equation, which has been simplified to the Bernoulli equation for a potential flow, merely serves to determine the pressure. So, the velocity is determined first by solving the Laplace equation, subjected to the normal velocity boundary condition and the Bernoulli equation is then used to solve the pressure in the flow, there are no viscous stresses. So, once you know the pressure in the flow, then you can integrate it over a solid surface to find, what is the net force on a body? For example which we will indicate a little later.

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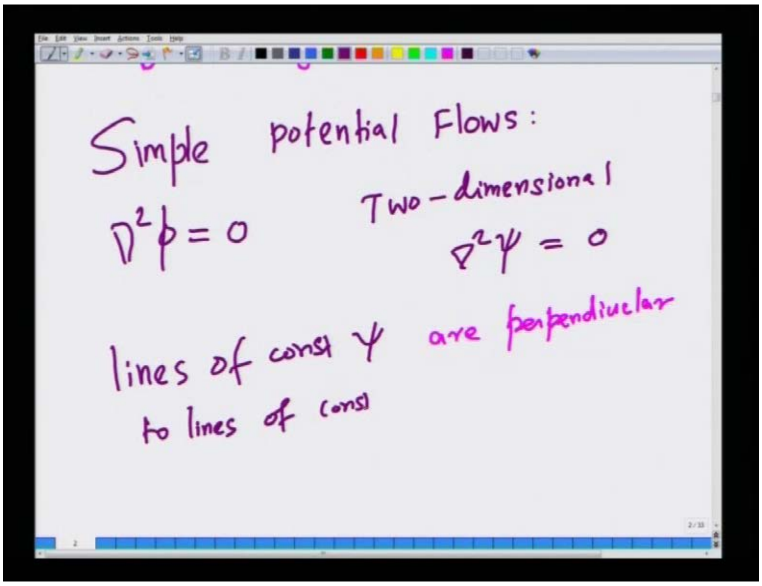

Now, the the context in which we started out discussing potential flows is that, we provided examples of some simple potential flows, so we started discussing some simple potential flows. Before that, we also discussed that just as this potential satisfies del squared phi is equal to 0, for 2 dimensional flows, that is when the flow is happening only in the x y plane, we also showed that the stream function also satisfies the Laplace equation.

And lines of constant stream functions, psi are perpendicular to lines of constant phi, that is, the stream lines and the equipotentials are orthogonal to each other, each and at each and every point in the flow. So, this is something that that we discussed, then we proceeded to do two examples, we did the (()), we found out what is the potential and stream function for what a uniform flow is.

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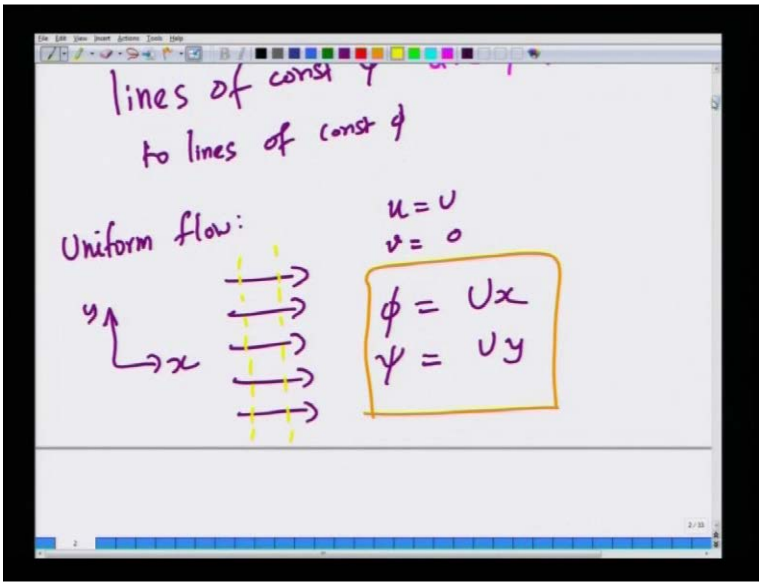

A uniform flow is essentially a flow, in the x y plane or the flow is constant in the x direction, there is no flow in the y direction. So, u equals U and v equals 0 and we found that, the potential is U times x and the stream function is U times y, we have already solved this. So, the potentials or lines, so lines of constant potentials are found by putting x constant values. So, you will find that, lines of constant potential or vertical lines along the y y direction, while lines of constant stream function are putting, are obtained by putting values of y, because psi is simply U y, which are just horizontal lines, which makes sense for a uniform flow.

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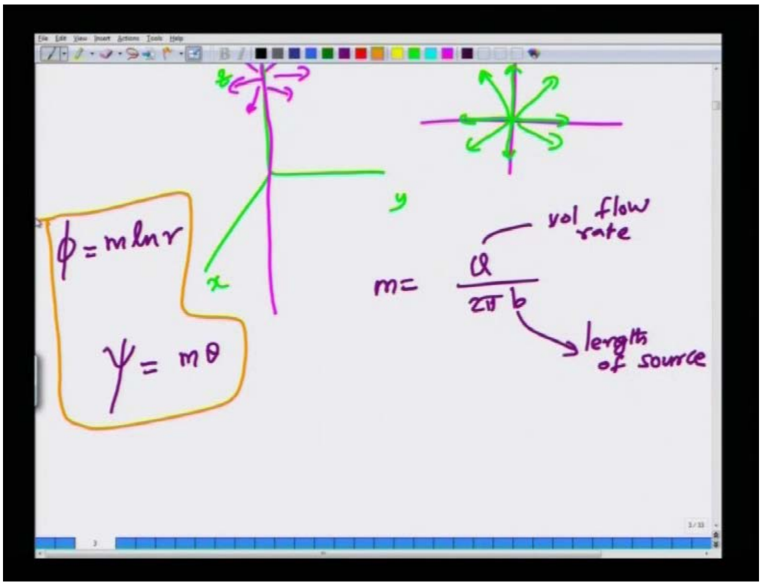

Then we also discussed, we also discussed line source, or sink which is essentially you have, the x y z plane. Imagine you have a source of fluid of mass, and where fluid is coming out radially, at each and every point. Since, the source length is very very large, you can imagine it to be like in the x y plane, it will appear like, you have a point source and fluid comes out radially at each and every point. So, this is the, and if it is a sink, if all the vectors are radially invert, it is the source of all the vectors are radially outward.

And for this problem, we found that the stream function velocity potential is m log r and this stream function is m theta, where m is nothing but Q by 2 pi b, where b is the length of the source and Q is the volumetric flow rate, this is called the source strength. If m is positive, it is a source flow; if m is negative, it is a sink flow. So, we have discussed this also, in the last lecture. We then began discus what is called a line vertex that is the topic that we going to start now.

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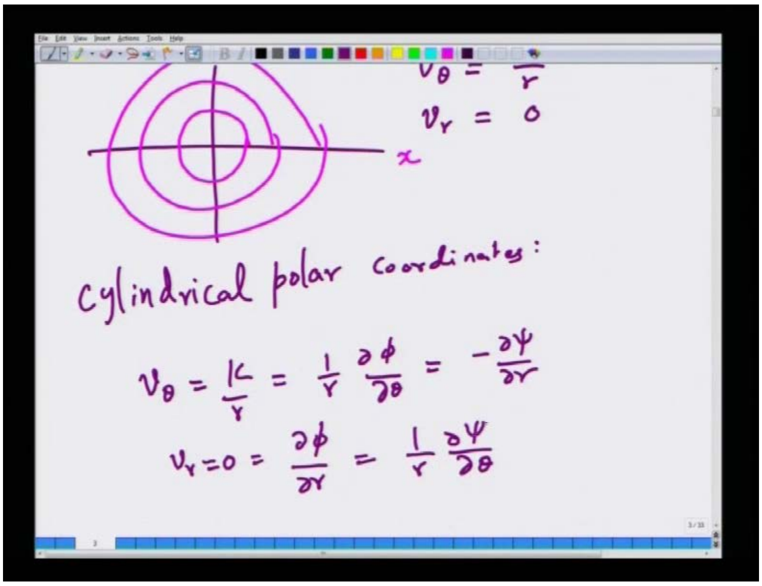

A line vortex is essentially, if you look at the 2 D plane, you have fluid flow completely in circular stream lines; stream lines are completely circular in the x y plane. So, you have v theta is some constant by r and v r is 0, now this is the description of the velocity field. So, from this, you can find out use cylindrical polar coordinates coordinates to find what is the stream function and what is the velocity potential velocity potential? So, v theta is K by r is nothing but 1 over r, partial phi partial theta is nothing but minus partial psi partial r and v r is 0 is partial phi partial r is 1 over r partial psi partial theta.

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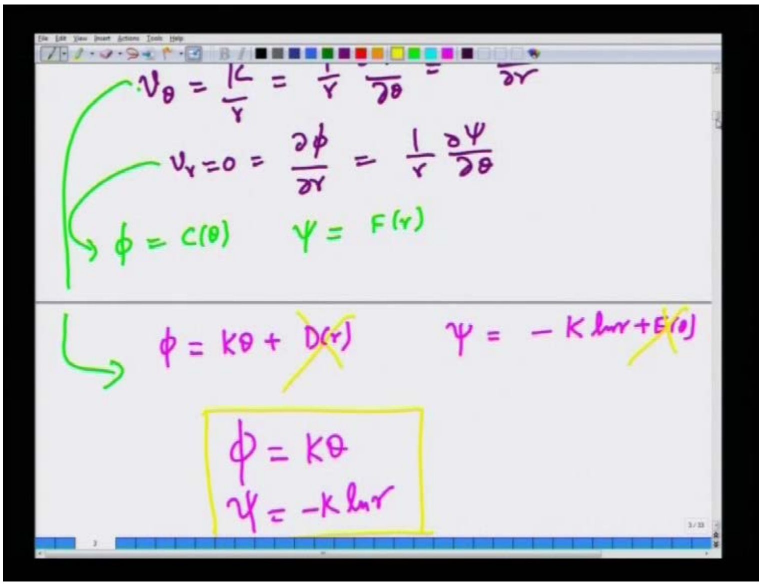

So, if you integrate the v r equation, you will find that phi; integrate this with respect to r. So, phi becomes a constant which could be a functional theta and psi, if you integrate this with respect to theta, you will find that it is a constant which could be a function only of r. Now, if we integrate, so that is upon integrating this equation, if integrate this equation, if we integrate that equation, we will get phi is K theta plus a constant which could be a functional of r and psi is essentially.

So, if you do this, psi becomes minus some other constant. So, sorry minus K log r plus some constant which could be a function of theta, should compare all this, you finally get this. So, these constant must be identically 0, since phi already K theta. So, let me 6 sorry, so phi is already, phi is a function only of theta so that, so this constant is identically 0, I am sorry likewise this constant, since i is the function only for this constant is 0. So, ultimately we get, phi K theta psi is minus K l n r, these are the potentials and stream functions for line vortex. So, if you look at the expressions, we can finally draw, what are the potential and stream functions.

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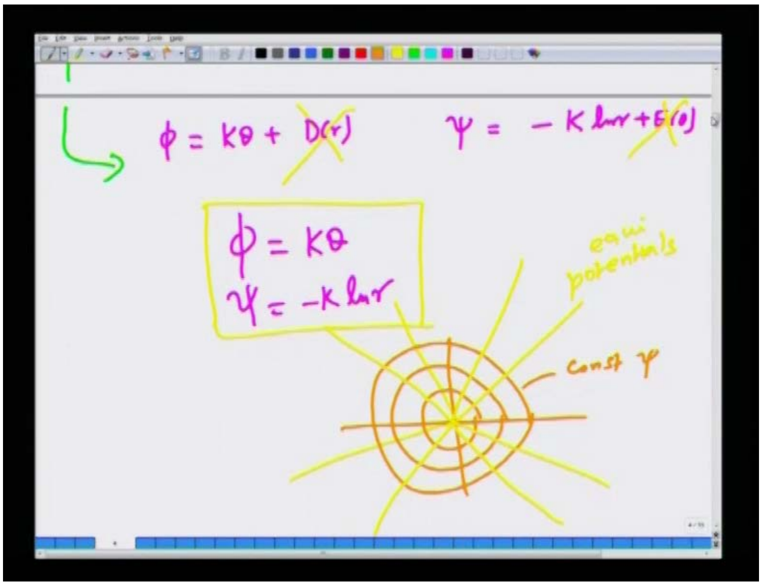

So, steam functions are lines of constant r, so they are circular two dimensions and potentials are lines of constant theta. So, they will be radially outgoing lines, these are potentials, equipotentials and these are constant psi lines. So, these are three fundamentals solutions of potential flows, simple solutions of potential flows. Now, we going to discuss a very important concept in potential flows, that is we going to discuss what is called super position.

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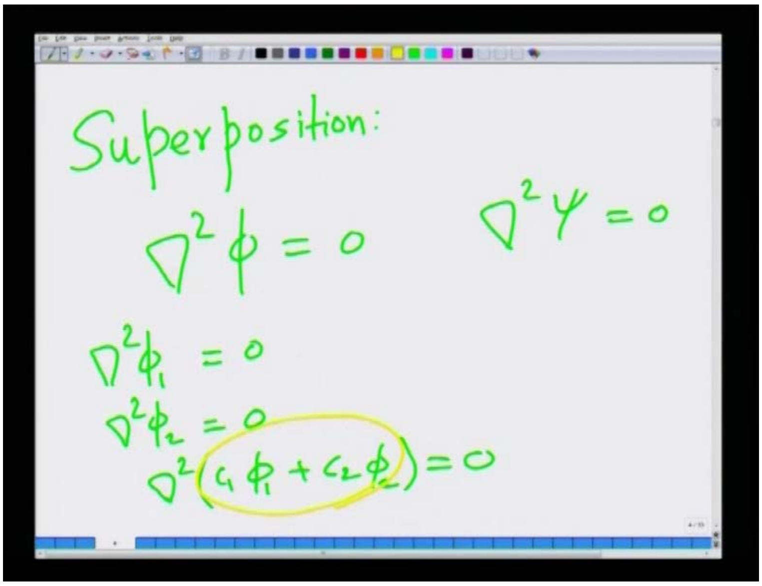

So, the Laplace equation, that governs the stream functions and potential are linear equations. So, if phi 1 and phi 2 are solutions of Laplace equations, then so is some constant times phi 1, some other constant times phi 2. So, if we have two simple solutions of the Laplace equation, you can linearly superpose them and the super position is also a solution to the Laplace equation. So, if we have two solutions to the Laplace equation, then the liner, any liner combination of these two solutions is also a solution to Laplace equation.

So, by generated, by combining all the simple solution that we have seen, we can generate newer solutions, which are also solution to Laplace equation, but we have to then figure out what to what physical contexts are the solutions of. So, that is the task that we have to complete, but all we we will definitely have a solution to the Laplace equation and then, we will find out to which problem it is a solution of it.

It is in fact, like the reverse of what we do normally, we have a problem and we find a solution, here it is almost the reverse where we find the solution by linearly combining various simple solutions and then, find out what is the problems for which, this is the salutation of, I am going to illustrate this with a couple of examples.

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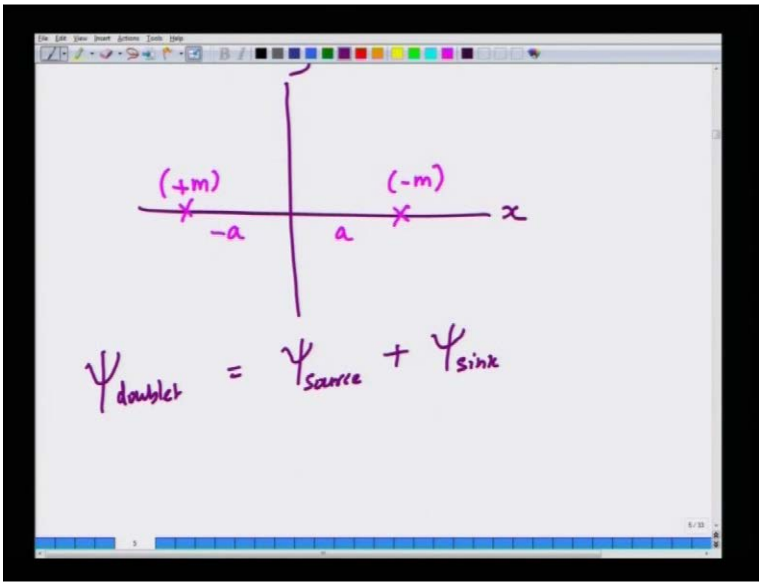

So, let me do this for what is called doublet or a die pole. So, essentially you have in the x y plane, you have a source at a distance of strength plus n, distance minus a from the origin in the x axis, and a sink of strength minus m. So, we want to know what is the solution for this problem, this is called a doublet or dipole, because you have a source and sink that are separated by distance 2 a. Now, by the argument I just told you, psi for doublet is essentially psi source plus psi sink, because if psi source is a solution of Laplace equation, size sink is a solution of Laplace equation, then a combination of that also solution is also Laplace equation.

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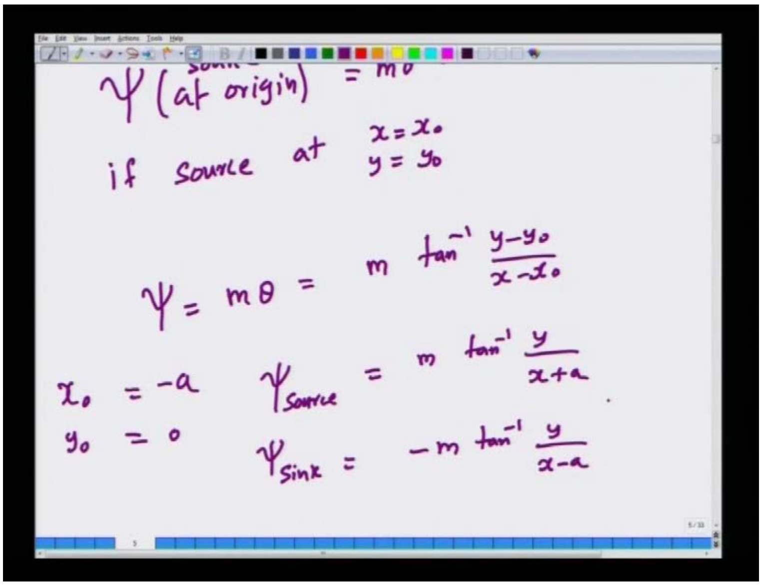

Now, we know that if psi is at the origin, sources is at a origin, psi is m theta, theta is tan inverse of y by x, in terms of the Cartesian variables y and x. If source, this is source at origin, if source is at any location x is, x equals x naught y equal y naught, then we can write psi is m theta is equal to m tan inverse of distance from the source becomes y minus y naught, and x minus x naught.

In this problem, x naught is x naught is minus a, the source is at a distance minus a, but why naught is 0, it s still at the origin, in the y direction, all along the x axis in the y direction. So, size source for our problem is essentially m tan inverse of y divided by x plus a, and psi sink is essentially minus m tan inverse of y divided by x minus a.

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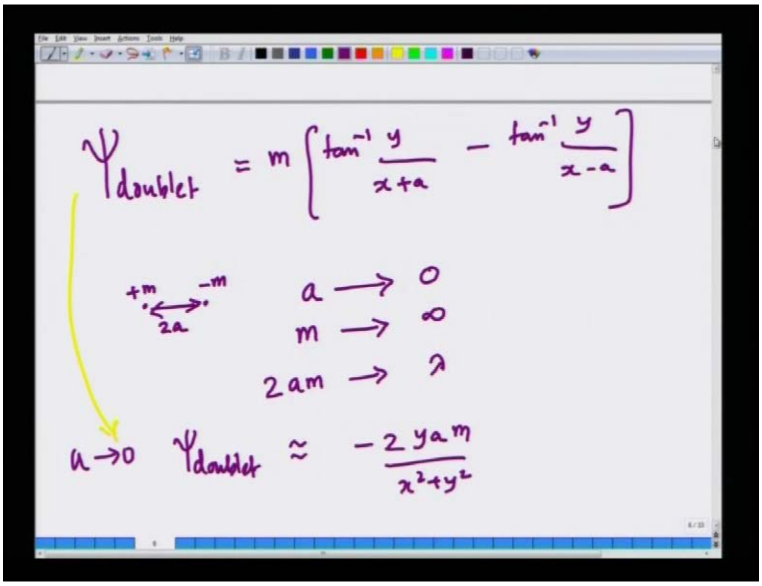

So, psi doublet which is essentially the super position of these two, is m times tan inverse of y divided by x plus a minus tan inverse of y divided by x minus a. So, linearly combining the two solutions for psi source and psi sink, we have easily obtained the solution for a doublet.

Now, we want to take the limit, so I will first rewrite this as, m times tan inverse of y by x times 1 minus a by x minus tan inverse of y by x times. So, if we multiply this by y by x, I will get, so let me just do this algebra. So, y by x times 1 minus a by x is y by x minus, so instead of doing this, I just delete this step, it is not required. We can take the limit; we want to look at the limit where the distance between the two sources, the source of string plus m and sink of sink minus m, we want to let the distance go to 0.

At the same time, but if the distance goes to 0, the source and sink will cancel each other, but in such a manner that, m tends to infinity, the sink strength tends to infinity such that, the product of this distance and the strength tends to two times that tends to lambda, this is the strength of the doublet. So, in that limit, we can simplify this as a tends to 0 psi doublet, this expression as a tends to 0, this becomes minus 2 in the limits as a tends to 0, becomes minus 2 y a divided by x square plus y square times m.

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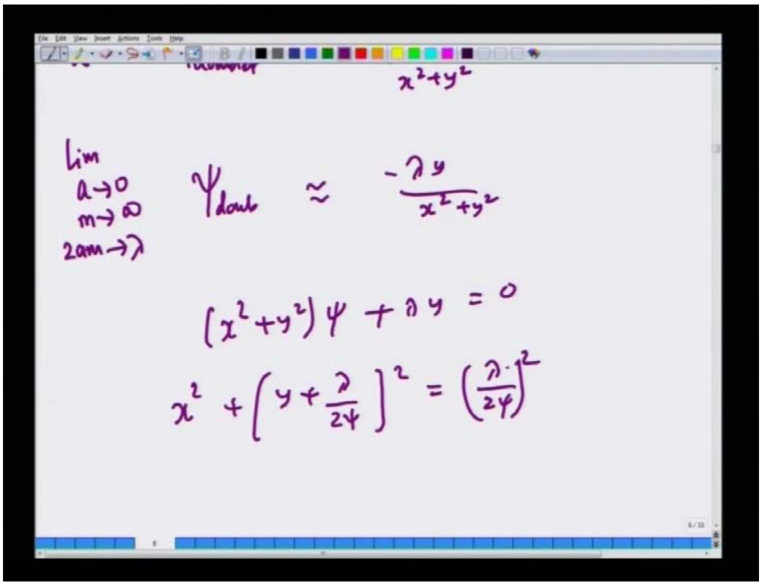

So, this becomes psi doublet as a tends to 0, m tends to infinity, 2 a m tends to lambda becomes minus lambda y divided by x square plus y square, or we can say that, we can rewrite this as x square plus y square psi plus lambda y 0, or x square plus y plus lambda by 2 psi whole square is lambda by 2 psi whole square. So, as becomes very very small and m becomes very very large, and such a manner that a times m is a constant, the doublet essentially becomes like this.

So, the stream functions are circles, you have to, so in order to plot a same function, you have to put psi equals to constant; or in order to plot the stream lines, you have to put psi equals constant and you have to simply draw the curve.

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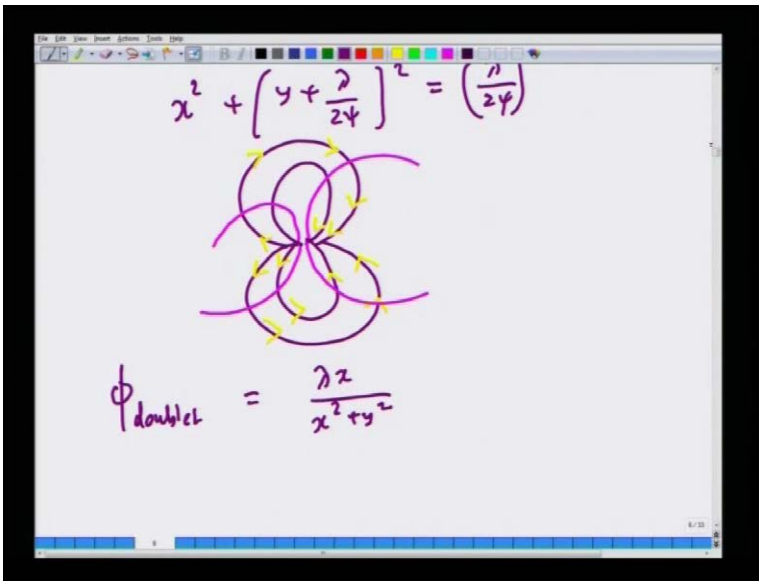

You will find that, these are essentially circles like this of, so essentially, the source will turn to push the fluid in this direction, and the sink will turn to attract the fluid in this direction. So, the direction of flow is like this, so source is trying to push the fluid like this, the sink is trying to attract the fluid and the equipotential will be perpendicular to this.

So, we have to draw circle, I mean circles that are perpendicular to at each and every point and so on. So, those are the equipotentials, and the potential are also easily derived for the doublet. So, by superposing the two solutions and getting the limit as a tends to 0, you will get this is lambda x by x square plus y square and these are also circles, but in such a manner that, they are orthogonal to the steam lines, the equipotential also circles.

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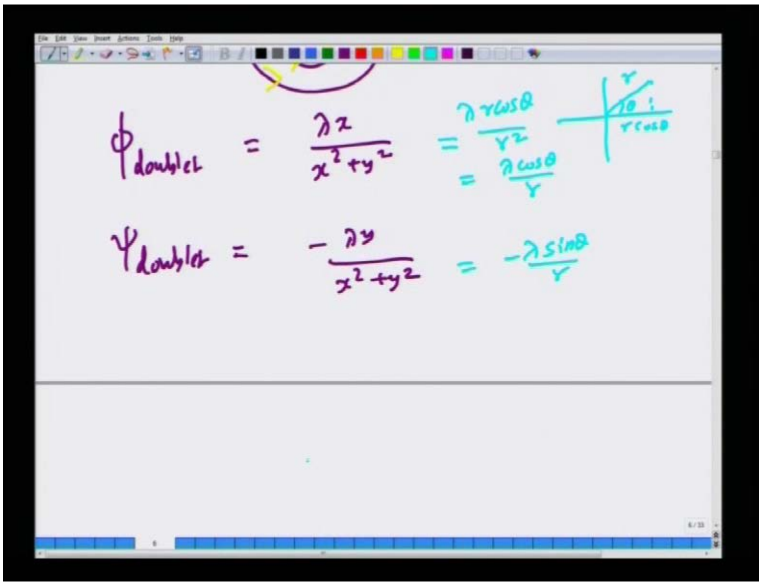

Now, so we have these two expressions psi doublet and phi doublet, psi doublet is minus lambda y divided by x square plus y square. Then from geometry, we can write this in cylindrical polar coordinates as, so this is x by x square plus y square is nothing but, so if you look at the polar coordinates, you have r theta. So, this is x is r cos theta, so lambda r cos theta divided by r square. So, this becomes lambda cos theta by r, this becomes minus lambda sin theta by r. So, these are the expressions for doublets, the steam functions and velocity potentials for a doublet.

So, the doublet is again solution, that is a mathematical abstraction, but it is nonetheless solution to potential flow, in which case wherein, you have a source of fluid and a sink of fluid in such a manner that of identical strengths, but source as positives values and sink has negative values, in such a manner that you bring the source and sink together, but the distance, as the distance goes to 0, the magnitude of the source are sink strength also goes infinity in such a manner that, a times m, that is a product of these two tends to a constant lambda, twice of that, two times, a times m tends to a constant lambda. And in that limit, you get what is called doublet or a dipole.

Now, all though the dipole by it self is merely a mathematical abstraction, we will soon see that the dipole plays an important role in finding, what is the flow pattern for, potential flow fastest cylinder.

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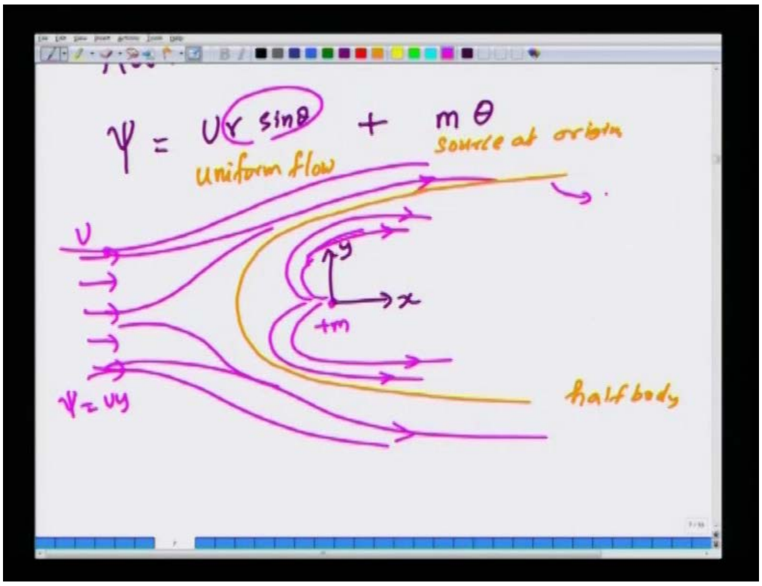

But before that, I want to discuss one important example that is, that of what is called Rankin half body. Suppose, you have a source at the origin, so in cylindrical polar source at the origin means the string functional is m theta and a uniform flow in the x direction.

So, let me draw the coordinate direction, you have x y, you have a source at the origin plus m and far away, you have uniform in the plus x direction of velocity u. So, the uniform flow has potential stream function u y, y is r sin theta. So, I am writing this as r sin theta. So, this is the uniform flow and I am adding it with source at the origin. Now, if I were to plot the steam lines, by putting in values for psi, we will find that you get far away, you get flow like this.

Now, close to the cylinder sorry close to the origin, you have the source which will try to push fluid in this direction, but they will be soon connected away by the uniform flow in the plus x direction. So, the steam lines will look like this, but there will be, so there will be another stream line like this. But there will be one stream line, which will we essentially like a close surface. Now, this stream line, so this is stream line that is completely like a surface, this is called a half body, because it is almost like a sort of an elliptically shaped body and there is fluid flow pass that solid surface.

Now, what is important to understand is that, whenever you have a stream line and you have flow fast, potential flow and there is a stream line, the stream line can be essentially treated as a solid body, because in, within the potential flow, within the realm of potential flow we can satisfy only the normal velocity condition and there is no flow across the stream line, there is no way we can satisfy the (()) velocity condition with in potential flow. So, if we have a close stream line like this that means, that fluid is flowing past such a closed event. So, you have flow like this, and you may have flow like this ok.

So, in fact, fluid is flowing past this shaped body, that is called the half body. So, what I am trying to say is that, by just merely super posing to simple flows, a uniform flow and a source at the origin, we are almost getting the potential flow solution to a flow past a solid body. Along the same lines, we are now going to understand, what is potential flow fastest cylinder? It is a very important problem.

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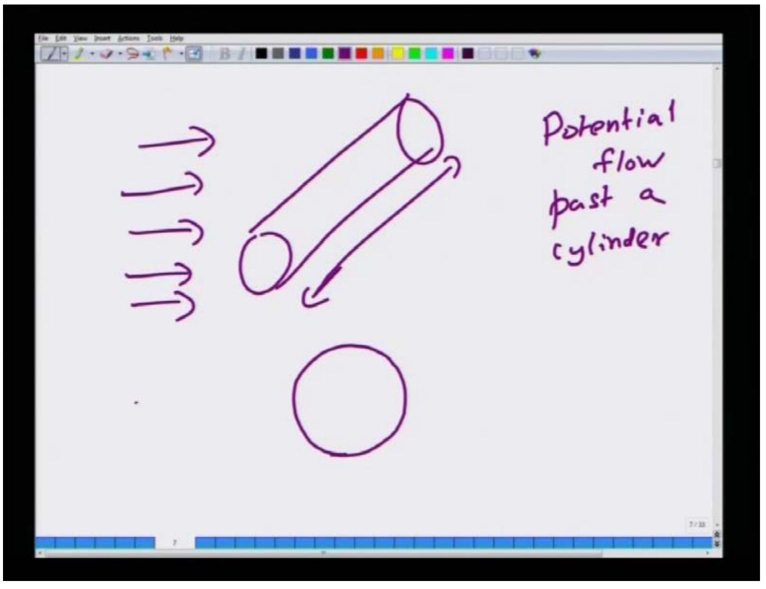

So, essentially you have a long circular cylinder and fluid is flowing like this, so potential flow past, how are we going to get this solution. So, essentially if this direction very long, we can just look on, look we can essentially simplify the problem to look like on the plane it is like a flow past a circle, how do you, how are you going to simulate this flow using the simple potential flow, using the simple potential flow solutions.

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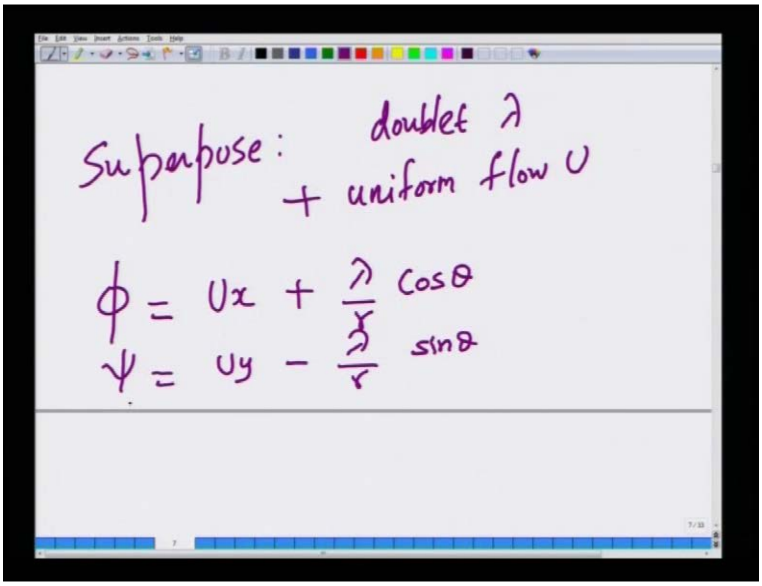

You superpose a doublet of strength lambda and uniform flow with velocity U. So, essentially use superpose these two, we know what is the uniform flow potential which is U x and we know what is the doublet potential or similarly, we know what is the uniform flow stream function, we know what is the doublet steam function, we just derived this. I am going to tell you.

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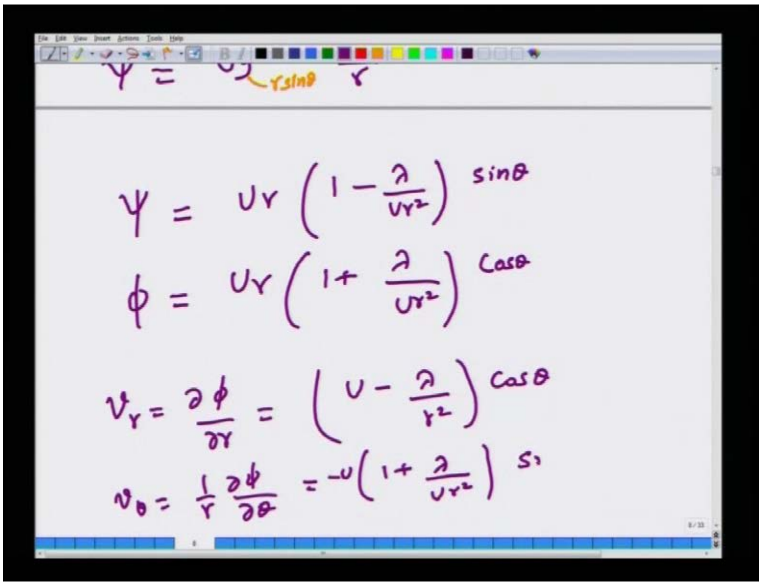

I am going to show you or demonstrative to you that these correspond to potential flow past a cylinder, how that I am going to do that, well instead of x, I am going to put r cos theta, polar coordinates; instead of y, I am going to put r sin theta. So, psi becomes U r times 1 minus lambda by U r square, if I take sin theta.

Now, and phi is U r times 1 plus lambda by U r square times cos theta. Now, we can ask the question, what are the velocity components? v r is partial phi by partial r is nothing but U minus lambda by r square times cosine theta, and v theta is 1 over r partial phi, partial theta which is nothing but minus U 1 plus lambda by U r square sin theta.

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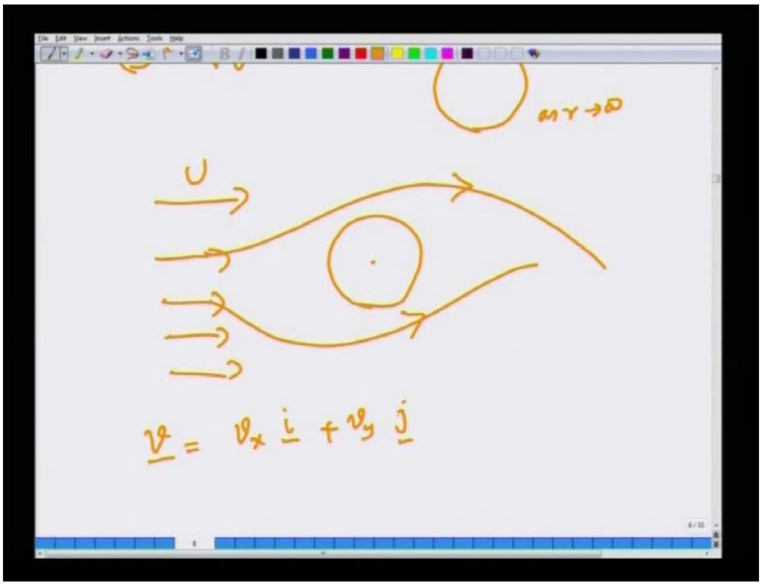

If we ask the question when is v r 0, v r 0 by looking at this expression when U equals lambda by r square, or theta becomes pi by 2, 3 pi by 2. So, in along those lines, v r is 0. So, either v r can be 0 when either this quantity is 0, when U is lambda by r square, or an cosine theta 0, cosine theta is 0 when theta equals pi by 2 or 3 pi by 2. So, when you look at this, this simplifies to when r equals the radial coordinate is root of lambda by U. So, when r equals, when the radial distance r equals a constant, that is which is essentially is shape of a circle in two dimensions, there is no normal velocity; v r is 0 along a circle.

That means, that there is a close stream line in the problem, in which v r is 0, that is that is the definition of a stream line, because you are essentially putting a constant value. So, you can also look at the psi value, when lambda when r is equal to square root of lambda by U, you mean you will seem that, you will see that the stream function value is 0. So, that is a line of constant stream function, which is essentially steam line, it is a circle. So, it is almost like flow past a circle, which is essentially 2 D flow past a cylinder.

So, by superposing uniform flow with the doublet at the origin, we have stimulated potential flow passed a circle for a cylinder. Now, far away as r tends to infinity, if you look at this expression, as r tends to infinity, v r becomes U cosine theta, that is the uniform velocity, radial component of the uniform velocity, but the velocity is v x times i plus v y times j, there are two components.

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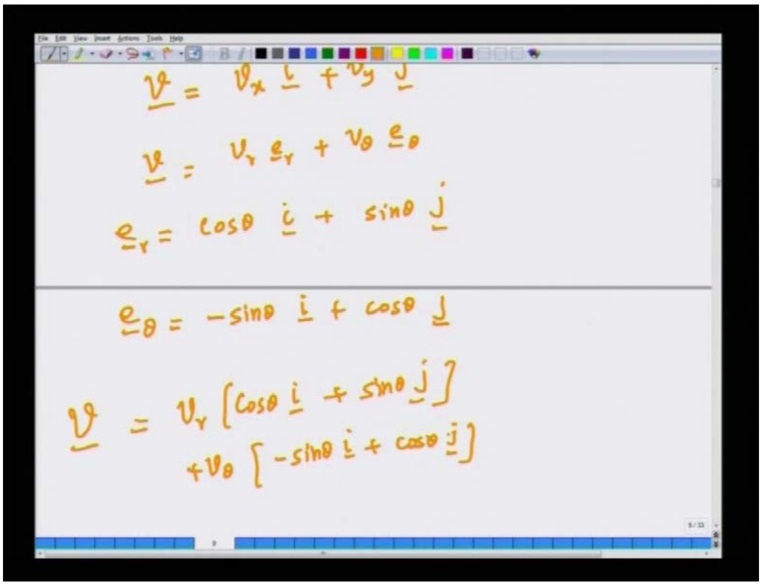

Or we can write it as, v r e r plus v theta e theta, but e r can be expressed in terms of e x and e y or i and j, using geometry cos theta times i plus sin theta times j and e theta is nothing but minus sin theta times i plus cos theta times j. So, we can write the velocity, we can plug this values of e r in terms of i and j out here and then, write the velocity in cartesian coordinates, which essentially becomes U.

So, v becomes v r times cos theta i minus sorry plus sin theta j plus v theta times minus sin theta i plus cos theta j. As R goes to infinity, we will find that the velocity vector is essentially, if you simplify this, you will find that this is U cos square theta i plus U cos theta sin theta j plus u sin square theta i minus U sin theta cos theta j as r goes to infinity. So, these two terms will cancel to give with the velocity to be just U times i it is the uniform flow. So, essentially we have solved for what is the velocity field for flow around a cylinder in the potential flow regime, by simply superposing the uniform flow far away with the doublet at the origin and that gave rise to essentially a close stream line of radius square root of lambda by U, where lambda is dipole strength and U is a strength of the uniform flow and this gave rise to flow past a cylinder.

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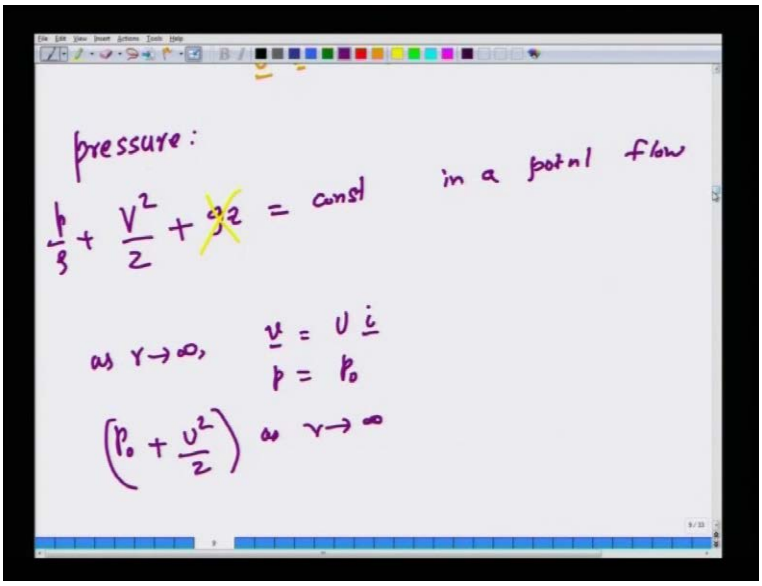

We will now compute what is the pressure force on a cylinder. So, having solved for the velocity field in the potential flow regime, for flow past a cylinder, a long cylinder, we are now ready to compute the pressure force. So, the pressure if you remember, should be calculated from the Bernoulli equation.

So, v square by 2 plus p by rho plus g z is a constant in a potential flow, that is the statement of Bernoulli equation for inviscid, incompressible, irrotational flows and if you assumed that the gravitational effects are unimportant, because everything as I mean, the the distances are very very small. So, there is no effect of gravity, so essentially, we will have p by rho plus v square plus by 2 is constant.

So, as r goes to infinity, v the velocity vector is U times i, we just derived that and the pressure is some constant pressure p naught. So, the constant that we want, far away is p 0 plus U square by 2. So, this is that constant, because you can fix that constant by looking at what the pressure and the velocities are at p by rho is U square by 2.

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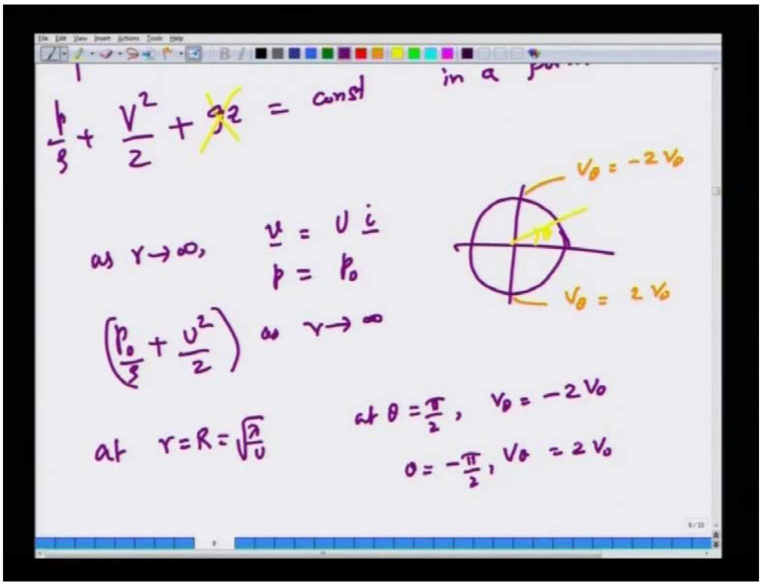

Now, we want to calculate what is the force on the surface of the cylinder due to pressure forces, at the surface of the cylinder, square root of lambda by U at theta equals, suppose you put a coordinate system, this is theta. Now, at theta equals pi by 2, v theta is minus 2 v naught, at theta is minus pi by 2, that is, if you go like this. So, here v theta is minus to v naught and out here, v theta is 2 v naught.

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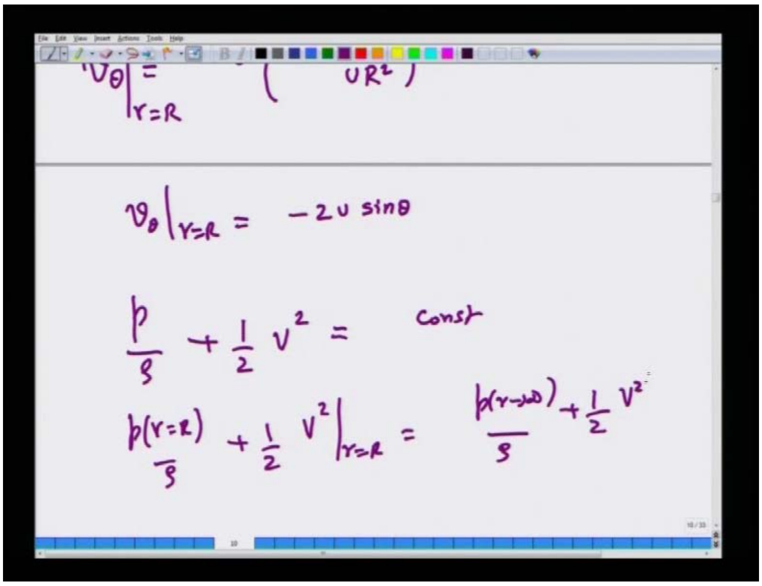

So, at r equals R, v r 0 by definition, because there is no normal velocity in to the surface of the cylinder that is how we identified that it is, there is no normal velocity at the steam line when R equals, r capital R equals lambda by U. So, the only velocity you have to worry is v theta, which upon putting r is the radius, you get minus U times 1 plus lambda by U capital R square sin theta. Now, r square is nothing but lambda by U by definition, that how we define the radius of this sphere. So, v theta evaluated at the radius of the cylinder I am sorry is minus 2 u sin theta. So, this is the point I was trying to, I mean make to you there is no v naught, it is just U, this is the free stream velocity far away.

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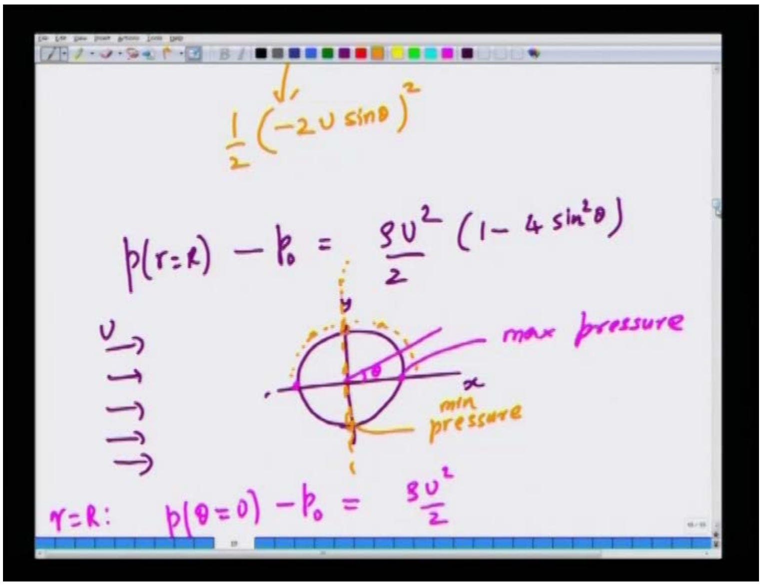

Now, we know that p by rho plus half v square is constant. So, p at r equals r, by rho plus half v square evaluated at r equals R is p at r tends infinity by rho plus half v square r tends to infinity. Now, this is nothing but p naught by rho plus half U square and p at r equals R, which is what we want plus half v square r equals R.

Now, this is nothing but half times minus 2 u sin theta, there is no r component, there is only theta component which we just evaluated at r equals the radius capital R. So, eliminating, therefore the pressure, p at r equals r minus p naught is rho U square by 2 times 1 minus 4 sin square theta, so this is the pressure. Now, we look at this from the context of the coordinate system, you have x y flow is far away in the x direction, and now, if we look at various points, look at this expression, this expression when theta is equal to 0, sin theta is 0 (Refer Slide Time: 41:59). So, let us assume this to be theta; when theta is 0, sin theta is 0.

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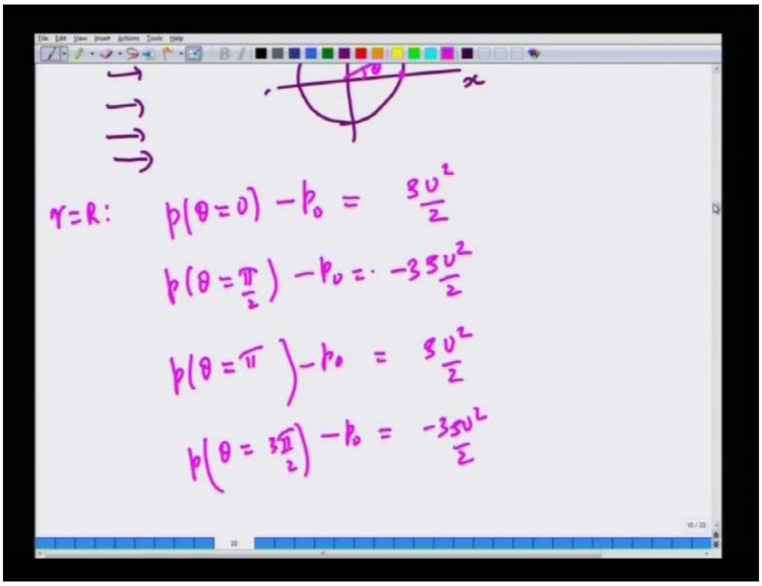

So, the pressure takes the value at, so pressure at r equals R is only a function theta, pressure at theta equal to 0 at r equals R. So, we are fixing r equals R, pressure at theta equal to 0 minus p naught, it is the constant pressure is rho U square by 2. Now, pressure at theta is pi by 2, when theta is pi by 2, sin theta is 1. So, this becomes 1 minus 4 which becomes negative, compared to the free stream pressure.

So, this becomes minus p naught is rho U square by 2 times 1 minus 4 which is minus 3. Now, and pressure at theta is, if you go around 3 pi by 2 minus p naught sin theta is again 0 at value of sorry not 3 pi by 2, pressure at the value pi sin pi is again 0, we will again get rho v square by 2 and pressure at the value of 3 pi by 2 minus p naught, we will again get minus 3 rho u square by 2.

So, the pressure is maximum at these two points and pressure is a minimum at these two points (Refer Slide Time: 44:05) and the variation of pressure over the surface is symmetric, there is complete symmetry about this, there is complete 4 half symmetry about the half of the cylinder is a complete symmetry of the pressure variation, because pressure is an even function of theta, because you have 1 minus 4 sin square theta.

So, it is an even function of the theta. So, there is a complete symmetry of pressure. So, the pressure value here will be the same as pressure value here, because of sin square theta. So, it will be the same if you are whether, if we are here or here. So, now once you find what is, so pressure is completely symmetric in the about this, this axis it is called 4 half symmetry.

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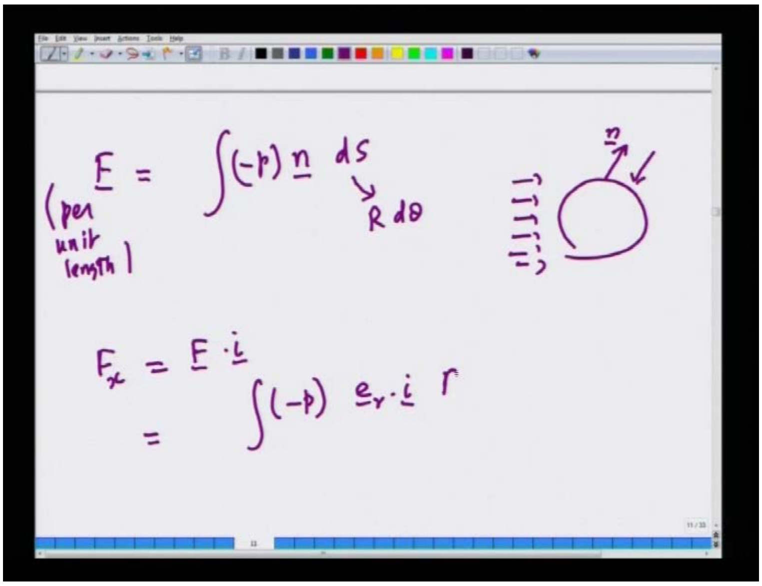

Once you have found what is the pressure, we can find what is the force, the force on the object is only due to the pressure force, pressure exerted by the fluid on the surface acts inward, because if the outward unit normal is n, inward normal is pressure acts in the direction of minus n. So, d s, d s is nothing but R d theta, this is the force per unit length of the cylinder, because is it is a long cylinder, so this is force per unit length.

Now, by symmetry since the flow is in the x direction, we can imagine that the only force should be in the x direction, because there is nothing in the y direction to suggest that, there will be a net force in the y direction. So, by symmetry we can say that, the only force that we have to calculate it is in the x direction is minus p. Now, n is nothing but e r dot i and then d s is R d theta, where r is the radius of the circle.

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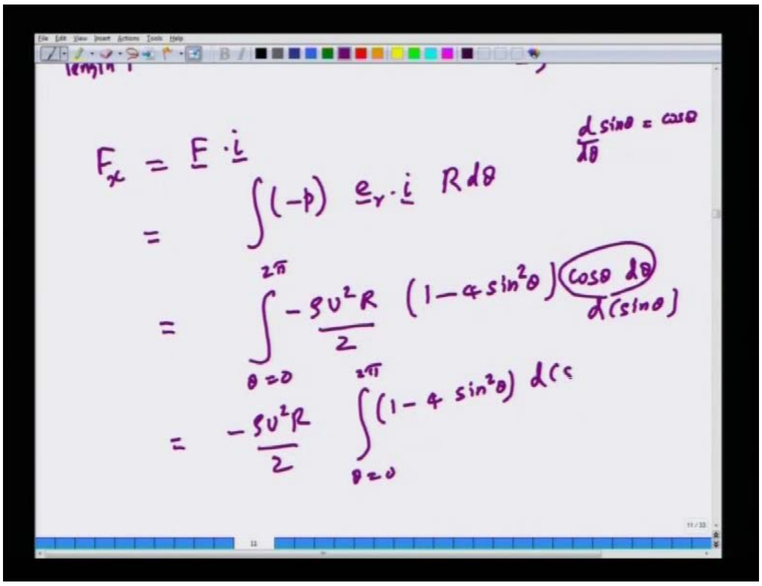

And the pressure is, simply minus rho U square R by 2 times 1 minus 4 sin square theta times cosine theta d theta. So, the x component of the force is nothing but minus rho U square and sorry. So, before I do that, I will look at this expression, so theta is going from 0 to 2 pi. So, this is equal to minus rho U square R by 2, theta is 0 2 to pi 1 minus 4 sin square theta, d d theta of sin theta is cos theta. So, instead of cos theta d theta, I will write d of sin theta, so it is d of sin theta. So, you have a very simple integral, wherein you have an even function on sin theta, an even function of argument and theta goes from 0 to 2 pi.

So, if cos theta is written as x d x by d theta is essentially sin and theta d theta. So, the x by d theta is sin theta or d theta is d x by sin theta. So, you can evaluate this as a integral by using this change of variable and we can show that f x is identically 0 and similarly, we can also compute if you do not invoke symmetry, we can also compute the force in the y direction. It becomes rho U square R by 2 1 minus 4 sin square theta time sin theta d theta and that also become 0, so F y it is also become 0.

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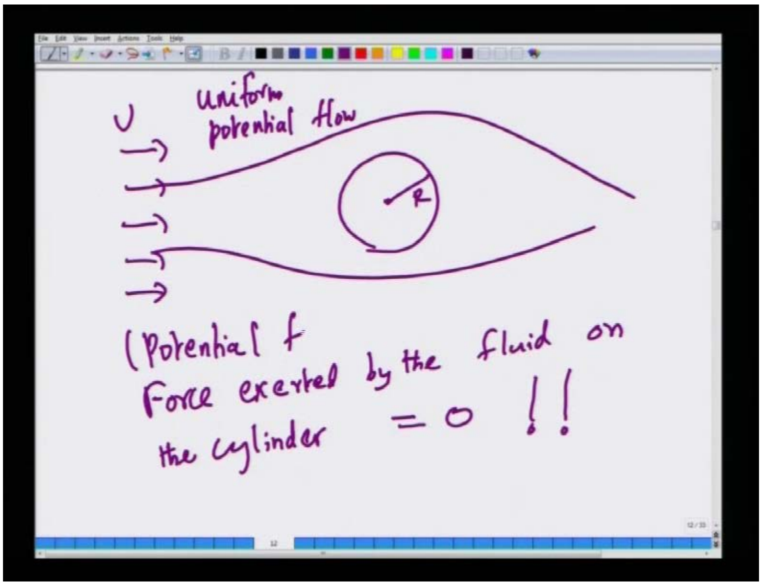

So, if we have flow past uniform flow, past a circle as cylinder of some radius r, what we are finding, you have uniform potential flow (No audio From 49:01 to 49:07). In a potential flow, there are now viscous effects, because the viscous effects are set to 0, when we went from the Navier Stokes to Euler equation, and the only forces are due to fluid pressure.

So, the stress (()) has only the pressure contribution, there is no viscous stress contribution. So, the force exerted by the fluid on the cylinder is identically equal to 0. So, this comes as a surprise in the potential flow limit, for potential flow. This comes as a surprise, because excrements tell us that, even at very high Reynolds numbers, whenever you have potential flow past any object like a long cylinder or a sphere, you always find a finite force that is resist. I mean, if you move sphere or a cylinder at very high Reynolds numbers, then you do find that, there is force experienced by these solid objects.

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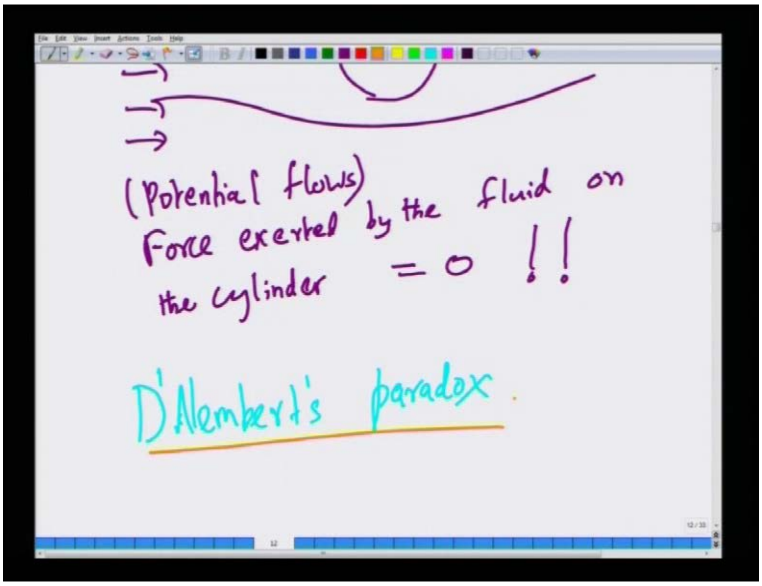

But when you try to treat that within the potential approximation, you find that there are no forces so that this appears like a paradoxical situation and this is called the D' Alembert's paradox that, the potential flow solutions give rise to zero force on solid objects such as cylinder, a long cylinder when you find the pressure profile and integrate the pressure, you find that the pressure is identically, the force due to pressure is identically 0.

So, this leads to a contradiction or a paradox, because experimentally these objects do experience a finite drag force, when they move in a fluid. So, there is a force, resistance force exerted by the fluid on the solid objects, but they come out to be 0, identically in the potential flow approximation.

So, this forces has to rethink our approximation itself, because as I mentioned in the beginning, when we went from the Navier Stokes equations, we neglected the viscous terms completely assuming that, they are multiplied by a small number 1 over Reynolds number, that assumption meant that gradients of velocities are uniformly small everywhere in the flow, and this also led to the sacrifice of the no slip condition, because we were not able to satisfy all the boundary conditions, on of the problem.

So, this already signals where we could have gone wrong, namely that when you neglect the viscous terms, probably the neglect is ok, when you are for away from a solid surface. But close to the solid surface, we cannot completely the neglect the viscous terms, because it is where the velocity will change from whatever it is there, little away from the solid surface to 0, because the no slip condition is always satisfied by fluid regardless of the Reynolds number.

It is a physical condition that is independent of the Reynolds number. No matter how low or how high viscosity of fluid is, the no slip condition is always satisfied. Therefore, one has to revisit our assumption of scarifying the no slip condition in the potential flow approximation. Now, that is what is done in what is called the boundary layer theory, wherein we now are going to say that, the potential approximation is not a bad approximation, when you are far away from the solid surface, but close to the solid surface, we have to re-invoke the viscous effects, even at high Reynolds numbers.

And when you invoke the viscous effects at high Reynolds numbers, by suitably approximating the viscous systems, then we are bringing the viscous terms back in to the governing equations and then, we have the ability to satisfy both the normal velocity and tangential velocity conditions and that region close to the solid surface where viscous effects dominate, is called the boundary layer.

Now, by including the viscous forces, then we now have hope of obtaining a finite drag, because now the net force on a solid surface is not because of just the pressure forces, but close to a solid surface where the boundary layer is viscous effects also become important. And therefore, we can now have hope of getting a non-zero force. Now, that is the subject of boundary layer theory, which I will start in the next lecture.