**Fluid Mechanics**

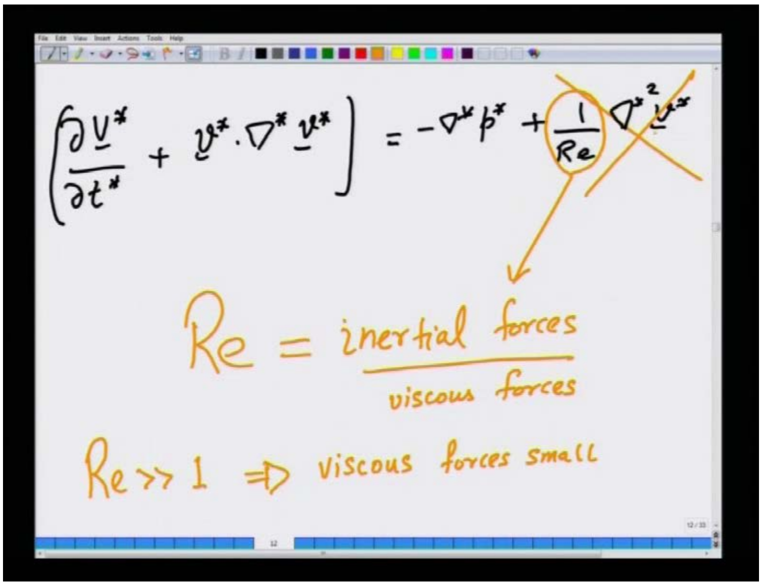
**Prof. Vishwanathan Shankar**
**Departmentof Chemical Engineering**
**Indian Institute of Technology, Kanpur**

**Lecture No. # 32**
**Module No. # 01**

Welcome to this lecture number 32 on this NPTEL course on Fluid Mechanics for under graduate chemical engineering students. The topic that we are currently discussing is fluid flow at very high Reynolds numbers. As I mentioned in the last lecture, the solution of the referential balances, namely the Navier Stokes equations are extremely difficult, when you consider the Navier Stokes equations in their entire form.

So, often it is necessary to make simplifications of the Navier Stokes equations in appropriate flow regimes. Many practical applications in chemical engineering and in other other engineering contest happen at very high Reynolds numbers, the fluid flow at such applications in such in such applications happen at very high Reynolds numbers. So, it often appears, it is often beneficial to look at what is going to happen to fluid flows at very high Reynolds numbers.

(Refer Slide Time: 01:14)

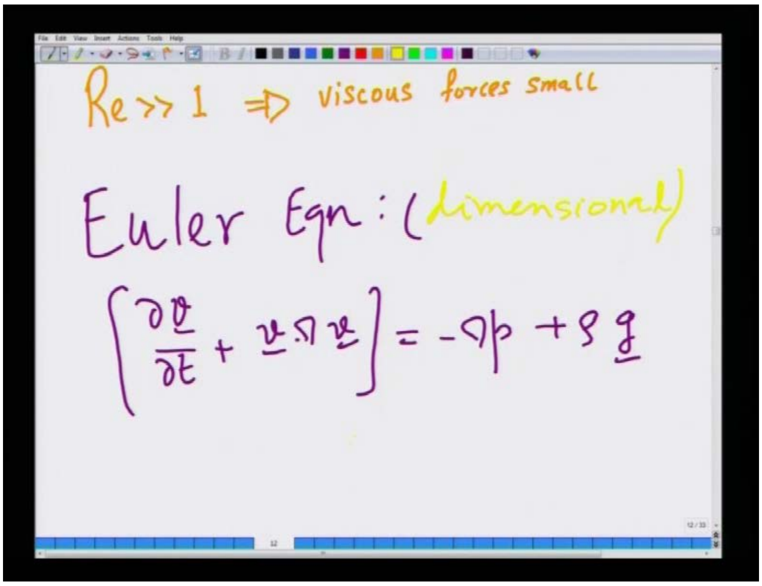


In order to do that, we first looked at the non-dimensional Navier Stokes equations, which look like this, plus v star dot del star v star, all the star quantities are non-dimensional, this minus del star v star plus 1 over a Reynolds number times del star square v star plus the non-dimensional gravity term, which is which is proportional. Let us ignore gravity for the moment and so, this is the non dimensional form of Navier Stokes equation and we know that the Reynolds number is a ratio of inertial to viscous forces; it is a measure of inertial forces in the system to viscous forces in the system.

And when the Reynolds is very large compare to 1, this implies that viscous forces are small, compared to other forces in the system. So, as a first approximation, we can think of neglecting the term multiplied by 1 over a Reynolds number, because 1 over Reynolds number is very small. And assuming that, this quantity that del star square v star is a reasonably well behaved quantity in the sense that, it is a non-dimensional quantity, so it should not be very large.

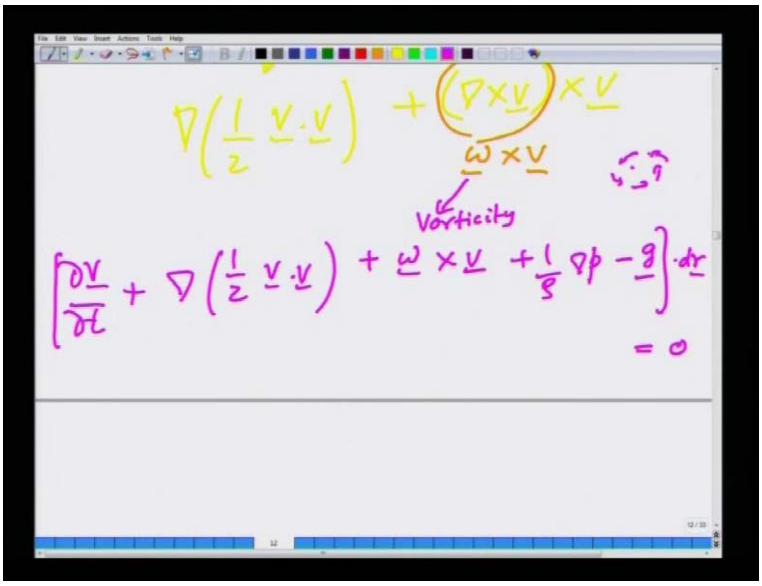
Then, we can neglect this term, there are issues with this neglect and this forms a very important part of discussion in fluid mechanics and I will come to that a little later, after 1 or 2 lectures. But right now, this is the most sensible thing that appears, this is the most reasonable things that appear plausible at this time.

(Refer Slide Time: 03:09)




So, we will go ahead with this assumption and the resultant equation is called the Euler equation, where you have thrown away the viscous term. So, we will have simply, this is the dimensional form of Euler equation and if there is gravity, you can put rho times g, this is the dimensional form form of Euler equation. Now, once you had the Euler equation, in the last lecture, we proceeded further.

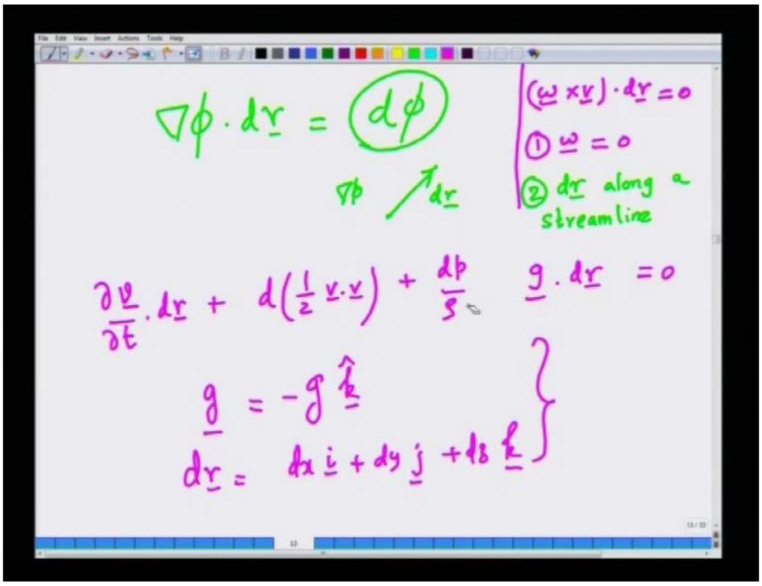
(Refer Slide Time: 03:50)




And then what we did was to rewrite this term, v dot del v term as del of half v dot v plus del cross v cross v, and del cross v was omega. So, this became omega cross v. So, once you substitute this entire thing, back in the Euler equation and if you dot the entire equation, with with an elemental displacement d r, we obtained (No Audio from 04:27 to 04:35) plus 1 over rho del p minus g dot d r is 0. Notice that, omega is called the vorticity in fluid mechanics; it is a measure of fluid rotation about a given point in the flow.

If there is vorticity, that means that about a point in the flow, the neighboring points under go a circular motion about a given point. So, there is, it is, it does not correspond to bulk rigid body like motion of the entire rotation of the entire fluid volume, but about a given point there is a differential, about a given point if you consider differential displacement vector, the neighboring points are moving in a circular motion about this point. So, that is the meaning of vorticity, non zero vorticity. If the vorticity is 0, at each and every point in the fluid, then such flows are called irrotational flows, we will come to that a little later.

(Refer Slide Time: 05:40)




Right now, from basics partial derivatives, if you have del phi dot d r, that will give you the change in the value of the function phi between the two points. Suppose, you consider two points which are connected by an elemental vector d r, if you take the gradient at this point and dotted with d r, this will give you for small d r, the change, the amount of change the function is going to incurve when you merge a tiny distance d r.

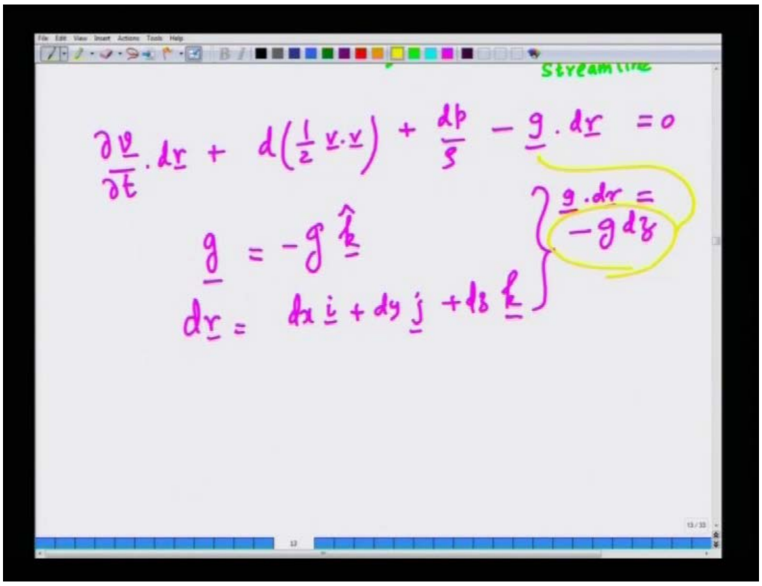
So, if I dot this with d r, I will get, the first term will remain as such, the second term will become d of half v dot v, the third term will become d p by rho, if you look at there are four terms 1, 2, 3, there are five terms. So, for the moment, we going to restrict ourselves to cases where omega dot sorry omega cross v dot d r is identically 0, only then the equations can be simplified.

I pointed out in the last lecture that, this can happen if either omega itself is 0, that is the fluid rotational or flows along the stream line; or if you consider d r, or d r along the stream line, that is if you are marching a long stream line, then that is true.

So, under these circumstances, either when the flow is rotational or when you are considering along the stream line, this term is identically not there, this term is not there. So, we are left with just d p by rho plus, you had g dot d r is 0. Now, let us assume arbitrarily, write g as minus g times k, this is pointing in the minus z direction and d r is nothing but d x times i plus d y times j plus d z times k.

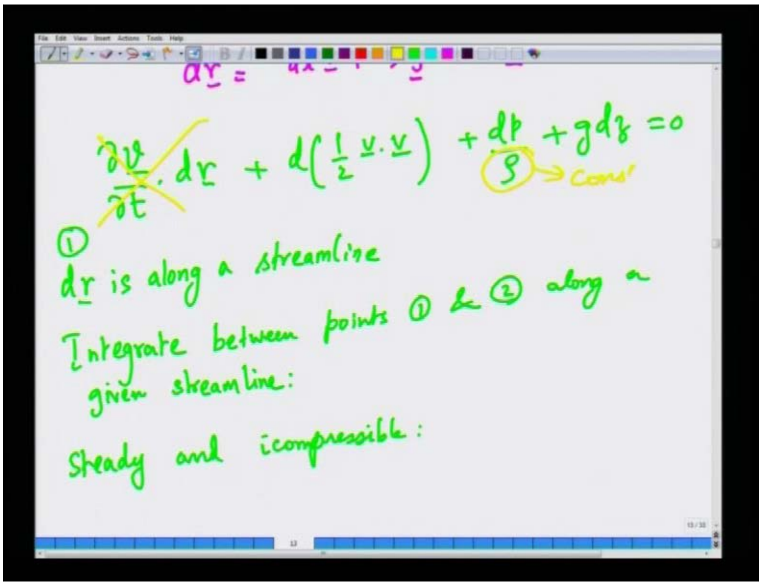
So, if I dot this two, I will get, g dot d r as, I am sorry there is some minus sign here, if I remember correctly, there is a minus g here.

(Refer Slide Time: 05:40)



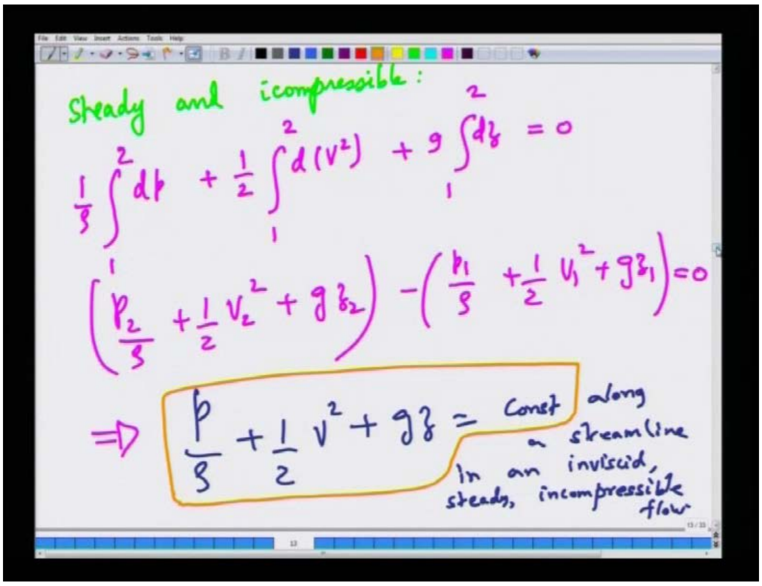

So, the minus should happen here. So, g dot d r will just become minus g d z. So, this is g dot d r is equal to minus g d z.

(Refer Slide Time: 08:15)




if I substitute this, back in here, the two negatives will get cancel to give you d v d t dot d r plus d of half v dot v plus d p by rho plus g d z is 0. Now, if I integrate, if I consider that d r is along the stream line, there are only two cases, we can consider, that is d r is along the stream line, or the vorticity omega is identically 0. So, d r is along the stream line, and I integrate the above equation between points 1 and 2 along the same stream line, points 1 and 2 along the same stream line, along a given stream line. Now, then further make the assumption that, the flow is steady and incompressible (No Audio from 09:26 to 09:34). If I make these two assumptions, then if the flow is steady, then this term goes away; if the flow is incompressible, rho is a constant.

(Refer Slide Time: 09:53)



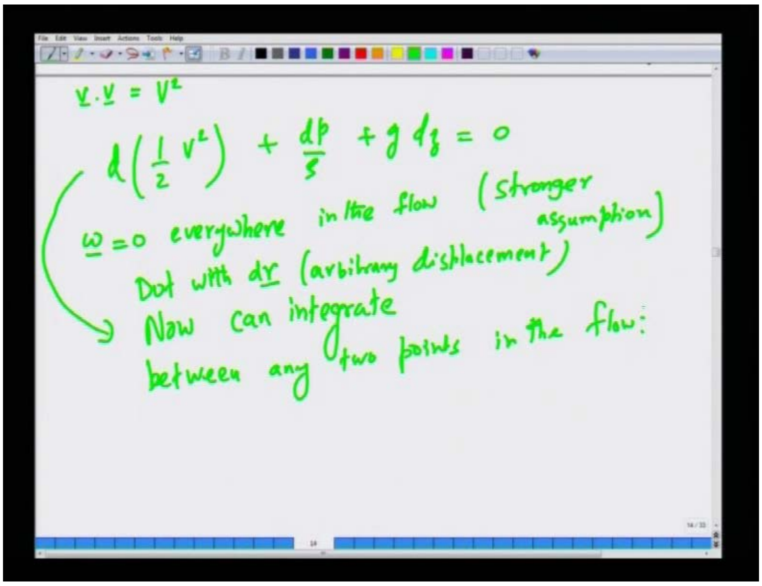

Then if I integrate along any two points, I will get integral form 0.1 to 0. 2, d p by 1 over rho d p, plus from 0.1 to 0.2 d of v dot v is v square and there is half and then, plus g d z is 0, this implies that p 2 by rho plus half v 2 square plus g z 2 minus p 1 by rho plus half v 1 square plus g z 1 is 0.

But since, points 1 and 2 are any two points, along the stream line, this implies essentially that this implies essentially that p by rho plus half v square plus g z is constant along the stream line, stream line in an inviscid flow, steady inviscid, steady incompressible flow, these are the assumptions we made to derive this. So, it is good to keep them in mind, because it is always important to know when such equations are applicable. So, what we have shown essentially is that, this is the Bernoulli equation.

So, strictly speaking the Bernoulli equation is valid for an inviscid flow, steady the, if the Bernoulli equation is written in this form that p, that is p by rho plus half v square plus g z being a constant along the stream line, it is valid for a steady incompressible inviscid flow. Now, we also found, we also had this other assumption. So, the first assumption, well we consider this first, d r along the stream line, you also had this as another possibility, that omega is identically 0.

So, let us go to this equation and integrate this now, considering the other assumption that omega is identically 0. So, will go back so will go back, to this equation and consider the assumption that, instead of considering along the stream line, we will consider omega is identically 0, so we are going to take this equation.

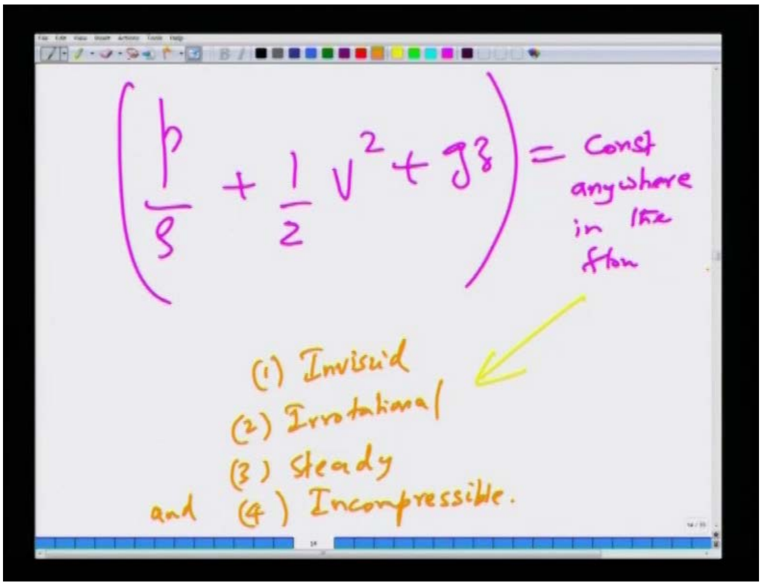
(Refer Slide Time: 12:28)




So, we again have d of half v square, v dot v is v square, it is the square of the magnitude of the velocity vector plus d p by rho plus g d z is 0, considering that vorticity is 0 everywhere, in the flow. Even even with this assumption, this equation is valid, this equation is valid for two different types of assumptions; one is that, the flow is the differential vector d r is considered along a stream line, then the term omega cross v dot d r is 0 or if you identically consider, this is the much stronger assumptions, this is the much more stronger assumption, because you are assuming that the flow is irrotational everywhere, but it also leads to lots of simplifications, this although it is a very strong assumption, it leads to a lot of simplifications.

So, if you assume that you will again get the same equation, but now you can integrate between any two points in the flow, because you can dot it with d r, now dot with d r and there is no necessity that d r is along a stream line, and d r is an arbitrary vector, displacement can integrate between any two points in the flow.

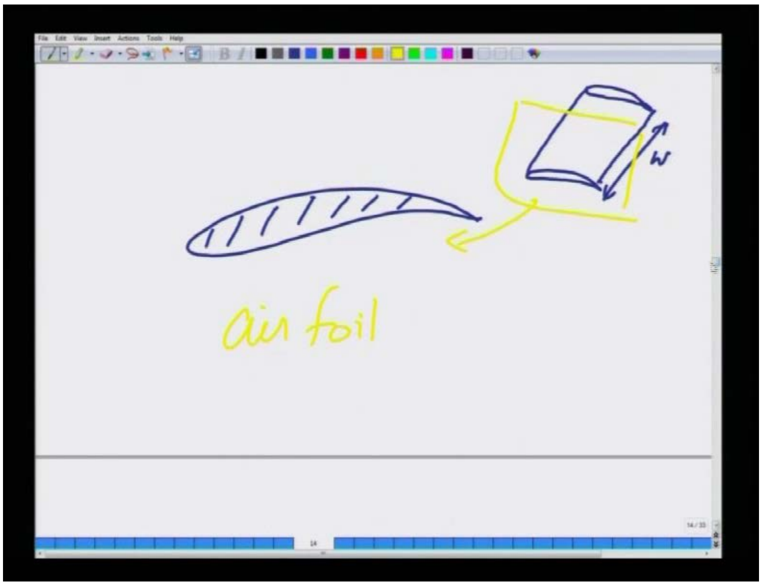
(Refer Slide Time: 14:13)




Then, again you will get the same equation, p by rho plus half v square plus g z is constant anywhere in a flow, in the flow, provided the flow is inviscid, irrotational, steady and incompressible. So, these are four, this is the same equation, the same type of equation results, but the type of assumptions that we made to get this equation that p by rho plus half v square plus g z is constant, being constant anywhere in the flow, there are completely different types of suspensions, basically you need not consider d r along the stream line, it can be any two points in the flow, but we are making a much stronger assumption that the flow is irrotational.

So, now we have to really worry about the following thing, that when can a flow we consider irrotational, because we are going to tell shortly, we going to see very shortly, that when the flow is irrotational, it leads to lots of simplifications in finding the velocity fluids, at high Reynolds numbers. So, when we can ask the question, when the flow is likely to be irrotational?

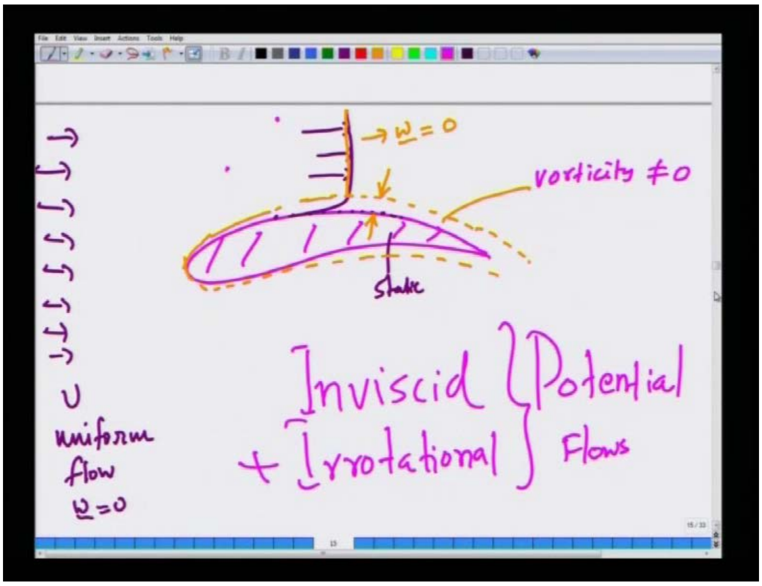
(Refer Slide Time: 16:04)




The answer turns out to be that, suppose you consider body shape like this, this is called an airfoil, this is a solid body, essentially it is a body that looks like this. And I am, since it is a long, this is very very long, width of the body is very very large compare to other dimensions, other dimensions are thin.

So, I am going to just draw cross section of this, I am going to take a cut across a plane and it is going to look like this, this is called an airfoil, essentially this is like cross section of an air plane wing, it is a model of a cross section of a air plane wing.

(Refer Slide Time: 16:45)



So, if we consider flow pass airfoil airfoil, faraway the flow is uniform, you have uniform flow. If you have an uniform flow, one can immediately find out that the flow is irrotational also, uniform flow has no vorticity. It is a kinematic feature of an uniform flow, if you have a uniform flow, regardless of whether you consider the fluid to be viscous or the ideal fluid, inviscid fluid, if you merely compute the vorticity, since there are no gradients in the flow, vorticity as to be identically 0, so vorticity is 0.

So, there is no vorticity when the fluid is flowing far away from the airfoil, the moment it reaches close to the airfoil, what is going to happen is that, in reality on the surface of the airfoil, there is a no slip condition. Let us assume the airfoil is stationary, the fluid is moving. So, on the surface of the airfoil, at each and every point on the surface of the airfoil, there is a no slip condition, which forces the fluid to retard to 0 velocity, that is it forces the fluid to reach 0 velocity. Now, little away from the surface of the airfoil, the fluid as to move, because you are considering a steady flow, fluid has to move pass the airfoil.

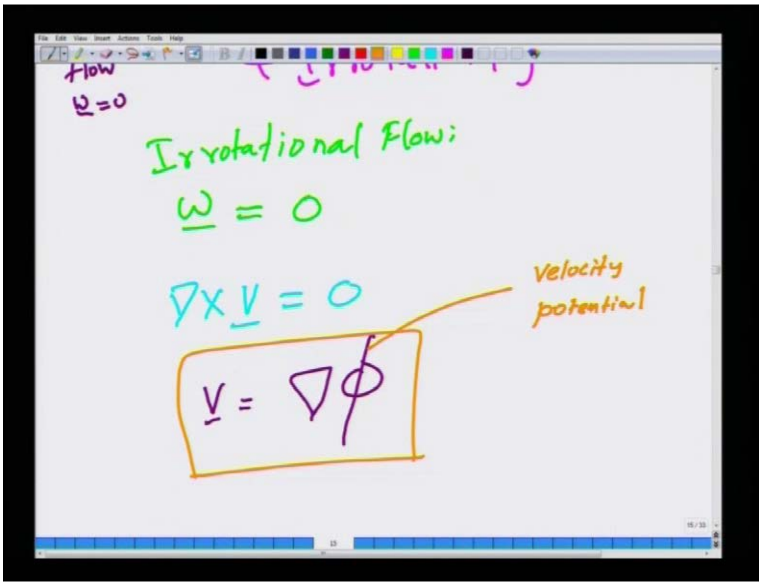
So, there will be a region where velocity goes from this uniform value to 0, on the surface of the airfoil. Now, that region it turns out to be, turns out that the region very thin, at least for objects like an airfoil, so again the flow is uniform here, roughly uniform here. So, we imagine that the vorticity in the flow is 0 here, but here with in this zone, close to the surface of the airfoil, there are velocity gradients and it is not immediately clear whether the vorticity is 0, vorticity is the curl of the velocity vector and curl as gradients in it.

So, whenever you have gradients of velocity, it is likely that there will be non-zero vorticity. So, the assumption that the flow is irrotational is well valid, when you consider regions away from the solid surface, but on regions close to solid surface, the vorticity is not 0. So, the irrotational flow assumption will breakdown in regions close to solid surface. In some sense, the solid surface, we can think of or even imagine the solid surface to be a source of vorticity, because by forcing the fluid to reach 0 velocity on its surface, it is creating or setting up velocity gradients and high regions of shear and such such that is the, those are the regions where the vorticity can be large.

So, we can imagine that vorticity is generated in the solid surface and it is swept away by the flow, faraway. So, there has to be zone where vorticity is confined and that zone is actually called the boundary layer, I will come to that a little later. So, essentially when you are away from the solid surface, it makes sense to assume the flow to be irrotational. And generally the flow, Reynolds numbers are high such that, you can ignore the viscous term. So, flow is anyway inviscid approximately speaking, far away from the solid surface and in addition, it is also a irrotational.

So, the thethe moment you a make the inviscid assumption, you will get the Euler equation, the moment you make the irrotational assumption, that is the vorticity is 0, then you get the result, the Bernoulli equation that p by rho plus half v square plus g z is constant across any two points in the flow, you need not follow stream line. Such inviscid and irrotational flows are called potential flows are called potential flows.

(Refer Slide Time: 21:14)




Now, the reason why they are called potential flows is not part to see, because when you assume the vorticity is 0, this is an irrotational flow, this is the kinematic constraint that is, we are simply assuming that there is no rotation, about a point in a fluid and at each and every point in the fluid, this is the purely kinematic condition, this has nothing to do with whether viscous forces are dominant or inertial forces are dominant.

But it so happens, that this assumption is more and more applicable at high Reynolds numbers, because if you consider like the examples I suggested, when you have flow past solid surfaces; little away from the solid surfaces, the fluid flow is uniform. And so, we can expect that the vorticity is 0 or negligible there, and all the vorticity is confined flow to the solid surface in a region called the boundary layer.

So, even though this assumption is a purely kinematic assumption, it has by itself, we cannot say whether this is a good assumption for viscous flows or inviscid flows. But by looking at the nature of why vorticity is generated, and the nature of flows at high Reynolds numbers, we can say that this assumption is more and more valid at higher and higher Reynolds numbers, especially in fluid flow past solid surfaces, because the vorticity is generally 0 in regions away from solid surface or small. So, it makes sense for us to assume that, the vorticity is 0 that is negligible.

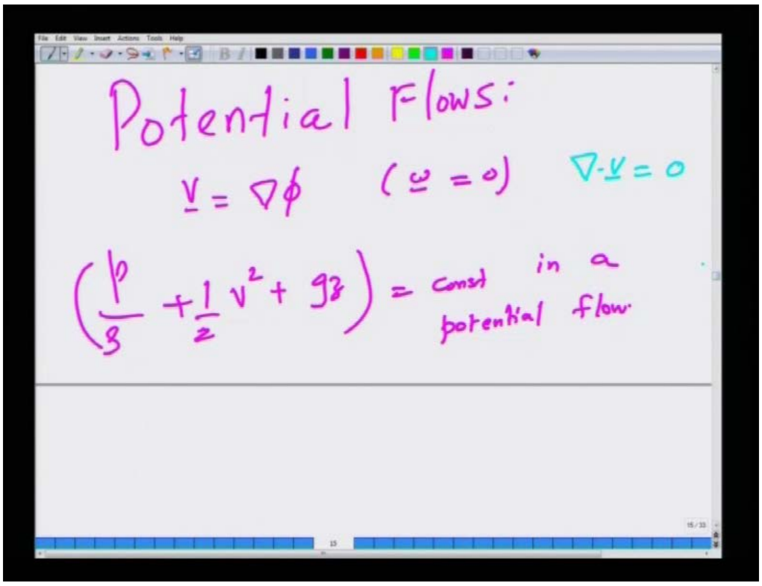
So, these are irrotational flows, whenever you have curl, vorticity is curl of the velocity vector, whenever you have a curl of vector field is 0, you can write the vector field to be the gradient of a scalar function. And conventionally, this is called the velocity potential and hence, inviscid plus irrotational flows are called potential flows, because the velocity is determined by a scalar potential. Now, what is the interesting is that, the velocity is a vector; it has three unknowns, three components, v x, v y and v z in Cartesian coordinates.

So, in order to solve for the velocity, you have to solve three equations, three coupled equations, Euler equations in general. But if the vorticity is 0, then we are saying that instead of solving for three velocity components, you can nearly solve for one function, instead of solving for three functions, that is a velocities of vector function of position variables for steady flows and the vector function has three components, v x, v y, v z and each of this is the position variables x, y, z.

So, instead of solving for three unknown functions, you can solve only for one unknown function, that is the velocity potential and once you take the gradient of potential, you will get back the velocity vector. So, that is a very very great simplification that happens when you assume the flow to be irrotational and as I have told you, irrotational flow is merely a kinematic constraint, but when is that irrotational flow valid, when is the irrotational assumption valid, it makes better sense to use it at higher Reynolds numbers, where you can treat the flow to be effectively inviscid, faraway, at least far away from the solid surfaces.

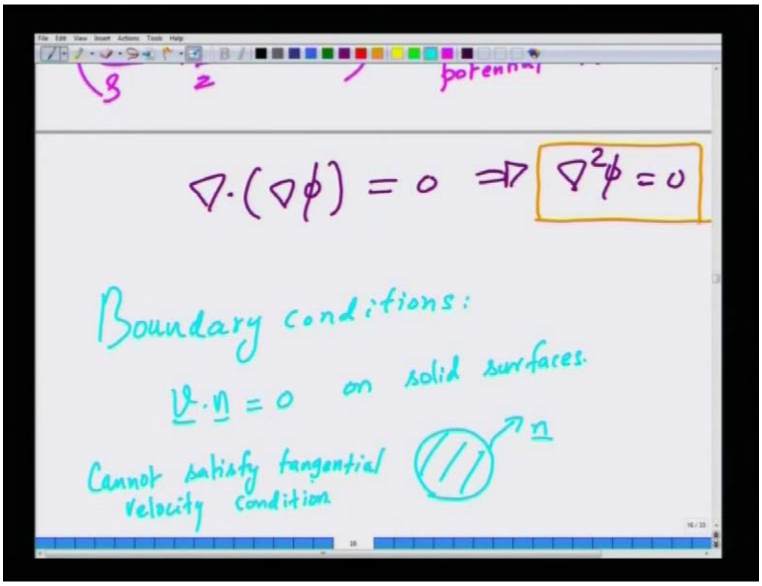
So, inviscid and irrotational flows are called potential flows, because the velocity is written as the gradient of a velocity potential, and such potential flows are applicable at high Reynolds numbers, in regions away from the solid surface. For for regions close to the solid surface, we have to worry about viscous affix and we will come to that, using the boundary layer frame work a little later.

(Refer Slide Time: 25:10)




So, essentially now what we are simplifying is that, we are going to now restrict our attention to potential flows, what I mean by potential flows are the following, velocity is written as gradient of potential. And since vorticity, that is vorticity is 0, curl of velocity is 0. So, we can write velocity as a gradient of potential, but and of course, by using the Euler equation, we also found that p by rho plus half v square plus g z is constant in a potential flow. Now, these are not all, because our flow is also incompressible. So, you also have in addition, the mass conservation equation, that del dot v is 0.

(Refer Slide Time: 26:10)




So, if you have v is del phi and substitute that in the mass conservation equation, you have del dot del phi is 0 or simply, del square phi is 0. So, the two condition constraints on the flow, that the flow is irrotational and the flow is incompressible means that, the velocity potential satisfies Laplace equation. And Laplace equation is one of the most well studied equation in any branch of physical science, because it occurs in not just in fluid mechanics, it occurs in heat transfer, mass transfer, it occurs in electrostatics, it occurs in quantum mechanics, in so on and so forth.

And so, it as been well studied and there are all lots of methods to solve this linear equation. So, that is the main advantage of working with high Reynolds numbers, potential flows, because because there is the velocity is written as gradient of potential and the potential satisfies Laplace equation through mass conservation condition, that del dot v is 0. Therefore, you have to solve del square phi is 0, there are many many solution techniques, analytical and series solution techniques, that are available to solve the Laplace equation, so that is the simplification that one obtains.

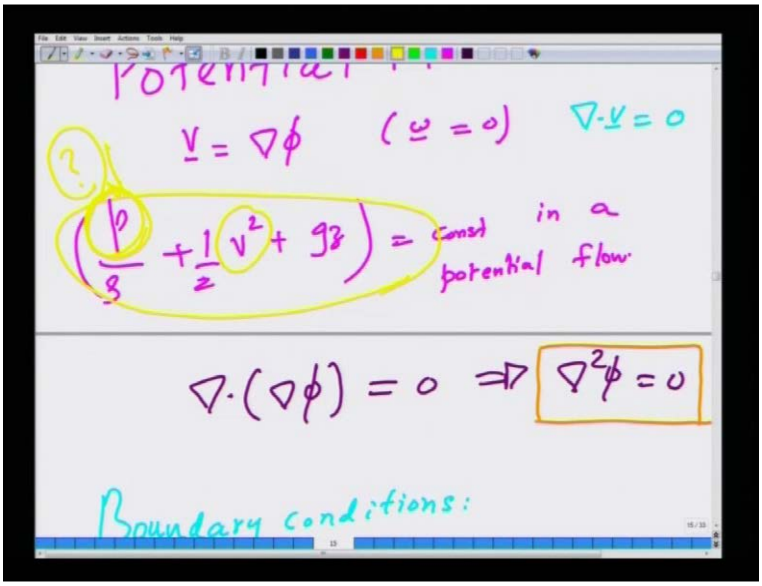
Now, there is one thing that we have to understand, that when we went, what are the boundary conditions (No Audio from 27:30 to 27:36) to solve solve this Laplace equation? When we went from the Navier Stokes equation to Euler equation, we drop the viscous term, the viscous terms, the viscous stress term had del square v, we had del square, that is second order derivative in space.

When we neglected that term, because it is multiplied by 1 over R e in a non-dimensional sense, we have lost the highest order derivative term in the governing equation, when we went from the Navier stokes to Euler equation. The Euler equation had simply v dot del v term and del v as only first order gradients in spatial locations. So, if we had just first order differentials in spatial locations, you you cannot satisfy both in principle, if you have potential flow past solid surface, it has to satisfy both the normal velocity equal to 0 condition and the tangential, if you in in principle, if you have flow faster solid surface at any Reynolds numbers, you have to satisfy both the normal velocity condition as well as the tangential velocity condition.

Now, the fact that we have lost the highest order derivatives, means that we cannot satisfy both the boundary conditions, for the simple reason that, that means we would over specify the problem, if you want to solve the Euler equation. So, we have to forgo one or the two conditions. Now, we will do the least of the, we will do way with least of evils that is, we will try to forgo the tangential velocity boundary condition, rather than the normal velocity condition, because if you say that v dot n is not 0 on a solid surface, that would imply that mass mass can flow in or out of the solid surface, depending on the sign of v dot n. So, that would violate mass conservation, that is amore serious violation and so, compared to that, this is lesser of the two evils, that is, instead of forgoing the normal velocity condition, we will say that we will forgo the tangential velocity continuity condition at a solid surface.

So, this is the major problem in making the flow or making the assumption that, the viscous terms are neglected. In that, we are not able to satisfy the no slip condition and it as major physical implications as we will see a little later. So, the boundary conditions are that v dot n is 0, on solid surface. Suppose, you have solid surface, then there is no, if the solid surface is stationary, then there is no normal component to the velocity, that is the only condition you can satisfy, and cannot satisfy tangential velocity condition (No Audio from 30:19 to 30:28). So, we can get the velocity field for a potential flow, incompressible potential flow by just solving the Laplace equation over the potential, and using the boundary condition that v dot n is 0, and then we left with the one more equation that is the Bernoulli equation.

(Refer Slide Time: 30:48)




Here, we still have this equation, that p by rho plus half v square plus g z is constant. Now, what is the use of this extra equation, we have not, we have already obtained the velocity profile without using the momentum equation.

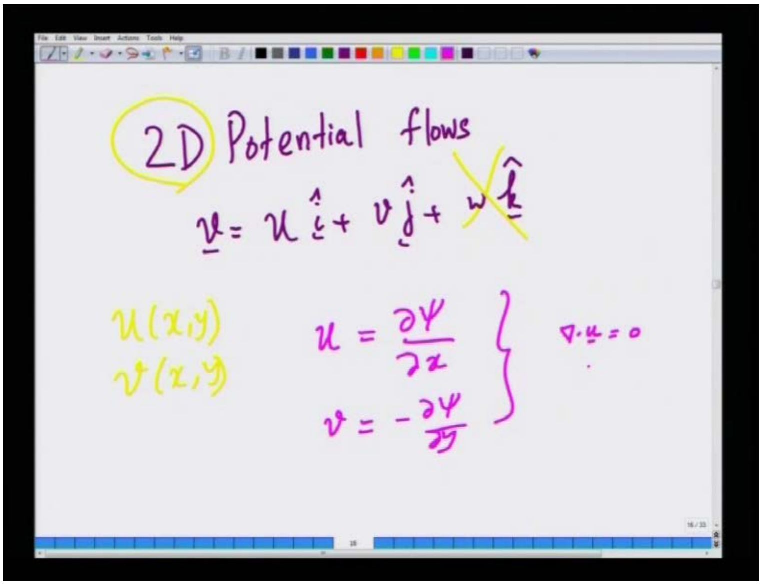
Remember that, this momentum, this is essentially the momentum balance is the Euler equation, rewritten. Now, the answer for that question is that, the Bernoulli equation serves to determine the pressure in a inviscid and irrotational flow. So, the procedure to solve potential flow problems is that, first use del squared phi is equal to 0, to along with the boundary condition that normal velocity as to be continuous, cross solid. Normal velocity is 0 on a stationery solid, and once you solve for the velocity field, you plug the velocity field back in here and find the unknown pressure through the Bernoulli equation.

And pressure is determined only up to a constant, because only gradients of pressure are important in any physical sense. So, pressure can be found only up to a constant in any flow, in incompressible flows. So, this is the one of the most important simplifications that one obtains by looking, by making the assumption that the flow is inviscid and irrotational.

Now, I am going to make one further assumption, potential flow assumption is not that the velocity is written as gradient of a potential has no bearing on whether the flow is 2 dimensional or 3 dimensional. It is merely a consequence of the fact that, the vorticity is

0 everywhere in the flow, but in order to make further simplifications, at least in this course, we will restrict ourselves to two dimensional potential flows, that is flow is only in the x y plane. The other direction z is so large that, there is no variation in the z direction.

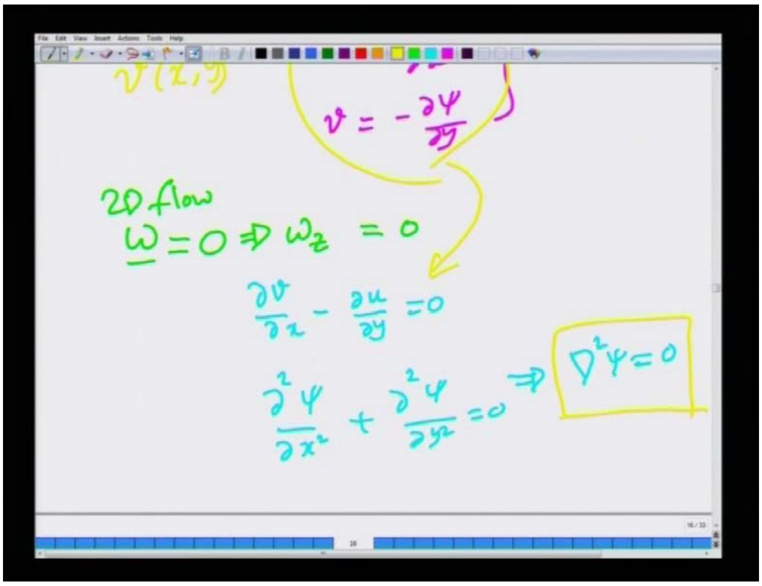
(Refer Slide Time: 32:41)




So, we will assume 2 D potential flows, from now on. When I say 2 D that means, that the velocity vector is written as u times i plus v times j plus w times k, when I say 2 D I mean that there is no w velocity. And there is no variation of u and v in the z direction, that is what we mean by 2 D and u is a function of x y and v is the function of x y, they are independent of z.

Now, whenever you have a 2 D flow, you can always write u as u is a, the stream function formulation, partial psi by partial x, v as minus partial psi partial y, because this automatically satisfies the, in continuity condition, that del dot u is 0.

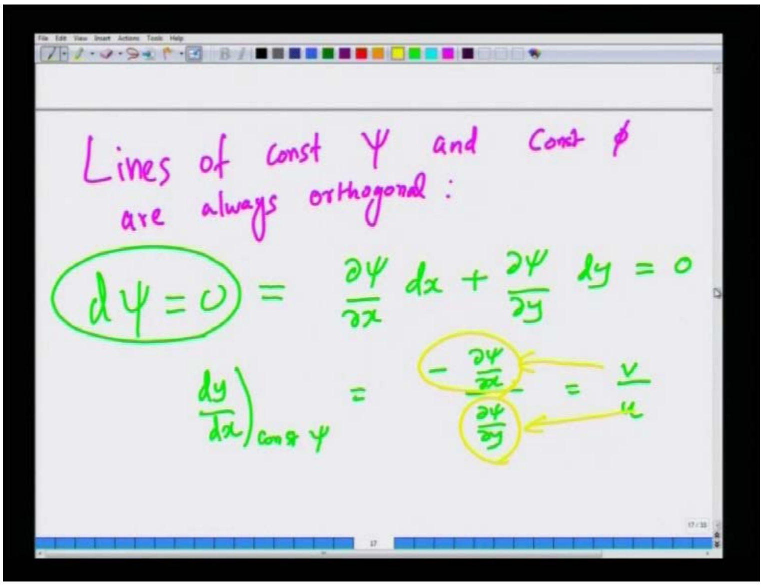
(Refer Slide Time: 33:51)




Now, this is not a all, because we are also saying that omega is 0. In a 2 D flow, where the flow is only in the x y plane, the only non-zero velocity is omega z and since omega is 0, means that omega z is 0, for a 2 D flow. For a 2 D irrotational flow, we have only one component of non-zero velocity that is omega z.

So, and since it is irrotational, that has to be 0. So, omega z is nothing but partial partial v by partial x minus partial u partial y is 0. Upon substituting, the definition of u and v in this expression, you will see that, you get partial square psi by partial x square plus partial squared psi by partial y squared is 0, or you get del squared psi is 0 in the two dimensions x and y. So, the string function for a two directional irrotational flow also satisfies Laplace equation, it is not just the velocity potential that satisfies the Laplace equation, the fact that the only velocity is omega z implies that, string function also satisfies the Laplace equation in a two dimensional potential flow.

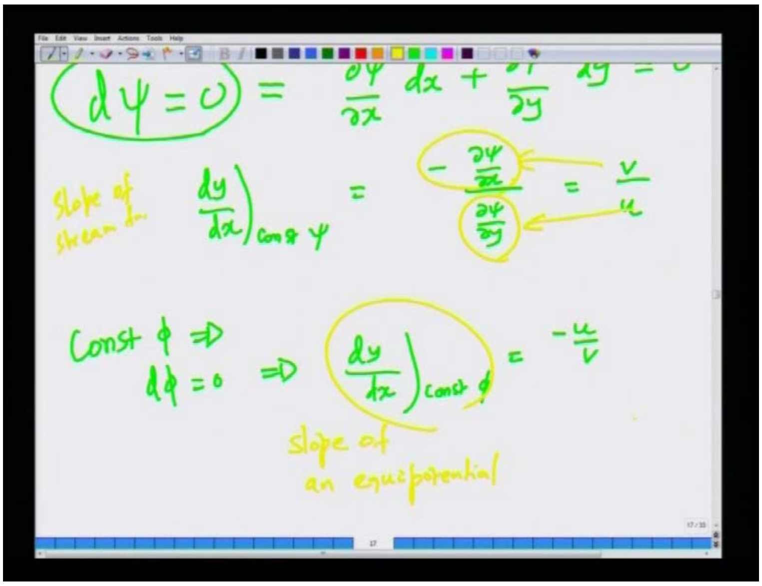
(Refer Slide Time: 35:13)




So, one can also show that, lines of constant psi, that is the stream lines and constant phi these are called equipotential, are always orthogonal, this is something that we can show very easily. So, constant psi means d psi is 0, d psi is partial psi partial x d x plus partial psi partial y d y from the basics of multi variable calculus, partial psi partial x is minus. So, we can also, since we can also rewrite this as saying that d y by d x, so d d psi is 0 means you are along the stream lines, because psi is constant, means you are along the stream line.

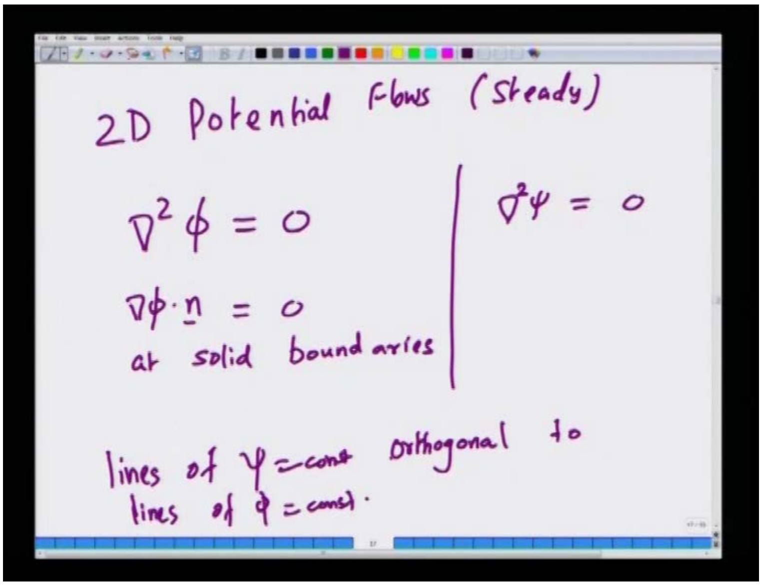
So, d psi is 0, means you are along the stream line. So, this is the slope of a stream line at constant psi, is nothing but d y by d x, is nothing but minus partial psi partial x by partial psi partial y, this is nothing but it is v by u, because this u is partial psi partial y v is minus partial psi partial x.

(Refer Slide Time: 35:13)




So, constant phi implies d phi is 0, this will imply that d y by d x, and the slope of then equipotential constant phi is minus u by v. So, if you see these two expressions, you have the slope of the, slope of stream function, this is slope of an equipotential (No Audio from 37:14 to 37:22) and equipotential. So, the two slopes, m 1 m 2 multiplied to minus 1; that means that at each point, the stream lines and equipotentials are orthogonal to each other.

(Refer Slide Time: 37:47)

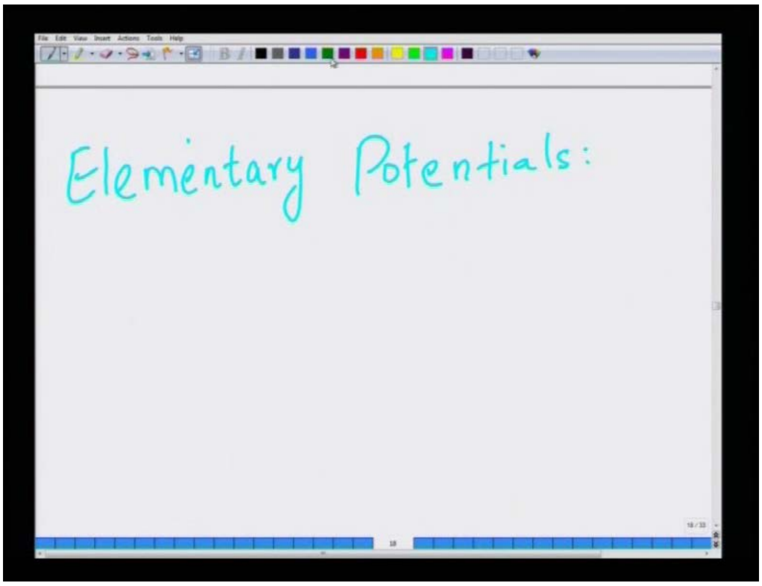


So, to solve for 2 D potential flows all we have to do, so we are restricting ourselves to 2 dimensional potential flows and the flows will be steady. So, what do you have to solve for, you have to solve for del squared phi is 0, with the condition that del phi dot n is 0 at solid boundaries. And we also showed that, for 2 D potential flows, since the flow is irrotational there is only one component of vorticity that non zero, that is the z component of vorticity.

So, we just showed that, del squared psi is also 0, where psi is a psi is a string function and we also showed that, lines of constant psi are orthogonal to lines of constant phi. So, that helps in visualizing the flow in a much easier way, as we will just show in some examples to follow with. So, now what is the strategy for solving potential flows, we have to solve these two equations. Usually what is done is that, we assume a given solution to the, so usually what normally we do in solution of Navier stokes is that, we have a problem and then we tried to solve for the velocity field, by using suitable boundary conditions in by solving the governing equations.

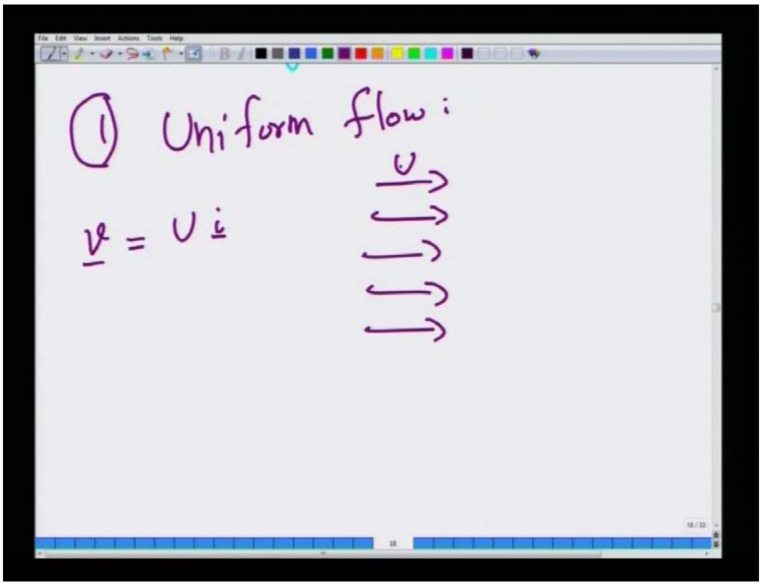
Now, here what we are going to do is that, we are going to assume some solutions and see, to which problem or what physical context does the solution correspond to.

(Refer Slide Time: 39:43)



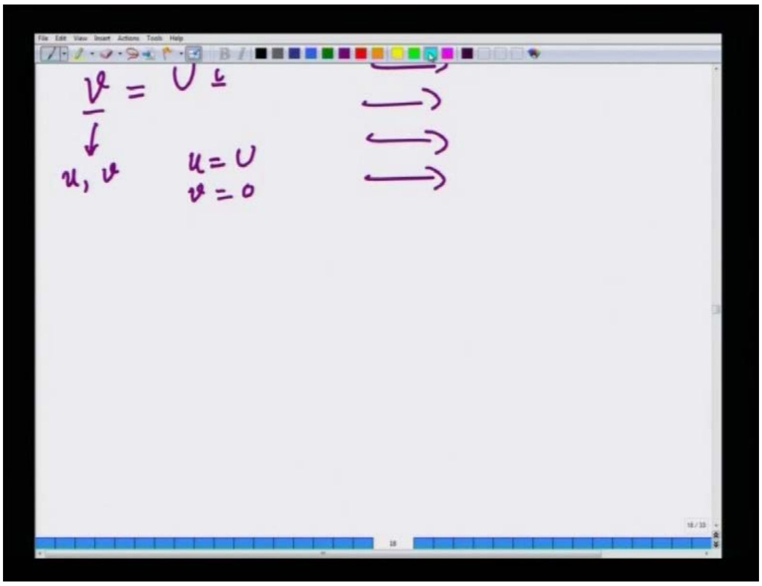

First, so we will treat some elementary potential flows, where we are going to just assume some solutions of Laplace equations and then see what flows they correspond to.

(Refer Slide Time: 40:00)



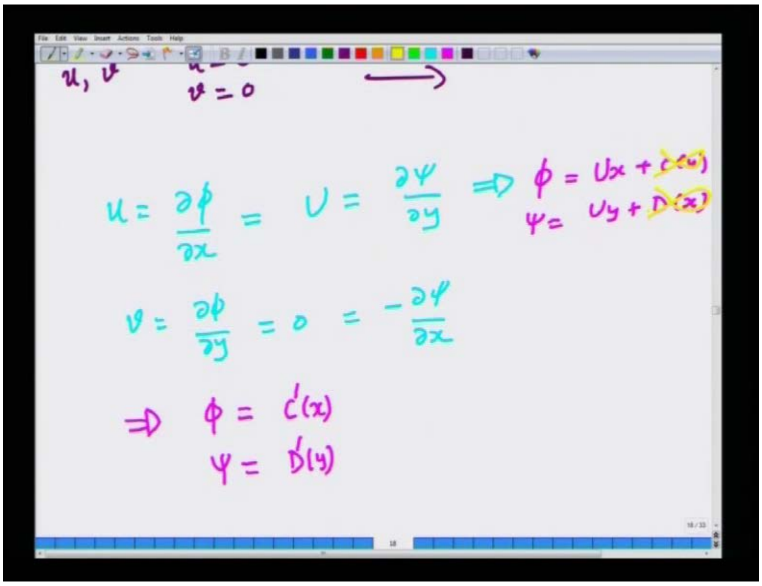

So, first I will consider uniform flow, I already told you that uniform flow as no vorticity, so it is an irrotational flow. So, an uniform flow, so you want to see, we want to see what an uniform flow is, and how it is represented by the velocity potential, what velocity potential corresponds to an uniform flow. So, consider the, a uniform flow in the x direction, there is a constant uniform flow in the x direction.

(Refer Slide Time: 40:00)




Now, so the velocity vector has two components; u and v, u is capital U, while v is 0.

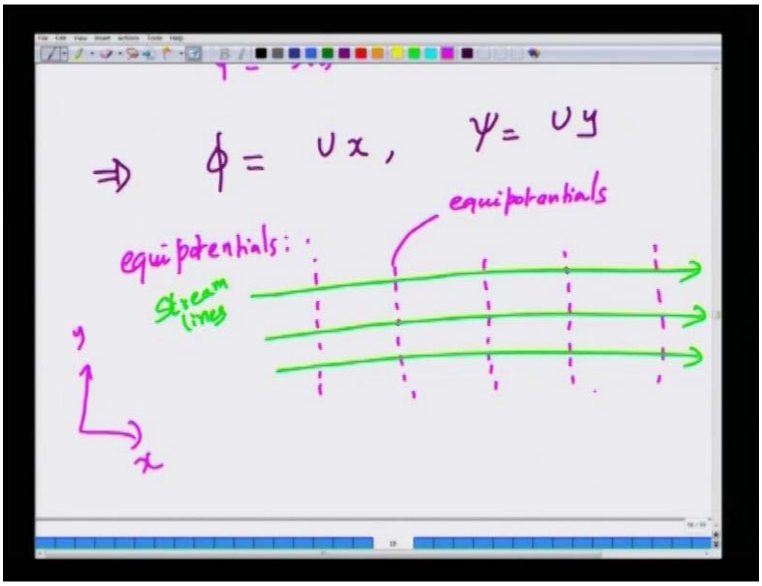
(Refer Slide Time: 40:57)



Therefore, you can write u is partial phi by partial x, is equal to u that is also equal to partial psi by partial y, v is partial phi by partial y, is 0, is minus partial psi by partial x. So, we can integrate these two equations partially, with respect to x and y. So, this equation will tell us that, phi is nothing but, if you integrate this partially with respect to x it is U x plus constant, could be a function of y and psi is nothing but U y plus some other constants which could be a function of x.

Now, if I do this equation, so phi is nothing but, if I integrate this equation, phi is nothing but a constant which could be a function of x, and psi is nothing but a constant which could be a function of y, it is called C prime D prime, distinguished from these two. So, if you compare these two equations, phi is U x plus constant function of y, but here phi is a constant only as a function of x. So, this constant C must be 0; likewise for psi, it is u y plus a constant of x and it is also a constant of y, so this constant of x is 0.

(Refer Slide Time: 42:32)

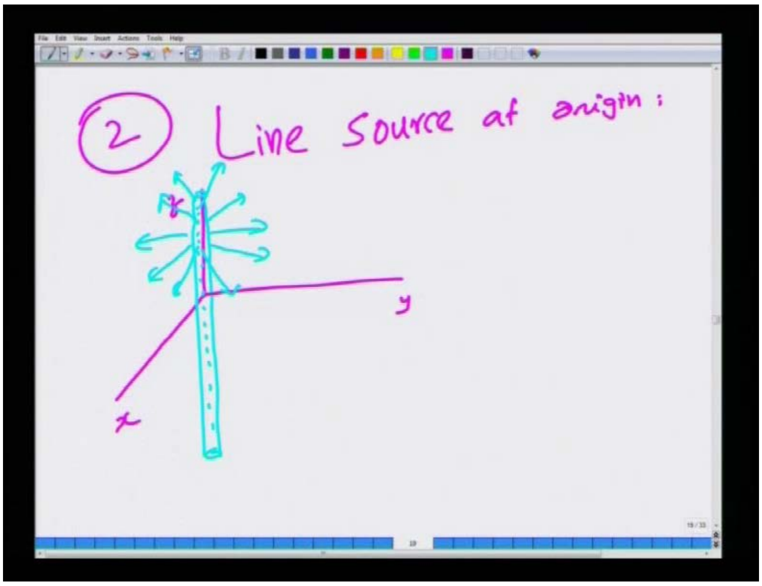



So, both these equations therefore imply that, phi is U x and psi is U y. When phi is U x, when psi is, when you phi is U x, you can plot at different values of, you can calculate the velocity vector in this in this simple way, that it is, at different values of, so what are equipotentials?

Equipotentials are values, where of lines where phi is constant, when is phi a constant, for each values of x phi is a constant. So, equipotentials will be, suppose suppose I put a coordinate system like this, x, y. So, equipotentials will be vertical lines, these are equipotentials. Now, the stream lines will be obtained by putting different values of y. So, there will be horizontal lines, the green line the green lines are stream lines, the pink lines are equipotentials. Just by plotting qualitatively, sketching lines of phi, constant phi and constant psi and since, it is an uniform flow, make sense of fluid flows from left to right along the x direction.

So, the stream lines will be along the horizontal lines, along the x direction and the equipotentials will be vertical lines, because the stream lines and equipotentials are always orthogonal to each other, that that is something that we just prove. So, uniform flow as velocity potential of U times x and stream function of U times y.

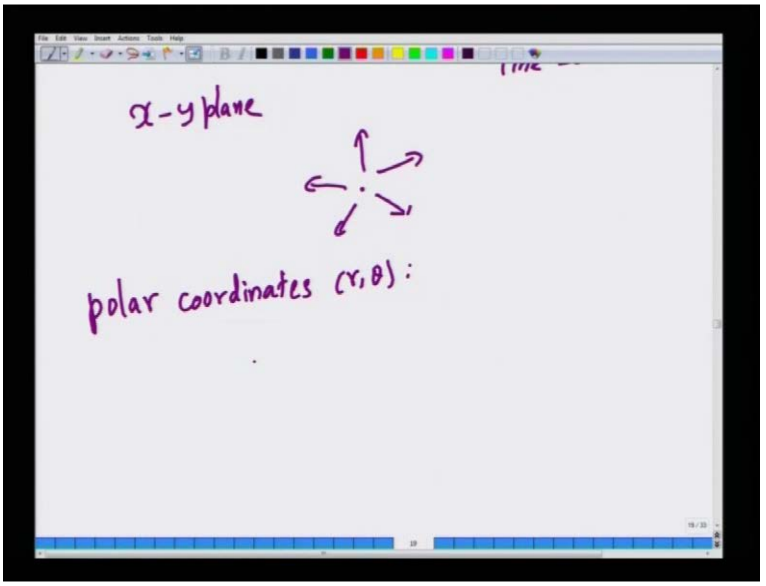
(Refer Slide Time: 44:35)




Now, another problem that we going to do, another model potential flow is, a line source at the origin. Imagine, that you have x, y, z; imagine that along the z axis, you have a very tiny tube, a long tiny tube from which fluid is flowing, imagine that this fluid, this tube is porous and so, fluid is flowing readily out.

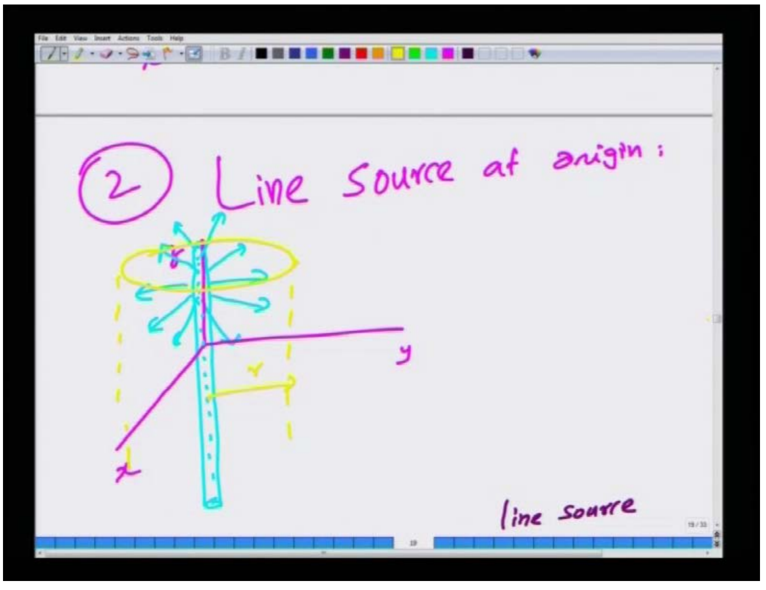
So, lot of holes in this tube, this is just a mental, it is just a model to describe, to tell you what a line source could be, in reality, we just abstracted to a mathematical idea. So, you could imagine that, you have a long pipe, thin long pipe with lots of holes and then lots of holes, and then imagine that fluid is flowing. So, it as to come out readily and so, the flow is in the x y plane, because there is no flow in the z axis, if along the z axis, if the length of the tube is very very large.

(Refer Slide Time: 45:57)



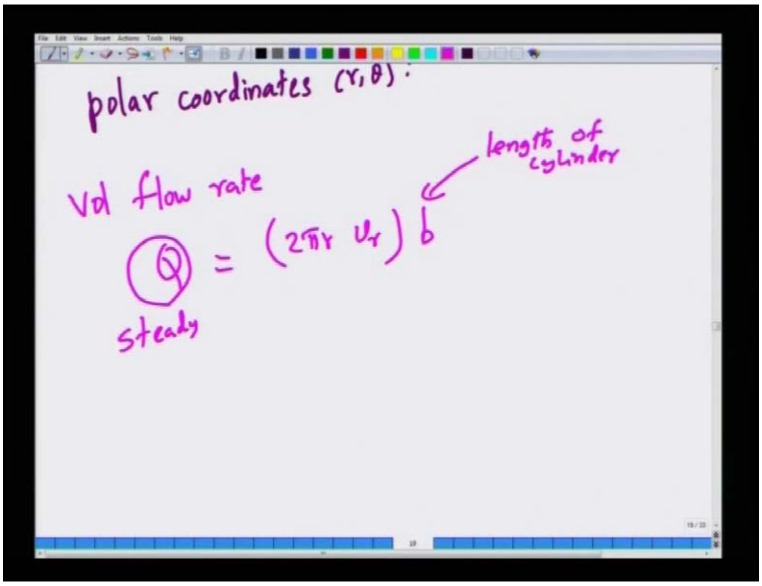

So, we are essentially worrying about flow in the x y plane. In the x y plane, if you look from the top, you look like there is a point from which fluid is fluid is flowing readily out, this is called a line source, it is called a line source. Of course, if you take a cross section, it will look like a point source in the x y plane. So, it is convenient to use planar coordinates polar coordinates, instead of x and y, it is convenient to r and theta.

(Refer Slide Time: 46:41)



Now, if you consider, if you go back to this picture, if you consider an outer cylinder of radius r from the center, and if you consider what is the volumetric flow rate that is crossing this outer cylinder.

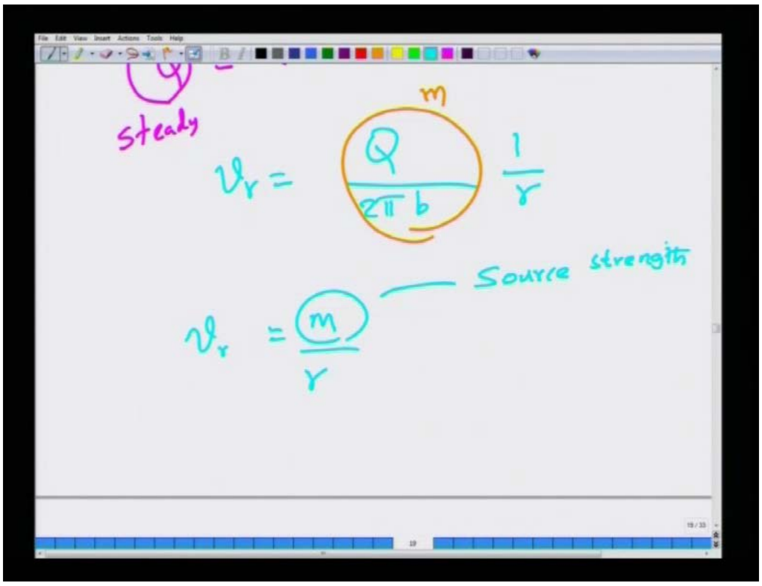
(Refer Slide Time: 46:55)




That is, volumetric flow rate Q is 2 pi r v r times b which is the length of the cylinder. So, this is the volumetric flow rate that flows, because fluid is flowing purely in the radial direction.

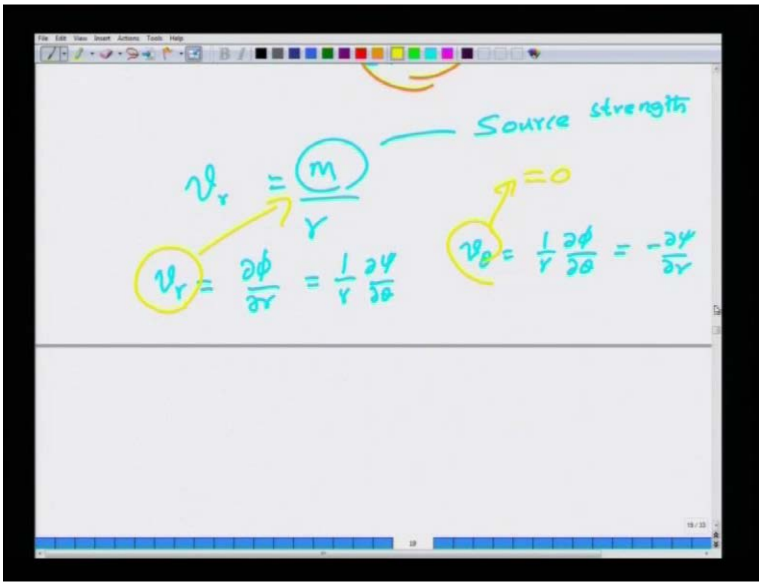
So, the velocity vector that is coming in is v r. So, 2 pi r is the, so you want to multiply that velocity by the area of the cylinder, area of the cylinder is 2 pi r times b, that is surface area of the cylinder. So, that will give you, what is the volumetric flow rate. Now, we are going to assume that Q is a constant for steady flow, a constant volumetric flow rate keeps coming out from this origin, where you had kept this line source of flow.

(Refer Slide Time: 48:00)



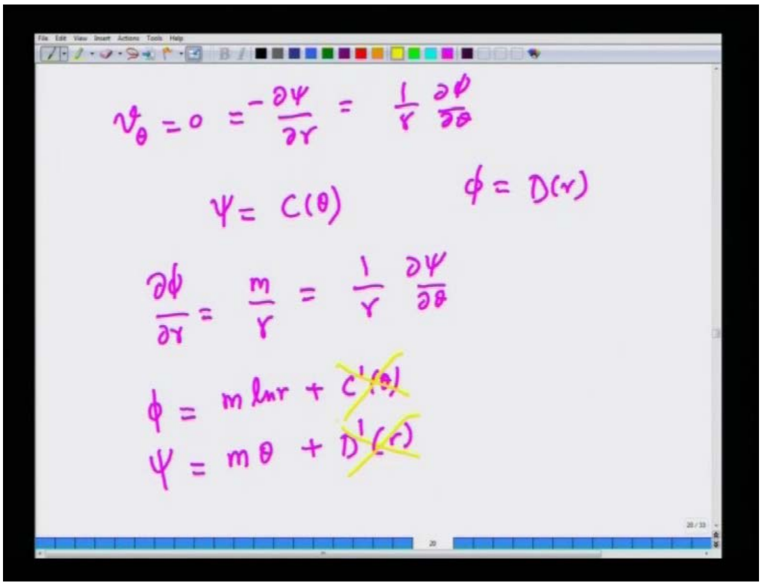

So, we can write, therefore v r is Q by 2 pi b times 1 over r. Now, this is denoted by the letter m, it is a constant. So, v r becomes m over r, where this is called the source strength, the strength of the line source. If m is the positive quantity, then fluid emerges out radially; if m is a negative quantity, fluid comes in towards the origin. We want to find for this lines source of flow mass, what is the velocity potential and what is the strength function?

(Refer Slide Time: 48:00)



So, v r in cylindrical coordinates, there is simply partial phi by partial r and v theta is 1 over r, partial phi by partial theta, v r is also equal to 1 over r partial psi by partial theta, v theta is also equal to minus partial psi by partial r. For our problem, v r is given by this, but v theta is 0, there is no flow in the theta direction, fluid is flows purely radial in the cylindrical coordinate system. So, we can use these, through equations to again integrate.

(Refer Slide Time: 49:27)

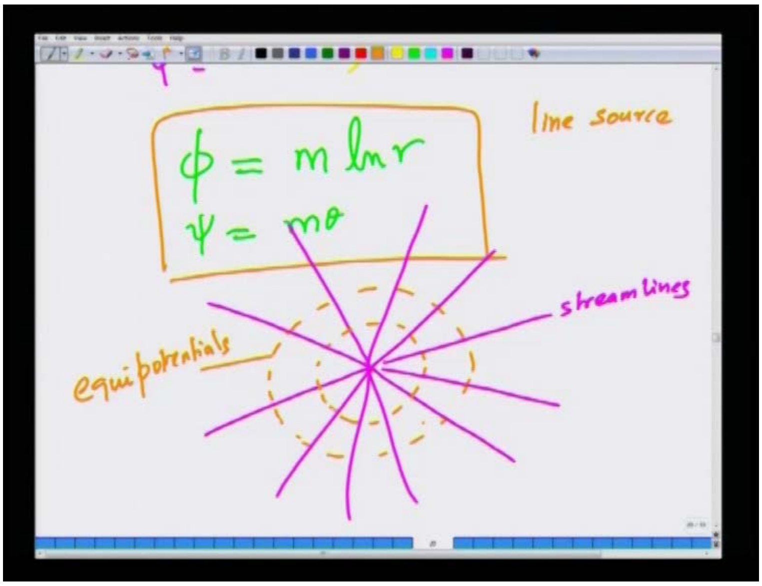



So, v theta is 0 that means partial psi by partial r is 0, because v theta is minus partial psi by partial r that is also equal to 1 over r partial phi by partial theta. So, if I integrate this with respect to r, I will get psi is some constant which is a function of theta and I will get phi is equal to, so m is 2 pi, m is 2 pi times v.

So, you have 1 over r partial phi by partial theta is 0, which means phi is a constant D which is the function of r. So, that means partial phi by partial theta is 0, if you integrate partially with respect to r, what you will get, partially with respect to theta, you will get a constant, that is a function only of r. Now, the other equation tells you that, partial phi by partial r is m by r and this also equal to, or plus 1 over r partial psi by partial theta.

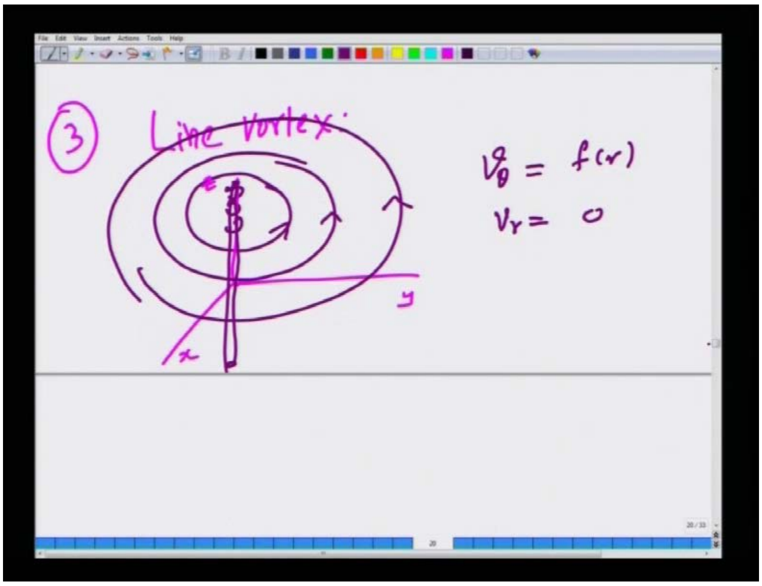
This implies, if you integrate this equation partially, phi becomes m logarithm of r plus some constant C prime, which is the function only of theta, and psi becomes, if I integrate this this 2 r will cancel, you will just get m theta plus D prime, which is a function only of r. So, if I compare these two conditions, psi is a function only of theta from this equation, whereas the function of theta plus r. So, it cannot be function of r, likewise this constant cannot be function of theta.

(Refer Slide Time: 51:31)




So, phi is m logarithm of r and psi is m theta, for a uniform source, for a line source. These are the descriptions of velocity potentials and stream lines. So, let us look at lines of constant r. So, lines of constants r are circles, so the equipotentials are circles. And lines of constant theta are radial lines; I am going to plot them in pink color. So, these are almost like spokes of a cycle wheel, these are stream lines, the pink ones are stream lines, and the orange ones are equipotentials, and it also agrees with our general result that, stream functions and stream line and equipotentials are always orthogonal to each other and that part is of course, brought out nicely here.

(Refer Slide Time: 52:51)




Now, the next simple simple example or illustration that I am going to do is, a line vortex. A line vortex is one in which you have again, if you put an x, y, z coordinate, Cartesian coordinate at z equal to 0, you can imagine that you can imagine that very long thread of fluid which is rotating at some constant velocity, there is purely circulating motion, because of that, there is circulating all the vorticity in the flow is confined only to the z axis.

So, far away because of this motion far away, there will be purely circular motion of the fluid, this is called the flow due to a line vortex, the flow is purely circulation. So, only v theta is there, and v r is 0, this is exactly the opposite of the uniform sorry, opposite of the flow which we just discussed, which is a line source problem where the flow flow is purely radial and there is no flow in the theta direction, here we are considering flow only in the theta direction and there is no flow in the radial direction. So, what we want to do is, use this idea to find out what the stream functions and velocity potentials are, and we will stop here and we will continue with this topic in the next lecture.