Fluid Mechanics
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Lecture No. # 30

Welcome to this lecture number 30 on this NPTEL course on Fluid Mechanics for under graduate chemical engineering students, the topic that we are currently discussing is Dimensional Analysis and how dimensional analysis is used in many engineering applications.

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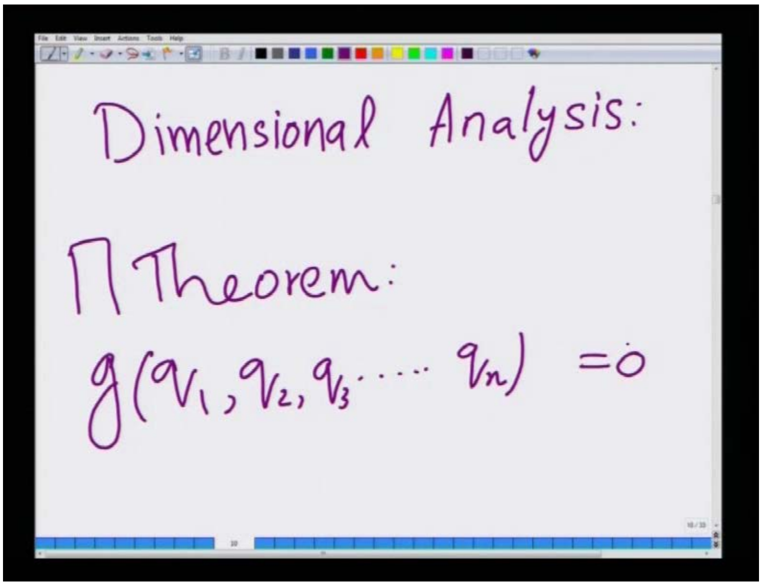

So, in dimensional analysis, we first discussed the Buckingham's pi theorem, which essentially says that, if you have a functional relationship among n-dimensional groups q 1 q 1, q 2, q 3, so on up to q n, we have a function relationship among this n-dimensional groups.

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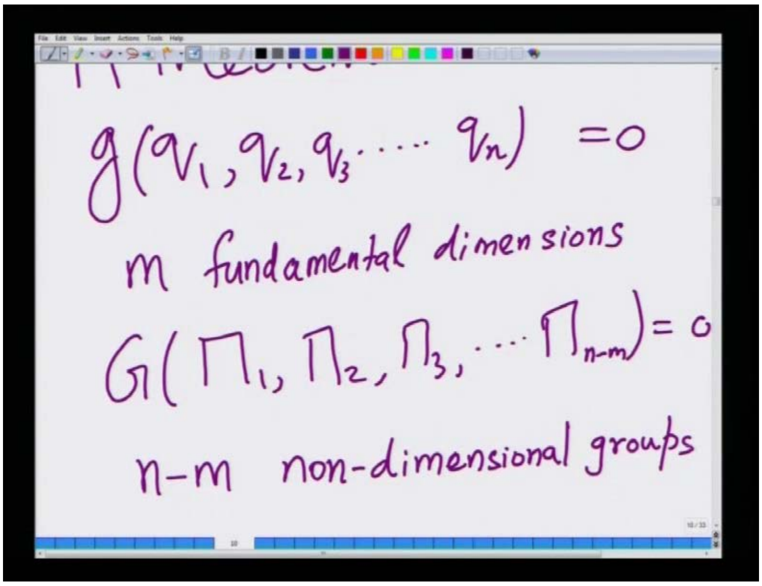

Then and if there are m fundamental dimensions in the problem, in the problem such as if typically in a mechanics problem it is mass, length and time, then there are there is another functional relationship among n minus m, non-dimensional groups called the pi groups. So, you are effecting reduction in number of variables, so there are n minus m non-dimensional groups or dimensionless groups, this is the essence of the Buckingham's pi theorem.

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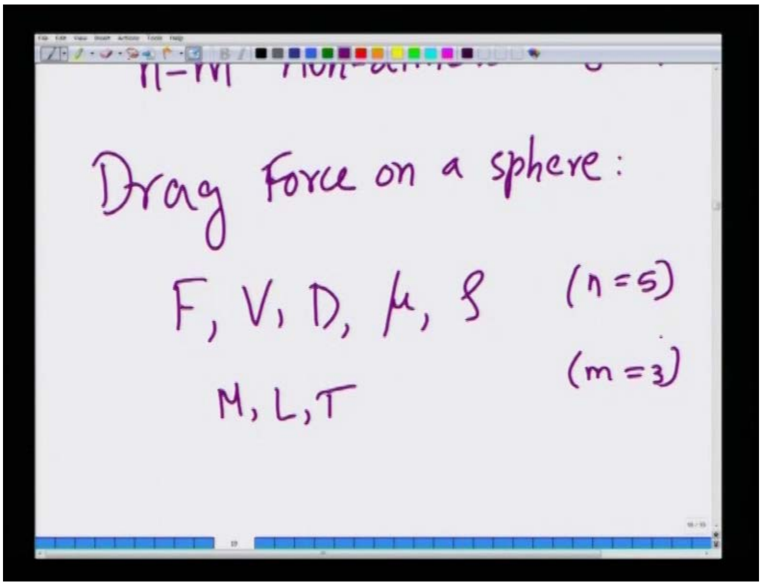

And then, we applied it to the specific case of drag force on a sphere. The initials variables of the problem that, we thought was relevant we thought were relevant, where the force, the velocity at which the sphere is moving, the diameter of the sphere, viscosity of the fluid and the density of the fluid, there are 5 variables and the fundamental dimensions contained in all these 3 5 variables are mass, length and time, so m is 3.

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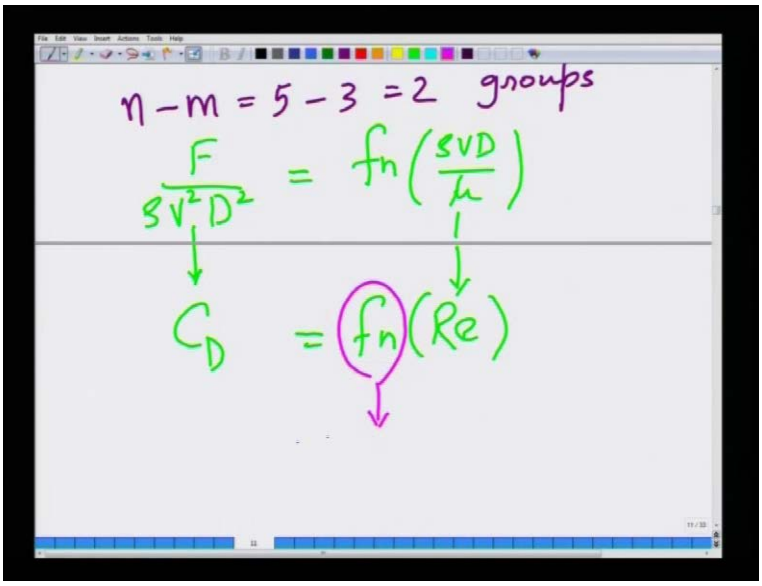

So, the pi theorem says that, there are two non-dimensional groups is 5 minus 3 is 2 non-dimensional groups and we found that, those two groups are F divided by rho V square D square is some function of rho v D by mu; and traditionally this is called the drag coefficient and this is the Reynolds number; so, what this is saying is that the drag coefficient which is a non-dimensional drag force is a function of the Reynolds number. Reynolds number is as I pointed out in the last lecture is a relative magnitude of inertial forces to viscous forces present in the system.

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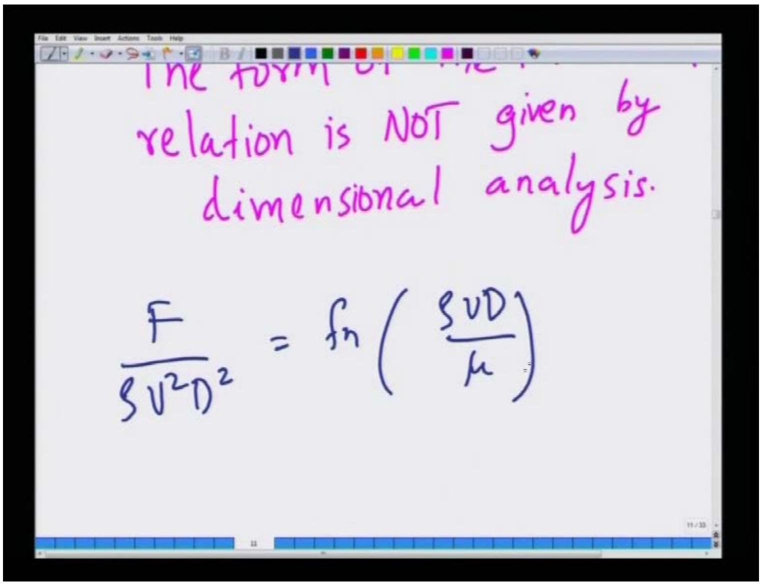

Now, what is important for us to understand is that, the form of the functional relation the form of the functional relation is not specified by dimensional analysis is not given by dimensional analysis. In order to find the functional form one has to do experiments, but I pointed out that, this actually leads to great simplification because, instead of varying all the original 5 variables to do experiments, in order to see the dependence of the force on various parameters such as, viscosity, density, diameter and so on.

We are now we are now we now have to vary only one variable, the Reynolds number and we can find the effect of the Reynolds number on the non-dimensional force, which is a drag coefficient and this actually is a condensed form of variety of experimental data on, so essentially we said that, F by rho V square D square is a function only of rho V D divided by mu.

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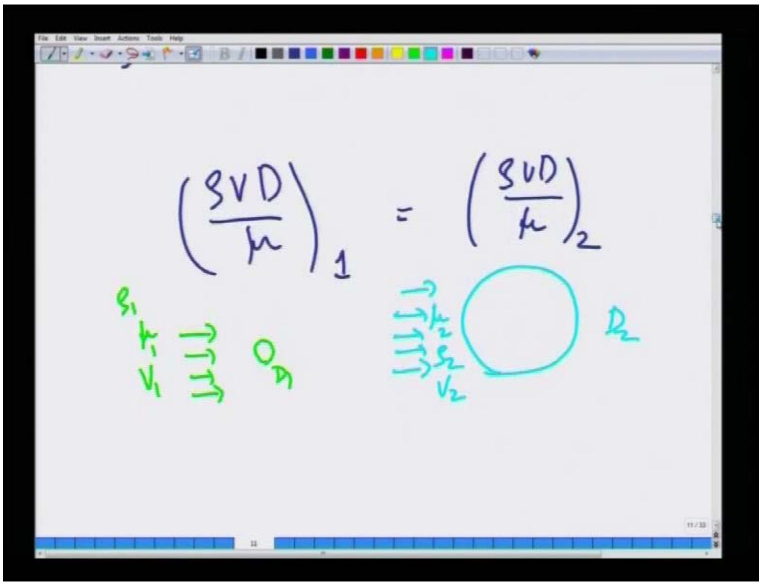

So, this means that, if you fix the Reynolds number for a given problem for two sets of problems, that is rho V D divided by mu for a second system, so here you have one system in which fluid is flowing passed a tiny sphere with some viscosity, the diameter of the sphere is D 1, viscosity of the fluid is mu 1, density of the fluid is rho 1. In another problem, you have a very large sphere, the diameter of the sphere is D 2, the viscosity of the fluid is mu 2, density is rho 2, the velocity at which the fluid is moving is V 2.

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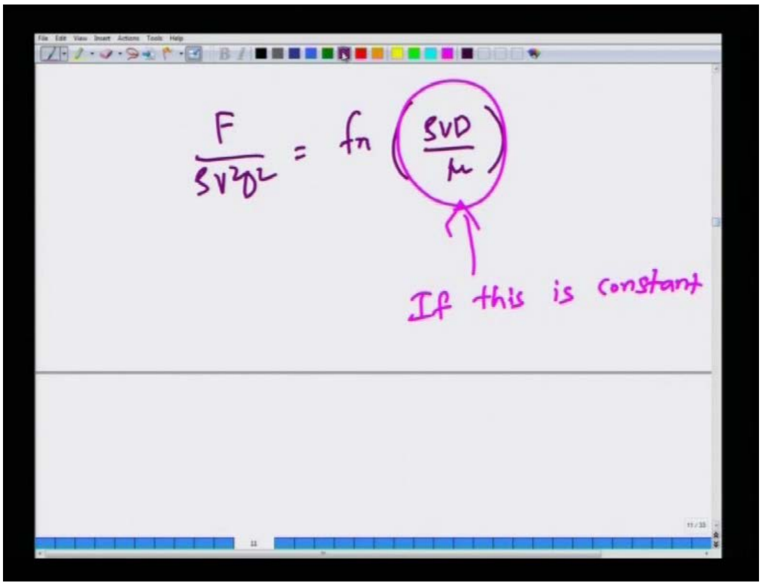

Then, what what this relation is saying is that, F by rho V square D square since is a function only of rho V D divided by mu, if I keep this constant, if this is kept constant, if this is constant then, if that if this is kept constant for two different systems.

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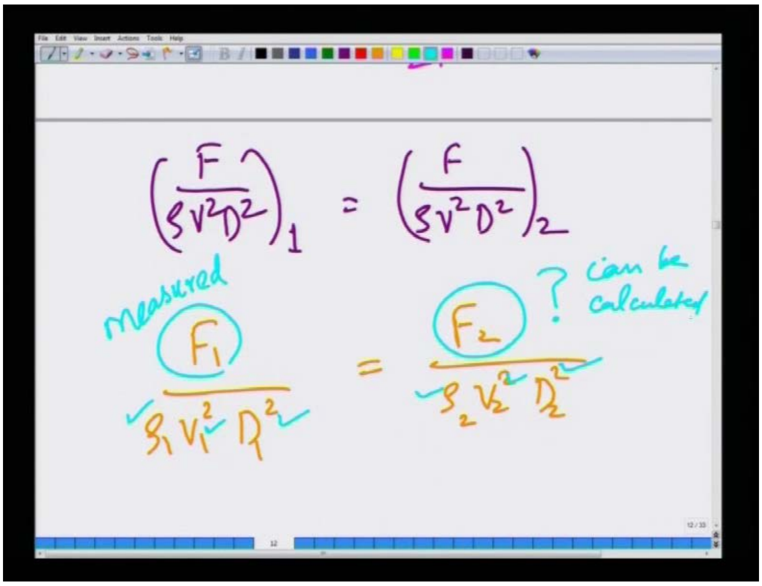

Then, F divided by rho V square D square for system 1 should be the same as F by rho V square D square for system 2. So, what is the advantage? The advantage is that, this means that F 1 by rho 1 V 1 square D 1 square is F 2 by rho V 2 square D 2 square rho 1 and rho 2. Now, suppose if you do not know this, let us say this is unknown and this experiment is difficult to perform in the lab, but we can always choose another system, where it is easier to do experiment; so this is measured in the lab for different system provided the Reynolds number for both the system is the same and since we know all the other parameters, this unknown can be calculated through this measured force for a different system.

So, this is a very very great simplification and this leads to the idea of scaling up and scaling down of various measurements. So, suppose you want to measure the drag force on a very very tiny sphere of let us say, 10 micron diameter and if you need that in an application, all you have to do is to, do the same experiment at the same Reynolds number in a in in in the lab as long as the Reynolds number for the lab experiment is the same as the case, where you have the real application.

Then, the non-dimensional force is the same, although the dimensional forces themselves are different, because all these parameters are in general different for the two cases. So, this leads to the notion of scaling up or scaling down from a model to prototype or a lab scale experiment to real real life experiment, real life situation. So, that is one major advantage of dimensional analysis by expressing your results in non-dimensional numbers, since non-dimensional numbers are scale free because, they are independent of units you choose to express them, they must be the same for geometrically similar systems.

Now, the next thing we did was to show that by non-dimensionalzing Navier-Stokes equations we again found that, the force drag force on object like sphere is merely a function of a Reynolds number, but we also found one additional piece of information by non-dimensionalzing the Navier-Stokes equation.

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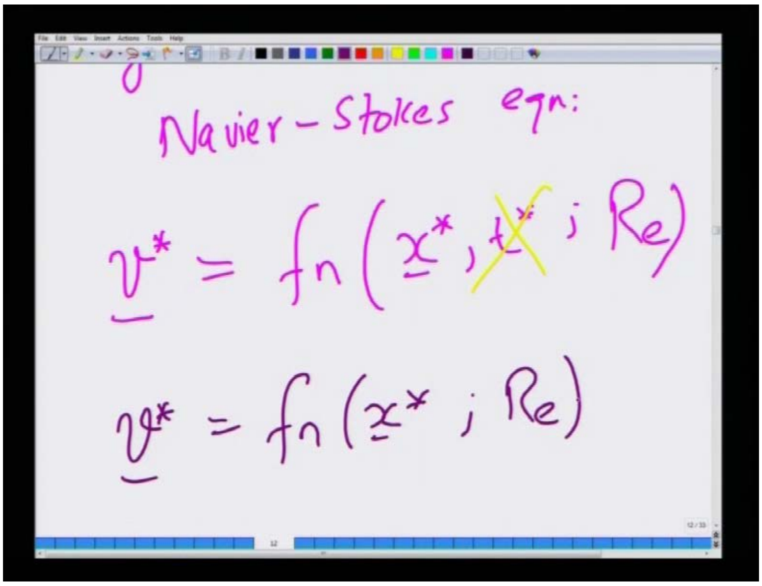

By non-dimensionalzing the Navier-Stokes equation in the previous lecture, lecture number 29, the momentum equations that is the Navier-Stokes equations, what we found is that, the non-dimensional velocity is a function only of the non-dimensional position, the non-dimensional time and the Reynolds number. So, if the problem is steady then, there is no dependence on time. So, the non-dimensional velocity is a function only of the non-dimensional position and the Reynolds number.

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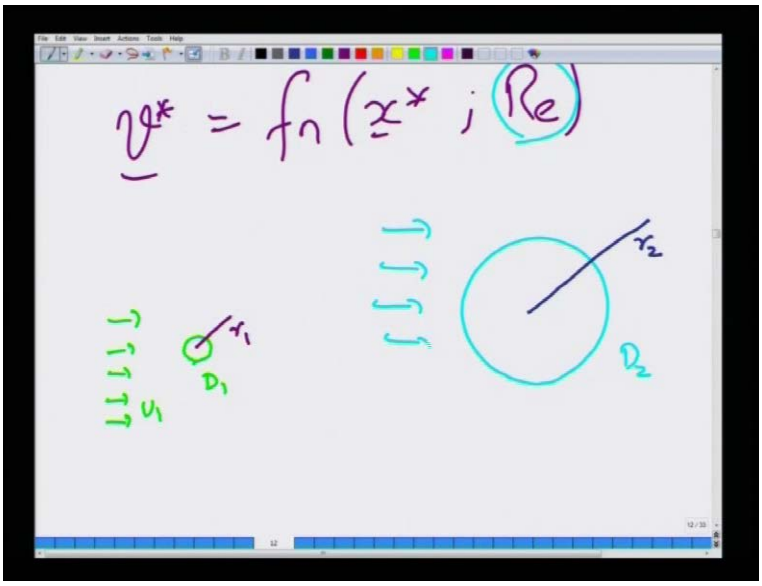

So, suppose you have two different systems you have a tiny sphere and then, you have a big sphere. Now, you want to know, what is the velocity at a point r 1 from the distance, from the center and from here the point r 2 and the diameter of this sphere is D 1, diameter of this sphere is D 2. Now, if you keep the Reynolds number same for the both this different system, then the velocity is a function only of the non-dimensional distance from the non-dimensional distance. And let us say the here the fluid is flowing with a velocity U 1 and here it is flowing with a velocity U 2.

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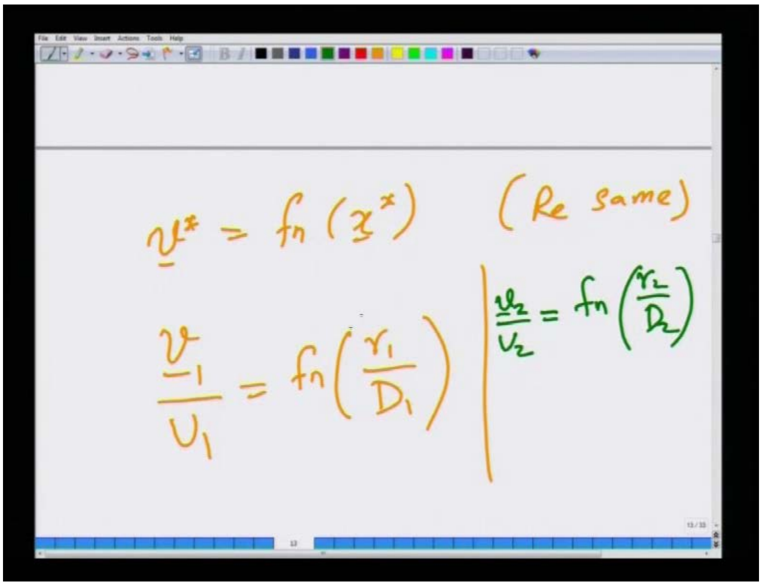

Then, what we are saying is that, v star is the function only of x star, R e same, if you keep the Reynolds number to be same in both the cases; that means, v 1 divided by U 1 is a function only of r 1 divided by D 1 and similarly, v 2 divided by U 2 is a function the velocity vector in situation 2 is a function only of r 2 by D 2, because Reynolds number for these we have managed to keep the same. So, what this means is that, if you look at these two plots as long as you looking at the same non-dimensional distance.

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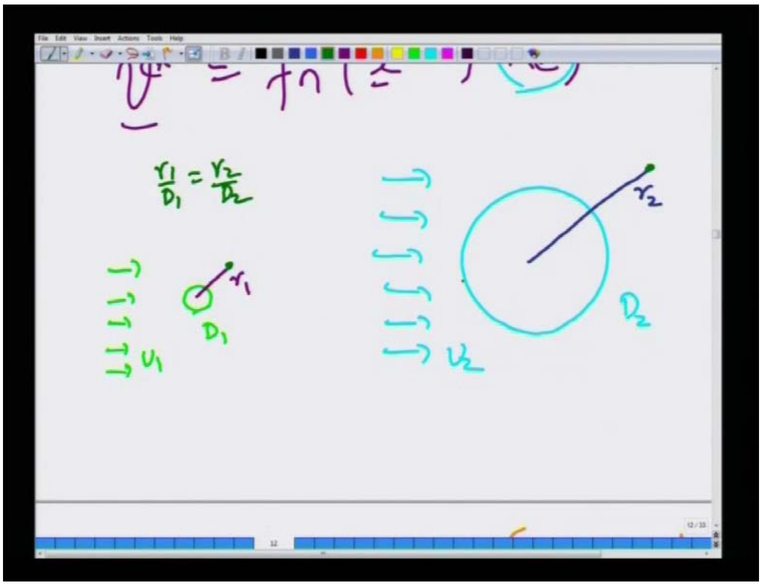

So, let us say r 1 by D 1 is that same as r 2 by D 2 then, the non-dimensional velocities will be the same in both non-dimensional positions. So, even if you want to know, what is the detail flow structure around for flow around of sphere for two different systems one in which you have diameter is D 1, another is in which you have diameter D 2, suppose you are able to do this problem in the lab through experiments or using a computer using computer stimulations you need not solve this problem separately, because that information is already buried in a non-dimensional form in this simpler form in this smaller problem.

So, the non-dimensional velocities at various points as various non-dimensional positions are identically the same, although the dimensional velocities are different, but they are easily scaled. Now, this is a very very important input that we get from the Navier-Stokes equation.

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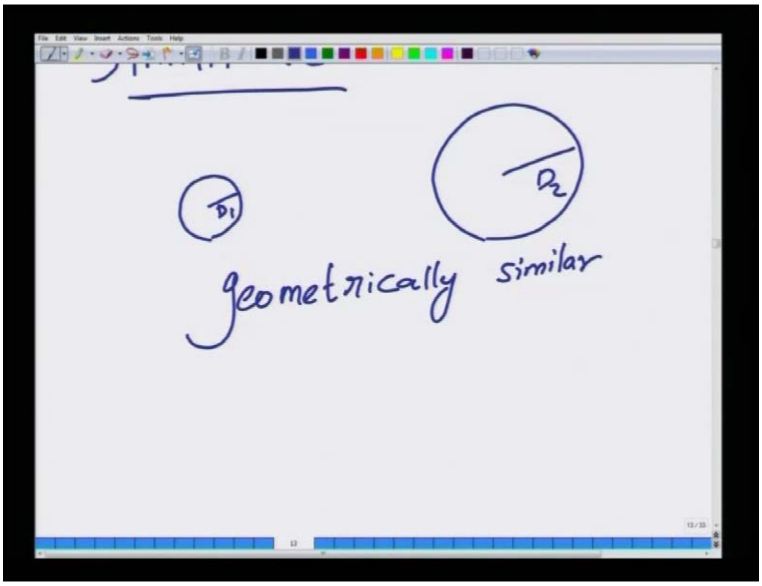

Now, the final application of dimensional analysis is similitude that is how to how to identify that two systems are similar such that, the non-dimensional groups are the same in both the cases. Suppose, I have a sphere for the problem that we just considered if you have two different spheres we know that, the only parameters present are the only length scale present are D 1 here and D 2 here. So, if you represent all length scales, if you non-dimensionalize all lengths with the respect to these diameters then, we know that the non-dimensional velocities will be identical at the same Reynolds numbers.

But suppose, so these two systems are said to be geometrically similar that is both are spheres, although the diameters of the spheres are different, but both are the same geometric objects. Now, I am going to change the problem slightly.

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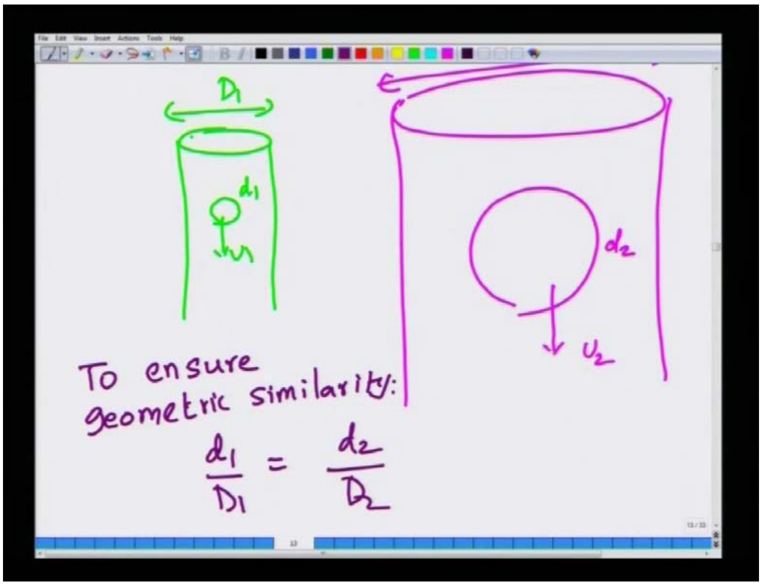

Now, instead of the sphere presenting present in an infinite fluid let us say, you worry about sphere of diameter D 1 moving in a pipe of diameter let us say let us call this sphere diameter as small d 1 moving in a pipe of diameter capital D 1 and you have another case, in which you have larger pipe in larger sphere diameter D 2 and it is moving with a velocity U 2, this is moving with a velocity U 1 and the diameter of the larger pipe is D 2 sorry this is small d 2, this is capital D 2.

Now, can we just say that at the same Reynolds number, the drag forces will be the same and the velocities will be the same at same non-dimensional positions, the answer is not so simple, because there is another length scale present in the problem that is the diameter of the pipe in which the sphere is moving.

So, in order for these two systems to be in order for geometric similarity to ensure geometric similarity all the length scales must have the same ratios that is you should have d 1 by D 1 is small d 2 by capital D 2 that is the ratio of the diameter of the sphere to diameter of the pipe through which it is flowing, through it is moving should be the same for both system 1 and system 2, if this is not satisfied then, these two systems are not geometric similar geometrically similar and you cannot expect the drag coefficient to be the same for both these cases at even at same Reynolds number, even if you assume that the Reynolds number based on the sphere diameter and so on, it is the same for the both cases, unless you ensure this geometric similarity you will not have the same drag coefficient.

So, in order for two systems to be similar in order for you to be able to use dimensional analysis to scale up from one size to another size, the first thing is to ensure is that, these two systems must be geometrically similar.

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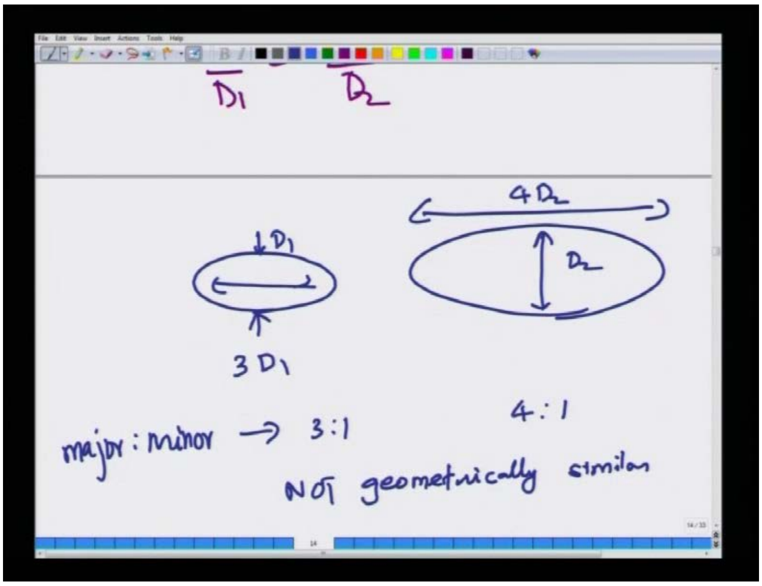

Another example of geometrical similarity is this, suppose you have flow passed in an ellipse let us say this is let say the minor axis is D 1, the major axis is 3 D 1 and in another case, the minor axis is D 2, the major axis is 4 D 2. So, the major to minor ratio is 3 is to 1 here and it is 4 is to 1 here. So, these two systems are not geometrically similar, again because the ratios of various lengths must also be identically the same in across two different systems only then, we are ensuring geometric similarity.

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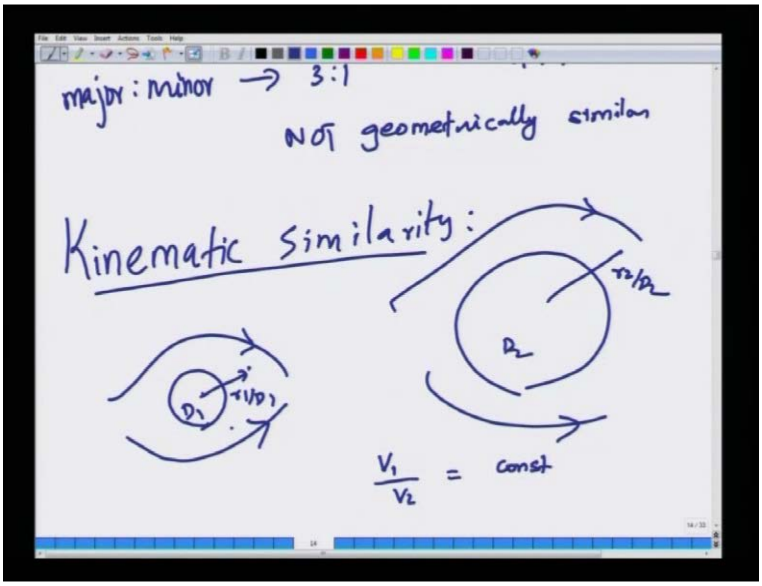

The next thing comes in kinematic similarity. Now, two flows are kinematically similar, if the velocities at various points, they they are pointing in the same direction and they differ by the same factor at various points. So, suppose you have flow fastest cylinder or a sphere sorry you have two different systems, now if you look at the same geometric geometrically similar point that is the distance r 1 by D 1 is the same as r 2 by D 2 let us called the diameter of this sphere D 1, D 2. So, that means you are looking at geometrically similar points, now if you look at geometrically similar points at such geometrically similar points kinematic similarity happens, when the velocities at these two points are let us say v 1 by v 2 are constant. And if you look at some other point, the ratio of the velocities must be the same constant.

So, two flows are kinematically similar, if the velocities at corresponding geometrically similar points are always related by a constant scale factor, then we can say that, these two flows are kinematically similar. Now, we also have done some analysis of the Navier-Stokes equations to find out; when two flows can be kinematically similar? We found that, when the Reynolds number of the two situations is the same then, two flows will be kinematically similar.

So, in order for you to ensure kinematic similarity all you have to ensure is that, the Reynolds number of these two situation must be identically equal, then the ratio of the two velocities at various points in the fluid geometrically similar points in the fluid will identically of by a constant scale factor.

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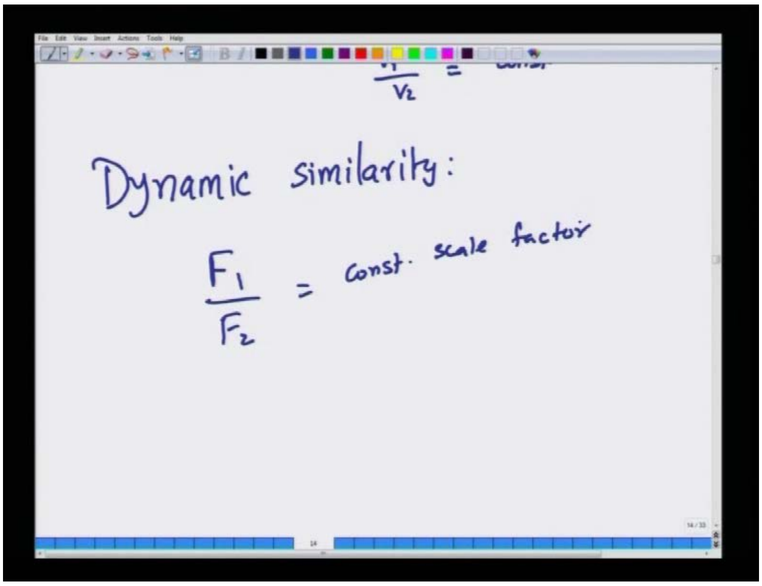

Finally, we come to dynamic similarity, in dynamic similarity the forces are of by a constant scale factor at all corresponding points.

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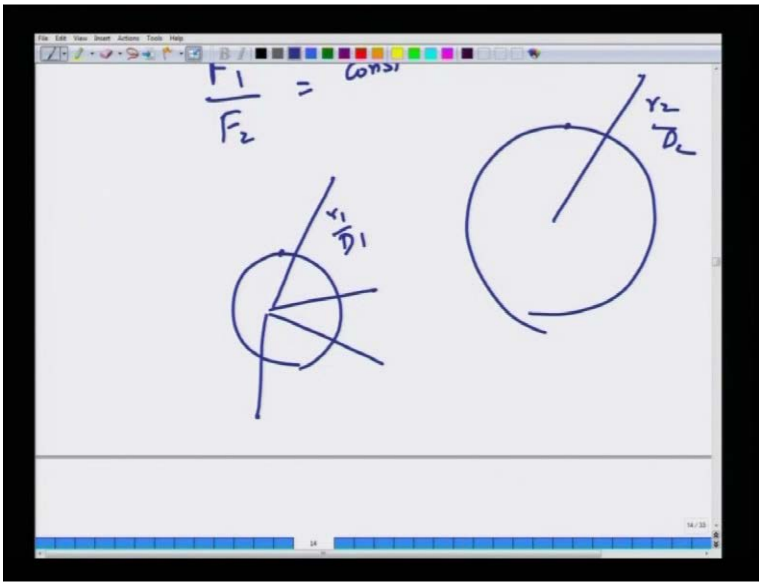

Suppose, you look at two spheres, if you look at geometrically similar points on the surface of the sphere or even interior of the liquid so long as, the ratio r 1 by D 1 is the same as r 2 by D 2 geometrically similar points, then the magnitude of the forces will be related, the direction of the forces will be the same and the ratio of the forces, magnitude of the forces will be related by a constant scale factor even at various geometrically similar points, even if you look at other points, there always be of by same constant factor. This is dynamical similarity.

Geometric similarity merely refers to the similarity of the shapes and also the ratio of the length scales between two different situations. Kinematic similarity means that, the ratio velocities at geometry similar locations must be the same and by an analysis of the Navier-Stokes equation we know that, kinematic similarity is ensured if you keep the Reynolds number of the two situations to be the same.

Finally dynamic similarity means that, the ratio forces must be of by a constant scale factor and if the Reynolds number is the same we know that, by integrating the velocity by integrating the stresses over the surface, then the forces will also come out to be a function only of Reynolds number. So, if we as long as you keep the Reynolds number to be the same, then dynamic similarity is also ensured. So, these are the three types of similarities that one often talks about, when you scale up or scale down experimental data using non-dimensional analysis. So, let me complete my discussion on dimensional analysis by summarizing through this example of drag force on a sphere.

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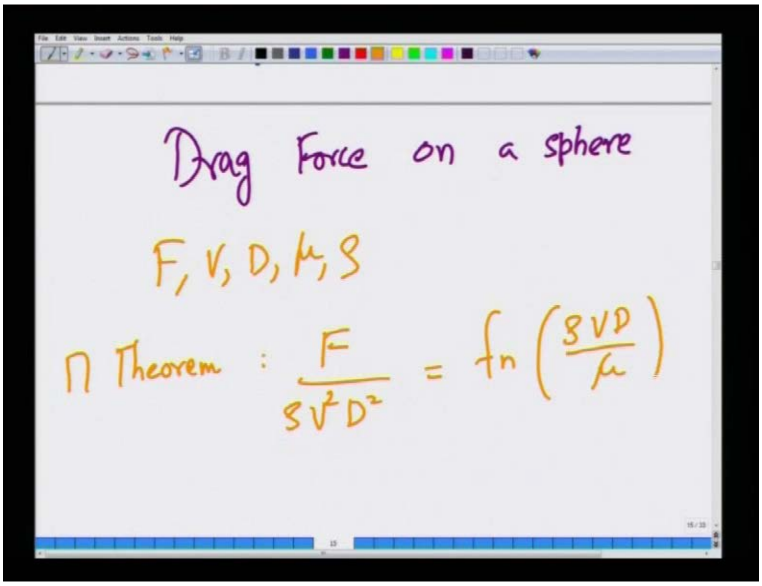

So, for drag force on a sphere we had five groups, five dimensional parameters upon using pi theorem, the pi theorem told us that, this can be written as F by rho V square D square is equal to a function only of rho V D by mu.

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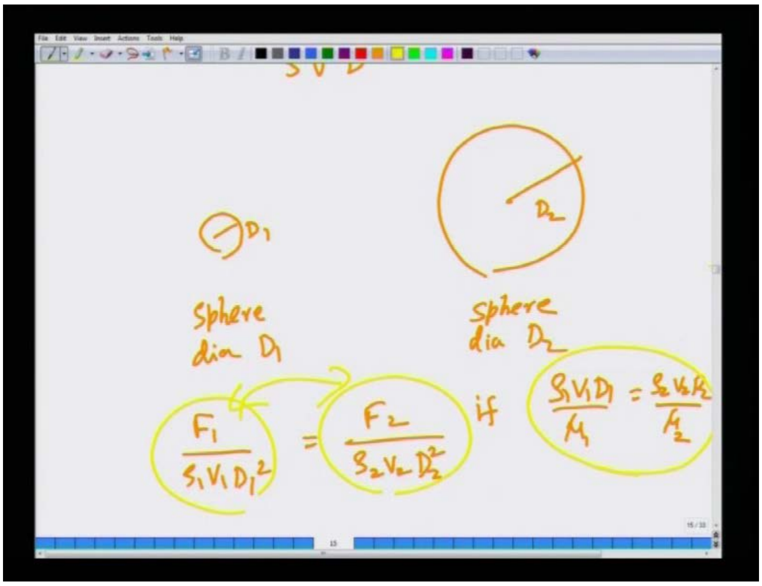

So, if you have two different spheres of diameter D 1 and D 2. So, both are spheres dia D 1, the diameter is D 2. So, this ensures geometric similarity then, we found that the forces D 1 square will be equal to F 2 by rho 2 V 2 D 2 square, if rho 1 V 1 D 1 by mu 1 is equal to rho 2 V 2 D 2 by mu 2. So, as long as you ensure the equality of Reynolds number between the two systems, the non-dimensional forces will be identically the same and this helps us in actually backing out forces, which are difficult to measure in the lab for a system for which you it is very difficult to measure forces in the lab.

We can relate that to another setting where in it is easy to measure the forces in a lab and then by suitably non-dimensionalzing we know that, the two forces non-dimensional forces are identical from which you can pack out the dimensional force. So, this is what I want to say about dimensional analysis. So, again to emphasize the power of dimensional analysis we told in the previous lectures that, there are three ways of analyzing flow problems in chemical engineering cross chemical process engineering; one is to use macroscopic balances or integral balances, another is to use microscopic or differential balances.

But, there are pros and cons, there is advantages and disadvantages of using macroscopic and microscopic balances because, while macroscopic balances are relatively simple, they need a lot of experimental data as a inputs, while microscopic balances while they are accurate, but they are extreme difficult to solve. So, there are this opposing and contrasting requirements or features of macroscopic and microscopic balances.

So, usually what is done in chemical process engineering in designing various equipments and unit operation is that, one often takes a course to experimental data and while doing experiments, the best way to understand and carry out experiments and understand them interpret them apply them is to use dimensional analysis for a variety of reasons that, we have been explaining in the last couple of lectures.

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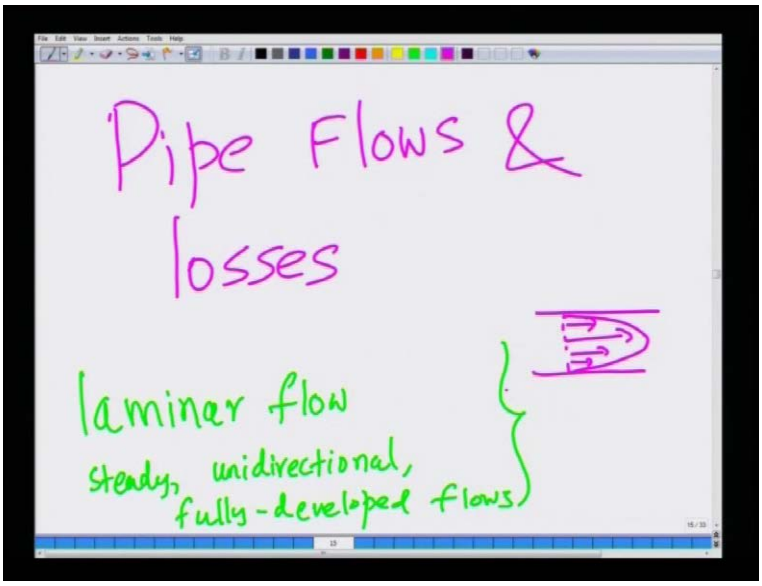

Now, I am going to go to a new topic that is pipe flows and losses, now we are going to focus on pipe flows and the losses encountered in pipe flows and so on. Now, so far what we have been doing is that, we have worried about laminar flow in a pipe; laminar flow in a pipe by laminar flow we mean simple steady flow, unidirectional flow and we have also restricted our attention to fully developed flows, under these restrictions we found the velocity in a pipe to be nice parabolic distribution with respect to the radial coordinate in a cylindrical coordinate system.

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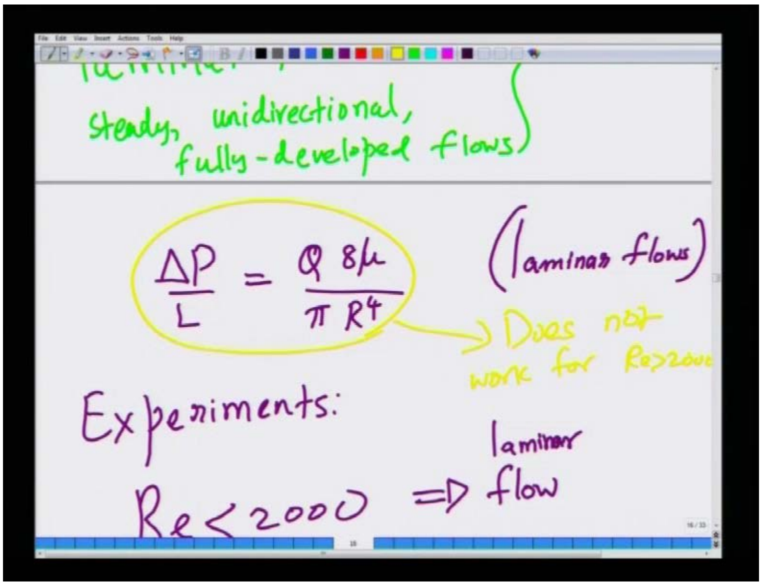

And using this we also derived the relationship between the pressure drop divided by the length to the flow flow rate is Q times 8 mu. So, this is valid only for laminar flows. And experiments tell us that experiments tell us that, when the Reynolds number is less than 2000 this relation is valid that is flow is laminar, but when the Reynolds number is greater than 2000, there is a transition to turbulence and this relation is not valid, this relation is not valid does not work for R e greater than 2000, but that does not that does not mean that, such situations are not encountered in industrial applications, in industrial applications one often sees that, the flows is in the turbulent regime.

Now, how are we going to then determine what is the pressure drop that is the required to make the fluid flow at a given flow rate in the turbulent regime, what are the options available for us we cannot solve this the differential balances, microscopic balances because, they are too complex, because turbulent flow as I have mentioned few lectures back is unsteady and it is three-dimensional.

So, in order to solve for a turbulent flow velocity profile you have to solve the Navier-Stokes equations completely that is a very very tall order that is a difficult task one cannot do that. So, one has to do, what is called one has to do experimentation. So, in order to do experiments to find out, what is the pressure drop in the turbulent region it is better we first write down the problem in a non-dimensional sense and express what are the important non-dimensional groups are characterized this problem.

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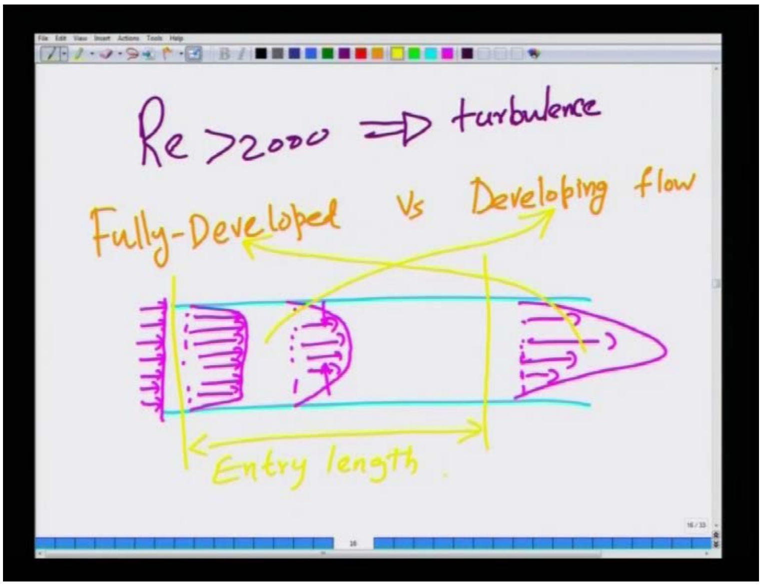

But before that, let me tell you a little bit about the fully developed flow assumption versus developing flow. Suppose, you have a pipe in which let us say the fluid is entering from the reservoir. So, initially you can assume the flow to be plug like that is the velocity is uniform, when it enters the pipe, the moment it enters the pipe, the pipe walls drag the fluid to 0 velocity. So, very close to the entry of the pipe, the velocity profile will be like this that is very close to the pipe walls, the velocity will be 0.

But, in the majority of the pipe, the velocity will be a constant plug like velocity, but as you proceed downstream the extent of the region in the pipe over which the velocity is nearly uniform will decrease and eventually and this happens this happens by diffusion of the momentum from along the radial direction by it just happens by momentum diffusion, momentum diffusions diffuses from regions of high shear rate to lower shear rate, high shear stress lower shear stress, shear stress are higher here. So, momentum diffuses in these two directions and it tends to finally, diffuse through the entire region of the pipe and you will get a parabolic velocity sufficiently downstream.

Now, the length required for this to happen is called the entry length and the velocity profiles in this entry length is called the flow is developing from an initial velocity profile, which is almost uniform to the eventual parabolic velocity profile, this is the fully developed profile, the parabolic velocity profile.

So, in any problem in any pipe flow problem, there will be a certain distance called the developing length or which the fluid velocity is developing from in its initial velocity profile at the entry to its eventual parabolic velocity profile and in this development length, the velocity profile is strictly not parabolic, it is in fact something else its very different and once you are away from the developing length, entrance length, then the flow eventually becomes parabolic.

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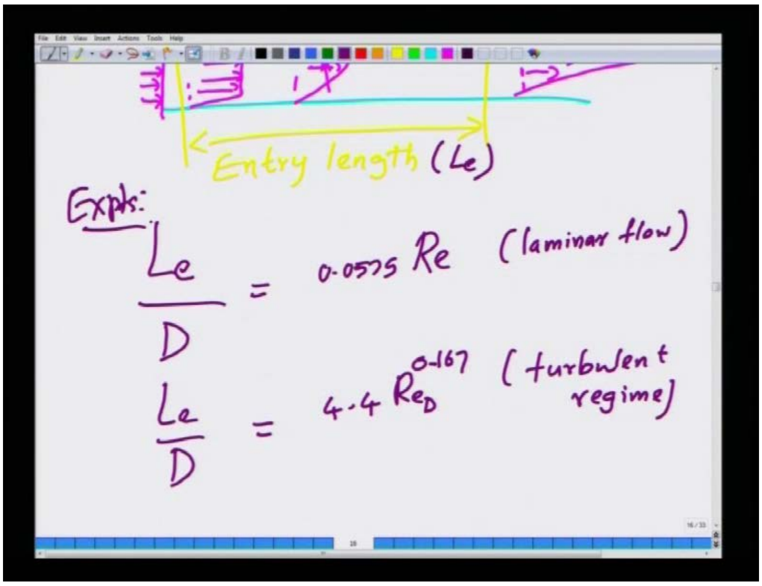

So, it turns out that the experiment tell us that, the entry length L e divided by the diameter of the pipe is approximately 0.0575 times the Reynolds number for laminar flow, that is as the Reynolds number increases, more and more length of the pipe more lengths of the pipes more distances required for the flow to become fully developed and and the same quantity for a turbulent flow is 4.4 R e D to the power 0.167 these are from experiments in the turbulent regime, these are experimental observations.

So, clearly there is a zone in the entry in the near the entry region of the pipe, where the velocity profile is not fully developed therefore, the parabolic velocity profile distribution is not valid in the entry of the pipe.

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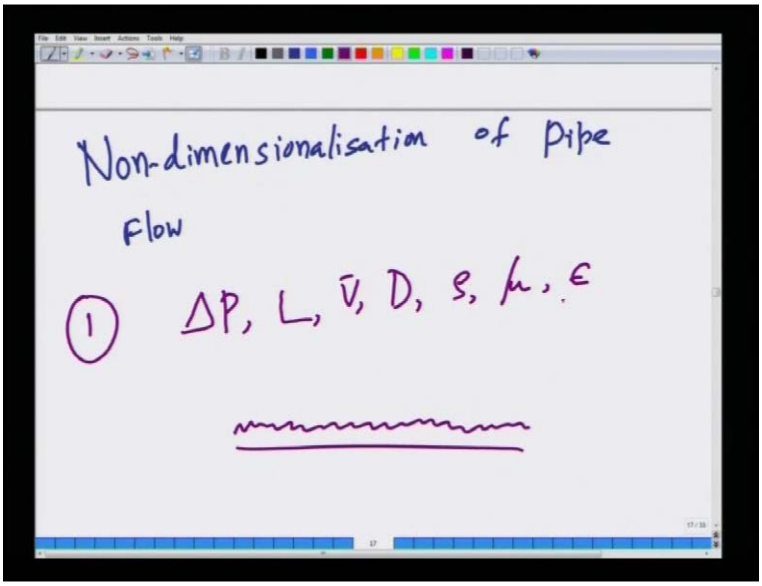

So, now let us proceed towards understanding the pipe flow problem in a non-dimensionlisation, non-dimensionlisation of the pipe flow problem. How do we do that? First, what are the variables? The variables are of course, you are interested in the pressure drop across the length of the pipe, L, the average velocity with which the fluid is flowing, the diameter, then the density of the fluid, viscosity of the fluid finally; the pipe walls will have some roughness and the root mean square. So, suppose you have pipe wall, the pipe walls are generally rough and the amplitude of this fluctuations is characterized by standard deviation and that is E, so this also the dimensions of length. So, these are pretty much what we can write down.

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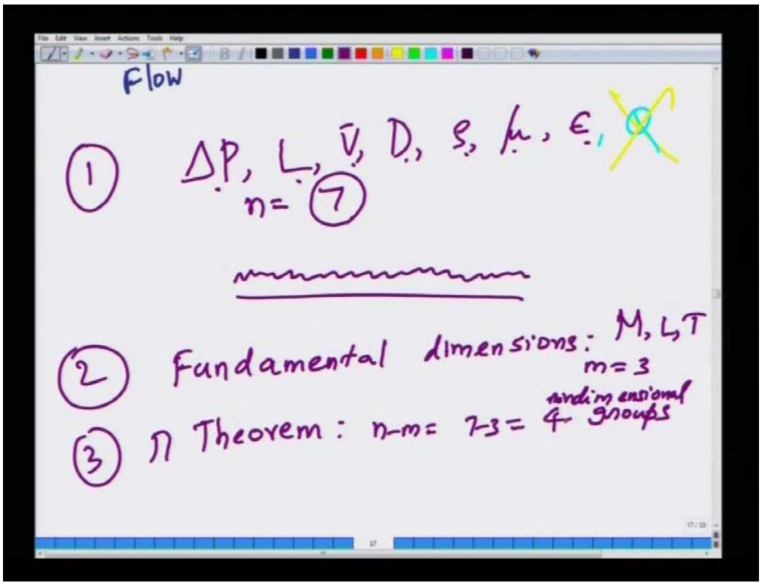

I am not writing Q because, Q is related the volumetric flow rate, because volumetric flow rate is related to the average velocity and the diameter of the pipe in a trivial way. So, you cannot over count variables like that, because once you have written the average velocity, then the volumetric flow rate and once you have included average velocity and the diameter of the pipe, then the volumetric flow rate is simply dependent variables on this two variables. So, there is no need to include such the redundant variables in our initial estimate of what are the relevant parameters of that affect the pipe flow problem and the fundamental dimensions are mass, length and time as in the most cases as in the most cases in mechanic.

So, pi theorem tells us that, there must be so there are 1, 2, 3, 4, 5, 6, 7 variables, so n is 7, m is 3, n minus m is therefore, 7 minus 3 is 4 non-dimensional groups, 4 dimensionless groups, non-dimensional groups.

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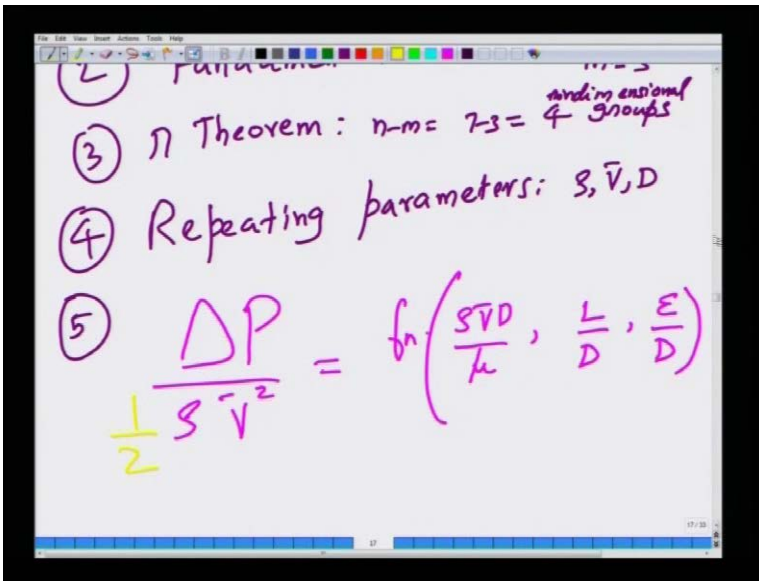

Now, we choose the repeating parameters as rho V bar D, where V bar is the average velocity of the flow in the pipe or the laminar flow in the pipe. Now, once you do that we can carry out the steps that I explained in the last lecture, I will not carry out those steps in detail here I will merely present the result.

Finally you will get delta P by rho V square is a function of rho V bar D by mu, the Reynolds number, the length of the pipe divided by the diameter of the pipe, then D non-dimensional roughness parameter epsilon by D, epsilon is a standard deviation of the root mean square fluctuations of the pipe roughness and it has dimensions of length. So, you non-dimensionalize that with the diameter of the pipe and traditionally just out of convention people put a factor of half here and this just historical practice. It is nothing we cannot say that, from the non-dimensional analysis just a matter of practice now this is valid always.

It is not a function of whether the flow is laminar or turbulent we have not assume anything like fully developed flow we have just merely did dimensional analysis we have merely done dimensional analysis on the pipe flow problem.

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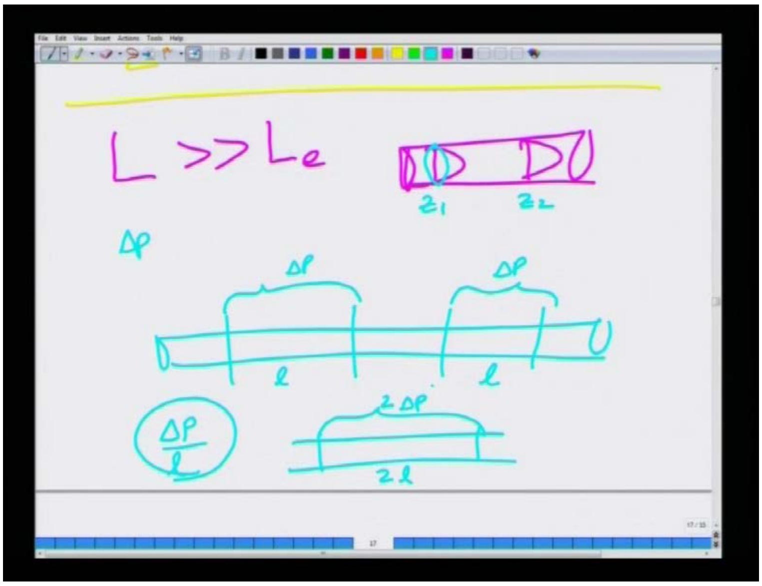

Now, suppose you want to specialize this now the length let us let us say the length of the pipe is very large compared to the entry length. That is now, the entry length is let us say just you know 100 of the length of the pipe. So, in a predominantly large region of the pipe lengths of the pipe, the flow is fully developed therefore, in such cases we can neglect the entry length and focus only on the fully developed flow region; in the fully developed flow region, the velocity profile is identical in two axial stations, the velocity profile at station x 1 is the same as z 1 is the same as velocity profile at station z 2.

So, the stresses encountered by the fluid due to the factor that as the wall present will also be identical therefore, the pressure difference across any two lengths of the pipes suppose I take a long pipe now I take a piece of length that is say small l and another piece of like downstream small l, if I measure the pressure difference, they will be identical at different sections of the lengths l.

So, I can say that therefore, in the fully developed region the it is it makes more sense for me to talk about delta P divided by l, rather than considered delta P and l separately this is because, if you consider a length of pipe of section 2 l, then the pressure drop across this will be simply twice delta P. So, it makes sense for us to worry about the ratio of the pressure drop across the length divided by the length itself, because that is a quantity that is constant and it is independent of where you choose to do the measurement.

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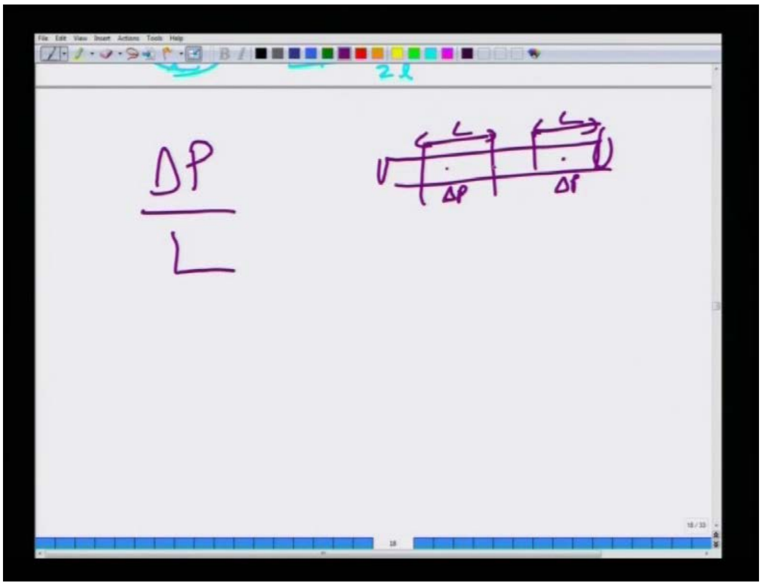

So, if you consider the fully developed section of a pipe, the relevant variable is not the pressure drop, there are no two independent variables pressure drop and length, but they always occur as pressure drop per length because, the pressure drop across different sections of the pipe will be exactly identical for different sections of the pipe will be exactly identical, if you consider a sufficiently a long pipe, whether length is large compare to entry length and if you consider a fully developed section of the pipe, the pressure drop will be because the fluid will see the same shear stress whether you are here or here. So, the pressure drop will be directly proportional to the length in the fully developed section of the pipe.

So, in our previous the general dimensional analysis without any assumption merely said this, but if the pressure drop is directly proportional to length, then this variable should come out, because we have said that in the fully developed region.

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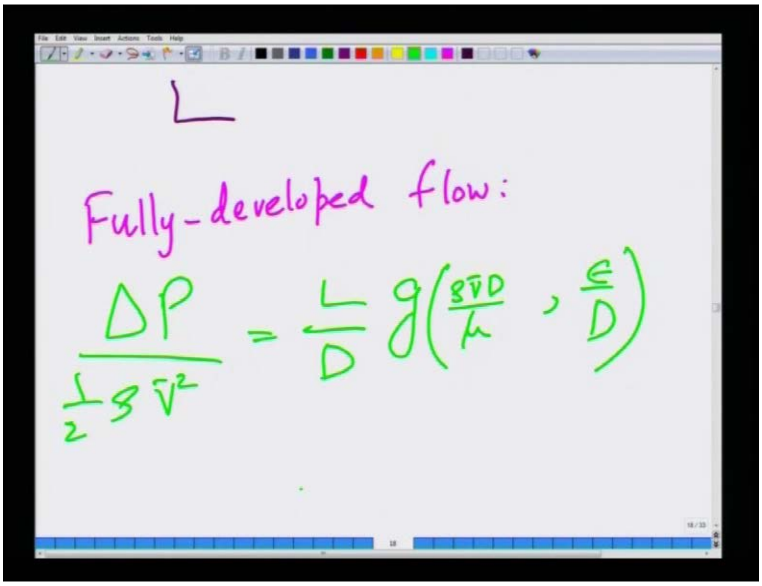

So, assume fully developed flow you will find that, delta P by half rho V square is L by D times some other function of rho V bar D by mu and epsilon by D.

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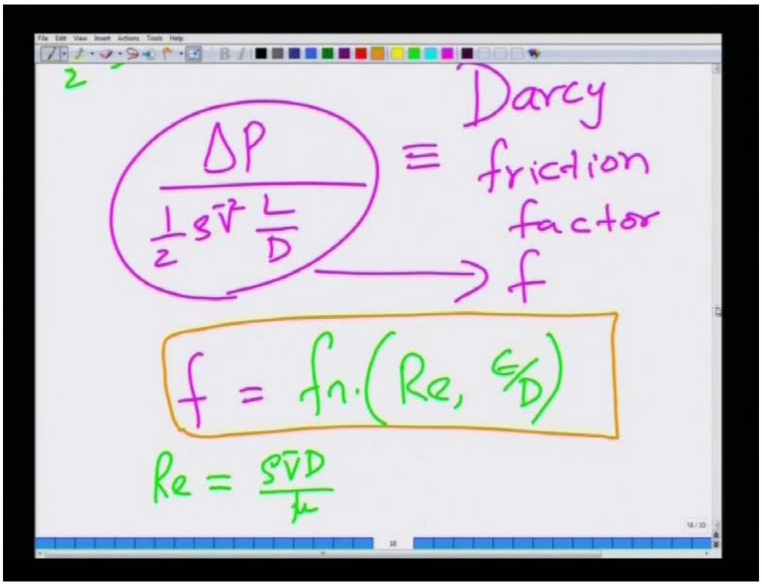

Now, traditionally this factor delta P now I bring this L by D to the denominator by half rho V square L divided by D this is called the Darcy friction factor. This denoted by the letter f just as the drag coefficient was a non-dimensional drag force, the Darcy friction coefficient friction factor is essentially a non-dimensional pressure drop in a pipe in a fully developed section of the pipe. So, this is denoted by the letter f. So, f is a function of R e and epsilon by D, where R e is a Reynolds number based on the average velocity of the pipe and the diameter of the pipe average velocity in the pipe and the diameter of the pipe. So, this is what dimensional analysis is telling is for the pipe flow problem that the friction factor is a function, the friction factor is essentially a non-dimensional pressure drop is a function only of Reynolds number of flow as well as the pipe wall roughness.

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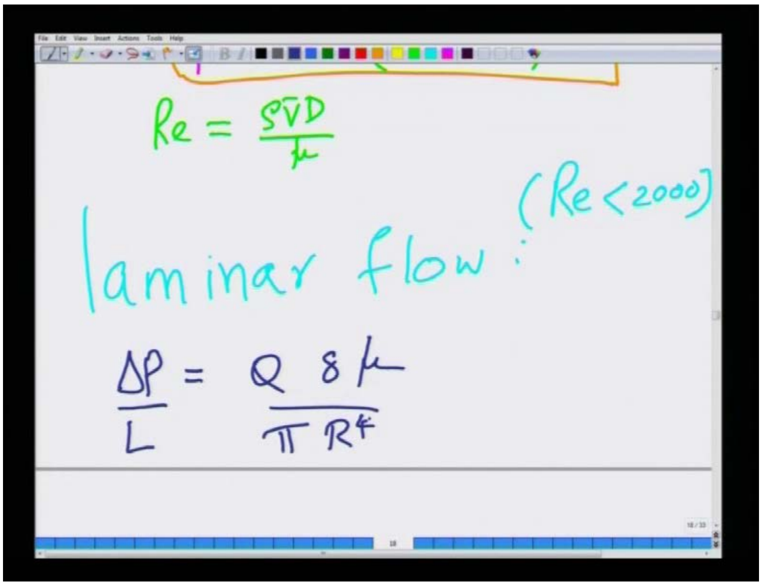

Now, for laminar flow we know from experiments at laminar flow is valid, when Reynolds number less than 2000. For laminar flow we already know, what is the delta P, how it is related to the average velocity, delta P is simply is equal to 32 let me just write down delta P in terms of flow rate first. So, delta P we wrote delta P by L is Q 8 mu by pi R to the 4.

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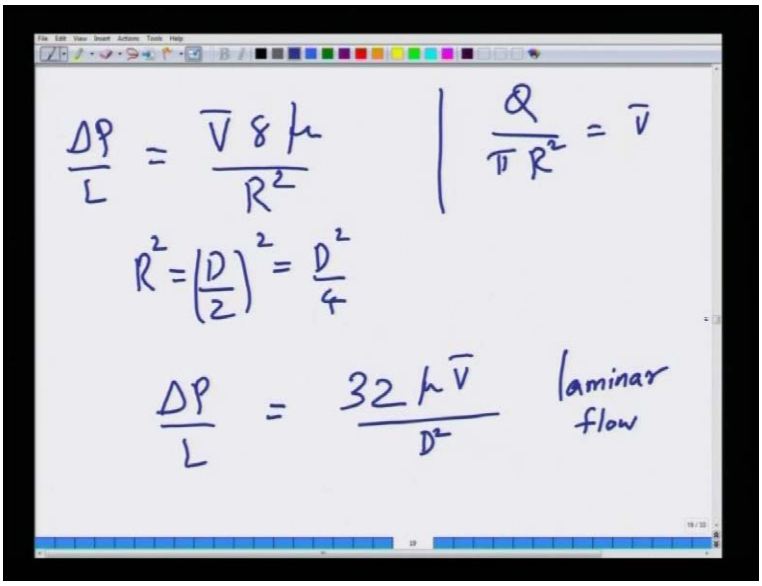

Now, I want to write in terms of average velocity, Q by pi R square is the average velocity. So, delta P by L is nothing but, V bar 8 mu by R square, now R is nothing but, D divided by 2, R squared is D by 2 whole square, D squared by 4, this implies delta P divided by L is nothing but, delta P by L is nothing but, 32 this is V bar is the average velocity that is V bar 32 mu V bar by D squared. So, that is the relation for laminar flow between the pressure drop and the average velocity, this is true of laminar flow.

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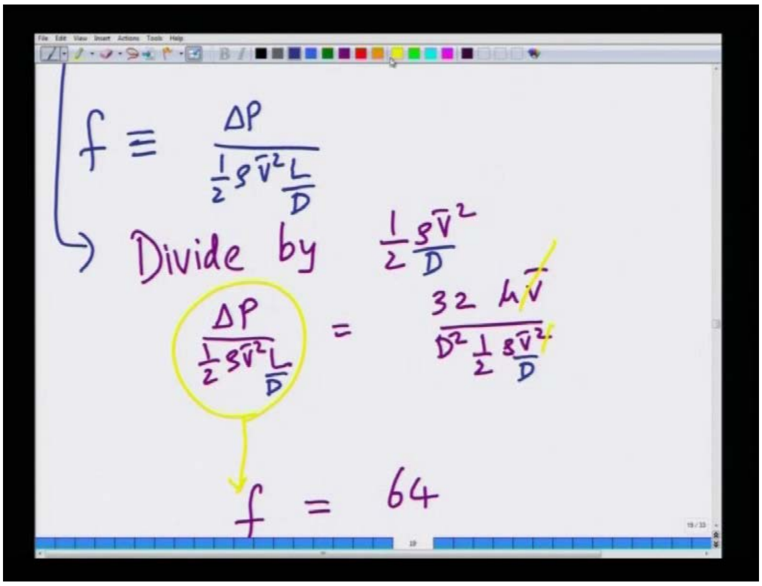

Now, the friction factor is defined as for in the general definition of friction factor is delta P by half rho V bar square L. So, I will divide I will take this equation and then, I will say divide both sides of this equation by half rho V square. So, on the left side I will have delta P by half rho V bar square L which is nothing but, the friction factor.

On the right side I will have 32 mu V bar by D square half rho V bar square this is nothing but, this expression is nothing but, the friction factor the Darcy friction factor is equal to for laminar flow we take this 2 above it becomes 64 we cancel a factor of V with the dominator and a cancel and then, there is also rho V square L by D, if you remember the definition of friction factor, so you have L by D. So, we have to divide by rho V squared by D on both sides, so this is L by D. So, you will have another D here and this factor of D square will go away with the factor of D here and leave you with D.

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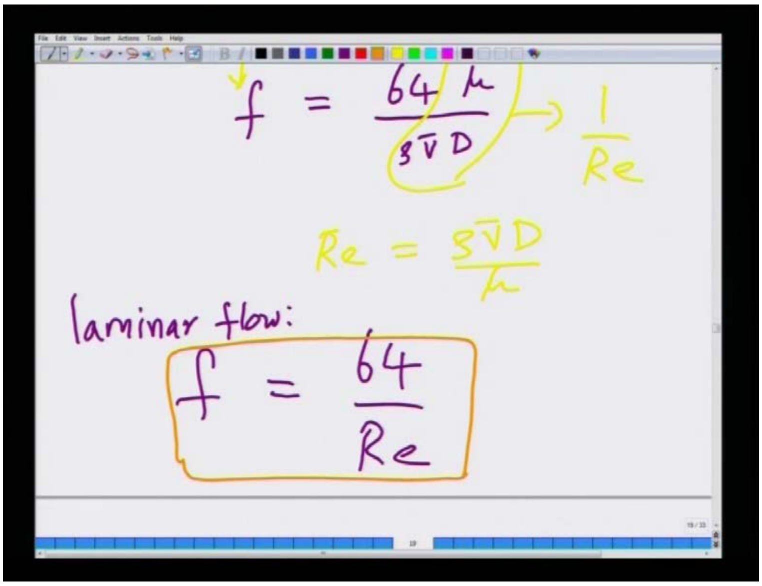

So, final answer will be you will get 64 mu by rho V bar D, but this is nothing but, the Reynolds number or 1 over Reynolds number, because Reynolds number if you remember is rho V bar D divided by mu. So, the friction factor, the Darcy friction factor for laminar flow, for laminar flow the Darcy friction factor f is nothing but, 64 by R e. So, this is an important result, this is not a new result this is merely a restatement of our old result that, the pressure drop how the pressure drop and flow rates are related for laminar flow, which we obtained by solving the Navier-Stokes equations after making suitable assumptions.

Now, we are merely repackaging that expression that result in terms of the friction factor. Now so, for laminar flow we know what the friction factor is going to look like it is 64 by R e. Now, I want to make a comment on the definition of friction factor the friction factor that we have defined is called the Darcy friction factor, there is another friction factor called the fanning friction factor, which is slightly different. So, let us call it f fanning essentially you will have instead of half here you will have I think 2 there. So, instead of half here you will have 2.

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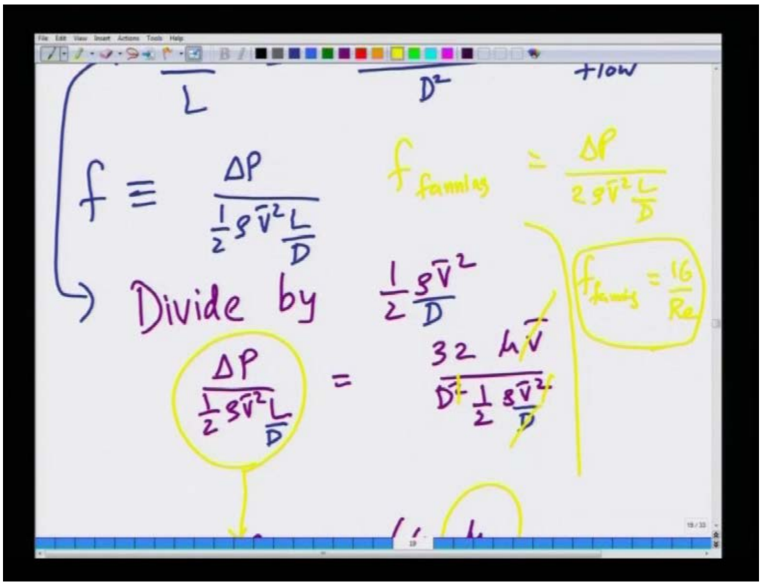

So, the fanning friction factor let me write it here is delta P by 2 rho V bar squared L by D, if I do the same thing for both sides of this expression for the laminar flow expression you will get f fanning is equal to 16 by R e. So, this is something that you have to keep in mind whenever you read textbooks or whenever you look up data you have to first understand the definition of friction factor because, there are these two commonly used friction factors in engineering literature the Darcy friction factor is defined in this fashion, fanning friction factors defined slightly differently and they are I mean it just a definition, but one has to be aware of that otherwise, the answers will be very difficult.

For example, the Darcy friction factor goes as 64 by R e for laminar flow, while the fanning friction factor goes at 16 by R e for laminar flow that just an aside. So, be aware of that. Now, let us stick to Darcy friction factor for the moment, there is nothing that is

(( )) to pick one or the other you can pick either one. So, let us pick the Darcy friction factor.

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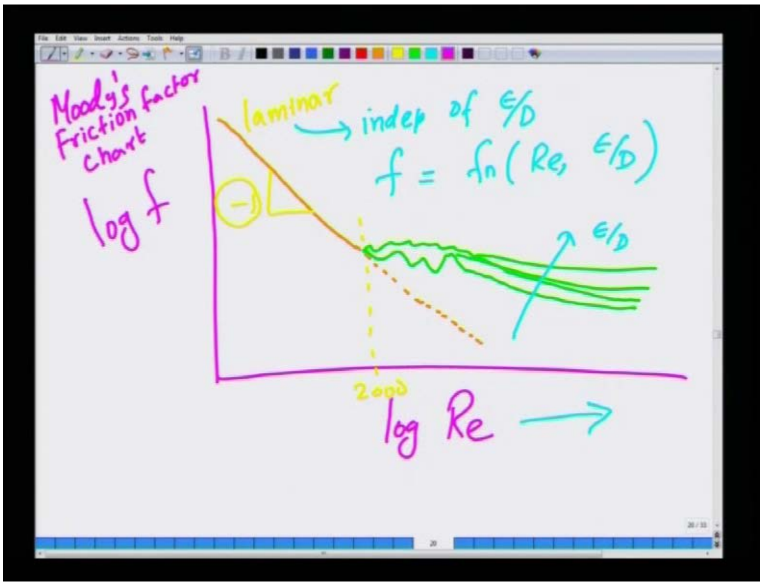

Now, if I aware to do experiments and plot in a double logarithmic plot log f verses log R e you know that, at low Reynolds number Reynolds number less than 2000, the laminar flow the flow is laminar. So, the friction factor goes as 64 by R e, if I take log on both sides it becomes a straight line in the double logarithmic plot. So, the slope of this line will be minus 1. So, this is the laminar flow region, but near about at Reynolds number around 2000 the actual curves, so let us extend this curve in this fashion, but in reality what will happen is the following? The flow becomes turbulent and the friction factor no longer becomes 64 by R e and it takes a completely different value.

So, this is a purely experimental result, if you were to do experiments at various Reynolds numbers that is at various velocities and measure the pressure drops at various velocities in a pipe then and then reformulate them and re-plot them in terms of a friction factor versus Reynolds number, which is a dimensionless representation of the same data then, what you will find is that when the Reynolds number is less than 2000, the friction factor in fact agrees with the laminar flow prediction, which is obtained from the solution of Navier-Stokes equations that is, it is actually 64 by Reynolds number, if it is the Darcy friction factor, but at Reynolds numbers greater than 2000, there is a complete deviation of the theoretical prediction for laminar flow from reality because, the flow has undergone a transition from laminar to turbulence.

So, this is something that is very important to understand that, the friction factor is merely a non-dimensional representation of the pressure drop in a pipe and the friction factor is a function of two non-dimensional variables, the Reynolds number and the wall roughness, epsilon by D. So, it turns out that for different wall roughness you have different curves for the turbulent flow, but in the laminar regime, the curves are independent of the wall roughness. So, the laminar flow turns out experimentally that, the friction factor in the laminar flow regime is independent of the roughness ratio epsilon by D, while in the turbulent regime you will have different curves for different values of the roughness parameter epsilon by D.

So, this is essentially what dimensional analysis is telling you that is f is a function of R e and epsilon by D. So, you are varying R e in the x axis if the different values of epsilon by D you get different curves in the turbulent regime, but they all collapse to the same curve that 64 by R e in the laminar region. Now, this is a very very important input in designing pipe line pipe flows and pipe line networks in many chemical process engineering applications because, suppose you want to know, what is the pressure drop that is required to make the fluid flow at a given flow rate then, in order to do that calculation it is very simple in the laminar flow regime, because you simply have to use the formula that we have already derived.

But, in the turbulent flow region you do not have a analytical expression all we have is this friction factor charge sometimes this is also called the moody chart, moody's friction factor chart chart. So, if you ask the question, what is the pressure drop that is required to make the fluid flow at a given velocity first thing you have to check is the Reynolds number, if the Reynolds number is less than 2000 you already have analytical expression based on the solution of Navier-Stokes equations, but if the Reynolds number is greater than 2000 that solution breaks down, because the flow as undergone a transition from lamina flow to turbulence and one has to use this friction factor chart to answer the question.

So, if you want to if you want to push the fluid with the given flow rate first thing to do is to convert the flow rate to average velocity by dividing the cross section area of the pipe, then calculate the Reynolds number, then use this friction factor chart and you should also know what is the roughness one should also be given the data for what is the roughness for the pipe, then you should look up the appropriate curve and find out the friction factor from which you can back calculate what is the pressure drop that is required to make the fluid flow in the turbulent regime. So, this friction factor chart is a critical input in many many many engineering applications involving pipe flows.

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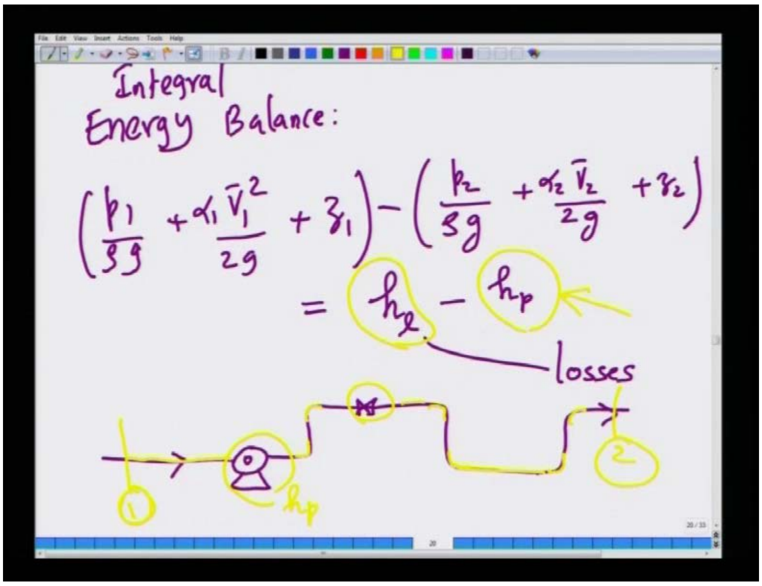

Now, we also discuss the energy equation, where we said that suppose you had this energy equation, integral energy balance. So, you have p 1 by rho 1 I am sorry p 1 by rho g plus you have an incompressible (( )). So, rho is constant alpha 1 V 1 square by 2 g plus z 1 minus p 2 by rho g plus alpha 2 V 2 square by 2 g plus z 2 is equal to many losses that are present minus the work done on the system on the C v by way of pumps or something like that.

So, you could have you could for example, have pipe line network involving a pump valves, bends and so on, that is trying to make the fluid flow at a particular flow rate. So, in principle you could write the integral energy balance between station 1 and station 2 and these losses will comprise of losses that involve flow through the pipe, then flow and then, there are bends which will have additional losses and you also have a pump through which you are putting energy in to the system constantly.

So, the pump will involve. So, the here is the work being continuously done on the C v through a pump through a shaft work. So, that will also be have to be taken into account. Now, these losses through straight sections of the pipes are calculated using friction factor charts because, if you have a straight section of the pipes the only loss that is involved is the viscous losses due to flow in a pipe, if it is laminar it is very simple, but if it is turbulent you have to use the friction factor chart. So, the first step for us is to relate the loss in a straight section of the pipe to the friction factor itself. So, that what I will do first and then, I will generalize the losses to include other losses in the in the system.

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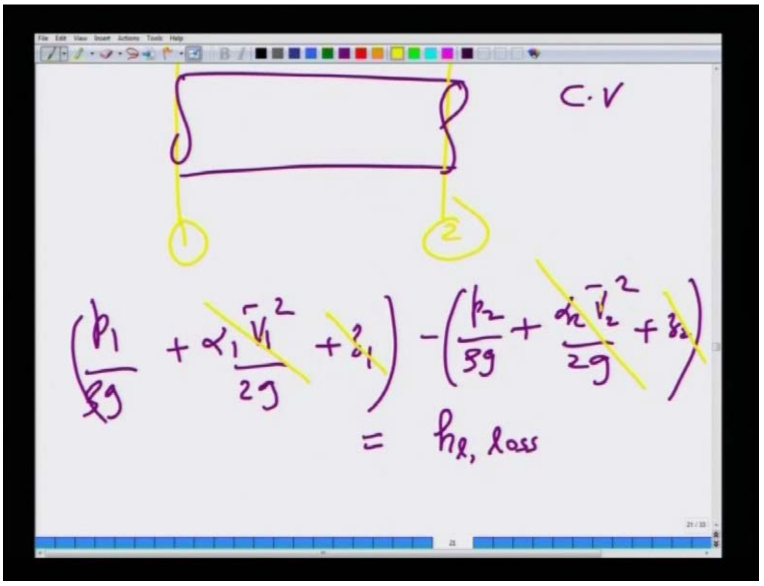

So, suppose you have a straight section of pipe, this is my control volume. So, I was straight section of pipe and I have control volume and I have station 1 and station 2, I want to write the energy balance between these two points. So, I have p 1 by rho g plus alpha 1 V 1 squared by 2 g plus z 1 minus p 2 by 2 rho g plus alpha 2 V 2 square by 2 g plus z 2 is equal to the only loss is the head loss in the pipe and that can be obtained from friction factor charts as I will just show you. Now, let us assume that the pipe is horizontal, so z 1 is z 2. Now, the velocity is the pipe is also of constant cross section. So, mass conservation will mean that V 1 is equal to V 2.

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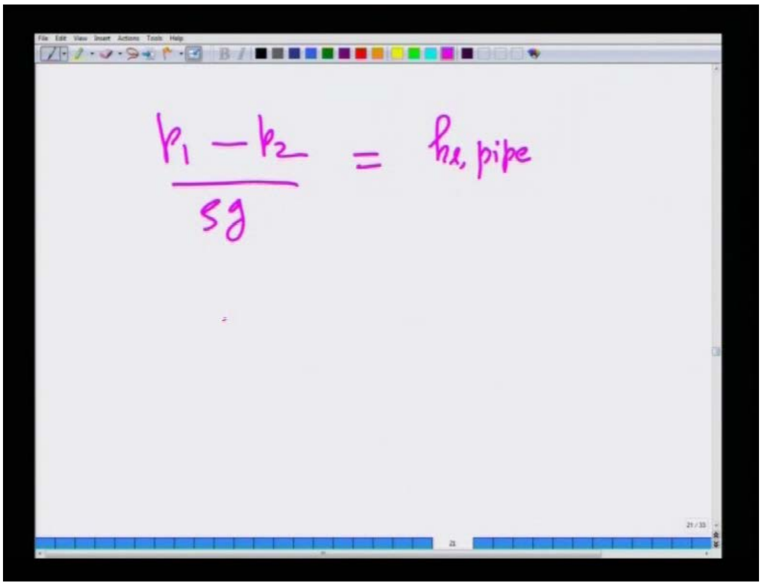

So, which implies that p 1 minus p 2 by rho g is h l in the pipe, the head loss in the pipe. So, let us write it as head loss in the pipe, now p 1 minus p 2 is delta p.

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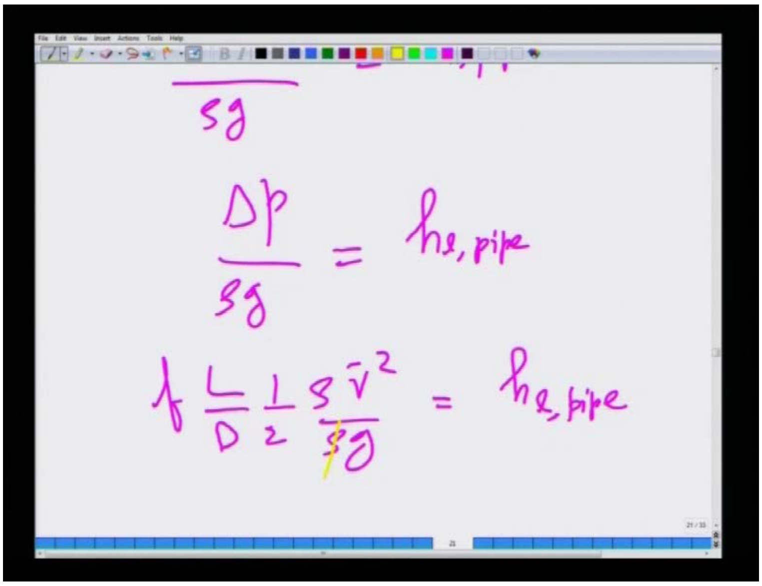

So, delta p by rho g is the head loss in the pipe, now I want to write delta p in terms of the friction factor. So, I will write this as f times L by D half rho V square by rho g after using the definition of friction factor is the head loss in the pipe. Now, I can strike off this factor this rho to give, what is the head loss in the pipe in terms of friction factor.

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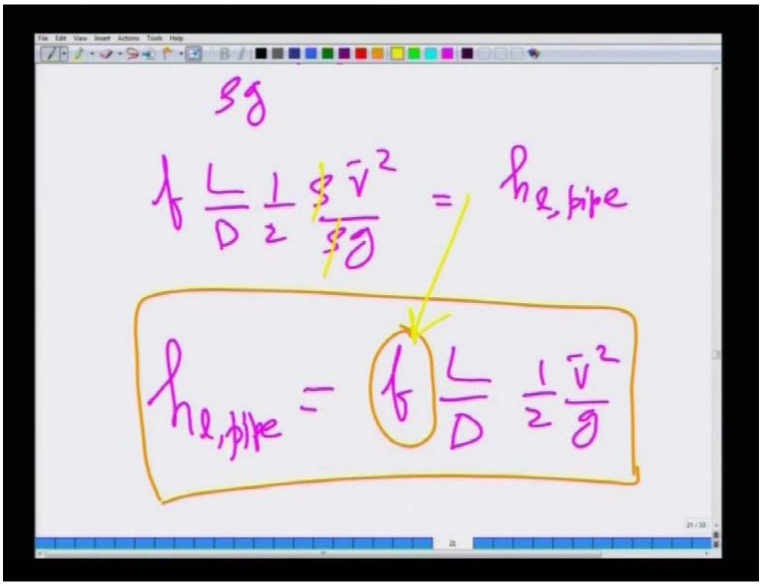

So, this means h loss in the pipe for a straight section of a pipe is simply f times L divided by D times half V square by g. So, if you tell me what is the friction factor, if you give me this input I can tell you what is the head loss that will happen in when fluid is flowing in the straight section of the pipe, now, we can actually build in more and more things now all we have done is to say that, when you have a complex piping network like this.

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You have many sections in which the pipe is straight, there is a straight section here, there is straight section here this, and there are many sections in which pipe fluid is flowing in a straight section of the pipe. In all these straight sections of the pipe, there will be viscous losses and therefore, you must compute those viscous losses by using the friction factor.

In the next lecture I am going to include the other losses such as losses through flow through valves, bends and so on. And also include the pumping head and then, that will lead us to an expression which will helps us solve say several problems for example, we could ask what is the rate at which you should do work on the pump in order in order that the fluid is flowing from 0.1 to 0.2 all these can be answered using the macroscopic integral energy balance with the aid of losses information from friction fraction charts. So, we will meet again in the next lecture and develop further.