Fluid Mechanics
Indian Institute of Technology, Kanpur
Prof. Viswanathan Shankar
Department of chemical Engineering

Lecture No. # 3

Welcome to this third lecture in this NPTEL course on fluid mechanics for under graduate students in chemical engineering. In the first two lectures, we introduced this course by telling you how chemical engineering is important, how fluid mechanics is important in chemical engineering in chemical process industries, and why chemical engineers have to have a thorough understanding of fluid mechanics in order to design many process many chemical processes better.

And we also told you the approaches that one normally takes in understanding fluid flows for chemical engineering applications. We said that there are macroscopic approaches where in one writes integral balances of mass, momentum and energy, and then there are microscopic approaches where one derives differential equations that are valid at each and every point in the flow.

And then when these two approaches are not workable in a very very complicated industrial setting, there is also the need for experimentation and there we said that dimension also will play an important role in using in properly designing experiments and using those experimental results for scaling up of results to from a lab scale to industrial scale.

So, after introducing the course, we also gave a brief outline of what are all the topics that we are going to cover in this course, and after that we introduced the system of units that we are going to follow; SI units that we are going to follow the SI units in this course which is the conventionally accepted units everywhere in this scientific world.

And we also told that if there are other systems of units that are being used in some context. Then, there are ways of converting from SI units to the other units if you need to convert them.

Finally, in the last lecture, we started a discussion on the frame work that we are going to use to analyze fluid flows in chemical process industries and that frame work is called the continuum approximation. ((No audio from 02:23 to 02:39))

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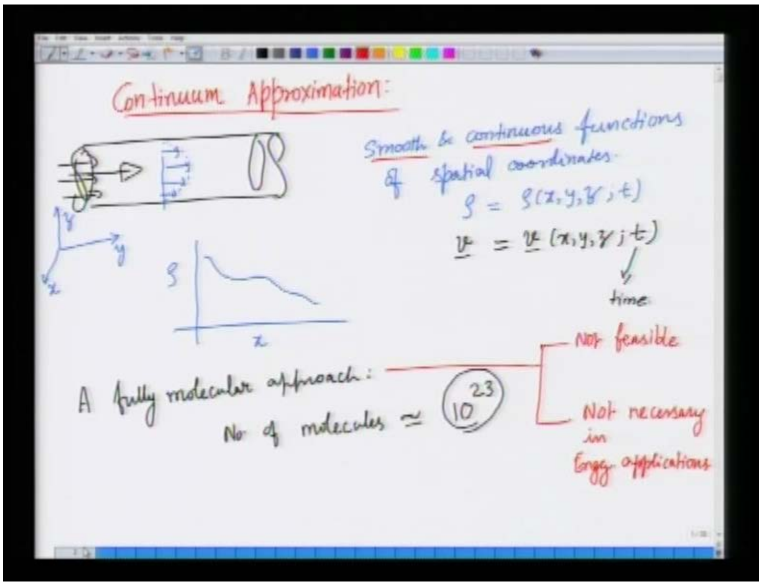

Continuum Approximation:

Smooth & continuous functions of spatial coordinates.

$\rho = \rho(x, y, z; t)$

$\underline{v} = \underline{v}(x, y, z; t)$

time

A fully molecular approach: No of molecules $\simeq 10^{23}$

Not feasible

Not necessary in Engg. applications

It is called the continuum approximation. What this approximation says is that in a flow for example, if you are interested in flow in a very long pipe just schematically shown. So, there is flow. You have flow in very long pipe. In principle, that continuum approximation says that you can define velocity at each and every point in the flow.

The continuum approximation says that you can define velocity, density, pressure or any fluid property, temperature as a smooth and continuous function of spatial co-ordinates. We will see that in order to analyze any problem in fluid mechanics, you have to first set up a proper co-ordinate system. For example here, a simple rectangular co-ordinate system. So, here for example, if you have density, density is a smooth continuous functions of x, y, z that the three spatial co-ordinates in the rectangular co-ordinate system and time.

So, the key thing is, it is a smooth function, continuous function that is defined at each and every point. So, all these quantities; velocity is a vector. It has both magnitude and the direction. Velocity is also a function of all the three spatial co-ordinates and t is time.

So, what the continuum approximation says is that you can plot all these properties if you take density as a function of x which says that you can plot as a function; it is a smooth function. So, the question is first of all, why is this approximation being made?

We saw that a fully molecular approach that one can conceive because we said that a fluid ultimately beat a liquid or a gas is ultimately comprised of molecules. So, one is tempted to first go for a fully molecular approach.

So, even if you consider any reasonable macroscopic volumes that are encountered in any practical application, you will have the number of molecules to be a very large number. It is of the order of Avogadro numbers 10 to the 23 in a very reasonable volume in any practical applications. So, this is a huge number. So, it is not possible to solve the motion of all these huge number of molecules to obtain for example, quantities of practical interest towards, such as forces that are experienced by a the walls of the pipe and so on.

For example, if you want to pump a fluid in this pipe, the reason why you need a pumping cost is because of the drag force exerted by the surrounding walls on the fluid flow. So, if you want to estimate or predict the pumping cause, you have to know you have to have an idea of these forces. So, ultimately these forces are due to the molecular interactions that are there, but it is simply not possible to compute the forces exerted by each and every molecule on the surface of the wall because simply because of the huge number that is present.

Even if you have that information ready, even if you have to solve for all these forces, such an information is not really required because all we require is the total force and we are not overly concerned in engineering studies has to a which molecule is exerting a force. All we are interested in design is the force that is being experienced by a solid surface for example.

So, a fully molecular approach we said in the last lecture is not feasible generally and even if it were there to be feasible, it is not necessary in most practical applications, in engineering applications.

So, the fully molecular approach is not necessary of feasible. So, we rule that out. Then the only other option is this continuum approximation where we completely ignore molecular details and treat the fluid as a continuous medium wherein with respect to a co-ordinate system, at each and every point in space, various quantities such as density, pressure, velocity, temperature, concentration; all these are defined at each and every point. And therefore, you can and these are assuming to this smooth and continuous functions of all these spatial co-ordinates and time. And this is the continuum approximation.

Now whenever we make an approximation on hypothesis, it is first useful to see the domains of validity of the hypothesis. So, how do we go about doing that?

Let us take again this case of a pipe flow.

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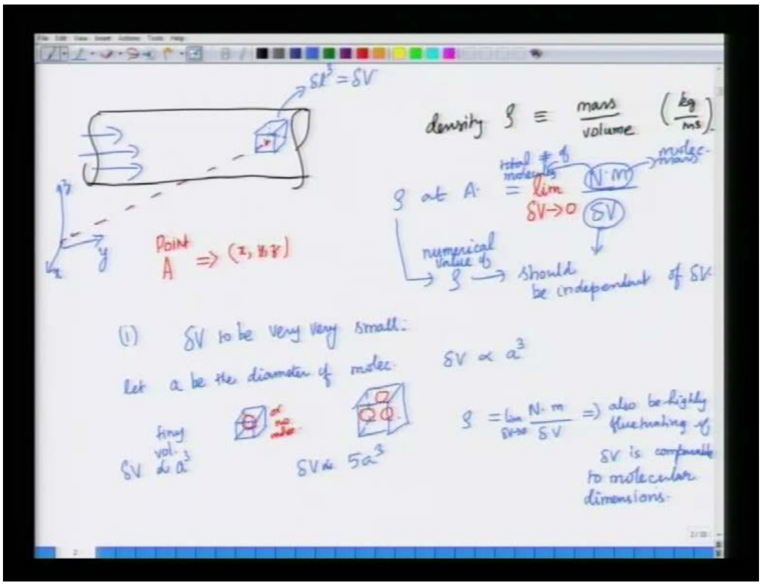

We have this pipe which is very commonly encountered in chemical industry, and let us say that we are interested in suppose we are interested in defining the density of the fluid at each and every point in the pipe. So, we first put a co-ordinate system. This is called a rectangular co-ordinate system. There are three mutually perpendicular axis.

So, you want to define in the continuum approximation within the continuum approximation; quantities like density at each and every point in the pipe. What do you mean by density at each and every point in the pipe?

Well, let us take a point. This point a is labeled by the co-ordinate x y z for example, if you have to construct a position vector. So, it will have a label x y z. So, you want to tell you want to see what is what do you mean by, we want to understand what do we mean by density at this point. So, how do we calculate density at a point? Well, a point really has no volume. So, we cannot dens… First before I say that, first what is density?

A fluid density, as we all know is denoted by the greek symbol rho is mass per unit volume of the… So, you take a particular volume, and measure the mass of the fluid present in the volume, divide the mass with the volume. So, this is the density, and in SI units, it is k g per meter cube. Those are the units.

So, here we are worried about density at a point. A point has zero volume. If there is zero volume, then there is zero mass. So, that does not make sense. So, what we mean the density at a point is that, you take a point, let us say a point here just for clarity, you take a point and construct a tiny volume around it. You construct a tiny volume. For simplicity, I am constructing a cubic volume. So, this cube has side delta l cube. So, that is the volume; delta v. So, you take any point in the pipe. So, that is fluid flow in the pipe.

You want to understand for simplicity let us say, density at a point. So, what is density at point a? So, let us call this a just because we are changed at. Let us call this point as a. So, density at point a, what is the density? Well what you will do is you will count the number of molecules in that volume delta v. So, we will take the delta v which is basically delta a whole cube in the denominator. You will count the number of molecules. Let us call it as n; capital n times the mass of each molecule. So, this is the total mass of, this is molecular mass. This is the total number of molecules in that volume. So, this will be the density at point a.

So, but, since we are saying the density associated with the point, we have to be careful in saying that this is in the limit as the volume delta v shrinks to a point; that is, tends to zero.

So, this is the way in which density is defined formally in a continuum approximation the to take a point, construct a tiny volume, count the mass or count number of molecules and find the mass, divided by the volume

Now, clearly we can up take. So, firstly, in order for this to make sense, this delta v; the definition a not for this definition to make sense, this density that we obtain, the numerical value should be independent of delta v. That is what you should we should expect because if we find if you calculate density at a point by considering one delta v, and if I calculate the density by considering a slightly different delta v, and if we come up a different answers, then density is not uniquely defined at that point.

So, in order for the continuum approximation to make sense, the density; the numerical value of density that we obtain should be independent of delta v. Now if that is the case, let us take the extreme cases. If that is the expectation that we have, then let us take delta v to be very small.

Now what do we mean by very small? Well, we know that a fluid is ultimately comprised of molecules. So, let us take, let a be the diameter of the molecules; that compresses the fluid. So, first is, consider delta v to be very small. If a be the diameter of the molecules, and delta v will be roughly proportional to a cube, there will be some numerical pre-factors. Those are not of importance to us in our argument.

So, if delta v is proportional to a cube, then you a have a very tiny volume that is proportional to, tiny volume that is comparable to molecular scales. So, if you consider a such a tiny volume, this is a tiny volume proportional to molecular volume, then at a molecular level, the fluid molecules are not stationary or static. They are at any non zero temperature, owing through thermal motion, they will be rapidly moving at very large velocities.

So, the presence or absence of a molecule is a purely probabilistic event. So, if you consider a volume at a if you take a point and construct a volume about that point and let if that volume is comparable to molecular dimensions, then whether there are molecules are not itself is a probabilistic event. So, this is true, even if the molecules, sorry, if the volume size is slightly larger, even if it is let us say, five times the molecular diameter. I mean just for the sake of argument.

If delta v is proportional to five times a cube, even so, then, the number of molecules that are present; so, here there could be either one molecule or no molecule. Here there could be let us say few molecules; three. If you have to consider the same volume element around that point at some other later time, this could have no molecules at all. So, the number of molecules in a very tiny volume that is proportional to the molecular dimensions is a highly fluctuating quantity.

Therefore, the density that you will get, which is the number of molecules times the molecular mass divided by delta v, in the limit delta v goes to 0, this will also be highly fluctuating, if delta v is comparable to molecular dimensions. So, clearly there is a lower limit as to how small your delta v can be because if your delta v is too small density, then t is not uniquely defined because it will fluctuate very wildly.

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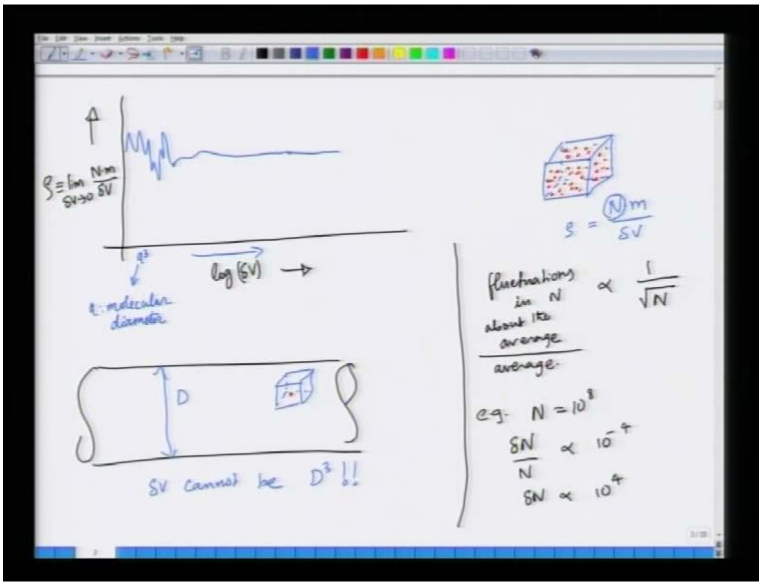

So, in the next in the next slide, what I am going to do is to plot, if you have to do this as a thought experiment, where you plot density which is defined as limit delta v going to 0 divided by delta v, now, since I am going to discuss wide variations of delta v, it is useful to plot density as a function of a logarithm of delta v in the x axis, and I am plotting the density in the y axis. Now what will I get? Now when delta v is proportional to a cube, very small, where a is the molecular diameter, then density will be a widely fluctuating quantity for the reasons I just mentioned.

Now as delta v increases in this axis, now; that means that your volume about that point is increasing larger and larger. So, the density that is defined as number and small molecular mass divided by the delta v, we will slowly settle to a constant value because if the volume becomes larger, a number of molecules that are present, which I am going to indicate now by several red dots, even though it is still a fluctuating quantity, whether you add one more or one less, should not largely affect the numerical value of density.

So, even though n is still fluctuating, the fluctuations cannot play a big role in defining the numerical value of the density.

So, as you increase delta v, you will find that the density will eventually settle to a constant value. Now this is precisely because of the fact that fluctuations in n about the mean, about the average, they are proportional to 1 over root n. This is a fundamental result and statistics and statistical mechanics. This goes by the name of central limit theorem.

So, if the fluctuations in the number of molecules in a volume, if they decrease as the volume becomes larger and larger and it decreases in the following way, decreases 1 over square root of n. So, if n is of the order of 10 to the 8, then the fluctuations will be of the order of 10 to the minus 4.

So, it means that it is so small. This is the fluctuations in the average relative to the average. So, divided by the average itself. So, if n is 10 to the 8, and let us define the fluctuations with delta n, then delta n by n will be proportional to the 10 to the minus 4 or delta n itself will be of the order of 10 to the plus 4 because n is 10 to the 8, example.

So, fluctuations are small when the size of the volume becomes larger and larger, but can we take the volume to be arbitrarily large. The answer is no because eventually, we remember the context was we to a pipe in the last slide, and we took a small point, construct a tiny volume about that point, and we are interested in obtaining a point wise value for the density.

Now clearly and trivially speaking, if this is the diameter of the pipe, your delta v cannot be proportional to or comparable to d cube because in which case, then there is no point. We cannot speak sensibly of a point wise variation of density. So, clearly there are restrictions as to how long, how large the volumes can be and how small the volumes can be.

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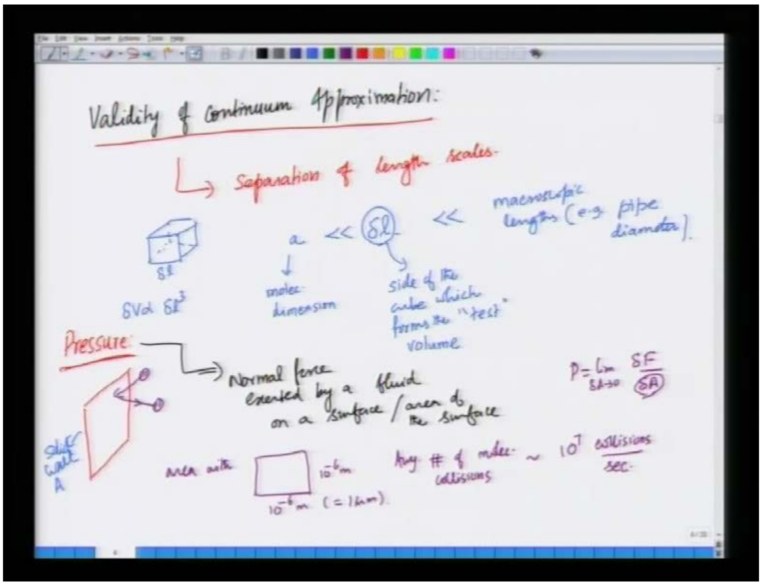

So, the continuum approximation works when we say… So, let us say validity of continuum approximation. ((No audio from 20:40 to 20:49)) It works well when there is a, we say separation of length scales. So, if you are if you take a point and construct a cube of size delta l, the volume is proportional to delta l cube.

Then, continuum approximation where in you can propose point wise variations of, you can define each physical properties at a point wise is valid, when delta l which is the side of this cube which makes the volume is large compared to the molecular dimension. This is the molecular dimension, molecular diameter. This is the side of the cube which forms the volume; the test volume.

And this should again be further; this delta l must be small compared to macroscopic lengths. For example, the pipe diameter in our illustration. So, this is when the continuum approximation is valid.

Now, this is not true only for density, even if you were to consider… so, we did this very briefly in the last lecture, let me remind you of that example. Even if you have to consider pressure, the pressure ultimately if you consider a flat surface, pressure is the normal force exerted by the fluid on a surface per area of the surface divided by the area of the surface.

So, if you consider pressure on a solid surface; this is a solid wall of area a, then why is there pressure suppose this solid wall is exposed to air, the pressure is that because of collisions of molecules that are present in the air, the gas with respect to the wall and the collisions means there is rate of change of momentum that whenever there is a collision of a molecule with a solid surface; that is, rate of change of momentum, that rate of change of momentum will manifest as a force on the surface and that force divided by area a is the pressure.

Now again the continuum approximation means that first of all, a continuum approximation works because even if you consider a area which is made of a square, with a side 10 to the minus 6 meter; that is, one micron, it is a very small; this is one micro meter. Even if you have to consider a tiny area, so, suppose you have to use a pressure transducer or pressure sensor, it will have a tiny area associated with it.

Even if you were to consider a tiny area which you would think so small that you can think of it as a point, within that area, some from elementary kinetic theory, one learns in physical chemistry classes, one can show that the number of collisions; the average number of molecular collisions is proportional to is of the order of rather can be estimated to be about ten to the seven collisions per second.

So, even in a given second, there are so many large, such a large number of collisions that the surface of the sensor which is trying to measure the pressure will sense only an average of all these collisions and it cannot resolve individual collisions. So, when you define pressure as force by this delta area in the limit as delta a goes to 0, so let me grace here. What does means is that as you decrease delta a from this area to a smaller area, you will still find the value of the pressure to be uniquely defined.

So, even if you change the probe dimension, as long as you are not encroaching on the molecular scales, you will still find that the pressure is uniquely defined and it is independent of the test area.

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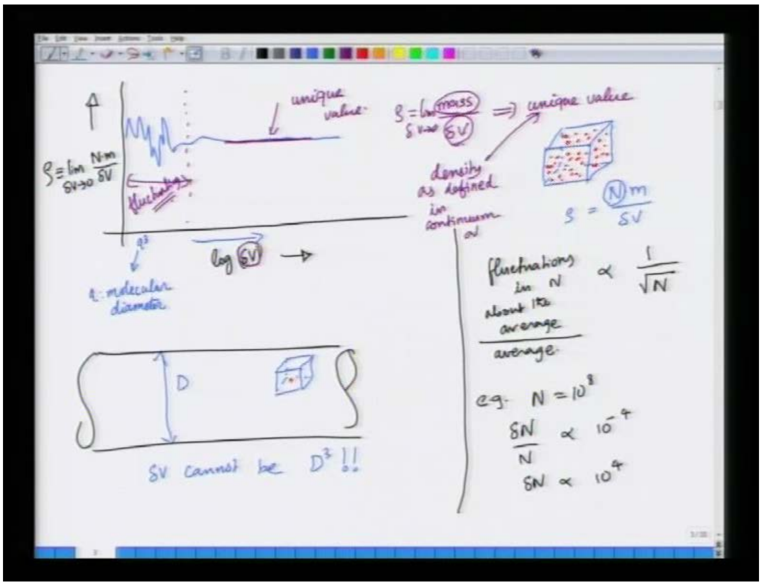

This is the same with density as well. In the previous example, we to different volumes and then we plotted; if you remember what we plotted was that we to different test volumes and we tried to plot the density that one would compute in this test volume as the function of the size of the volume.

Now for molecular, when the volume is proportional to the molecular dimensions, then you found that in this regime, the density is fluctuating, but when the volumes are such that they are large compared to the molecular dimensions, but small compared to macroscopic dimensions, then you will find that the density is a unique value; that is, density is mass present in that tiny volume divided by the volume.

So, it is defined as limit delta v goes to 0, sorry delta v goes to 0, mass divided by volume. So, as you shrink the volume such that still large compared to the molecular scales, the mass that is present in that volume will also decrease, but the ratio of these two will approach a unique value. That is the continuum density. This is the density as defined in the continuum approximation.

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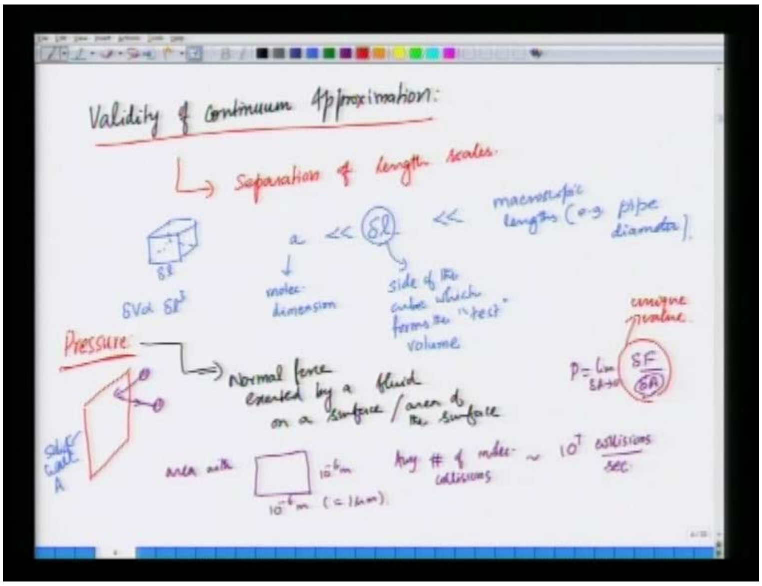

Validity of continuum Approximation:

↳ Separation of length scales.

$a \ll \boxed{\delta l} \ll$ macroscopic lengths (e.g. pipe diameter).

δ Vol δl^3

a → molec. dimension

δl → side of the cube which forms the "test" volume

Pressure → Normal force exerted by a fluid on a surface / area of the surface

$P = \lim_{\delta A \to 0} \frac{\delta F}{\delta A}$ unique value

area with 10^{-6} m × 10^{-6} m (= 1 μm).

Avg # of molec. collisions ~ 10^7 collisions/sec

And the same goes for the pressure which we discussed in the next. So, the pressure will approach a unique value irrespective of the area, if the area is large compared to the molecular dimensions. So, this is the sum and substance of the continuum hypothesis or approximation.

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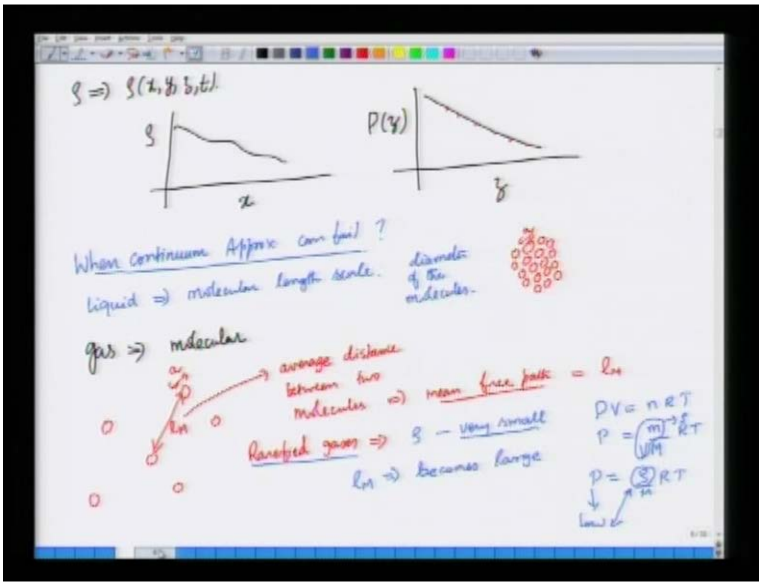

$\rho \Rightarrow \rho(x, y, z, t)$.

When continuum Approx can fail ?

liquid ⇒ molecular length scale. diameter of the molecules.

gas ⇒ molecular → average distance between two molecules ⇒ mean free path = l_m

Rarefied gases ⇒ ρ – very small

l_m ⇒ becomes large

$PV = nRT$

$P = \left(\frac{m}{VM}\right) RT$

$P = \left(\frac{\rho}{M}\right) RT$

low

Now once we define the continuum approximation, we can express density as a function of as a smooth function of x, y, z and time. So, as an illustration, I am just showing that density can be plotted as a function of x with one spatial co-ordinate as a smooth function and so is pressure as a function of let say z and so on. So, this is how once you define the continuum approximation, once we understand what we mean by values of density and pressure at each point, we can sensibly define these quantities at each and every point in the fluid.

Now, the discussion on the meaning of continuum hypothesis also tells us when continuum hypothesis can fail or continuum approximation can fail. Now the molecular length scale in a liquid, the molecular length scale is simply the diameter of the molecules that are present in the liquid because if you remember from your elementary physics or chemistry classes, a liquid is very dense. So, all the molecules are; so, this is diameter a, are fairly close by with very less, very little gap between them.

So, the molecular dense scale is that molecular length scale in a liquid is the diameter of the molecules, but if you consider a gas, the molecular length scale is not really the diameter of the molecule because a gas if you remember is very dilute.

So, the molecules are far well separated. Even though the diameter of the molecule is still a, the relevant length scale here in a gas is not the dimension though molecular itself, but its called the mean free path. So, let us denote it as l m. So, this is the average distance between two molecules before they collide. This is called the mean free path.

So, this is the average distance a molecule travels before it collides with some other molecule. So, this becomes is denoted as l m, this becomes the relevant molecular scale. Now, this mean free path can become substantially large for what are called rarified gases. Rarified gases are gases with very low densities. Rho is very very small.

So, this can be achieved if you know what the ideal gas law is; for example, p v is n r t, where n is the number of moles. So, this is the mass divided by the molecular mass r t, and I bring the volume in the denominator, mass by volume is the density. So, p is rho by m r t, where r is the gas constant and t is the temperature.

So, when can you get low densities? When the pressures are low; that means, the density is true because pressure is directly proportional to density in a gas.

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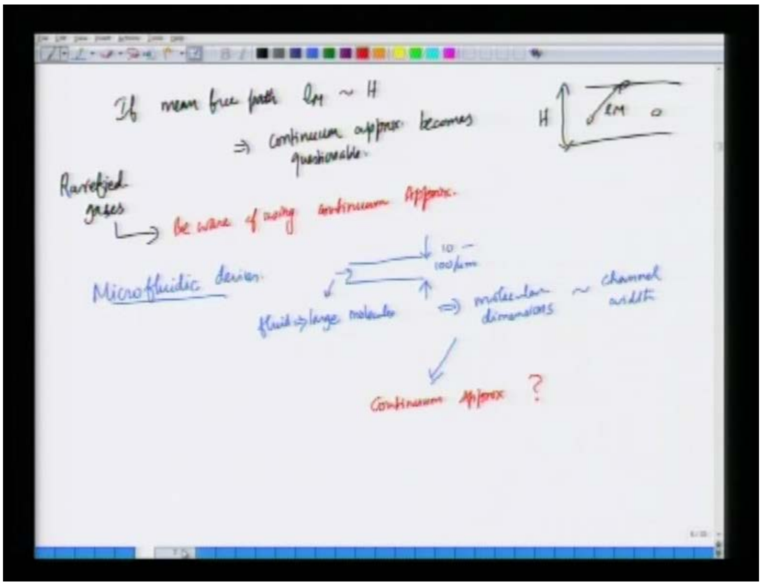

If mean free path $l_M \sim H$

$\Rightarrow$ continuum approx. becomes questionable.

Rarefied gases

$\hookrightarrow$ be ware of using continuum Approx.

Microfluidic devices.

fluid $\Rightarrow$ large molecules $\Rightarrow$ molecular dimensions $\sim$ channel width

Continuum Approx ?

So, densities; these are not… These are encountered in some applications when the pressure is so low, the density becomes very small, then the mean free path becomes large. If the mean free path becomes very large, by very large we mean that, the mean free path; if mean free path becomes comparable to the macroscopic dimensions.

For example, you have flow of a rarified gas in a channel with diameter or length h, and let us say the mean free path is of the order of h, then you will find that continuum approximation fails or it becomes questionable because you can no longer define sensibly point wise quantities in this case. So, in rarified gases, one has to be careful we should I mean this is a warning. So, beware of using the continuum approximation.

Another context is in applications, there are in recent years, there are what are called micro-fluidic devices which are used in several biological and bio-technological applications. These are made of channels whose the gap in which fluid is flowing, is itself of the order of let us say 100 microns or 10 microns.

If the fluid which is flowing is comprised of molecules which are not small, which are if the fluid itself has large molecules like polymeric molecules, then the molecular dimension, then the molecular dimension becomes comparable to the channel dimensions. Channel width.

So again, continuum approximation becomes questionable; the use of continuum approximation. So, there are cases where the length scales; the molecular length scales becomes comparable to the macroscopic scales as in rarified flows or micro fluidic flows.

In such cases, it is not obvious that whether one can successfully use continuum approximation, and there are always other more fundamental approaches where one takes a full molecular approach, full molecular detail, but those are; obviously, much more involved. And they are specific to the kind of problems that one addresses.

But in general, continuum approximation can be safely used in many engineering applications and almost all engineering applications, conventional engineering applications where the macroscopic scale is very large compared to the pipe diameter or the tank conveys of fluid is stored or if you have flow passed sphere, the diameter of the sphere or which the fluid is flowing. If those macroscopic dimensions are very very large compared to molecular dimensions, which is normally a valid in engineering circumstances, then one can safely apply the continuum approximation.

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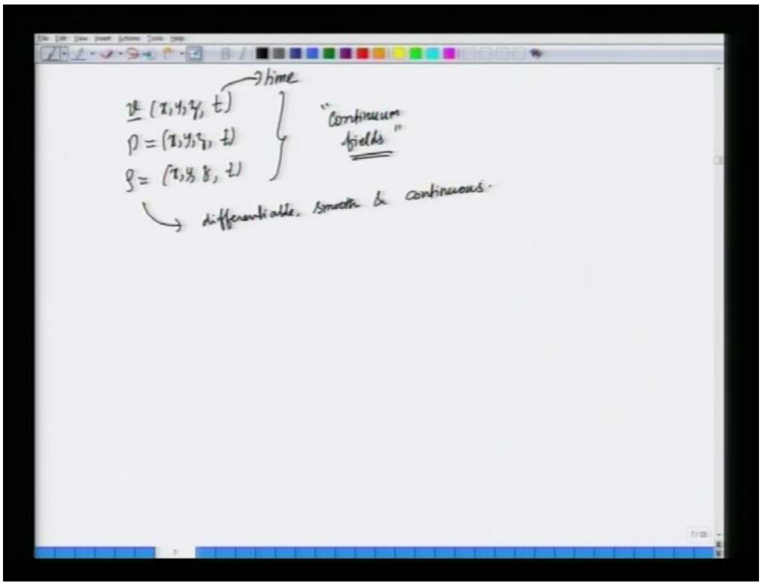

So, this completes my discussion of what the frame work that we are going to use. So, once you use that frame work; the continuum frame work, we will define velocity as a function of all the three co-ordinates in the fluid in the co-ordinate system and time, pressure as a function of all the three co-ordinates and time, and density as a function of smooth functions of all the three co-ordinates and time. These are called fields; continuum fields. So, this goes by name of fields. So, and we assume that they all can be differentiated easily. They are differentiable, smooth and continuous.

Now that we have understood the limitations and validity of continuum approximation, it is a right time to define what a fluid is. So, in order to understand what a fluid is, it is useful to contrast the mechanical behavior of fluid with that of solid. So, let us begin with so that we can compare and contrast the true behavior how a fluid responds under applied forces, and how a solid responds in the applied forces.

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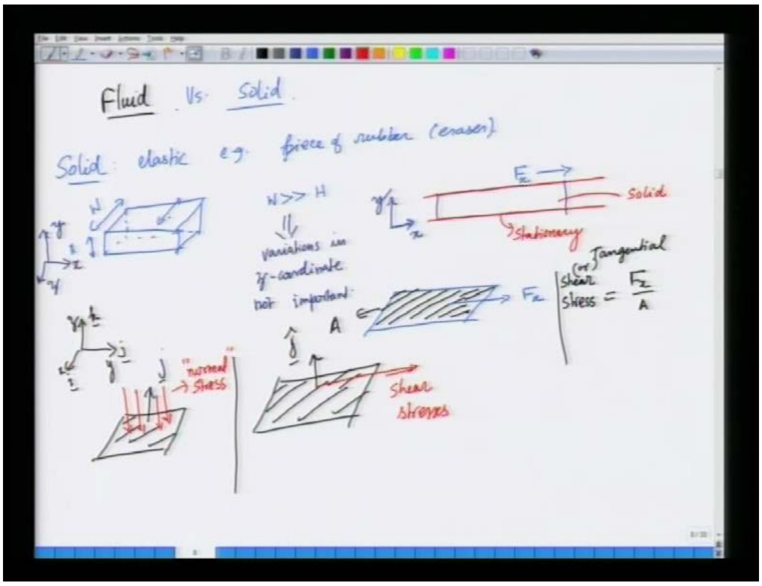

So, let me say, fluid verses solid. So, let us first consider the case of solid which is elastic in nature. For example, a piece of rubber; a piece of rubber that one uses to erase. So, example is eraser.

So, imagine let us take a block of solid. So, this is a block of solid. This is a piece of rubber that we want to use in our thought experiment. So, this dimension, let us say w is very large compared to this dimension which is h, w is very large, it is about let us say twenty or fifty times h. So, we need not worry about variations in the third direction. So, let us just consider the solid to be in the plane. So, let just consider the piece of solid here.

And of course, I am putting a co-ordinate system as I surely not analyze any problem; we have to first set up a co-ordinate system. So, this means, the z is the co-ordinate that is coming out of the board. So, variations in z co-ordinate are not important. So, we are considering a solid. Therefore, and we are taking a cross section at any plane perpendicular to the z direction. So, it will be in the x y plane.

Now imagine that this piece of rubber is placed between two plates. The top plate, this is the bottom plate, and let us says the bottom plate is stationary. This is a solid and the top plate, you are applying an external force and this force is being applied in the x direction.

This is the x co-ordinate. The force is being applied in the x direction. Now in mechanics, in both fluid and solid mechanics, if you have a surface, remember I am just drawing here the cross section at any plane that is perpendicular to z. This force is actually applied on a surface, on the top surface of this piece of solid, and let if the area of the top surface is a, then this f x by a is called the stress.

Now, one normally distinguishes between two types of stresses. This is called the shear or tangential stress because this is tangential to the surface on which the force is acting. So, if you have a surface, this is the surface. The surface has unit normal let us say j, j is the normal in the… So, if you have a co-ordinate system, i j and k are the three unit normals in the x y and z directions. So, unit normal of this surface is j.

But the direction of the force is parallel to the surface and not in the direction of the normal. It is perpendicular to the normal, its parallel to the surface. So, these are called shear stresses. So, we are applying a shear stress; a force that is applied parallel to the surface per unit area.

Now if this is in contrast to, suppose if I have a surface again in of course, unit normal is in the j direction, and if I apply a force like here, this is a compressive force. This is called a normal stress because the force is applied along the normal. So, here it is applied in the direction opposite to the normal. So, whether the magnitude; the direction of the force can be either in the plus j or minus j direction; that that is immaterial, but essentially it is a force in the direction of the normal. So, it is a normal stress whereas, a tangential if the force applied tangential. It is called shear stress.

So, this is the experiment we are doing. We are taking a piece of rubber and then we are placing it between two plates. The bottom plate is stationary, top plate you are applying a tangential force and tangential force per unit area is a stress.

Now, if you have to do this experiment, what is going to happen? That is the thing, that is the question we are going to ask.

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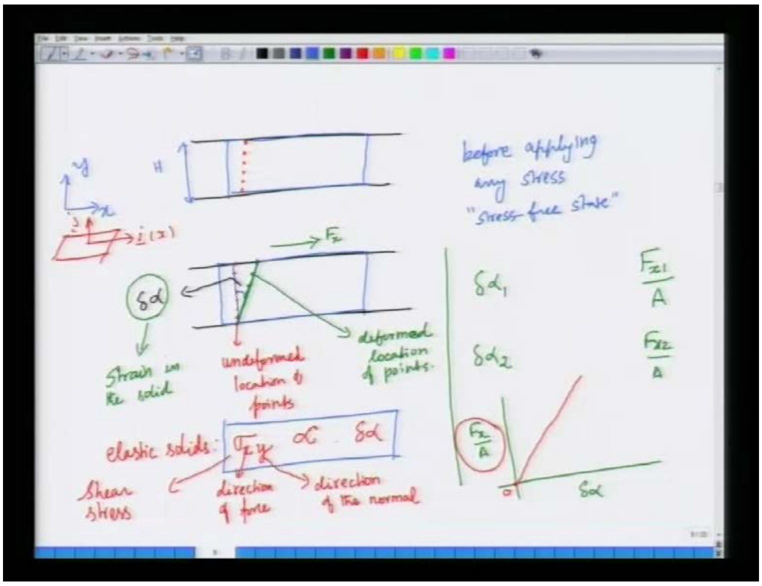

So, you have these two plates, and you have a material. The thickness of this material is h and as you know, this is the x y plane. We are not worrying about the variation in the z direction.

So, you have let say before applying stress any stress, the solid material is in the stress free state. What we do is that we take this piece of material which is not being acted upon the any stress; it is in a stress free state. So, let us apply, let us mark various points with the colored with the color dye or whatever.

Let us imagine that we can mark points on a cross section of the solid with some colored material and imagine what will happen to this, the same system, when you now start applying stress. ((No audio from 42:51 to 43:01)) So, this is the location of the points in the undeformed state or the stress free state; undeformed location of points

Now, you are going to apply a stress in this direction. Now there is no stress that is being acted in the bottom direction, bottom plates sorry, and some stress is being applied in the x direction in the top plate f x. So, what will happen to these points?

In a solid, a solid responds to apply forces by undergoing some deformations. So, all these points will move. By deformation we mean that, these points will start moving from the undeformed locations.

So, the green points refer to deformed location. We say that the solid undergo, solid responds to applied stresses by undergoing deformation. By deformation we mean, this various points that were there in the solid before the application of stresses will start moving to some other new point, will move to some other new point. But they will stop moving because, after you start applying stress, once the solid starts deforming, internal stress is developed in the solid which will resist further deformation. Solid will not deform continuously. The points will deform to some extent and this stop, and this is a very quick process in a solid.

So, it deforms and then it stops deforming, and the deformed location of points is given by the green points. Now, so let me do this in a more quantitative way. So, you have the undeformed thing and the deformed thing. So, this angle, since there is no force in the bottom plate, no force, so, this point will remain here itself whereas, this point would have moved because you are applying a force here, a tangential force here.

Now, this deformation can be characterized by an angle delta alpha because if you apply a very small force, then that means that the amount of deflection of this blue line; of this let us draw it with a green line.

The amount of deflection that you will experience is very small. So, this delta alpha is a measure of what is called strain in the solid because a solid undergoes some deformation upon in response to the applied stresses, but it does not continue to deform, it merely stops deforming because of the internal stresses.

So, in a solid, if you do experiments, suppose you have to do an experiment. Let us say you are applying f x 1; a force f x 1 and you divide by the area or is process force is applied, and then you will find that if you have to do this experiments…, so, let us let us write this in the left side, in the right side.

So, suppose you have to apply a force f x 1, and you are applying it over an area, and if you measure that deflection delta alpha, if you apply a value of f x 1, let us say you find a deflection delta alpha 1, and f x 2 and you will find some other deflection delta alpha 2, and if you were to plot all these, you will find that f x by a is proportional to delta alpha. You will get a straight line. When there is no force, there is no delta alpha. So, it starts with 0 and it is a linear. It is a straight line, it is directly proportional.

So, you will find for elastic solids, if you have to do this experiments for purely elastic solids, this f x by a is called the stress, and stress is denoted by the greek letter sigma and you will have x being the direction of the force which is one subscript and y is the direction of the normal over which the force is acting.

Remember that we are having the surface whose unit normal is in the j direction, and the force is acting in the i direction which is x. That is why there are two indices; one is a direction of the force, this is called a shear stress.

This is a shear stress because the force is acting tangentially to the surface. It is not acting in the normal direction of the surface. So, shear stress force per unit area. You will find that this is proportional to since I am using alpha, so, I do not want to use… So, let me use… This is the proportionality sign. It is proportional to delta alpha.

So, this is what one will find, if you are to do this experiment for a solid. The stress is directly proportional to strain in a solid.

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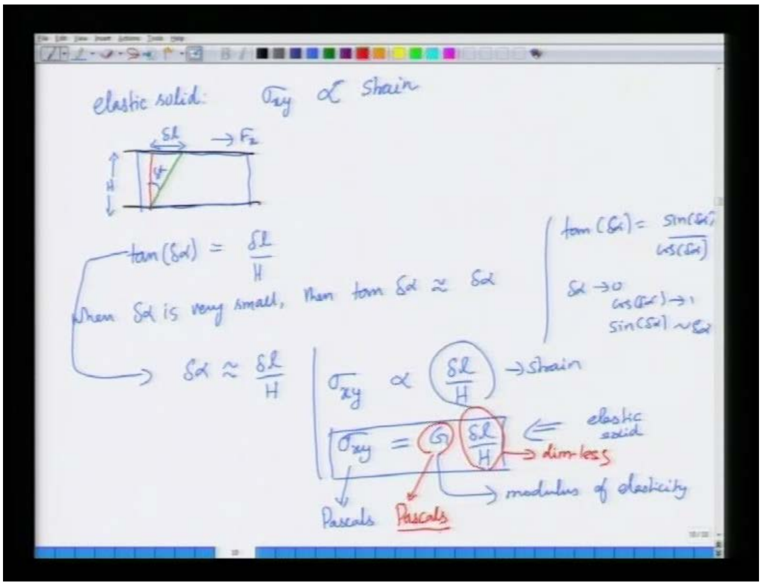

elastic solid: $\sigma_{xy} \propto$ Strain

$\tan(\delta\alpha) = \frac{\delta l}{H}$

When $\delta\alpha$ is very small, then $\tan \delta\alpha \simeq \delta\alpha$

$\delta\alpha \simeq \frac{\delta l}{H}$

$\sigma_{xy} \propto \frac{\delta l}{H}$ → Strain

$\sigma_{xy} = G \frac{\delta l}{H}$ ⇐ elastic solid

$\frac{\delta l}{H}$ → dim-less

G → modulus of elasticity

σ_{xy} → Pascals

G → Pascals

$\tan(\delta\alpha) = \frac{\sin(\delta\alpha)}{\cos(\delta\alpha)}$

$\delta\alpha \to 0$

$\cos(\delta\alpha) \to 1$

$\sin(\delta\alpha) \sim \delta\alpha$

So, for an elastic solid, stress is proportional to strain. So, let us draw this picture again. So, you have these two plates and you have a piece of solid; rubber let us say, and then you found that before deformation, things were in the red line. After deformation, things were in the green line. And this angle is delta alpha. Now this distance is h. Let this displacement be delta l.

So, this is the force f x that is acting on a surface whose unit normal is in the y direction. Now by geometry, tan delta alpha is delta l by h, but when delta alpha is very very small, for small forces, you will expect that the deflection delta alpha and displacement l is small is very small, then tan delta alpha is roughly equal to delta alpha.

Remember that tan delta alpha is sin delta alpha by cos delta alpha, and as delta alpha becomes small or tends to 0, cos delta alpha tends to 1 and sin delta alpha is proportional to delta alpha; it goes as delta alpha.

Therefore, tan delta alpha will go as delta alpha. So, this equation means that delta alpha is approximately delta l by h. So, sigma x y is proportional to delta l by h. This is called the strain and the constant of proportionality is called the modules of elasticity. This is for an elastic solid.

This is an equivalent of Hooks law of elasticity. It is essentially some statement of Hooks law of elasticity. For elastic solid, the stress is directly proportional to strain and the strain is measured by this deflection delta alpha and the constant of proportionality is called the elastic modules or modules of elasticity. Now stress has units of pascals, because its force per unit area. This is dimensionless. This is unit less or dimensionless because this is ratio of two lengths. This has no unit, it is a pure number. So, g will have units of pascals.

Now this is what a solid is. A solid responds by undergoing some deformation or strain impressed upon application of some shear stress. And the stress is directly proportional to the strain in solid.

In the next lecture, what we will do is we will contrast the mechanical behavior of fluid upon application of external stresses, and see how it is different from a solid. So, we will see you soon in the next lecture.