Fluid Mechanics

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Lecture No. # 29

Welcome to this lecture number twenty nine on this NPTEL course on fluid mechanics for under graduate chemical engineering students. The topic we are currently discussing, is dimensional analysis.

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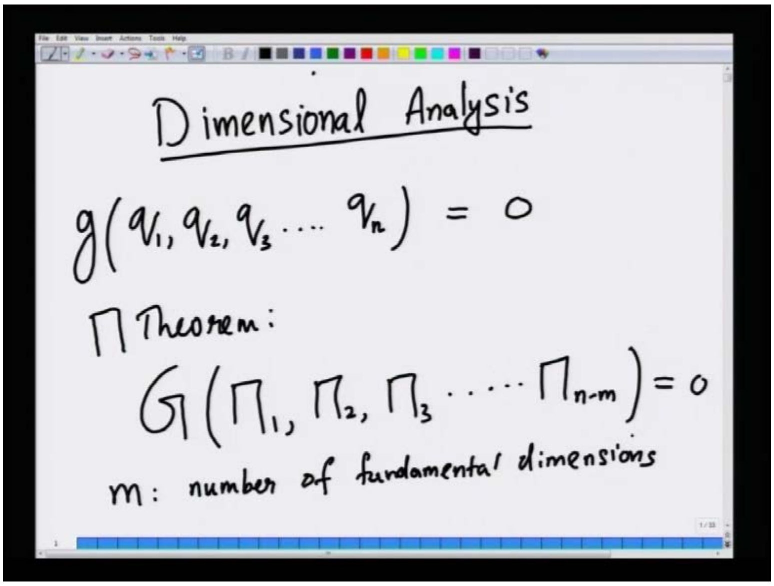

We pointed out in the last lecture that dimensional analysis affects a reduction number of in the number of variables, among few dimensional variables. Suppose, you have a set of dimensional variables let say q 1 q 2 q 3. So, on up to q n there are n dimensional variables. Then the pi theorem guarantees that there is an functional relationship. Let us say capital G, among n minus m non dimensional groups that are made out of these n dimensional variables. Where, m is the number of fundamental dimensions in the problem dimensions in the problem. So, these could be mass, length and time in most cases. But if you have heat transfer you could have mass length time and temperature as well.

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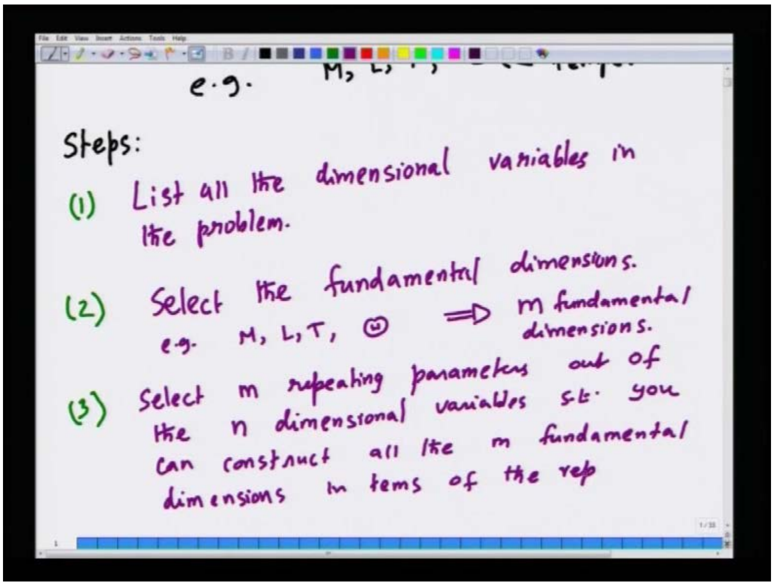

So, example M L T and if you have heat transfer this is the dimension of temperature. So, the phi theorem essentially guarantees, that you can reformulate a relationship between n dimensional variables into a reduced set of variables. They are these variables are non-dimensional groups sometimes called as phi groups. Now, we also outlined the set of steps that one has to follow in order to effect this non dimensional accession into pi groups. So, steps: the first step is you first list all the dimensional variables in the problem, relevant dimensional variables in the problem.

As I mentioned in the last lecture, admittedly this involves some physical intuition, some engineering judgment as well as some experience that helps in deciding, what are the various relevant physical parameters that affect a given quantity? For example, in the case, of drag force on a sphere we mentioned in the last lecture, that one expects the drag force to be dependent on the velocity at which the sphere is moving. The diameter of this sphere the viscosity of the liquid in which it is moving, and the density of the liquid. We have neglected other parameters such as surface tension at the interface between liquid and solid. Hoping that is not a relevant variable for this problem.

Now, if indeed these additional variables affect the functional relationship that will show off when we actually, do experimentation. So, the first step is to list all the dimensional variables in the problem. The second step is to select the fundamental dimensions. And this is usually in fluid mechanics problems mass length and time instead of mass one could use force as a fundamental dimension, but one could use mass length and time. And if you have heat transfer in your problem, if u have heat transfer and fluid flow, then you have the temperature theta as well.

The third step is now, you have to set select and let us say there are m fundamental dimensions in the problem, m fundamental dimensions. If it is M L and T, it is just three m equals three. In general you may have m fundamental dimensions. Now, you have to select m repeating variables you have to select m repeating parameters or variables dimensional parameters out of the n dimensional variables, null variables. Such that you can construct all the m fundamental dimensions in terms of this variables. Since, in terms of the repeating variables, I will illustrate this soon with an example.

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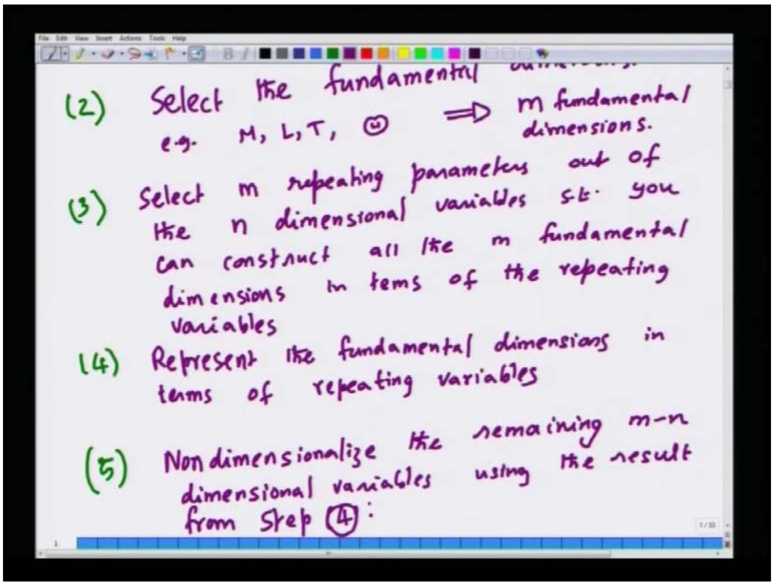

The fourth step is now to represent the fundamental dimensions in terms of repeating variables. And the fifth step is to finally, do the non-dimensional accession, non dimensional is the remaining, lies the remaining m minus n variables, m minus n dimensional variables using the result from step for step four.

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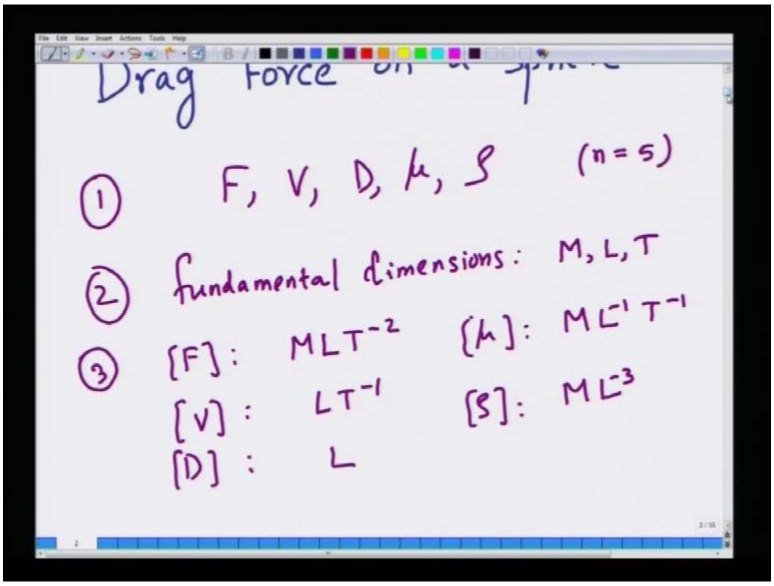

So, this is the essential procedure. Now, I am going to illustrate this for the special case of drag force on the sphere. On a sphere, we already found out what are the relevant variables? The relevant variables are the force, velocity, diameter, viscosity and density. So, there are n equals 5 variables. And the second step is to find out, what are the primary dimensions present among this five variables? The fundamental the word I used is fundamental dimensions. If you can list down all the dimensions of these variables you will find that the fundamental dimensions are mass, length and time, which I will do now? What are the dimensions of various variables? Well force, the dimension of force is mass times acceleration.

The dimension of velocity is length by distance, The dimension of the diameter of the sphere is simply length, The dimension of viscosity as we have seen is m times inverse length times inverse times the dimension of. And finally, the dimension of density is mass per volume. It is M L to the minus 3.

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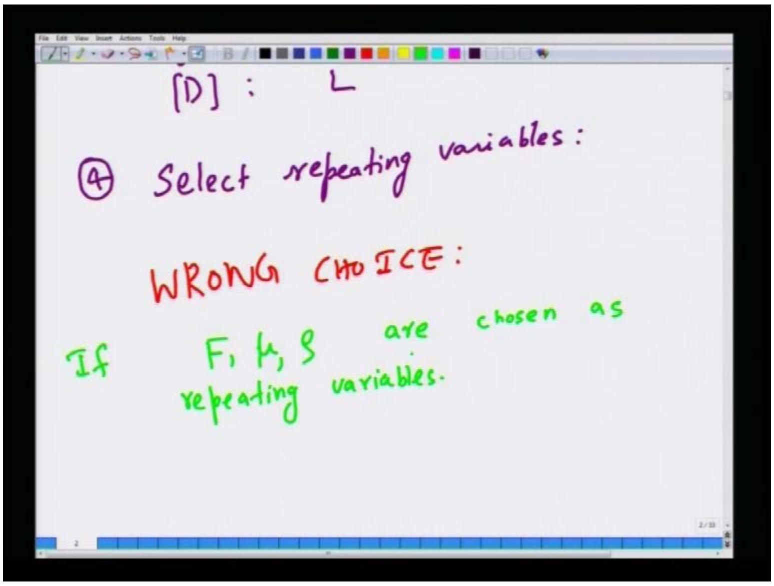

So, if you can see all the variables, you will see that you have M you have L and you have time. There is no other fundamental dimension present in this problem. The fourth step is to select repeating variable or repeating parameters such that you can construct all the three fundamental dimensions based on these three repeating parameters. Now, there is a element of judgment involved here. For example, I am going to first make a choice and show that it does not work and then I will do the right choice. All I am trying to say is that you cannot use any three variables.

So, first let me flag off by saying wrong choice and then I will show why it is wrong? So, let us say we are going to choose that three repeating variables as F, mu, rho. If are chosen, if F, mu and rho are chosen as repeating variables. Then all we want to do is now represent mass, length and time in terms of these three variables.

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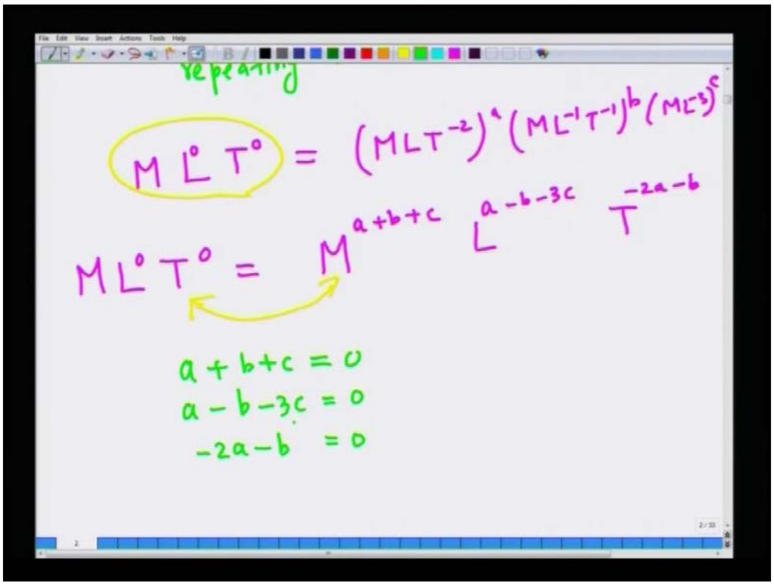

Now, let us see, whether we can represent mass in terms of this three variable. So, mass times length to the power 0 times time to the power 0 must be represented as the dimension of force, which is M L T to the minus 2 to the power a times mu to the power b, which is the dimension of mu is M L to the minus 1 T to the minus 1 to the power b times rho power C. If I can find a combination of a b and C, such that this is satisfied; that means, that we are found a way to non dimensionalized mass with a combination of F mu and rho.

Now, how are we going to proceed further? We going to equate in the first let me write down expand this powers out I can write this is m to the power a plus b plus C, L to the power a minus b minus 3 C ,T to the power minus 2 a minus b. If this two equations are equal, dimensionally equal same then the exponents first match. So, we will get the following equation for the exponents. If we will find, we will get three equations for the three unknowns. If we find a solution, a non-trivial solution to this set of three equations. That means we will be able to write mass in terms of the three repeating parameters F, mu and rho.

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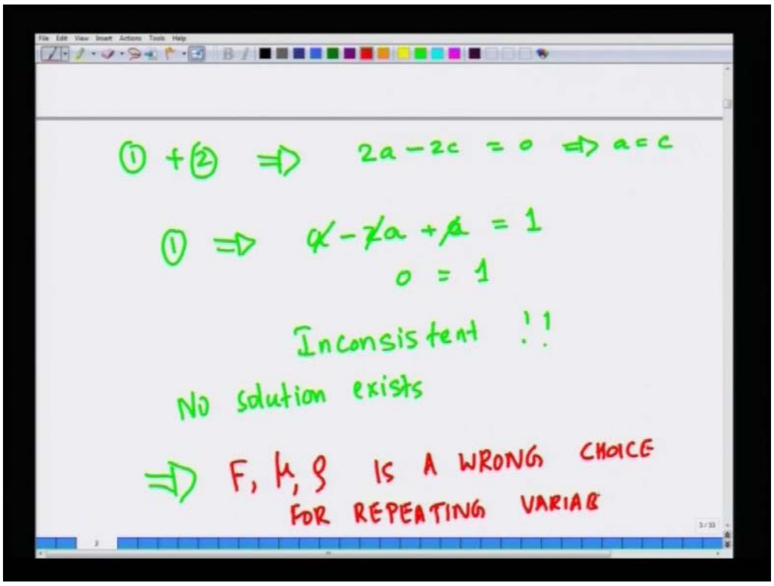

Now, let us try to solve this. Third equation tells you that b equals minus 2 a. Let us call this 1 2 3. Now, 1 plus 2 implies 2 a minus 2 C is 0 or a equals c. So, equation 1 then implies a plus b, b is minus 2 a and a equals C plus C is a is equal to 0 can we just see this I am sorry this is there is an error here a plus b plus C is 1, because the exponent of m is 1 so this is 1. So, a plus b plus C should be one. So, this should be one, but this is 0 so, this is inconsistent. The equation are inconsistent; that means, no solution exist, which implies that F, mu, rho is a wrong choice for repeating variables. So, it is a wrong choice for repeating variables. Therefore, we have to choose some other set of variables.

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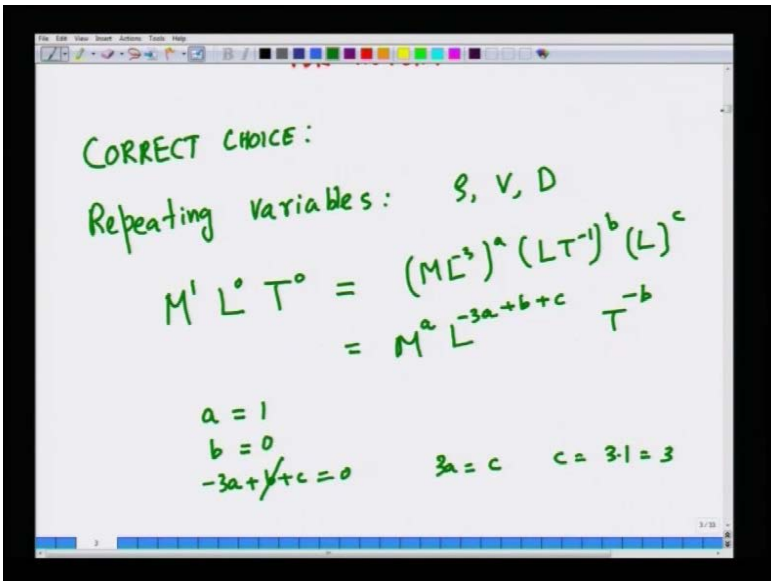

So, let us now let me write down correct choice one of the correct choice is could have others also. So, all I am trying to get at by doing this wrong example, is that you cannot choose any three parameters. And say I will construct all the fundamental dimension out of them, because as I have just shown it is not always possible. So, we have to choose those three variables from which we can construct all the three fundamental dimensions. So, now let us now try to select the repeating variables as rho, V and D. Now, let us see whether, we can get a mass out of rho, V and D, m to the 1 L to the 0 T to the 0 is M L to the minus 3 times a L T to the minus 1 times b times L to the power c.

So, you have this is equal to M to the power a times L to the power minus 3 a minus or plus b plus C times T to the power minus b. So, by comparing you will find that a is 1, b is 0 minus 3 a plus b plus C is 0, but since b is 0 you have 3 equal C or C is three times 1 is equal to 3.

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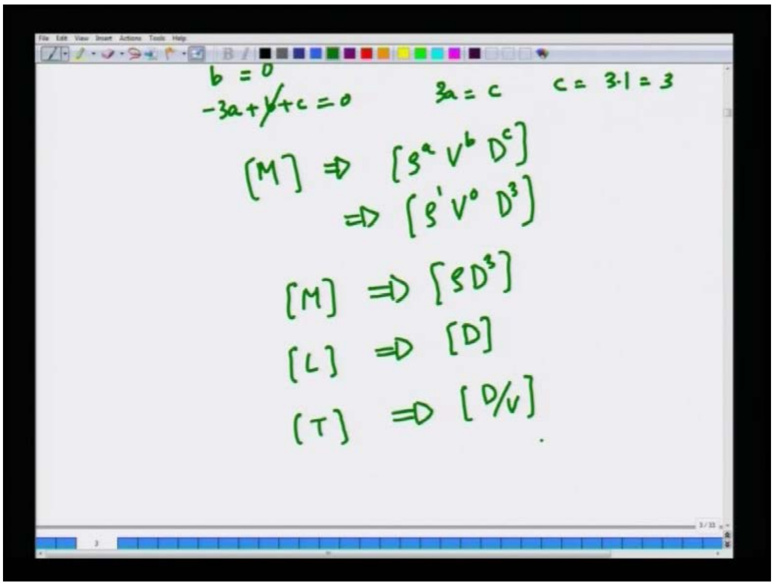

So, what we find is that the mass dimension can be obtained by writing rho to the power a times, V to the power b times, D to the power C. Which is rho to the power 1 V to the power 0 and D cube? So, this means that you can write get a dimension of mass by multiplying rho by D cube that makes physical sense, because rho is mass by unit volume and you are multiplying by D cube, which has dimension of volume. So, you will; obviously, get a mass dimension. Likewise, you can get the other two dimensions in the following way. So, this is mass dimension so, let the length dimension is trivial, because you can choose D as a length dimension and time dimension we can just choose D by v so, D by V is a times scale.

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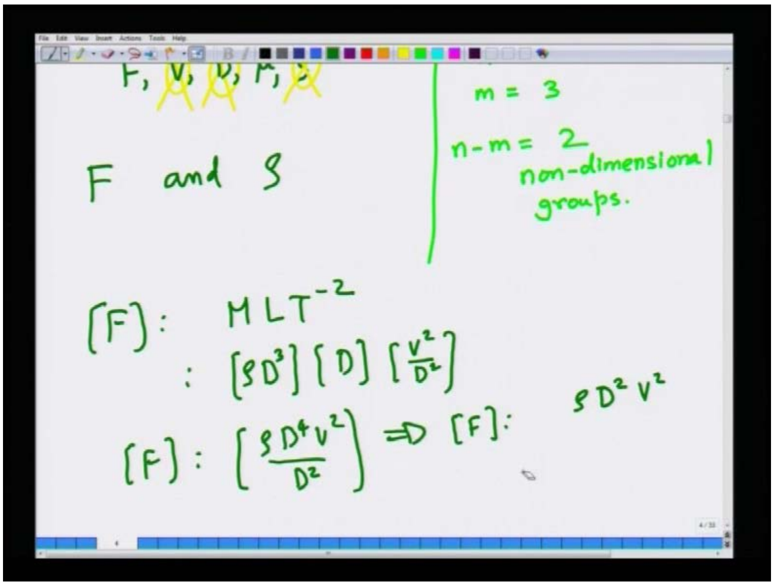

So, with these three now, we can none dimensionalize the remaining three variables. So, remember we had a total of six variables F, V, D, mu, rho. And we have already chosen V, D and rho as repeating variables. So, they are already chosen as repeating variables. So, we had a total of n is five variables and you had three fundamental dimensions. So, the phi theorem says as there must be two non-dimensional groups n minus m as 2 there must be two non-dimensional groups. So, the two non-dimensional groups must be obtained by non dimensionalizing F and rho.

So, the dimensions of F are M L T to the minus 2. So, mass is rho D cube, in our in our repeating variables. You can get the dimensional mass by rho obtained ,multiplying rho by D cube dimensions of length is D dimensions of time is essentially D by V. So, we have time inverse square. So, we have V square by D square.

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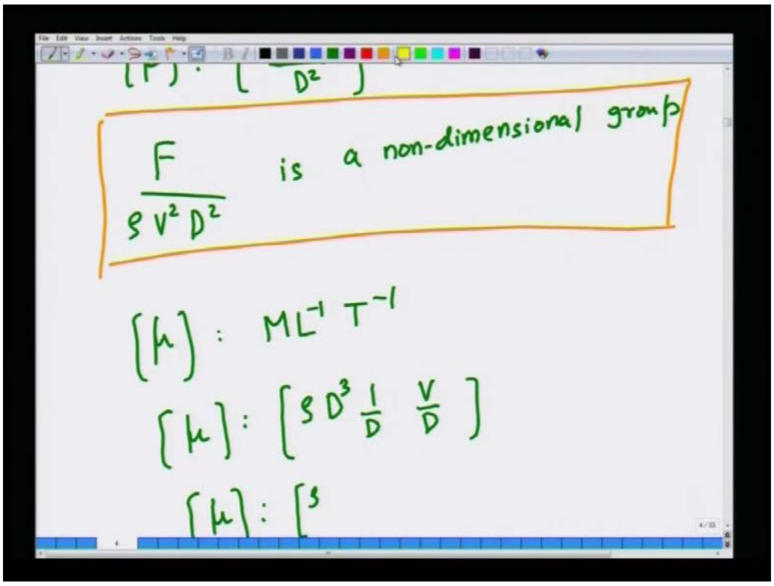

So, force can be non dimensionalize by the dimensions of force will be the same as rho D to the 4 V square by D square or the dimensions of force will be the same as dimensions of rho D square V square. So, you can non dimensionalize force by rho V square D square is a non-dimensional group as one of the non we have to find two non-dimensional groups out of the five physical variables. This is one non dimensional groups. What is other non-dimensional group we have to non dimensionalize? So, we have non dimensionalize the force now. So, we have to still none dimensionalize viscosity.

So, how we are going to do that? You write the dimension of viscosity dimensions of viscosity is M L to the minus 1 T to the minus 1. So, the dimension of viscosity will be the same as rho D cube that is mass length is 1 over D t is V over d so, T inverse is D over v. So, the dimension of viscosity will be the same as now I cancel this D and this d. So, it is rho D cube, this M L to the minus 1 T to the minus 1, T to the minus 1 is actually, V over d. So, let me do this again, T to the minus 1 V over d. So, you will have 2 D is cancelling D cube to give D square here to get finally, to get rho D V.

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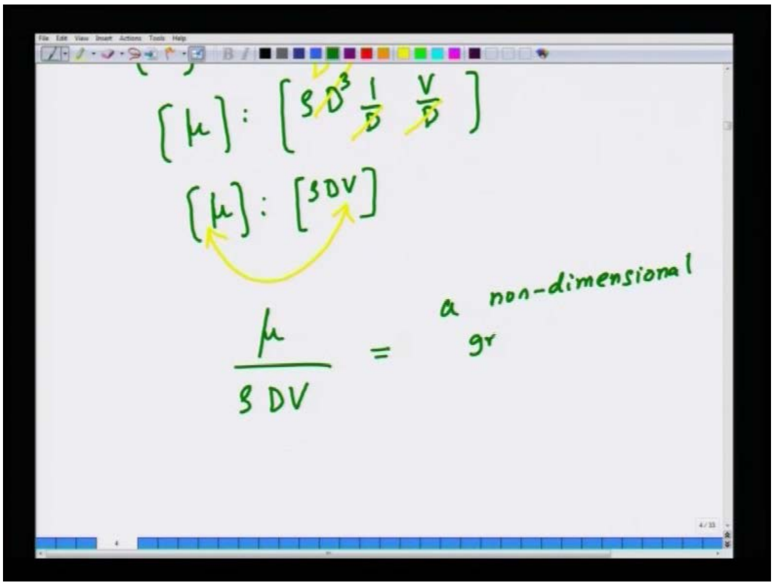

So, the dimension of view, mu the viscosity is identical to the dimensions of rho D V. So, if I divide mu by rho D V is a second non dimensional group.

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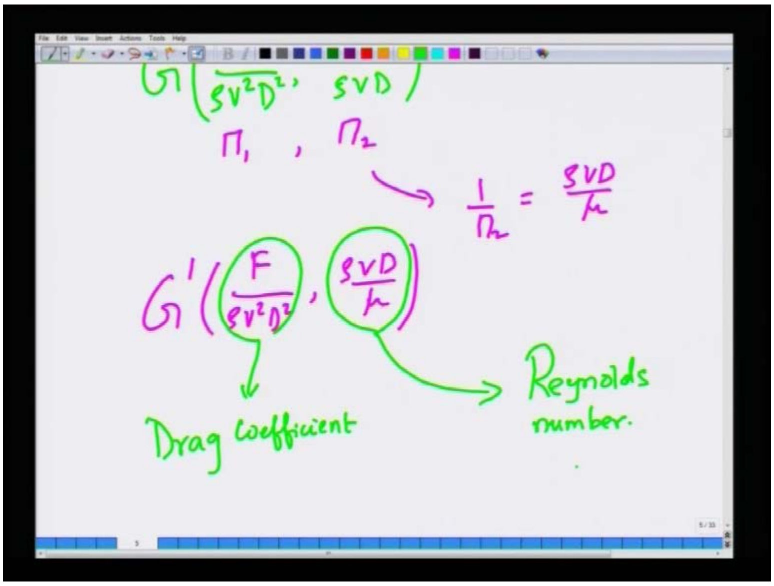

So, we have finally, come up with, we had initially a dimensional relationship between five dimensional variables. We have now reduces it to a functional relationship between two non-dimensional groups mu by rho V D. Now, one important thing is that it does not matter where so, let us call this as phi 1 we can always write, instead of writing mu rho V D we can also write 1 over phi 2 is rho V D by mu. You will get some other function relationship between these let say g prime of F by rho V square D square. And essentially you have two non-dimensional groups and you cannot construct one out of the other they are two independent non dimensional groups from the original set of five parameters.

Now, in fluid mechanics this is called the drag coefficient. These non-dimensional force drag force is called drag coefficient. And this non dimensional group rho V D divided by mu called the Reynolds number. Now, what is the interpretation of Reynolds number? We will come to that shortly, but essentially what we have tried to show is that by doing this non dimensional accession. We get a two non-dimensional groups and instead of writing the functional relationship among five dimensional variables. We can now write it as a functional relationship among only two non-dimensional groups.

And I have already, pointed out the advantage of using dimensionalize groups in representing physical data experimental data. Because that immediately allows for scaling up or scaling down as long as u are matching the non-dimensional groups from the model as well as the prototype. This we have pointed out extensively in the previous lecture.

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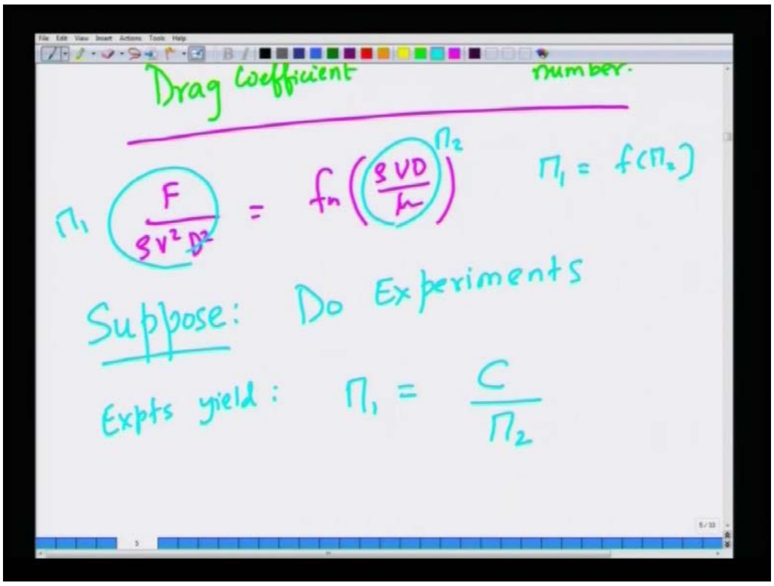

Now, I am going to make an additional comment suppose, we do a experiments. So, essentially we find that F by rho V square D square is a function of rho V D by mu this is what our dimensional analysis is telling you. Suppose, you do experiment tell us that the following result, that is let us call this phi 1 and let us call this group phi 2. So, we expect phi 1 is some function of phi 2. Suppose, the experiments tell you that the relationship between this two groups is such that phi 1 is some constant by ph2, phi 1 is function of phi 2. So, that is some constant divided by phi 2. Suppose, the experimental data tells us this after doing experiments you find a phi 1 is some constant by phi 2.

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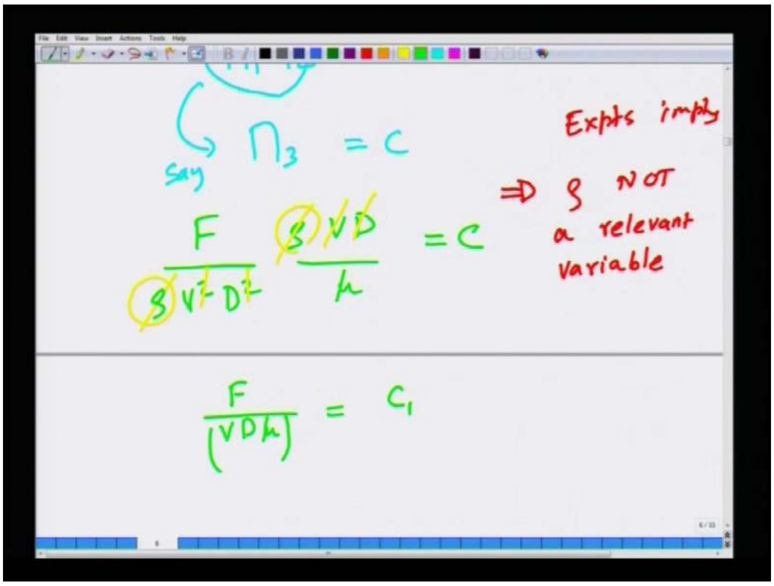

Now, what this means is that phi 1 times phi 2 is a constant. So, this is phi 1 times phi 2 is another non dimensional group, that is actually, we can call this phi 3. Let say is a constant. What is this phi 3? We can write the two non-dimensional groups F by rho V square D square times rho V D by mu is a constant. Now, I am going to cancel the two densities. And, what you will find is that F divided by and we will cancel 1 length dimension will cancel 1 velocity to give F divided by V D by mu is some constant. So, what this means, is that rho, which was thought to be a physically relevant variable originally, which was thought to be a physical relevant variable originally. It is dropping out of the functional relationship as given by experiments.

So, the experiments are telling as that rho is not a relevant variable. This implies experiments imply rho not a relevant variable. This, that is rho V D by mu, that is not an relevant variable. So, this is an important relation that one gets that just by finding out. So, mu should come here, that was the mistake I was making. You should just come here. So, F divided by V D mu is a constant; that means rho is dropping out of our entire picture. So, rho is not a relevant variable. So, this teaches this tells us a very important lesson that based on our intuition; we may choose a set of variables and precede dimensional analysis and in this simplest case, for drag force on a sphere. We found that we have two dimensionless groups; the drag force, which is F divided by rho V square D square. And then you have the Reynolds number which is rho V D by divided by mu.

Now, if the experiments are telling us that the drag force is proportional to 1 over Reynolds number that what we are essentially saying, that phi 1 is some constant divided by phi 2. Then by rearranging phi 1 and phi 2, that is we can re express that relation phi 1 times phi 2 is equal to a constant. That means, that phi 1 times phi 2 is redefined as a new non dimensional group phi 3 and that phi 3 has no density in it. So, this in essence this implies that density is not a relevant variable, at least as for the experiments done in a particular regime.

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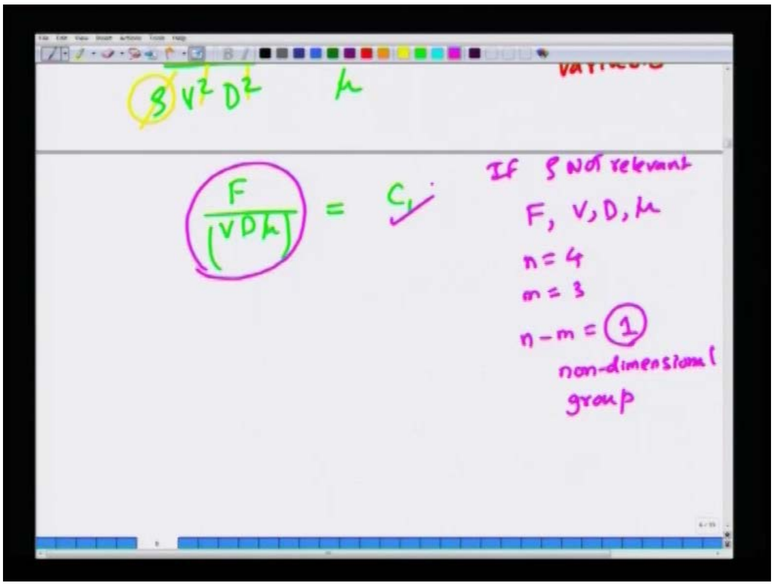

So, if rho is not a relevant variable, you had only if rho not relevant you have only four variables F, V, D, mu. So, n is for you have three fundamental dimensions you have therefore, n minus m is only one non dimensional group as for phi theorem. Which means, that you have this and that non dimensional group is just F by V D times mu. So, when rho is not a relevant variable then there is only non-one non dimensional group. If there is only one non dimensional group, that there is no functional relation that non dimensional group must be a mere number. That means, constant C 1, the constant C 1 is the mere number. So, this new non dimensional group must be constant if rho is not a relevant variable.

Now, we will come a little bit later, but physically this happens, when the viscous affects are dominant compare to the fluid inertial affects. Then row then see us to be a physically relevant variable we will point this out a little later when we talk about drag forces on particles and so on. But right now, all I am trying to say is that, when we use or physical intuition and judgment to come up with the set of variables, physically relevant variables. But if one of them is indeed not a relevant variable, then that will actually show up an experiments. That is a point I am trying to drive across at this stage of course, we will come to a drag forces little later also.

Now, the next topic I am going to do is to motivate dimensional analysis, not from the point of your experimentation. But I want to enquire into the physical meaning of the non dimensional groups such as the Reynolds number. Right now, the Reynolds number emerge purely from dimensionally non dimensionalizing a set of an a problem, a set of variable such as force, velocity and so on. If we non dimensional viscosity, we obtain the Reynolds number. Is there are a fundamental physical significance for the Reynolds number? The answer is yes. And the answer comes by non dimensionalizing, the now navier stokes equations.

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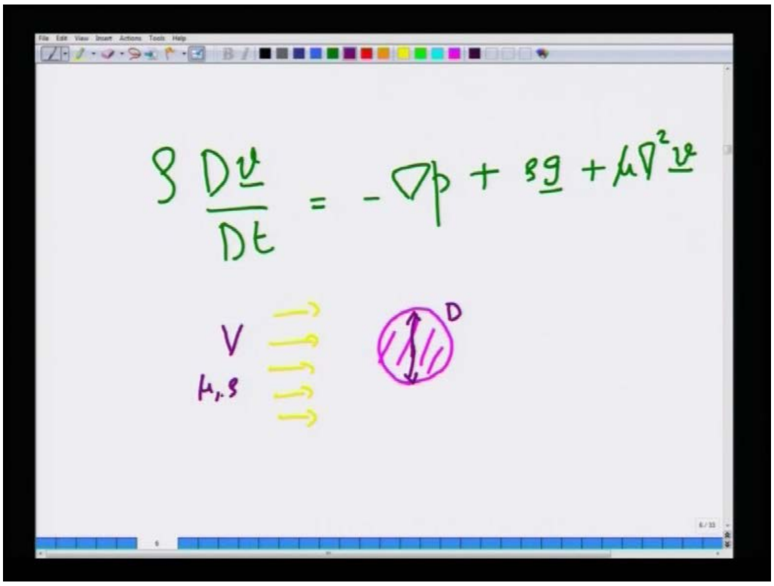

So, we want to know non dimensionalize the navier stokes equations are simply written as rho times the substantial derivative of velocity is minus the gradient of pressure plus rho g plus mu del square V. Now, all these parameters have dimensions. Now to be concrete to give you a physical idea let us, say we are worried about flow past as sphere, the problem for which we just carried out dimensional analysis. We have you could either consider a sphere moving at a constant velocity in a stationary fluid. Otherwise, stationary fluid or you could consider the sphere to be a stationary and you could have the fluid flowing past a sphere far away.

The fluid could have uniform velocity; both are equal and formulation, because you could always change the reference frame from a stationary reference frame to a constant to a reference frame that moves with a constant velocity. So, what we are going to do is? With this problem in mind we are going to non dimensionalize navier stokes equation. So, what are the physical variables you have the velocity V at which the fluid is flowing far away. You have the diameter of the sphere you have the viscosity of the liquid density of the liquid.

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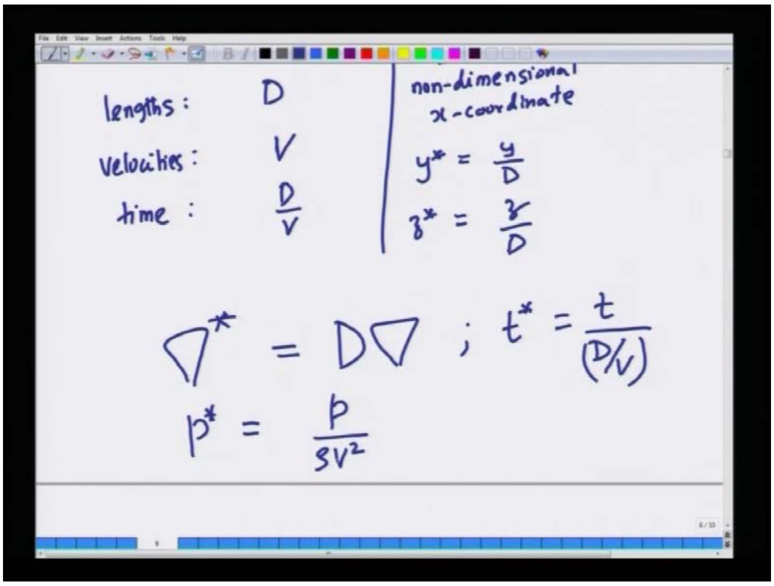

Now, these are the parameters that we have, and we want use this parameters to non dimensionalizing navier stokes equation. Now, first links all links that occur in navier stokes equation. That is a natural length scale; they must be non dimensionalized by the spheres. All velocities can be non dimensionalized by V and times by D by V. So, we are going to define x star is the non-dimensional x variable the x coordinate. Similarly, y star is y divided by D, z star is z divided by D. Now, we also have gradients in navier stokes equations. Now, the non-dimensional gradients, gradients have dimension of one over length.

So, they are obtained by multiplying the dimensional gradient by the length scale D. Now, times the that occur in the navier stokes equations you divided by the non-dimensional time and pressure already from Bernoulli equation. We know that p by rho and V square have the same dimensions. So, we will use this inertial scale for the pressure. Remember the Bernoulli equation is valid in the strict limit when the fluid has no viscosity in viscid limit. The frictionless limit and in the Bernoulli equation, the only forces that are happening, other pressure forces the inertial forces due to kinetic energy of motion and the gravity of force. So, we can choose the inertial scale rho V square to non-dimensional pressure.

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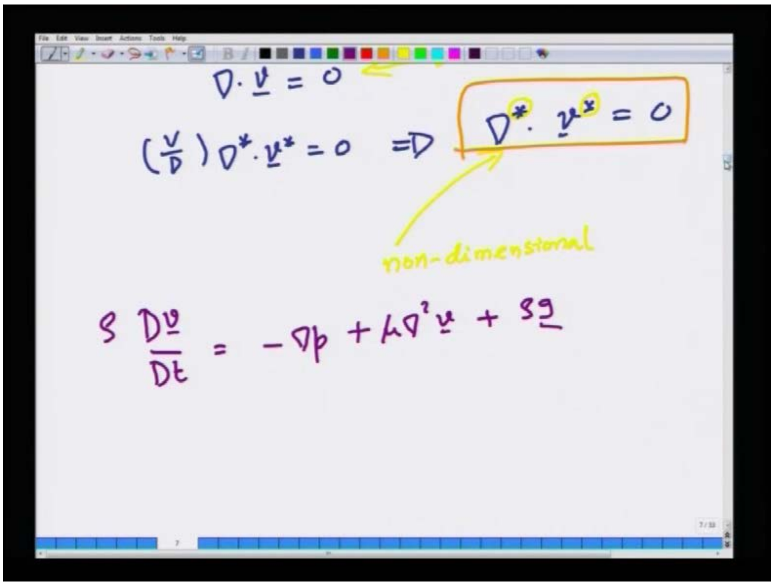

So, when we do all this we have the continuity equation del dot V 0. Now, we get D times V times del star dot V star 0. This implies that V divided by D this implies that continuity equation simply becomes del star dot V star is 0. This is the dimensional continuity equation. This is the non-dimensional continuity equation and as you have as you would have realized here. Now, all the non-dimensional variables are denoted by an asterisk or a star. This is the non-dimensional mass conservation equations, differential mass conservation equation.

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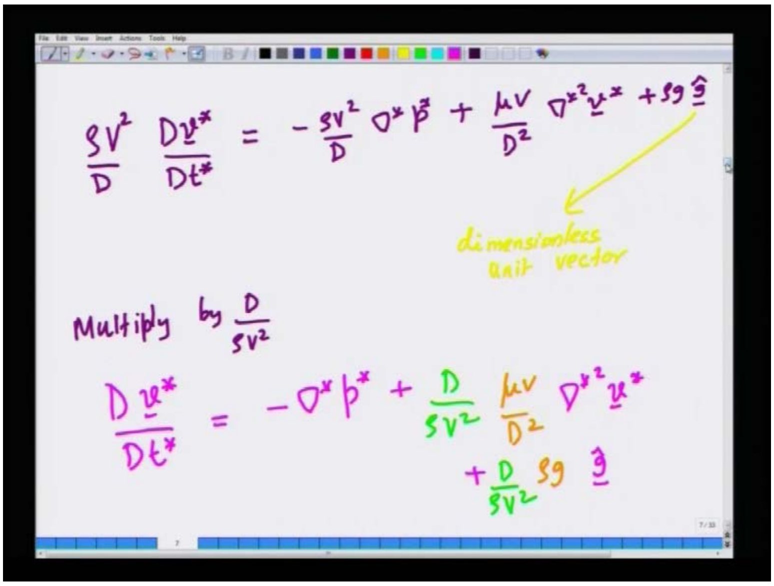

Likewise, we can non dimensional the momentum equation. You have rho D v D t, this is the dimensional equation is minus del p plus mu del square V plus rho g. Now, instead of velocity I am going to write as D V star and instead of time I am going to write t star, t is nothing but t star times D by v. So, is minus rho V square by D del star p star plus mu V by D square del star square of V star plus rho g. We will keep rho g as such right now, only thing I will write, as I will write g as the acceleration due to gravity vector times a unit vector, which is dimensional less. So, this rho g times g cap, g cap is a dimensionless unit vector.

So, we are going to now multiply the entire equation by D square by mu V. So, I am going to multiply this entire equation by this or we can multiply the entire equations alternatively by D by rho V square, that will be much simpler for us. If you do that, now the left side will be D v star by D t star is equals to minus del star p star plus you get D by rho v square times mu v by D square times del star square V star plus rho g times mu v by D square rho g times or D by rho V square times rho g times D. Dimensionless gravity unit vector that points in the direction of gravity.

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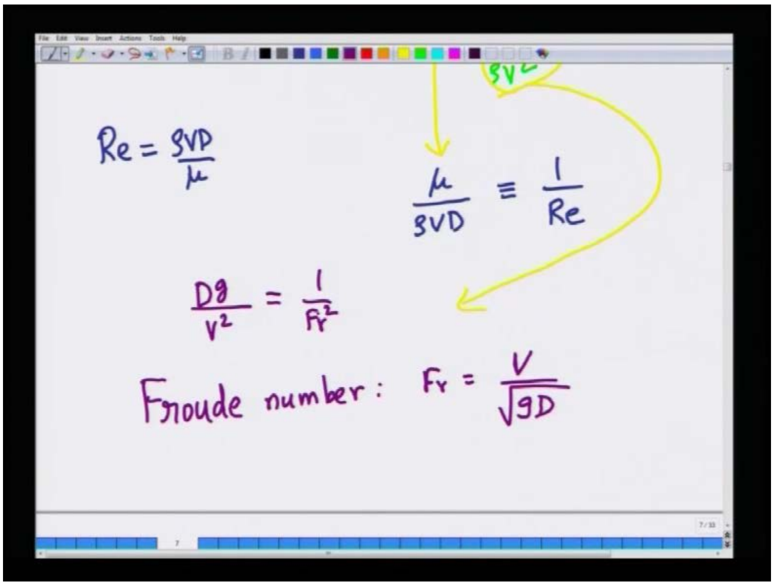

If you consider this combination, this is essentially mu by rho V D, which was 1 over Reynolds number. Because Reynolds number, if you remember we just define, Reynolds number as rho V D divided by mu that is one non dimensional group. Now, the other non-dimensional group comes from this part, this is nothing but D by D g divided by V square. That is the other group that comes about, but traditionally a Froude number is defined as F r is V by square root of g D. So, what you have is 1 over F r square 1 over Froude number square. So, if you look at this equation all this terms of non-dimensional, this is non dimensional, this is non-dimensional, this is non-dimensional. And these, two groups that multiply the viscous term and the gravity term, they are also non dimensional. One is Reynolds number and other is related to the Froude number.

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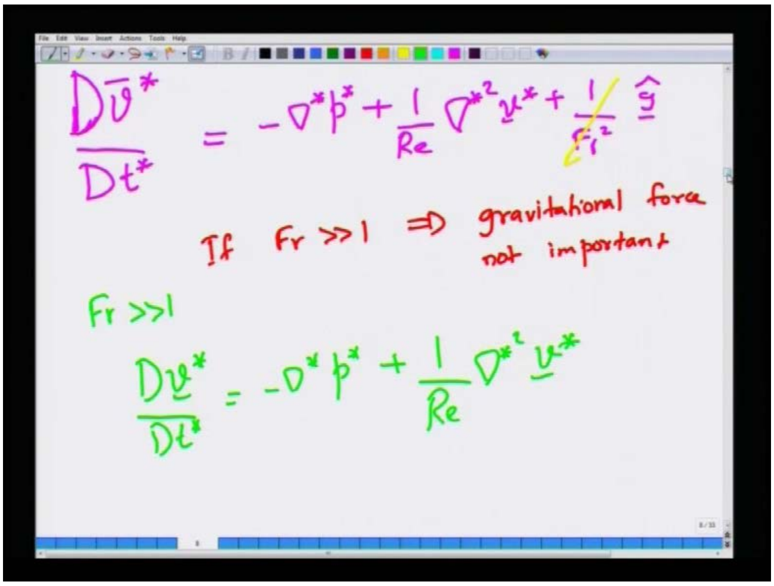

So, we can write the non-dimensional relation non-dimensional navier stokes equation as D t star is equal to minus del star p star plus 1 over Froude square times. The let us first write the viscous term 1 over Reynolds number times del star square V star plus 1 over Froude number square times the acceleration due to gravity vector. Now, if gravity is not important that is if F r is large compare to 1; that means, that gravitational force is not important. Because F r is very large compare to one so 1 over F r square will be very small compare to 1.

So, in the limit of large Froude number, what we are say the Froude number? If you see this can be written as is V. You can multiplying divide by, Froude number is essentially a ratio of inertia to gravity. How do you find that? You can simply multiply both the numerator and denominator by so, let us consider Froude square. Froude square is V square by g D, this is nothing but if u multiply and divide by rho you have rho V square by rho g D. This is the measure of inertial force and this is the measure of gravitational head. So, the Froude number is a ratio of inertial forces to gravity forces.

So, when the Froude number is large compare to 1, then the inertial forces are much large compare to the gravitational forces in a problem. While, if the Froude number is small compare to 1 it is either the gravitational forces become very important. So, if you consider Froude number large compare to 1, then one can neglect this term. So, the navier stokes equation will then become simply for Froude large compare to 1 is minus del star p star plus 1 over Reynolds times del star square V star.

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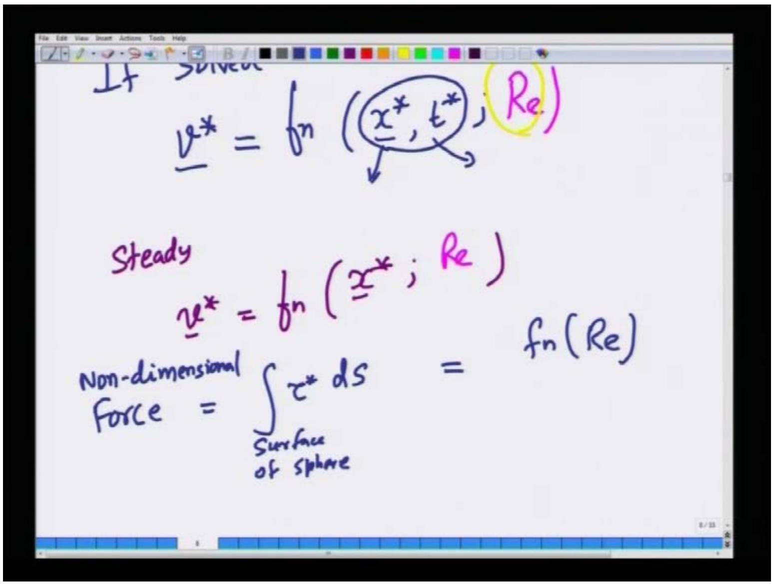

So, if we were to solve this if you solve this then the non-dimensional velocity filed will be a function only of the non-dimensional position. The non-dimensional time and Reynolds number there is no other parameter in these equations, because everything is non dimensional zed. So, there is no geometric parameter like the length of pi or anything length, diameter of sphere or anything, because you have already non dimensional zed. So, the non-dimensional velocity can be a function. Only of these three non-dimensional variables, out of which these are this is a physical coordinate along the various points in space. This is time, this is non-dimensional time.

So, the only physical variable that is under our control is actually, the Reynolds number. This is the only a physical parameters that is under control, this is a geometric parameter in the sense of how far you are from the sphere and this is an non dimensional time. Suppose, you have steady flow fastest sphere steady flow then the velocity at any point in the fluid will not be a function of time will be a function of only various special coordinates and Reynolds number. Now, if you were to find the drag force, from this, force is equal to the integral of the shear stress, viscous shear stress. The non-dimensional force will be an integral of the viscous shear stress over the surface of the sphere, roughly speaking.

Now, this is evaluated at the surface of this sphere; that means, that you are integrating over various positions. So, the non-dimensional drag force will be a function only of Reynolds number. This is exactly what we found even in pure dimensional analysis without a worrying about navier stokes equations, but the navier stokes equations also tells the same thing.

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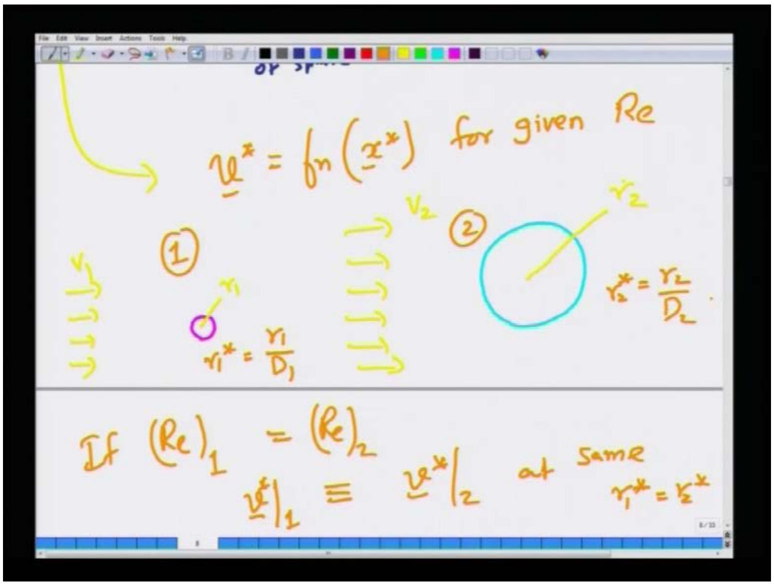

Now, another important lesson from this a result is that, at a given Reynolds numbers the velocity is a function only of for a given of the non-dimensional positions x star. So, what this means is that? Suppose, we have two systems you have a tiny sphere and then you have a big sphere and you have uniform flow passed it far away with some velocity V 1 here it is V 2. Now, if you were to ask, what the velocity in the fluid is? At a distance x 1and here at a distance let us say r 1 from the centre of this sphere and at a distance r 2 from the centre of the sphere.

Now, if Reynolds number is same for this problem. If r e for system 1, this is system 1, this is system 2, this same as r e for system 2. Then the non-dimensional velocity for system 1 will be identically equal to the non-dimensional velocity for system 2. At same non-dimensional distance r 1 star at same r 1 star is nothing but r 2 star r 1 star is nothing but r 1 star is nothing but r 1 divided by D 1. The diameter of the tiny sphere r 2 star is nothing but r 2 divided by the diameter of the biggest sphere. If you ensure that r 1 star is r 2 star, then the non-dimensional velocity at a given non dimensional position is the same, if the Reynolds numbers is same.

So, if you have to do experiments or numerical simulations not just to find integrated quantities like drag forces, but even point wise variation? This non dimensionalization of the navier stokes equation is telling us that it is not necessary to solve this two problems individually separately. If you want the, if you want the information at the same Reynolds number then the velocities of these non-dimensional velocity of this two different systems will be identically equal. If you are looking at the same non dimensional position, which is r divided by d.

So, this is a very very powerful result, because this means, that you do not have to do competition or experiments for these two problems separately. Once you non dimensionalized them, once you solve one problem you have solved a whole infinitely large number of geometrically similar problems and that is a very very important simplification.

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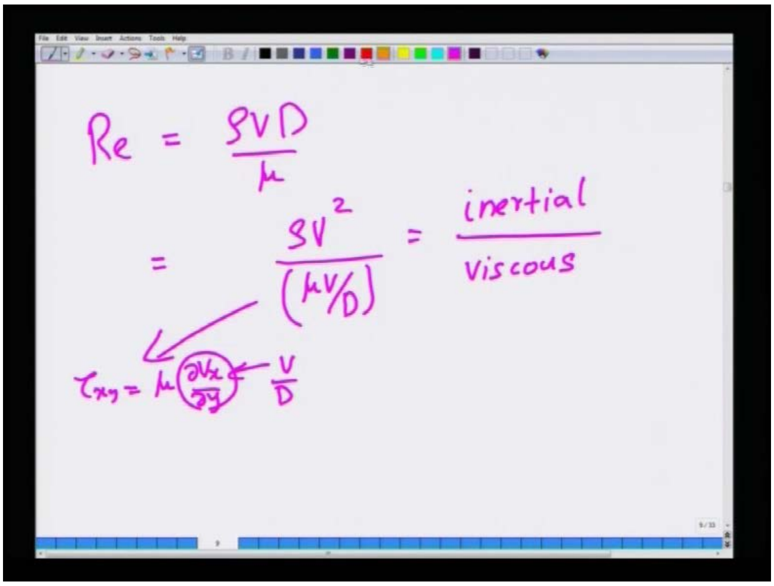

Now, let us also come to the interpretation of Reynolds number, which I promised few minutes back. Reynolds number is written as rho V D divided by mu, which can be written as rho V square divided by mu V divided by D. This is the ratio of inertial forces in the fluid, remember that rho V square is the summation of fluid inertia also comes in the Bernoulli equation to the viscous forces in the fluid. Remember that the Newton's law of viscosity tells you that tau x y is mu times partial V x by partial y in the simplest case. If you estimate this as V divided by D then mu V divided by D is a typical estimate of viscous stress present in the fluid so, inertial forces or inertial stress to viscous stress.

So, roughly speaking Reynolds number is ratio of inertia to viscous forces in the fluid and it is a pure number. If the Reynolds number is very large compare to one, then one can anticipate that inertial forces will dominate the flow. And if Reynolds number is very small compare to one, then one can guess that the inertial forces will be small compare to the viscous forces. This is another important thing that come across in non dieselization.

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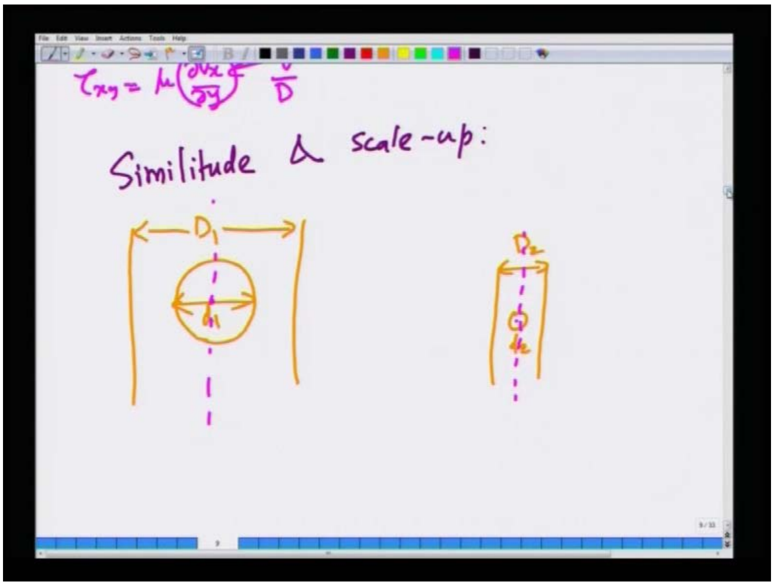

Now, finally, we will discuss, similitude and scale-up. I pointed this out earlier itself, but let me also discuss this again, in the context of an example suppose; I have the motion of a sphere with diameter D and D 1. So, let us call the diameter of the sphere small d 1 and the diameter, it is let us say it is moving in a pipe of diameter capital D 1 and you have another system a tiny sphere of diameter small d 2 moving in a pipe of diameter capital D 2. Let us say that the sphere is the center of the sphere in the center of the axis of the pipe are identical in both cases.

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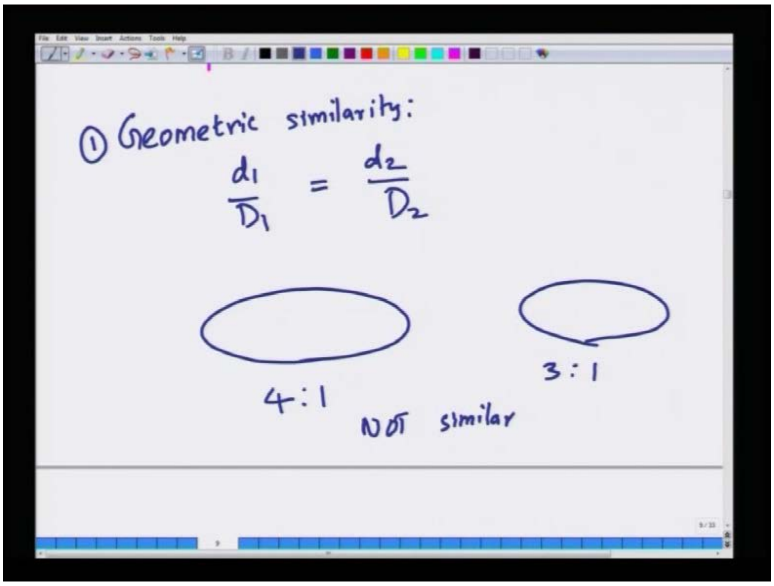

Now, when are these two systems dimensionally similar, when are these two system similar. First, requirement is geometric similarity that is you must have by geometric similarity. We mean that you should have a cylinder a mean, a pipe here and sphere here. You should have pipe here and sphere here, not just that, but the ratio of D 1 to D 1 should be the same as the ratio of D 2 to D 2. So, unless this happens, these two systems are not geometrically similar. More over we have assume that the centre of the sphere and the centre of the axis of the pipe are coinciding in both the cases. If it is if let us say in one of the cases it is eccentrically placed again, geometric similarity is violated.

So, we can say that these two systems are geometrically similar only when all the length scales in the problem have the same ratios. So, we should have first of all the distance of the centre of the pipe and the distance, this the centre of the pipe and the centre of the sphere to coincide on the same axis. And we should also have the ratios of these two length scale to be the same. Otherwise, geometric similarity is not possible. Now, that is one important similarity, the second requirement is kinematic similarity. So, if this is not the case, then you will not have geometric similarity. Another illustration of geometric similarity, suppose you have an ellipsoid. Suppose the major and minor axis or by the ration of four is to one.

You have another ellipsoid you have three is to one then this two or not geometrically similar. So, all the link scale ratios must be the same between two systems, when some things are geometric similar. And there are two more similarity namely kinematic similarity and dynamical similarity. I will come to that in next lecture and these are very powerful ideas that help us in scaling up and scaling down various experimental data from a laboratory to a real industrial device. We will stop here at this point and we will continue in the next lecture.