Fluid Mechanics
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Lecture No. # 28

Welcome to this lecture number 28 on this NPTEL course on Fluid Mechanics for chemical engineering undergraduate students. So, far in this course we have discussed two major approaches towards analyzing fluid flow problems that, occur in chemical engineering.

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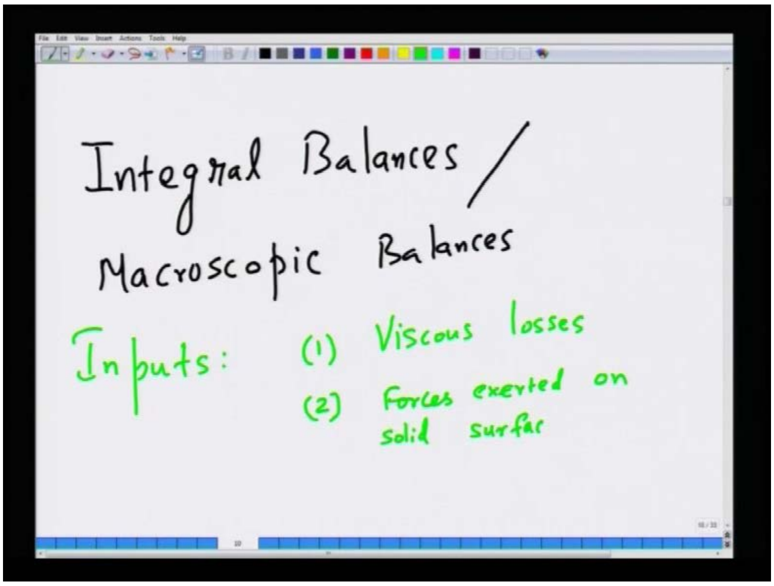

One is the integral balances integral balances or microscopic balances. Here, we write integral balances of mass momentum and energy over a control volume; and when we do this we can choose the control volume to be to comprise of various flow equipments such as, pumps or pipes or compressors or valves and. So, on and we can do this microscopic balances, but to solve this microscopic balances importantly, we need some inputs. The inputs to solve the microscopic balances are viscous losses, viscous losses that occur, in various parts of the system like flow in pipes or flow through a wall or flow through a sudden expansion, there are always viscous losses; we need that as an input and we also need the forces exerted on solid surfaces as an input.

Unless we provide these as an input to the microscopic or integral balances, we may not be able to solve them fully accurately. But in many occasions, because we do not have a knowledge, accurate knowledge of this, we can neglect this and get a approximate answer to various flow problems; and this we highlighted in some of the examples that we did before, but these are not entirely accurate, because we have neglecting some important factors such as, viscous losses or forces viscous forces exerted by the solid drag forces exerted by the solid on the fluid surfaces since we neglect them, we may get only approximate answers.

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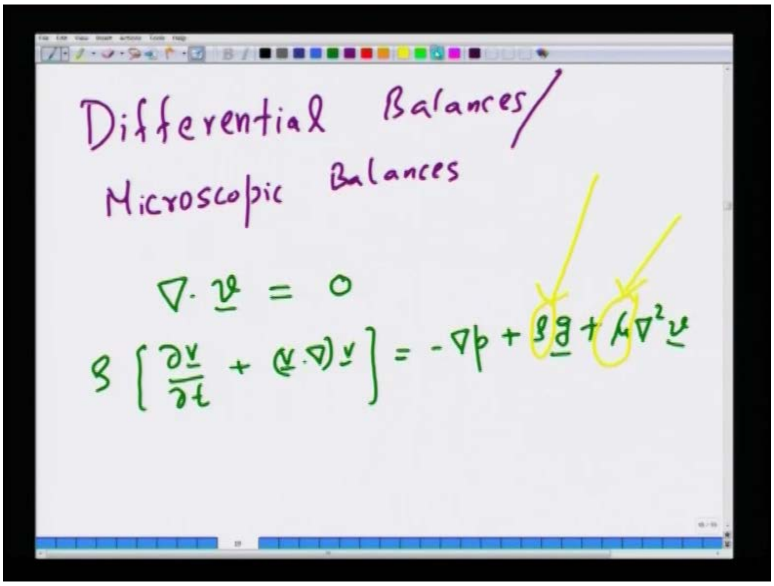

But this is one extreme, the other thing that we did was differential balances or microscopic balances or microscopic balances. Now, the end result of all the analysis on in microscopic balances can be summarized in a very simple way for an incompressible Newtonian fluid and it is deceptively simple, because even though we can write down the equations in a very simple way for the flow of an incompressible liquid.

The solution of this, so called navier stokes equations are notoriously difficult, because in a very very general setting, the navier stokes equation are non-linear coupled partial differential equations and solving them in their most general end form is very very difficult task. So, we had to end up making simplifications, such as flows only in one direction, the flow is steady and the flow velocity varies only in one direction and so on.

We had to make several simplifying assumptions and we can get the solution. But the advantage of differential balances is that, you do not need any input other than saying that what is the viscosity of the fluid and what is the density of the fluid. And once you know what is the viscosity as an input from an experiment and density as an input and once you provide suitable boundary conditions to the flow in principle one can solve them, but in practice the solution is very difficult; so, we had to make all these simplifying assumptions, but we also pointed out that the solutions, that one gets by making this simplified assumptions are not unique.

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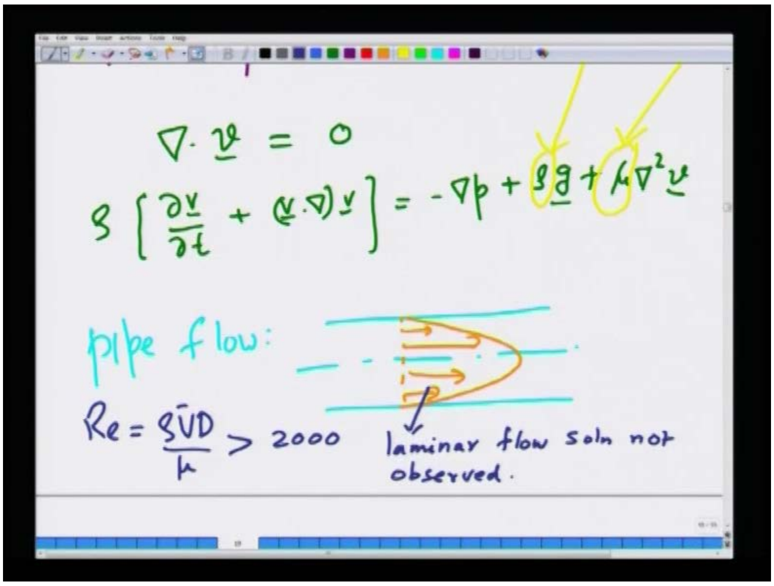

For example for flow in a pipe, we obtain a solution called the hagen poiseuille velocity profile using the differential balances, so this is the pipe, but the solution that we got is not always observed in reality, because the assumptions that we make fail when the velocities are sufficiently high or when the Reynolds number is greater than 2000. This simple solution, which is obtained by solution not observed in experiments.

Now, therefore, even though for Reynolds number smaller than 2000, we can get an accurate description of how much pressure drop is required to make the fluid flow at a given flow rate, we cannot have the same luxury, when the Reynolds number is greater than 2000, because the simplifying assumptions that we made to solve the navier stokes equations, they are not correct.

The flow becomes more complicated or specifically turbulent, so it does not enough to solve the navier stokes equations for such simplifying approximations, one has to in principle one has to solve full navier stokes equations, which is a very very difficult problem. In principle, one has to use very sophisticated numerical logarithms and high performance computers to be able to solve the navier stoke equations in turbulent regime.

So, we have these opposing requirements on the one hand integral balances are relatively simple to solve, but they need some input such as a viscous losses, while differential balances, they do not need any inputs, external inputs only you have to give is the viscosity of the fluid and the density of the fluid through a single experiment; and then everything is specified, but their solution is very very difficult unless one is prepared to go to very sophisticated computation.

So, but in industrial practice in chemical engineering, often the equipment the process equipments that one has they are exceptionally complex. So, one cannot often one cannot always hope to use differential balances to solve for example, the flow problems in chemical engineering.

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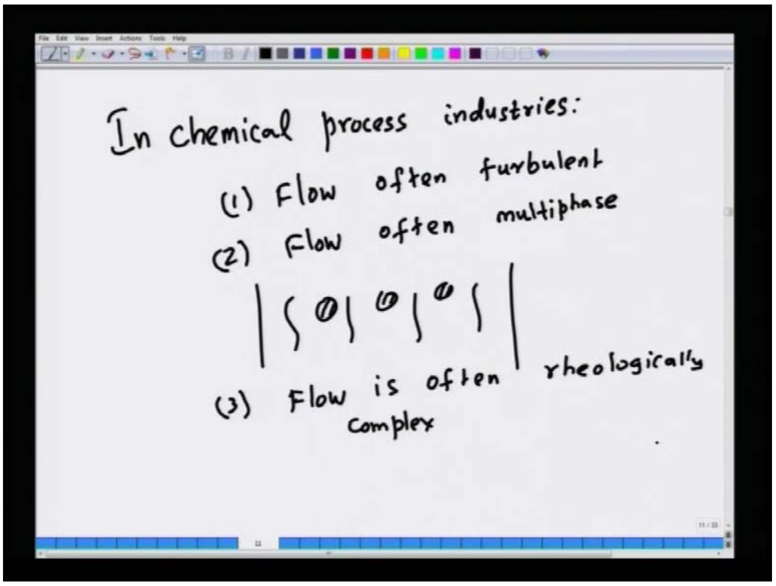

Although one can make an attempt it is not always feasible, the main reason is that in chemical engineering applications, the flow in chemical engineering or chemical process industries, flow is often turbulent and flow is often multi-phase by which, I mean you may have fluid flow I mean a flow of suspension of solids and fluid together. So, and we may have to estimate what is the pressure drop to make such a complex suspension flow in a pipe and for which, we cannot use the differential balance, because differential balances work only for a simple Newtonian fluid.

So, and the flow is often theologically complex, even if it is effectively a single phase flow, you may have flow of a polymer molten plastic which is a very very complex liquid in terms of its rheology you may not described as simple Newtonian fluid. So, it is not sufficient for us to just use the differential balances in many industrial setting; although we will hope to use the differential balances as much as possible because as they are very very accurate than the in the results, but the solution of the differential balances is very very difficult.

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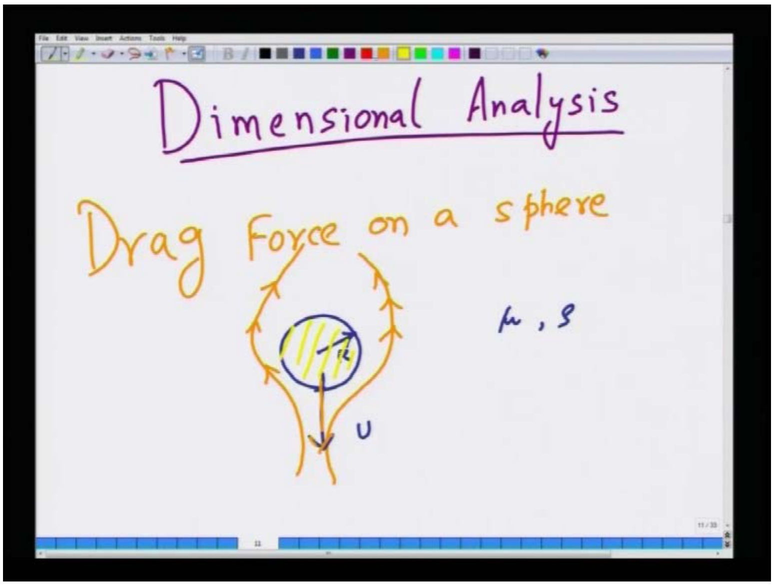

So, the third way of analyzing problems is through experimentation in industrial design. So, we have to carry out experiments for example, if we say fluid is flowing in the turbulent regime in a pipe, we cannot know what is the flow rate that is required what is the flow rate that will come out if you apply a given pressure drop for laminar flow we know the answer, we already solved the answer from differential balances.

For turbulent flow we do not know the answer from first principles to from the navies stokes equations. So, if you want to design a pump to make a fluid flow at a given flow rate or if you want to predict how much fluid will flow, what is flow rate of fluid for a given pressure drop in the turbulent regime, then clearly we have to resort to experiments.

Now, doing experiments in the laboratory is often simplified and it is much made much more organized and logical by doing, what is called dimensional analysis and that is topic of our discussion today. So, in the absence of any solution to a problem through differential balances, we have to resort to experiments; and experiments are made much much more logical and organized and simplified, if we use the principles of dimensional analysis; and that is the goal of our current discussion to illustrate the principles underlying the underlying dimensional analysis and their applications.

Now, I am going to illustrate first, the nature of dimensional analysis by using a very simple problem, the problem is drag force on a sphere. Imagine a sphere moving with a constant velocity, you have a sphere of radius r a nice rigid sphere and it is moving with a constant velocity u in a fluid and the fluid has viscosity mu and density rho.

In many applications, we want to know, what is the force, resistant force that is experienced by this sphere? Due to the fact that the sphere is moving through a fluid, it has to…

When the sphere is moving through a fluid, it has to push the fluid around it in order to move and when the fluid solid surface such as this, this is a solid and the fluid flows around the solid surface, there is a relative motion between the fluid and solid, that means, that they are viscous stresses, whenever there is a relative motion between two fluid elements or between a fluid and solid elements, there are viscous stresses, and when you consider the sum total of all viscous traces exerted by the fluid on the solid surface, you get the net force that opposes the motion of the sphere, that is called the drag force, that is the drag force.

The drag force on a solid substance objects such as, a sphere is the net force that opposes the motion of the solid surface, a solid object, because when the solid object moves through a fluid, it has to push the fluid around and when the fluid moves around the solid's surface it exerts viscous shear stress and that is the concept of the drag force.

Now, suppose you want to know, what is a drag force on a sphere and what type of experiments that we should do? So, let us first do a simple let us first estimate this problem.

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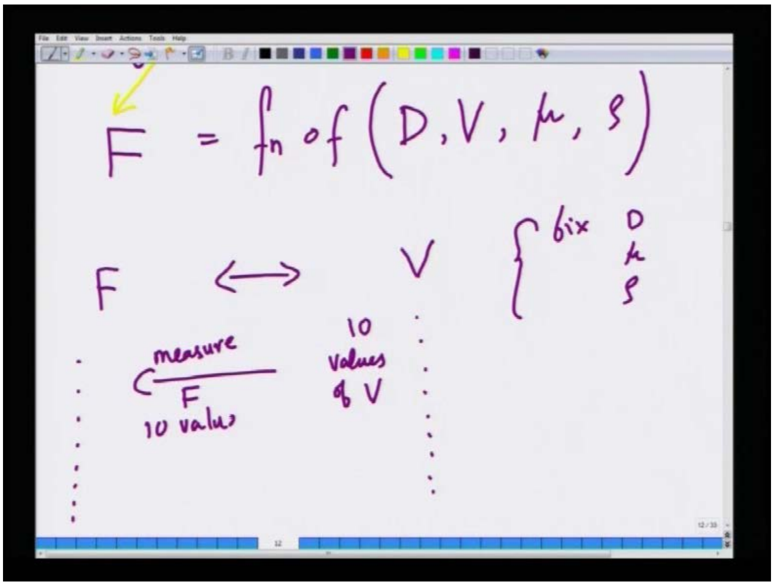

Suppose, you want to know drag force on, you want to have experiments that correlate the drag force on a sphere, on the various physical parameters, that affect the problem. So, first we want to ask, what are the variables in which the drag force should be a function of? So, let us denote the drag force to be F the drag force is denoted by the symbol f. If F is a drag force what is it a function of? It is a function this is of various variables.

We may imagine that it is the function of the radius r or let us say diameter D, D is the diameter of the sphere, we may imagine that it is the function velocity of the sphere, because this sphere moves with a higher velocity we may expect that the force experienced by the sphere is different compared to when its moving at lower velocity, we may imagine that it is a function of viscosity mu, density rho; we may have other variables such as the surface tension or interfacial tension between fluid and solid, but this is a judgment that one has to make.

What are the variables on which, the given physical observer such as drag force should be function of?

You have many variables that affect the problem, but we expect at this point of time that, these are the most relevant physical variables upon which, the drag force will be a function of. And if there are more variables in the problem then, it will show up later while doing experiments. So, these are the variables that, we think are the most important variables that will affect the drag force on a sphere.

Now, if you want to find out how the drag force is going to vary with all these parameters, you have four parameters, the diameter of the sphere, the velocity of the sphere, it is called the velocity V. So, we have four parameters upon which the drag force can depend on and we want to find the parametric variations of the force on all the four parameters. Now, let us imagine how we will do the experiments in a simplistic way.

Well first, we will say that in order to find how F is affected by v, you fix D mu rho and let us say we measure 10 different values of velocity, we have 10 different values of velocity of V of V and we measure 10 different values of F, correspondingly 10 values. Then, so all we would say now all we would have is for a given diameter viscosity intensity how does the force vary with the velocity.

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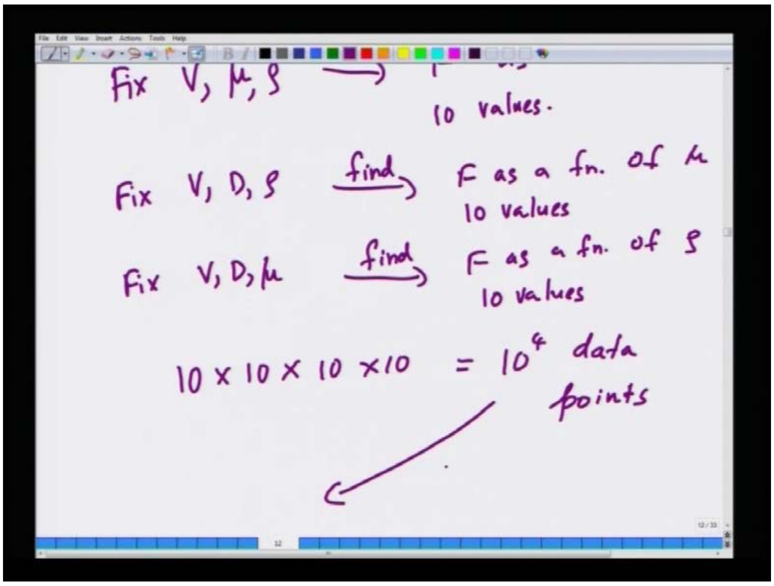

Now, you may say that well, now we want to know how the force varies with the diameter, so you fix the velocity, viscosity, density and then find F as a function of of D, again you may have 10 different values.

So, to find how F depends on just two parameters such as velocity and density, you already have 10 different values of velocity and 10 different sorry velocity and diameter, you already have 10 different values of velocity and 10 different values of diameter you already have 100 data points.

So, you may even it is not even easy to plot such 100 data points as a function of these two variables diameter and velocity, but you have other parameters, you have F as suppose you want to find the force, how the force depends drag force depends on the viscosity, so you have to fix the velocity diameter density and then find F as a function of viscosity, again you have let us said 10 different values.

That is you may choose 10 different fluids of different viscosities and then find F and finally, you may have to find how to F here varies as a function of density. So, you fix velocity diameter and viscosity and find F as a function of rho again you may have 10 values say. So, if you consider all the data points that, you want to generate to find how F depends on these four parameters namely the diameter, velocity, diameter, viscosity and density you have 10 times 10 times 10 times 10, 10 to the 4 data points

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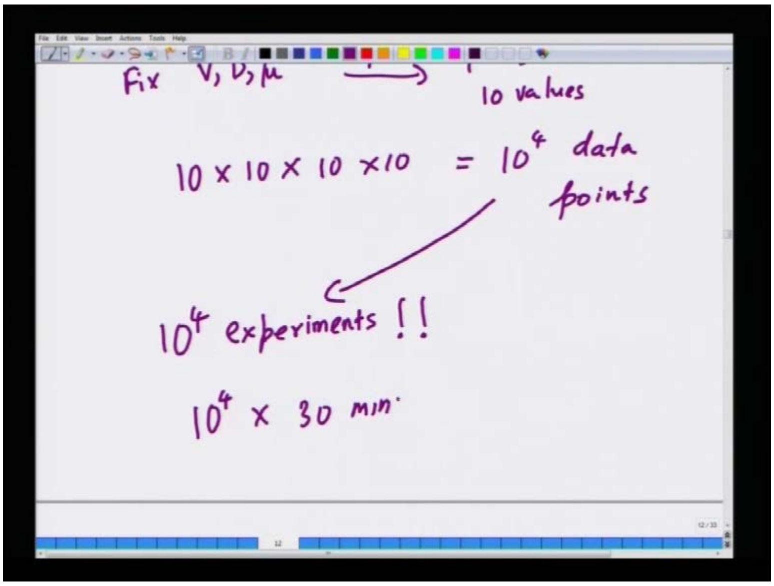

So, So, you have to do 10 to the 4 experiments to find all these 10000 experiments, which is extremely tedious and cumbersome and even if you generate those 10 to the 4 data points, how are going to make sense out of this 10 to the 4 data points? Because you have four independent variables upon which F is function of upon which F depends on, so how are we even going to plot, such a large number of data points with the in which F depends on four independent variables, so it is not at all an easy task.

Secondly, doing 10 to the 4 experiments is also not easy, because if each experiments takes about 30 minutes, then you can say you can say that you can estimate that 10 to the 4 experiments will take 10 to the 4 times 30 minutes, which will be a very very large number of time it is about 2 years.

So, to find how the drag force depends on four variables and with just 10 data points for each variable, we already find that we have to do many many experiments with order of 10 to the 4 and it is going to take a long long time to complete the data. Now, even after completing the experiment making sense out of the data in order to see any trend or to analyze the data is also extremely difficult and cumbersome. So, clearly there has to be better way to do experiments and this is where dimensional analysis plays an important role.

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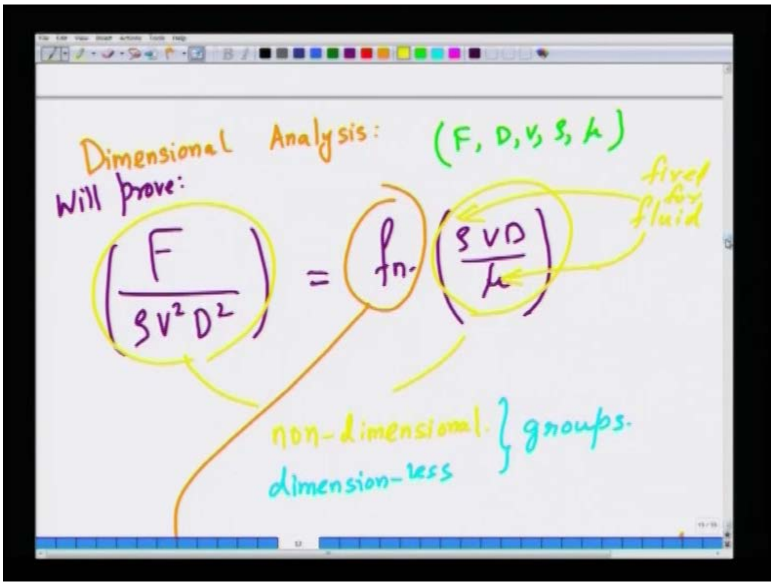

What dimensional analysis does is that, I am first going to make a claim and then I will of course, show how this comes about what dimensional analysis tells is that for this drag force problem, if you do dimensional analysis correctly then, you can write the functional relationship between, you have the 5 variables F, D, V, rho, mu.

The functional relationship among these five variables can be written as a functional relationship between just two groups, but these groups are non-dimensional. You can show, we will show, we will prove that, F this functional relationship among these five variables can be written as V square is a function of rho V D by mu, these two groups are non-dimensional these two groups are non-dimensional.

By non-dimensional, we mean that, that if you check the dimensions of this groups they will be mere numbers, they will not have any physical dimensions such as length, mass or force or acceleration anything associated with them, they will be pure numbers such pure numbers are called non dimensional groups or dimensionless groups, they are also called dimensionless groups.

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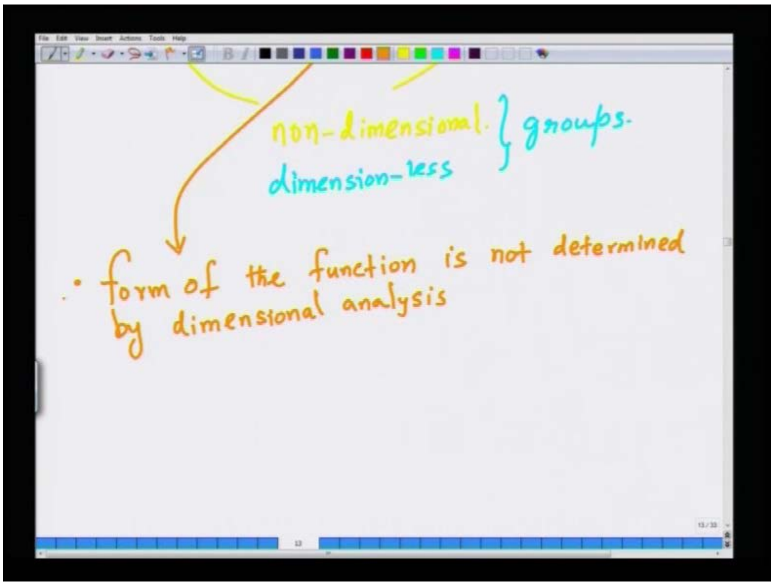

We have some some terms, which are purely non dimensional or dimensionless and we will later show that the relationship between all these 5 variables the force, velocity, diameter, viscosity density can be written as a non-dimensional as a relation between just two non-dimensional groups, which is a great simplification.

Now, we will also point out that, the form of this function the form of the function is still undetermined is not determined by dimensional analysis. The form of the function is not determined by dimensional analysis. We still have to do experiments, but remember that, in contrast to doing 10 to the 4 data points or 10 to the 4 experiments, we simply have to to do, just you have to find the functional dependence of one variable on another variable, one group on another group we simply have to do ten experiments.

So the functional relationship among 5 variables is now reduced to functional relationship among only two non-dimensional groups. Now, first of all it leads to greater, I mean great amount of simplification because, firstly the number of experiments that you have to do decreases by an order of three out some angle from 10 to the 4 experiments, we are now to only 10 experiments, which are fairly easy to do in a laboratory.

Secondly, in the previous case if you want to know how force depends on viscosity, we have to do 10 different fluids, because you have to find 10 different fluids of 10 different viscosity and only then you will be able to find how force depends on all these 10 different viscosities or 10 viscous fluids or 10 different viscous fluids.

Now, here since only we have saying that there is a relationship among these two groups, we can choose the fluid to be anything that is conveniently available to us, for example we may use water or air which is conveniently available. So, we can use the same fluid, we can just use the same sphere and we can make it fall at a different velocity move at different velocity or we can use the same velocity and so, when you choose the given fluid fixed for a given fluid.

The viscosity and density are already fixed, if you choose let us air or water, so the only parameters you have in order to generate 10 data points are either the diameter of the sphere or the velocity at which the sphere is moving. So, depending on experimental convenience we can choose either one of this and generate 10 data points just by varying let us say the diameter of the sphere.

Let us say we cannot change the velocity for some reason. We have experimental set of where the sphere is moving only at a given velocity let us say 10 meters per second or 1 meter per second. Then, the only variable that we have to vary is the diameter and let us say we choose steel balls of different diameters 10 different diameters. So, we can achieve, we can easily carry out this experiments and write everything in terms of this non dimensional groups.

Now, once you find this non dimensional functional relation between this non dimensional group which is a non-dimensional force and this non dimensional group which is essentially the Reynolds number, which we pointed out in the context of transition from laminar to turbulent in a pipe.

We will come to this shortly the interpretation of Reynolds number, but once you find this function relationship between two these two non-dimensional groups, then in principle you can find the force the dependence of drag force on any sphere in a fluid with any any property, any velocity, any viscosity and any density and this sphere velocity can also anything and sphere diameter can be anything.

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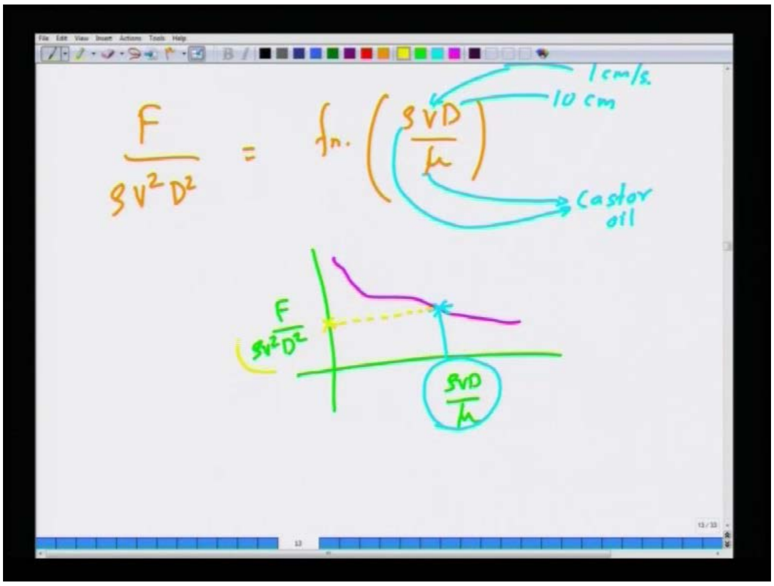

So, but so, let us say now, we have found that F by rho V square D square is a function of rho V D by mu and let us say we have generated, how F varies with F by rho V square D square varies with rho V D by mu. Let us say we have this data points, let us say this is the data experimental data. Now, if somebody comes and tells you that, they have the motion of a sphere and let us say we have generated this experimental data with water and a steel bowls of some let us say millimeter diameter.

If somebody comes and says that, they want to find out the drag force on a sphere of a diameter, let us say 10 centimeters and the more the fluid in which it is flowing is not water, but it is very viscous oil. Then, how are we going to do that how are we going to compute the drag force.

All we want to know is what is the velocity at which the sphere is moving? We need to know this, because if somebody wants to compute the drag force on an object, they should first tell you what is the velocity at which the object like the sphere is moving? So, somebody is telling you that a steel ball of 10 centimeter of diameter is moving in a very viscous liquid like castor oil.

And diameter is 10 centimeters and the velocity is given to us, let us say it is 1 centimeter per second. Then, we can compute the Reynolds this group, because we know the density of the once you say it is a given oil then, viscosity and density are fixed, we know the diameter of the sphere, we know the velocity of the sphere.

Then, we know this group, so let us say that value of the group is here. Now, we have to simply walk across to the y coordinate and find what is this group?

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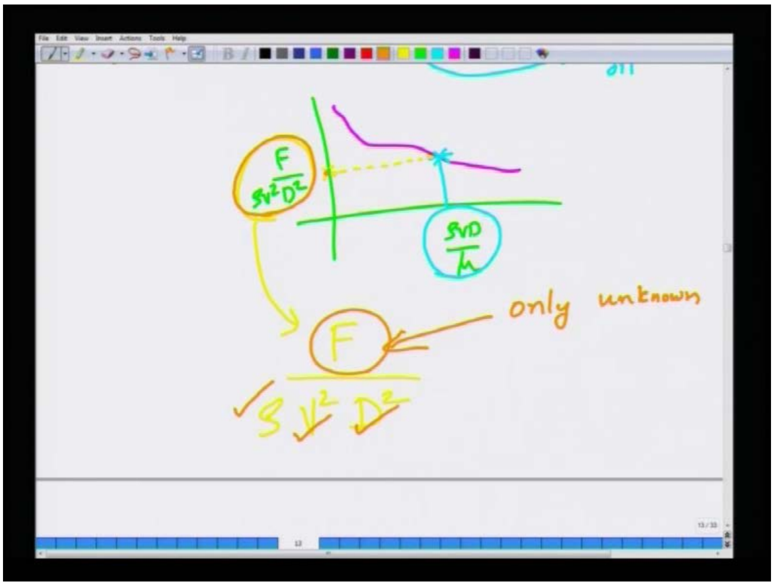

So, what this graph is going to tell you what is F by rho V square D square? But we already know what is rho which is the density of the castor oil which, already know D which is the velocity with which the big sphere is moving and we also we already know the diameter of the sphere and this is the only unknown, which can be computed.

Because, we already know what is this combination F by rho V square D square from this graph, but this graph was obtained for a completely different system. It was obtain for motion of let us say 1 mms or 5 mms steel ball in water and that is how you made this graph by suitably non dimensional zing the variables.

But now, once you have done that that graph that you obtain and which you plotted in terms of these non dimensional groups is valid for any problem provided the fluid is Newtonian. And you can find out the drag force on any object of any dimension moving through any fluid with any viscosity. So, that is the power of dimensional analysis, that first of all it is able to reduce the amount of experimentation.

Secondly, now you are able to do experiments on things, that are easily available in a laboratory, let us you have a very tiny steel ball of 5 m diameter and you have water readily available in a laboratory. You can do the experiment, but (()) or you have to report the experimental data or reanalyze or regroup the experimental data in terms of these non-dimensional group F by F divided by rho V square D square and rho V D by mu.

Once you do that, then the results of such a graph are valid for any sphere of any diameter moving with any velocity in any fluid, so long as the fluid is Newtonian because in your lab you have used water as the only as the as the test fluid to get these results, so this is once very very important simplification that one gets.

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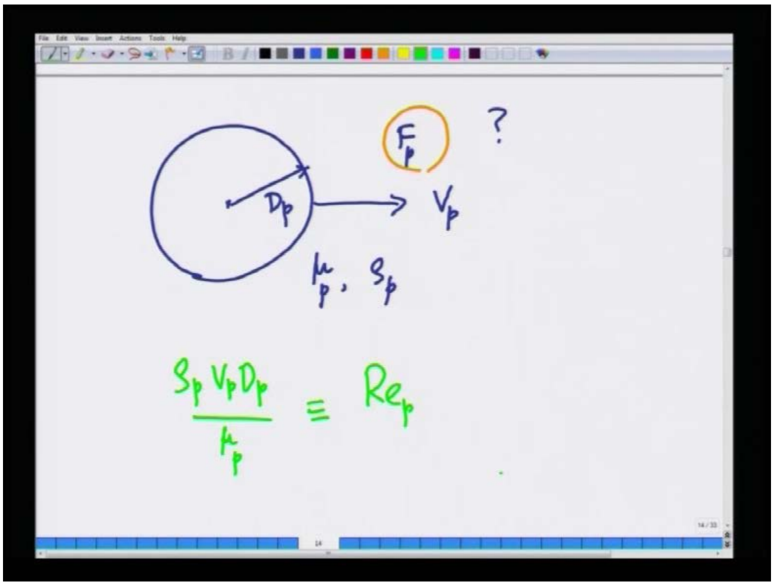

Now, suppose let us say you do not have the data for F, you have not done many experiments to get this data, but still you you want to know, what is the drag force on sphere with diameter D 1, let us say D prototype and the velocity of this sphere is V prototype moving in a liquid of viscosity mu prototype and density rho prototype.

What is the drag force experienced by this? Let us say, this is a very very huge sphere moving in a very viscous liquid, we want to know this, but we do not have access to the data like before, we do not have have this non dimensional relationship between F by rho V square D square and rho V D by mu. Suppose you will not have this how are we going to estimate this drag force, we want this, we want to compute this in some application.

Now, in order to this we can scale down the problem in the following way. First compute the Reynolds number rho p V p D p of the prototype, this is the non-dimensional group call the Reynolds number which I have we which we have encountered before, first compute the Reynolds number of the prototype.

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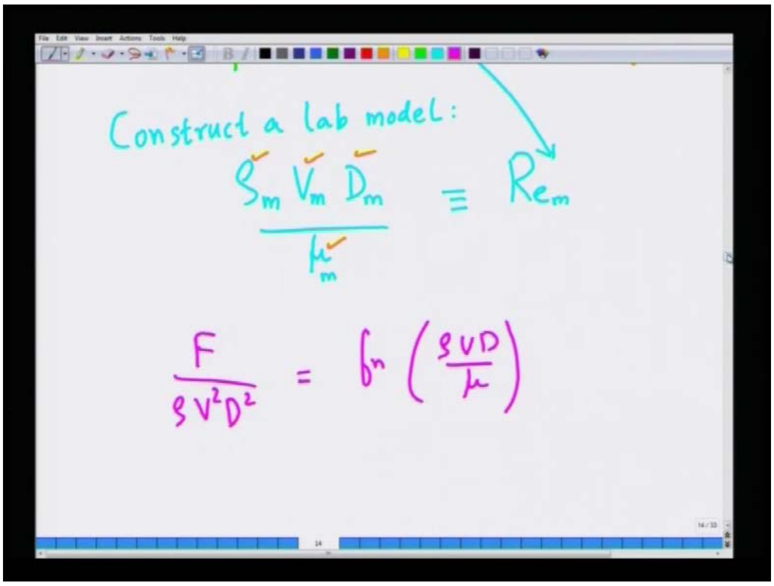

Now, construct a laboratory model construct a laboratory model in which, you choose a liquid that is conveniently available to you a model liquid rho m with density m, you choose a velocity that is feasible to you, choose a sphere of diameter that is feasible to you in such a manner that this Reynolds number is identical to the prototype Reynolds number.

R e model should be equal to R e prototype that is you should choose these values rho V D mu such that, with the Reynolds numbers of the model and prototype are the same. Then, what dimensional analysis is going to tell you, which we will show shortly that, F by rho V square D square is a function only of rho V D by mu.

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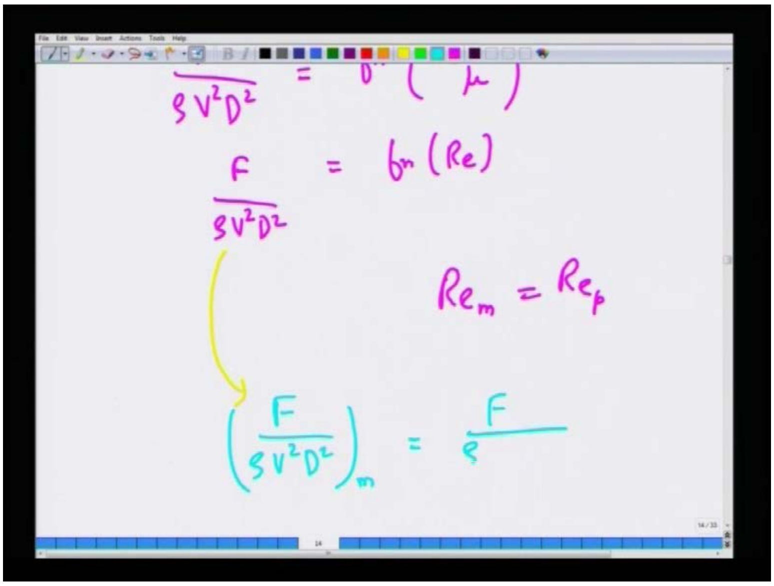

If that is the case, then once you fix the Reynolds number. So, the non-dimensional force drag force on an object is a function only of Reynolds number. Since we have fixed the Reynolds number of model to be the same as the Reynolds number of prototype, what this equation is telling us is that F by rho V square D square for the model should be the same as F by rho V square D square for the prototype.

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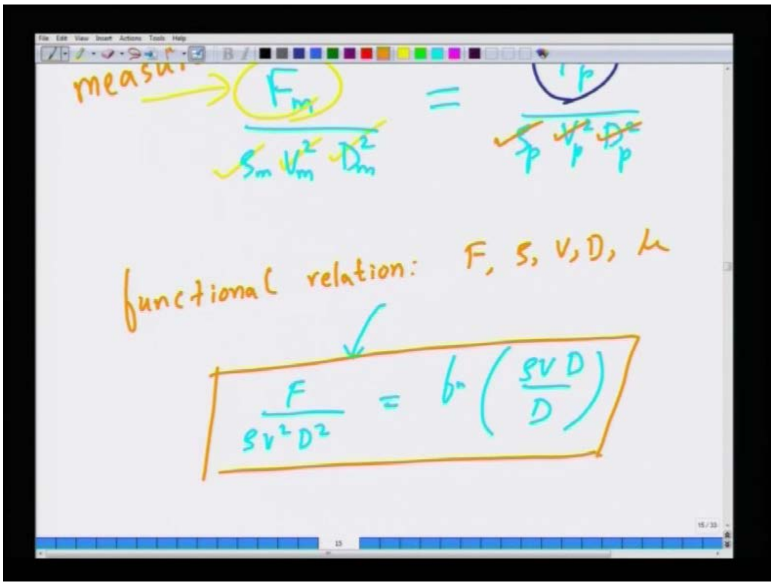

Now, for the model we can measure the drag force, so this becomes F model by rho model V model square D model square, so you measure F for the model, you measure this, you measure this. Now, this should also be equal to F prototype by rho prototype V prototype square D prototype square. Now, we know, what is the density of the liquid in the prototype in the real situation, we know, what the velocity with the sphere is moving, we know, what is the diameter of the prototype sphere particle. So, since and we also know, the density of the model liquid D velocity at which you are making the sphere move in the model and the diameter of the sphere, so the only unknown can be computed.

So, you need not have the entire data as to how F by rho V square D square depends on the non-dimension group rho V D by mu that is Reynolds number. So, long as we know what is the Reynolds number at which the prototype is operating and if you match the Reynolds number of the prototype with the Reynolds number at which, you are carrying out experiment in the in the lab and in which you can easily measure the force, then it is much easier to compute the force that will be experience the drag force that we experienced by the prototype particle just from this relation.

And why is this possible all this is possible is, because of dimensional analysis because dimensional analysis tells us that, the functional relation between F rho V D mu can be written as F by rho V square D square is function of rho V D by mu, this is made possible by dimensional analysis.

So, first thing many many simplifications come by dimensional analysis, one is the notion of reducing the number of experimentation to obtain a functional relationship between many variables, secondly, once you once dimensional analysis tells us that, the relationship between the drag force and various parameters is only through this non dimensional form then, in order to find the force on a real situation, a prototype situation where as a larger sphere is moving let us say through a very viscous liquid, all we have to do is find the Reynolds number of the prototype.

Now, do a similar experiment in the lab, by matching the same Reynolds number, but by using fluids and spheres that are readily available to us and velocities that are easily feasible in the lab. As long as you match the Reynolds number then the non-dimensional force on the model on the prototype must be identical, because at a given Reynolds number there is a unique relation between these two non-dimensional groups.

So, once I know what is the non-dimensional force, this group on the model then, that should be the same on the prototype then by knowing the velocity prototype diameter of the prototype and density of the prototype liquid, we can calculate what is the drag force experienced by a prototype. So, this is the most important lesson that one gets from dimensional analysis. So, far we have told you the advantages and consequences and applications of dimensional analysis, but we have yet not yet told you how to achieve this relation, this non dimensional relation.

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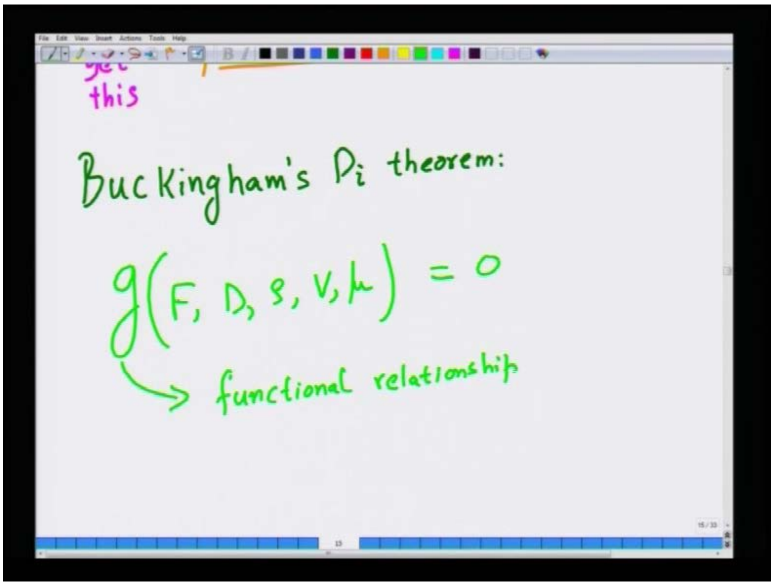

How to get this? Now, that is what we are going to do next and that is through a principle called Buckingham's pi theorem. Suppose, you have a functional relationship among 5 variables, let us say the functional relationship is denoted by this small letter g, so we have this 5 variables force, diameter, density, velocity, viscosity. And this is the function we want to determine through experiments. In general this is the functional relationship (No audio from 39:23 to 39:33).

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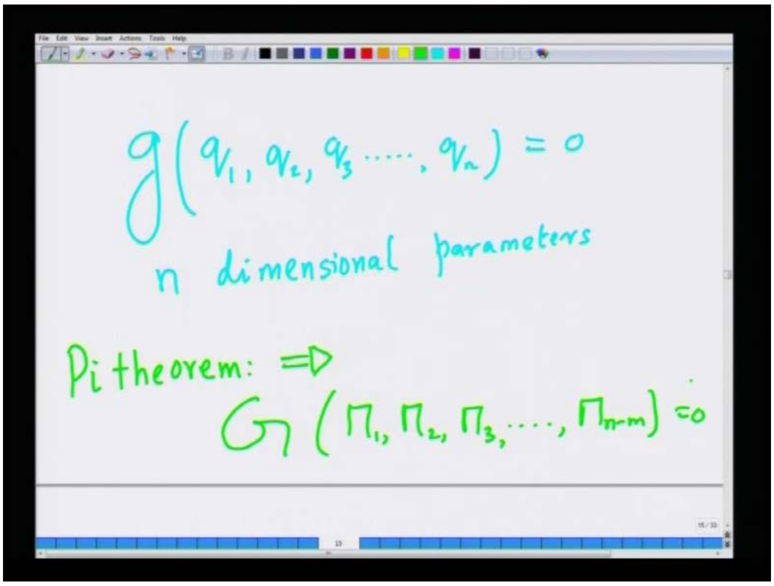

In general I can generalize this to a functional relationship among n variables, so now I am going to generalize the discussion and not restrict ourselves to the drag force, but of course, from time to time I will draw comparisons between the general formulation as well as the specific problem of drag force on a sphere.

So, in general let us say there are q 1 q 2 q 3. So, on up to q n variables and so, there are n dimensional parameters and you want to find a relationship among these n dimensional parameters. Now, what the pi theorem tells is is that, you can write this functional relationship among n dimensional parameters as a functional relationship among n minus 1 non n minus m non dimensional groups.

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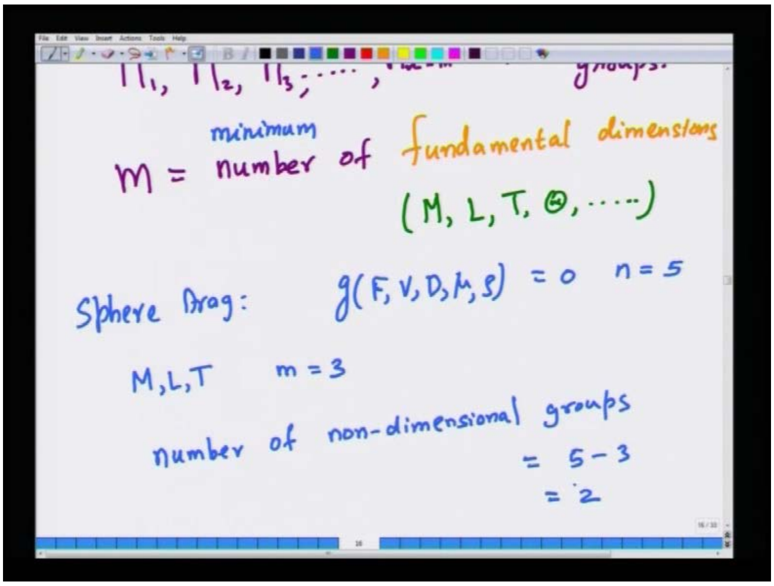

Where pi 1, so let me explain the notation pi 1 pi 2 pi 3 and so on up to pi n n minus m, they are non-dimensional groups, they are non-dimensional groups they are dimensionless groups in other words; and m is the number of fundamental dimensions in the problem.

What do you mean by number of fundamental dimensions? The fundamental dimensions is in any physical problem are mass, length, time, temperature and so on, so these are the fundamental dimensions, present in a problem in a physical problem; so, an n m is the minimum number of fundamental dimensions fundamental dimensions present in the problem, used to describe all the parameters.

So, let us now go to the sphere problem sphere drag problem, you had how many variables you had g as a function of F V D mu rho is 0, so you had n equals 5 variables and the fundamental dimensions are mass, length and time. If you look at dimensions of all the parameters then, you will find that there are only 3 fundamental dimensions, so m is 3 with these 3 fundamental dimensions, we can describe the dimensions of all the variables, there is no need to go.

This is the minimum number of fundamental dimensions, that is that are used to describe all the variables dimensions of all the physical variables present present in the problem, so that is 3 for this case, so what pi theorem is telling us is that the number of dimensionless or non-dimensional groups is 5 minus 3 is equal to 2 and those two groups an illustration or an…

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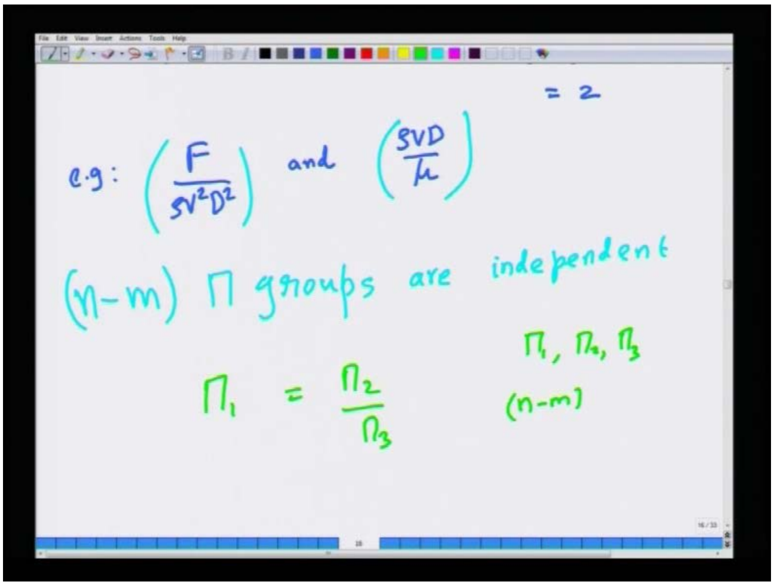

An example of those two groups is what are what we wrote before F by rho V square and rho V D by mu, examples of such two groups, non dimensional groups are these things, are these two groups.

So, the pi theorem for the problem of drag force on a sphere tells us that, the functional relationship among 5 dimensional physical variables namely F, V, rho, mu, D can be represented as a functional relationship among only two non-dimensional groups and the pi theorem will also tell you, how to derive those two non-dimensional groups.

So, but again the pi theorem or dimensional analysis does not tell you what is functional, what is the nature of the function between the two non-dimensional groups, that is not told by dimensional analysis, that still has to be carried out, that still has to be determined only through experimentation, but it does the dimensional analysis does result in a great amount of simplification by reducing the number of variables in the problem by from 5 to 2, initially we had 5 dimensional variables, now we have only two dimensionless or non-dimensional variables.

Now, these n minus m pi groups the non-dimensional groups are called sometimes called as pi groups are independent, in the sense you cannot get one pi group by combining the other pi groups, once you have found n minus m pi groups for example, here you have these two pi groups, you cannot get one pi group from another they are functionally related, but you cannot just manipulate one from that, they are independent because ultimately you will have to experiments to find how this is function of that.

All I am saying is that you cannot write in a more complicated problem, you cannot write pi 1 as pi 2 by pi 3, because you have pi 1, pi 2, pi 3 as three independent groups, you cannot get one in terms of other readily. So, So, all these three all in general n minus m groups are independent groups in that one cannot write one in terms of the other. Now, suppose you do find that after doing experiments.

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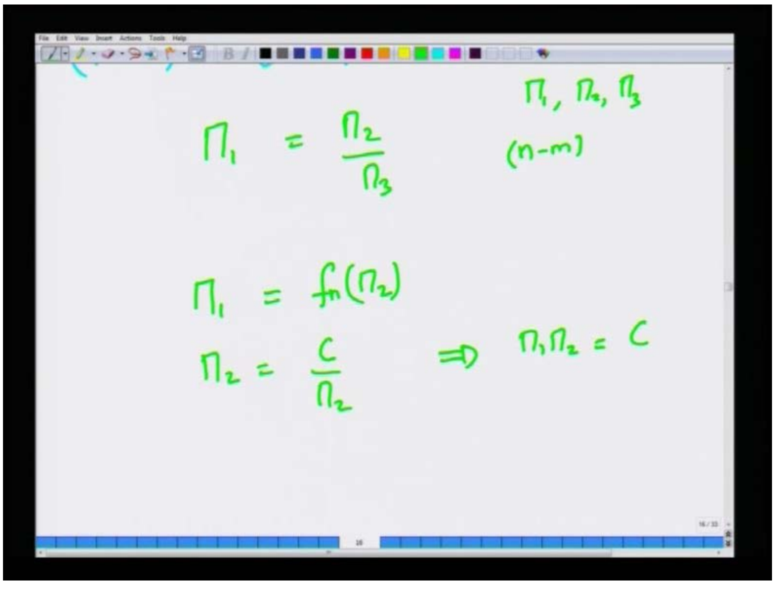

Suppose, you find that you have two only two groups and let us say you have pi 1 as a function of pi 2 and after doing experiments, you find that pi 1 is equal to 1 over pi 2, this implies that pi 1 pi 2 is equal to 1 or some constant over pi 2, is this constant. We will show later that this implies that you have overestimated the number of physically relevant dimensional variables in the problem; and one of those dimensional variables will automatically drop out of the functional relationship if it, so happens.

If the physical circumstances are such that one of the variables is irrelevant then, you will find that these two groups are not they are not simply they are very very simply related and you can combine these two groups to get a third non dimensional another non dimensional group and you will find that, one of the dimensional variables becomes an irrelevant variable in the problem, so I will point point this out little later when we actually when we implement dimensional analysis.

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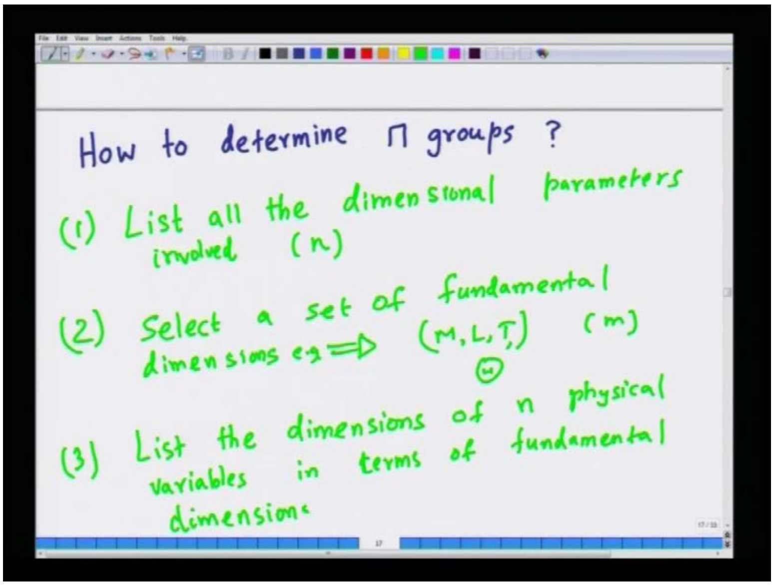

But now let us go through how to determine the pi groups, so all I have said is a claim. All I have said is a claim, that given a set of m dimensional variables and you have given a set of n dimensional variables and you have m fundamental dimensions minimum number of fundamental dimensions required to describe the dimensions of those n dimensional variables then Buck Ingham's pi theorem tells you that, you can reduce these n variables to n minus m non dimensional groups, now and these groups are often called as pi groups traditionally, historically.

How are we going to determine these pi groups? So, there is a well set methodology set of rules, that one follows and one can easily get the pi groups, let me go through it step by step. First list all the dimensional parameters involved in the problem in the problem, let it be n. Now in the case of drag force past a sphere we said that force is the of course, objective is to get the force, it could be function of velocity, diameter, viscosity, density; but we neglected the interfacial tension and we probably neglected the specific capacity things like this because, we felt that those variables are irrelevant to the problem.

Now, how are we going to check whether the hypothesis is correct, it will be proved right by experimentation only and right now it is purely a physically motivated guess that we are saying that these are the physical variables that are relevant to the problem.

Now, initially the students may find it difficult to find out to to judge, what are the physically relevant variables to the to a given problem, but it is a matter of physical and engineering judgment as to what these are, but with practice and with experience then one gains enough experience to write down, what are the relevant dimensional groups that that affect the given problem.

Now, the next step is select a set of fundamental dimensions, for example mass, length and time. Mostly in our case, we can choose mass length and time as a set of fundamental dimensions; occasionally people may use instead of mass a force as a fundamental dimension, but for all practical purposes you have to use only mass length and time in fluid mechanics problems.

Suppose, you are doing heat transfer then you may have to add temperature as a fundamental dimension because within the realm of classical thermodynamics temperature is another fundamental dimension. Of course, if you go to molecular theories such as statistical thermodynamics or kinetic theory then, temperature is essentially measure of average kinetic energy of molecules, so that becomes not a fundamental dimension, but within the realm of continuum classical thermodynamics, it is a fundamental dimension, so in heat transfer we may have to use temperature also.

Now, let let this be m, the number of fundamental dimensions. Now, we have to the third step is to list the dimensions of the n physical variables in terms of fundamental dimensions. So, we have to write, what is the dimension of suppose you choose mass length and time, we have to first write what is the dimension of force in terms in m l t dimensions and so on.

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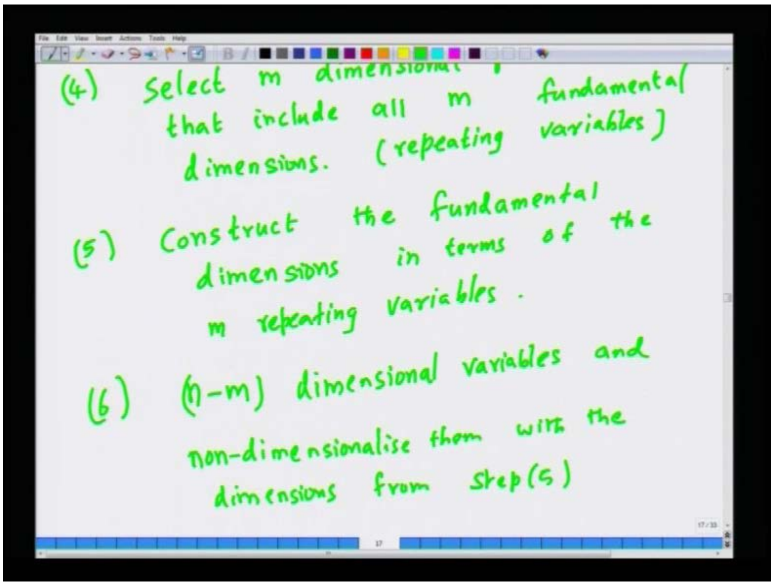

Now, the forth step is to select m parameters, dimensional parameters, that include all fundamental dimensions all three dimensions. Not all three in general all m and of course, m is less than n, because there are n numbers of physical variables; and out of these n numbers of physical variables you are choosing m physical variables, which have all the three fundamental dimensions.

Now, the next step is to construct the fundamental dimensions, in terms of the m, these are called the repeated variables, so let us give a name to them these are called the repeating variables, in terms of the m repeating variables, once you do this you take the remaining n minus m dimensional variables and non-dimensionalize them non-dimensionalize them with the dimensions constructed out of from step 5.

You have already constructed all the physical fundamental dimensions in terms of m repeating variables, so all you have to do is take the remaining variables and none dimensionalize them and finally, check whether if you have done everything correctly this n minus m dimension groups will be purely non-dimensional, they will not have any dimensions. We will stop at this point and we will illustrate this method in the next lecture for drag force fastest sphere. Thank you.