Fluid Mechanics

Prof. Vishwanathan Shankar
Department of Chemical Engineering
Indian Institute of Technology, Kanpur

Lecture No. # 27

Welcome to this lecture number 27 on this NPTEL course in Fluid Mechanics for chemical engineering undergraduate students. The topic of discussion is the application of differential mass and momentum balances to various simple flows that is, the solution of Naviar strokes equations to various simple flow in various simple flow geometry such as flow in channels and pipes.

In the last lecture lecture number 26, we started the discussion on study laminar flow in a pipe and we will complete the discussion in this lecture.

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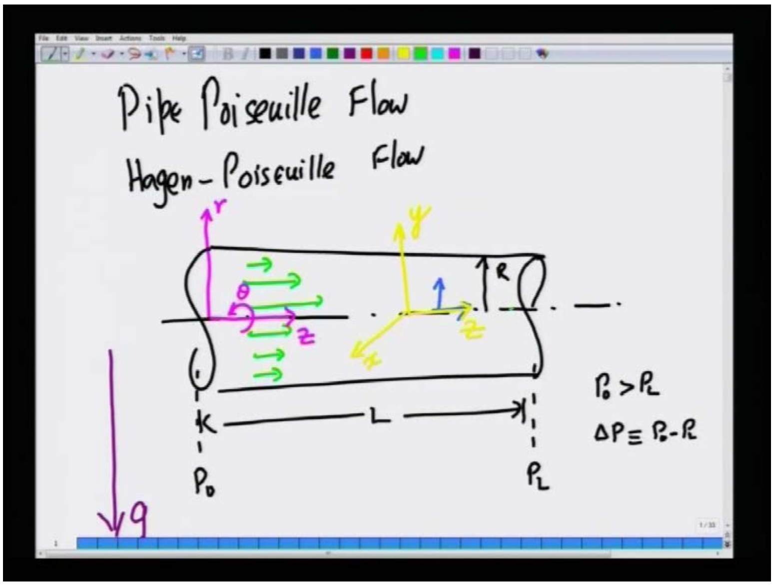

So, just to remind you we were considering the (()), so called Pipe Poiseuille flow or the Hanger-Poiseuille flow, essentially you have a pipe of some radius R and let R be the radius of the pipe and let us see across the length of the pipe, there is a pressure difference P 0 P L, P 0 is greater than p l and we will define delta P as P 0 minus P L.

So, due to this pressure difference across the ends of the pipe, there will be flow in the pipe and we want to know for example, what is the velocity distribution of the fluid in the pipe as well as what is the… Suppose if I want the, suppose the if I want a given suppose if I apply given pressure drop what is the flow rate, volumetric flow rate volumetric flow rate come out of this pipe for a given pressure drop these are the questions of practical importance in many many industrial applications.

Now, before I proceed with this problem, first thing is to put a coordinate system and I told you in the last lecture that, this cylindrical polar coordinates is very convenient in this particular geometry, if you align this z coordinate along the access of the pipe and r coordinate along the radial direction of the pipe, so theta coordinates of a cylindrical polar coordinate system is going around the axis.

Now, you can imagine in the relation to a Cartesian coordinate frame, suppose if I draw the Cartesian coordinate frame in yellow color in the same, so the z will be pointing in this direction, you can imagine the x y plane can be kept orbitrary, can be oriented orbitrary, but suppose the pipe is oriented horizontally in such a manner that gravity is acting perpendicular to the pipe we can imagine placing the y coordinate in the direction opposite to gravity and therefore, that will automatically fix the x coordinate, because the once you fix the two coordinates then the third coordinate is automatically fixed in a right angle in an orthogonal coordinate system.

So, I am intentionally aligning y in the direction opposite to gravity of course, you could it anyway, but just a matter of convenience, so that is the Cartesian coordinate, but we are going to work with the cylindrical coordinate system. In a cylindrical coordinate system any you are essentially interested right now, in the variation in the x y plane because things are independent of the z directions. So, essentially I you are interested in the variation in the x y plane. So, the r coordinate vector will, so you can orbitrarily focus on any plane and the r coordinate will simply be the distance from the center to the to the distance from the center along the radical direction of the pipe.

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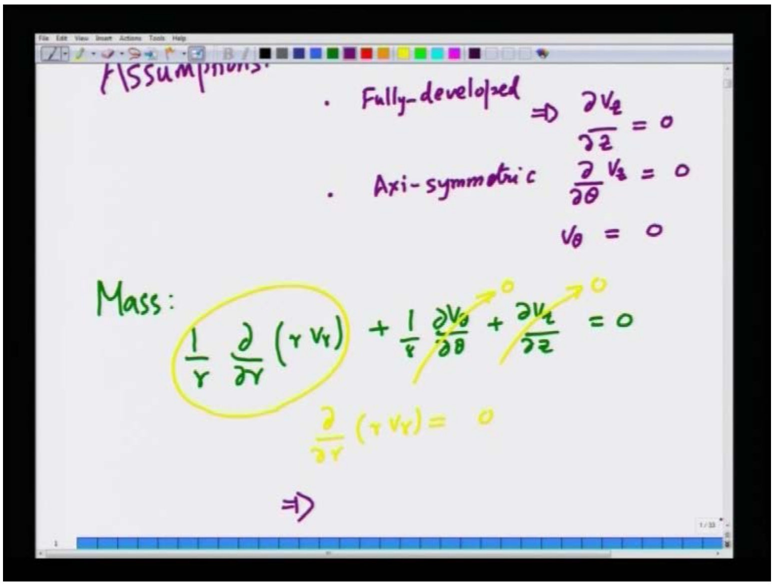

Now, we will make the assumptions as usual, the key assumptions are that the flow steady, if the applied pressure drop is steady, then it is reasonable to expect that the flow study, the flows fully developed which implies, so let me also write down what this mathematically means? Means that the local time derivative of any quantity the partial derivative of the any quantity with respect to time is 0 flow is fully developed implies that, there is no variation of the flow velocity. The flow velocity is in the z direction because you are applying the pressure drop in the z direction, because the fluid the fluid is going to flow in the z direction.

So, there is no variation of the z velocity in the z direction, so that is the fully develop flow assumption and the flow axis symmetric, which implies that, there is no variation of the flow velocity z in the theta direction also it means that, V theta is 0, because if if there there is no driving force in the theta direction for driving a flow along the theta direction and if there is theta velocity, that is going to break the symmetry along the axis symmetry along axes symmetry along the theta direction.

With this assumptions the continuity equation or the mass conservation equation, let us write it as the mass conservation equation essentially it it became plus.

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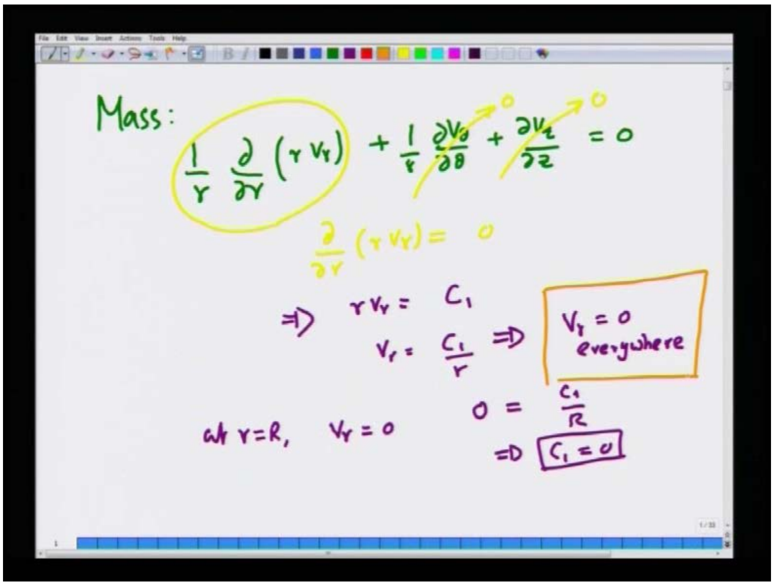

So, there is no variation of z velocity in the z direction, there is no theta velocity axes symmetry so this implies that partial by partial r of r v r is equal to 0 or this implies that r v r is a constant. Let us say c 1, so v r is by c 1 by r at r equals r, then radius of the pipe the boundary condition is at v r is 0. So, if you put 0 is c 1 by r which implies that c 1 has to be 0 if the constant c 1 is 0 this equation tells you that v r is 0.

Everywhere in the domain it is not just 0 on the walls it is 0 everywhere in the domain. So, the continuity equation automatically tells you that if you assume that the flow is axes symmetry and fully developed then, there is no reason for us to have a velocity in the r direction in the normal direction, so that is the input from the continuity equation. The z, the momentum equation there are two components the there are three components the flow component, which is the most important and the other two components which are in the r and theta direction in the r direction.

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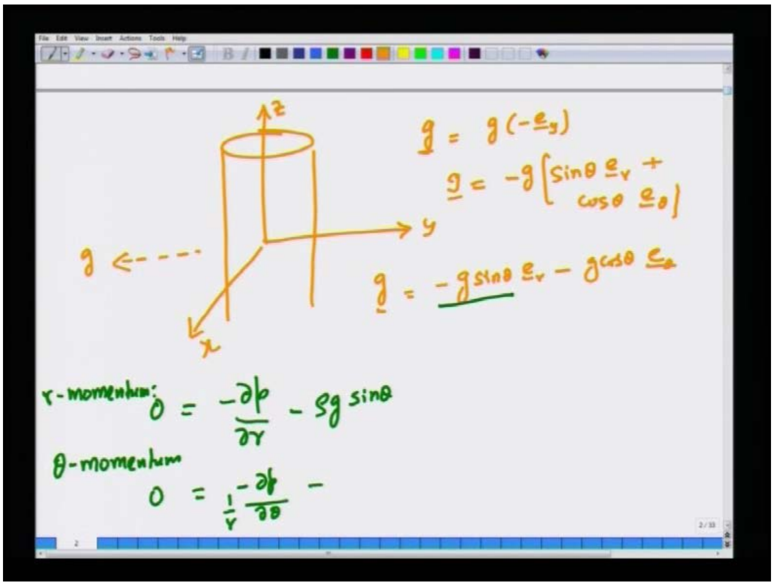

So, let me just redraw the figure for you. You have a pipe and we have aligned, gravity you have aligned y like this and x like this and gravity in the minus y direction and flow in this z direction, so the gravity vector is pointing in the minus e y minus y direction. So, this is the gravity vector, this is acceleration due to gravity magnitude in the direction of the vector is minus e y.

Now, I can write e y resolve y in terms of e r and e theta as follows, e y is simply minus e times sin theta e r plus cos theta e theta, so g becomes minus g sin theta e r plus g cos theta e theta. So, if I write down the momentum balance in the r and theta directions, we will get 0 is minus partial p partial r, this is the r momentum minus partial p partial r minus the component of gravity in the r direction which is g sin minus g sin theta and similarly theta momentum equation theta momentum equation is minus partial p 1 over r partial theta minus, I think this is minus g cos theta, because there is a minus sign overall therefore, it is minus g cos theta, so it is minus rho g cos theta.

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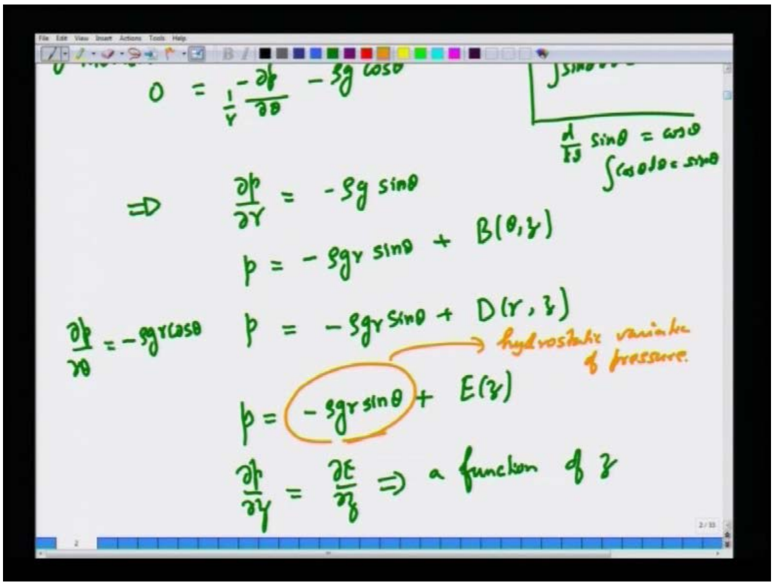

If I integrate the first expression with respect to r you will get minus partial p partial r or partial p partial r is minus rho g sin theta. Now p if I integrate partially with respect to r I will get minus rho g r sin theta plus a constant which is a function of theta, if we integrate the theta momentum I will get partial p partial theta is minus rho g r cosine theta.

So, if I if I integrate this with respect to let me redo this again if I integrate this the r momentum with respect to if if I integrate r momentum equation with respect to the partially with respect to the r direction. So integral of sin theta d d theta of cos theta is minus sin theta, so integral of sin theta is minus cosine theta plus a constant, so this there would not be a minus I am sorry this is a integrate only with respect to r.

So, I will sorry I am not integrating with respect to theta we will come to that later, but the key thing is that, this constant will be a function of both theta and z, because I am only partially integrating with respect to r likewise if wee integrate this with respect to theta minus I will get minus rho g r and d t theta of sin theta is cos theta, so integral of cos theta d theta is simply sin theta.

So, minus rho g r sorry sin theta plus some constant d of theta z and if I compare these two sorry d of r z, because I am integrating this with respect to theta, so the constant can in general be a function of r and z, so that if I take the partial derivative of this with respect to theta I get back this. Now, if I compare these two relations I can say that p has to be minus rho g r sin theta plus this constant can be a function of theta and this not to be a consistent this has to be just a constant of z sum of z.

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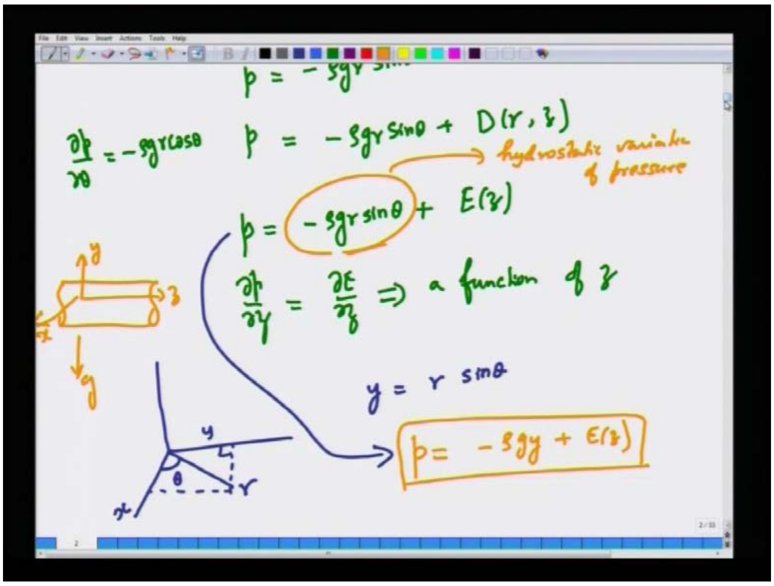

So that, partial p partial z will be a function only of z this is merely the hydrostatic this this hydrostatic variation of pressure in the y direction that is because in cylindrical coordinate system if I have r and theta y is nothing, but y is nothing, but r sin theta. So, this is x this is y if I want to know what this is in terms of r this is x is r cos theta y is r sin theta, so if I rewrite the pressure pressure becomes minus r sin theta is y rho g y plus e of z, this essentially means that if, since I have aligned the pipe in such a manner align I have, align the pipe in the manner such that gravity is like this and y is like this there as to be n z is like this and x is the direction again perpendicular to gravity.

So, it is clear that there cannot be pressure variation hydrostatic pressure variation in the x direction it has to be only in the y direction, so that comes through from our analysis that the pressure varies only along the y direction.

Now, let us and the most important input from this momentum balance, the normal component of the momentum balance is that is this that the pressure gradient in the z direction is the function at of most of z it cannot be a function of r and theta.

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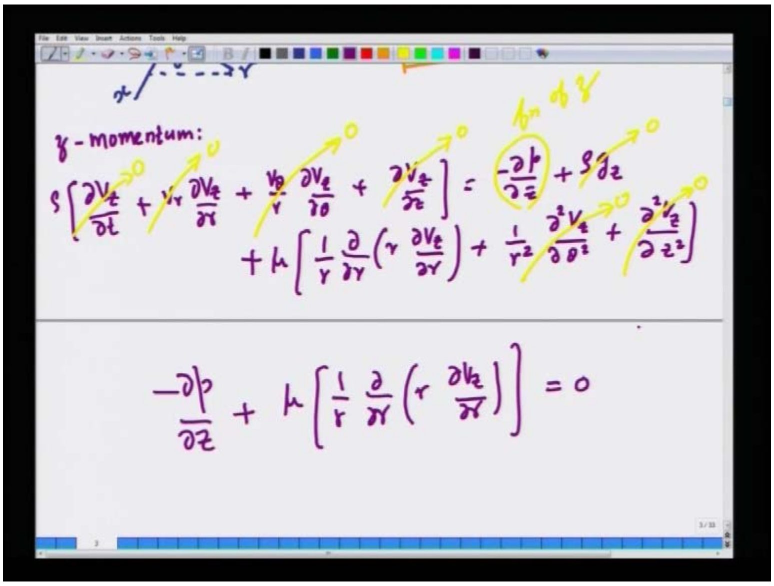

Now, let us go to the x momentum equation sorry or the z momentum the direction of flow is z in the cylindrical coordinate system, so you will get various terms like this rho partial v z t d t partial t plus v z or theta z. So, v r partial v z partial r plus v theta by r partial v z partial theta plus partial v z partial z is minus partial p partial z plus rho g z plus mu 1 over r partial partial r of partial r partial r plus 1 over r square partial square v z partial theta square plus partial square v z plus partial z square, this is the entire momentum equation in the z direction.

Now we will knock off terms steady flow means this is 0 there is no r velocity this is 0, the axis symmetry means there is no theta variation this is 0, fully developed means this is 0. So, again if you go in the right side of momentum balance there is gravitational force along with the flow direction, because gravity is oriented perpendicular, so that is 0 and d p d z is of course, present and it can at most be a function of z we have just seen at seeing this from the r component the r component of the momentum balance function of z and here axis symmetry means this is 0 fully developed means this is 0. So, essentially we are left with we are left with minus partial p partial z plus mu times one over r partial partial r of r of partial v z partial r is 0.

Now, we will use the same logic that we use for plane poiseuille flow flow in this gap between two rectangular surfaces to parallel plates, which is also driven by pressure driven which is also driven by pressure difference across the ends of the two ends of the channel two ends of the channel.

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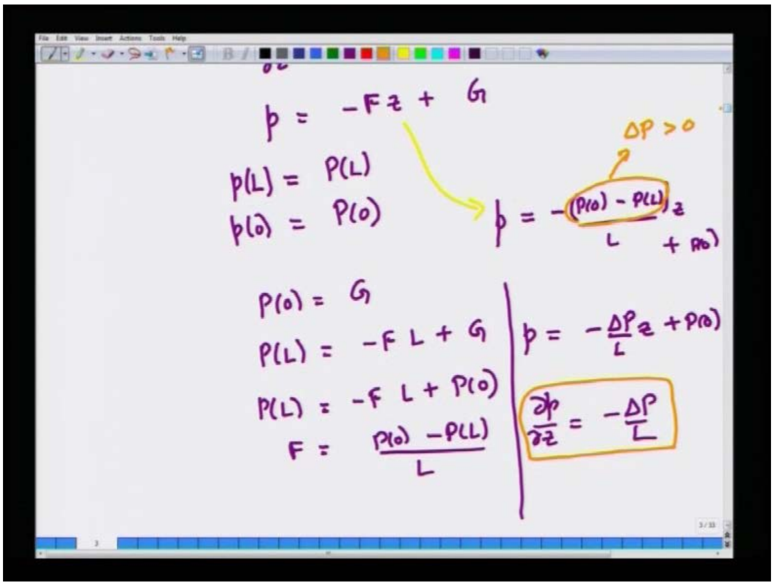

If this is the function only of z only a function of z and this is a function of r only because v z can be a function only of r it is independent of theta it is independent of z, so each has to be a constant, so what is that constant that constant, so minus partial p partial z is some constant let us say f. So, p is minus f times z plus some other constant let us say, g we can fix the constant by saying that P at 0 P.

We can fix this two constants by saying that P at z equal to L is P at L P at z equal to 0 is P at 0, we will find then, that pressure is nothing but, so we will eliminate the two constants, so P at 0 is G and P at L is minus F time's L plus G, so p at L is equal to minus F time's L plus P 0, so F is nothing but P 0 minus P L divided by L. So, pressure therefore, becomes P is minus P 0 minus P L divided by L times z plus G is P 0 from this equation.

Now, from our earlier definition P 0 minus P L is delta P it is greater than 0, because the pressure in the entrance of the pipe is larger than the pressure at the exit of the pipe. So, P becomes minus delta P by L z plus P 0 and partial P partial z which occurs in the momentum balance becomes minus delta p by L this is an important input to the z momentum balance from our analysis.

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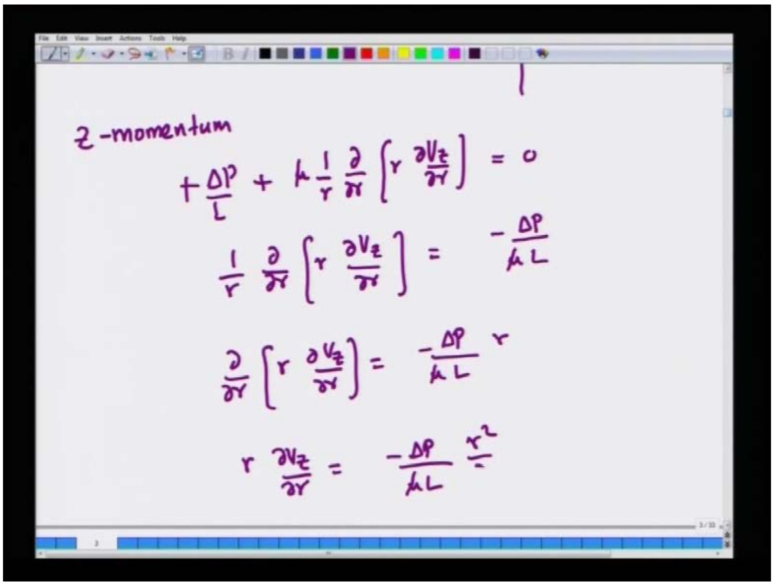

So, the z of momentum balance after all the simplifications becomes minus delta P by L plus, so you have P d P d z is minus delta P by L, so, minus of d P d z will become plus delta P by L. So, so you have plus delta P by L plus mu 1 over r partial partial r of r partial v z partial r is 0 this is the z momentum.

Equation if I take this rather side I will get one over r partial partial r partial v z by partial r is equal to minus delta P by mu L if I integrate this if I take this r to the other side I will get minus delta P by mu L r if I integrate this once I will get r partial v z by partial r is minus delta P by mu L r square by 2 plus a constant c 1 I bring the r this r to the denominator to the other side.

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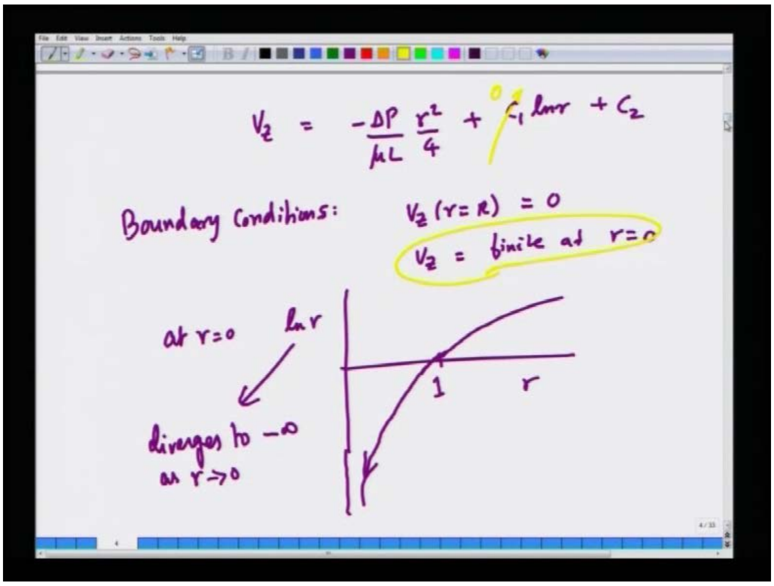

If I do that and then I will get if I and then if I integrate the equation I will get v z is minus delta P by mu L times r square by 4 after integrating plus c 1. Once this r comes here I will get one over r here and c 1 by r here and this r will disappear here, so I will get c 1 log r because you are integrating one over r it becomes log r plus c 2. Now the bounder conditions to fix this two constants r the following namely, that v z at r equals capital r is 0 no slip condition and v z is finite at r equals 0.

At r equals 0 if I look at logarithmic natural logarithm of r is a function of r it goes to 0 at r equals 1 increase, but for r less than 1 the natural logarithm keeps on going towards minus infinity as r tends to 0, so log r has a property, that it diverges to minus infinity slowly as r tends to 0. Now if i want v z to be finite, then I cannot have c 1, because if there is c 1 then v z will become infinitely large, logarithmically infinitely large as r goes to 0. So, c 1 has to be 0, so v z therefore becomes, v z therefore becomes minus delta P by mu L r square by 4 plus c 2.

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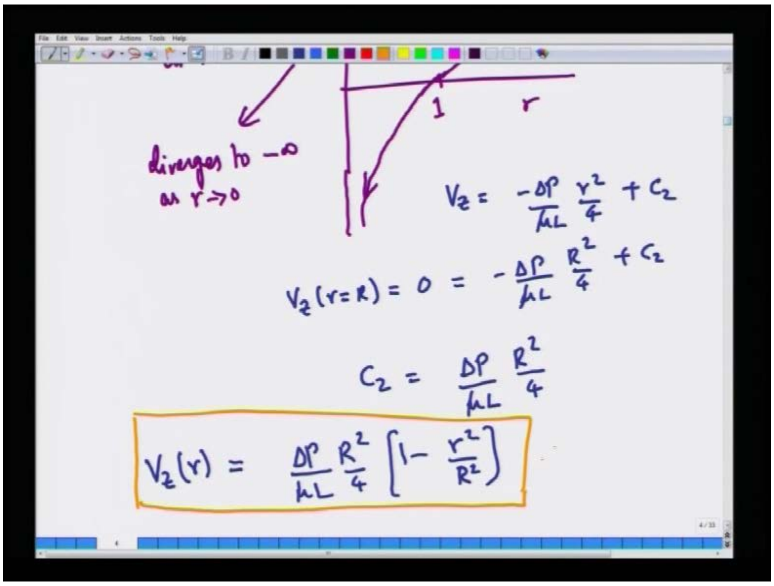

Now I can evaluate c 2 by saying that v z at r equals capital r is 0 is minus delta P by mu L capital R square by 4 plus c 2, so c 2 becomes delta P by mu L R square by 4. So, v z of r is nothing but delta P by mu L R square by 4 times 1 minus r square by R square this is the final expression for the velocity profile in the pipe, just as in the channel.

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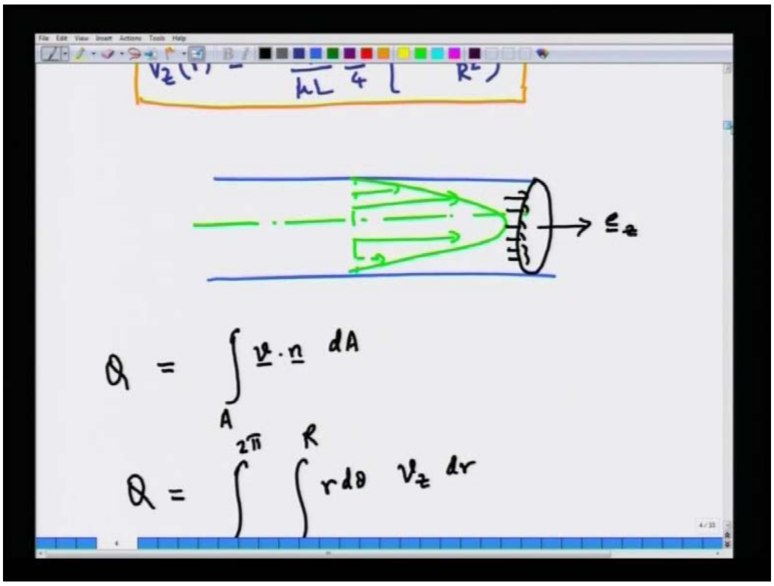

The velocity profile is parabolic about the axis ok, so the velocity is 0 at the wall and maximum at the center of the pipe. Now we do the same exercise that is carry out the calculate the volumetric flow rate and so on, we can do that in the following manner.

So, to calculate the volumetric flow rate across the pipe all you have to do is calculate the velocity vector dotted with the, suppose this is the then, flow direction is e z. So, you want to calculate volumetric for it on a surface whose unit normal is e z, so take the velocity vector dotted with n integrated over the cross section of the pipe which which is a circle integral over the area of the pipe, cross section area of the pipe. So, this becomes therefore, integral theta equals is 0 to 2 pi r is 0 to R r d theta v z d r this is the volumetric flow rate.

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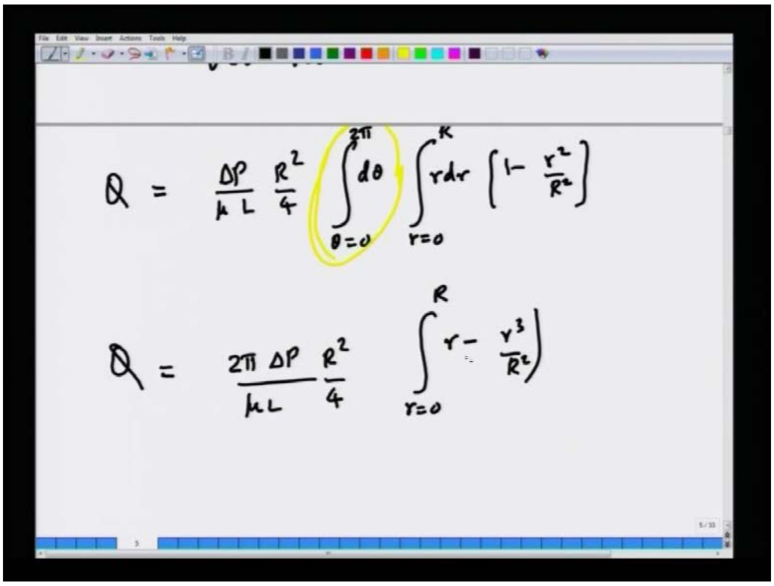

So, volumetric flow rate is nothing but, now I am going to substitute of v z which is delta P by mu L r square by four integral theta is equal to 0 to 2 pi d theta integral r equals 0 R r d r of 1 minus r square by R square. Now if you notice this this integral is independent of theta, so I can do theta integral trivially it will give you a factor of two pi ok.

So, Q is 2 pi delta P by mu L R square by 4 integral r equals 0 to R r minus r cube by R square d r that is the value of the integral.

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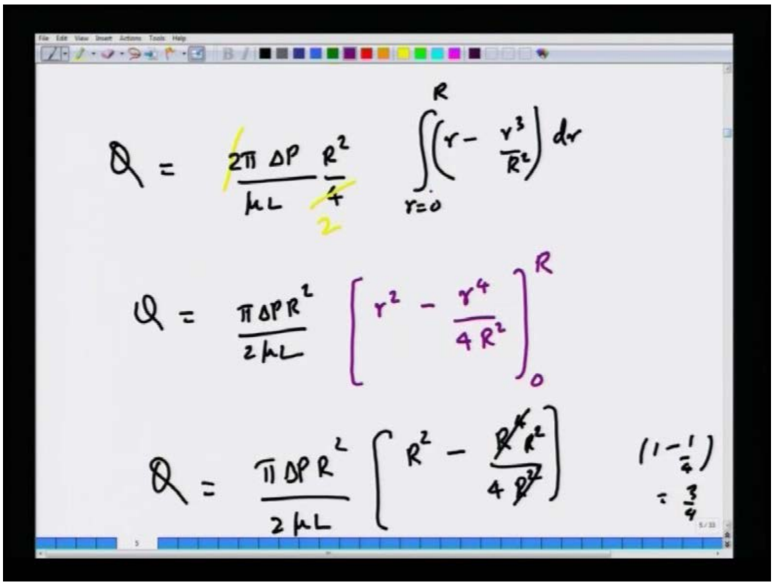

So, Q is now pi delta P R square by 2 mu L, I am going to I have cancelled this 2 with 4 to get a factor of 2 which is what I am carrying over here, then if I do this integral I get times r square minus r to the 4 by 4 capital R square 0 to R, so therefore, Q becomes pi delta P R square by 2 mu L times capital R square minus R square by 4 R to sorry R to the 4 by 4 R square. So, this will cancel to give you an R square here, so I will get Q is, so I will get a factor of 3 over 4, so I will get Q is nothing but, pi delta P R square.

So I mean, now I can write it as R to the power 4 by 2 mu L times 1 minus 1 by 4 is 3 over 4, so 3 over 4, so Q becomes delta P R to the 4 by 8 mu L pi delta P R to the more R to 4 by 8 times mu times L, so this there was a mistake this integral is R square by 2, so you have an R square by 2 here.

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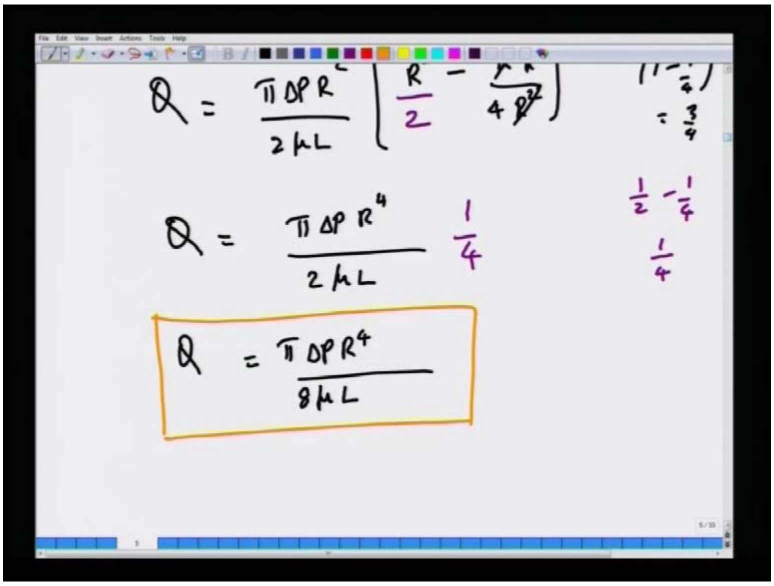

So, you get 1 by 2 minus 1 by 4 which this 1 by 4, so you get a factor of 1 by 4 here, so you get 4 twos are 8, so this is the expression for volumetric flow rate in a pipe when there is a laminar flow. So, the volumetric flow rate is directly proportional to the radius of the pipe as the 4 th power we can invert this relation before you do that the volumetric flow rate is inversely proportional to the viscosity that is for a same pressure drop.

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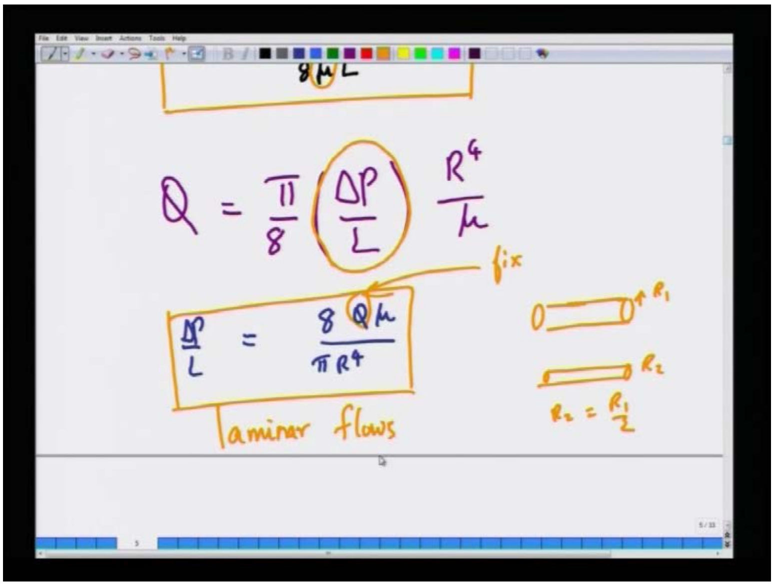

Suppose I write this expression in the following way Q is pi which is a constant delta P over L which is the pressure drop or unit length across the pipe pi by 8 is a constant to be precise then I can write this as R to the 4 by mu. So if I impose for a given for a given pipe length, if I impose a same pressure difference across the length of pipe, so delta p by l is a constant then the flow rate increases with increase in the radius and it decreases with increase in viscosity because it is difficult to push a liquid with higher viscosity, because it offers more resistance to flow, so it certainly inversely proportional to the viscosity.

Now I can of course, invert this relation and say delta P by L is 8 Q divided by pi R to the 4 8 q mu divided by pi r to the 4. So, this implies that if I want to fix a given flow rate for a fixed volumetric flow rate then the delta P by L it increases as you decrease the size of the pipe by radius as 1 over R to the 4, because if you if you want to reduce the pipe dimension if everything is kept constant, that is if I have the same fluid and if you want to pump the same fluid with the same flow rate, if you decrease the pipe, but by by a factor from R 1 to R 2 where R 2 is R 1 by 2, then delta P by L will increase by a factor of 1 over 2 to the 4 that is 1 over 16.

So, it will it will increase by a factor of 16 times when you decrease the pipe diameter by when you half the pipe diameter, so this is a major result, but this again to reemphasize this is valid only for what are called laminar flow laminar flows that is as I told you in the last lecture also, the fact that we are able to obtain one solution to the Naviar strokes equations after making several assumptions does not mean that, one can necessarily get these solutions in experiments.

So, the way to check whether this solution is valid or not is two carry out experiments by making flow, fluid flow with a given flow rate and measuring the pressure drop or vice versa and checking whether the observed results for the measured pressure drop agrees with this predicted value, predicted expression if it does not then that means, that the assumptions that we have made while deriving this equation Ah deriving this expression they are wrong.

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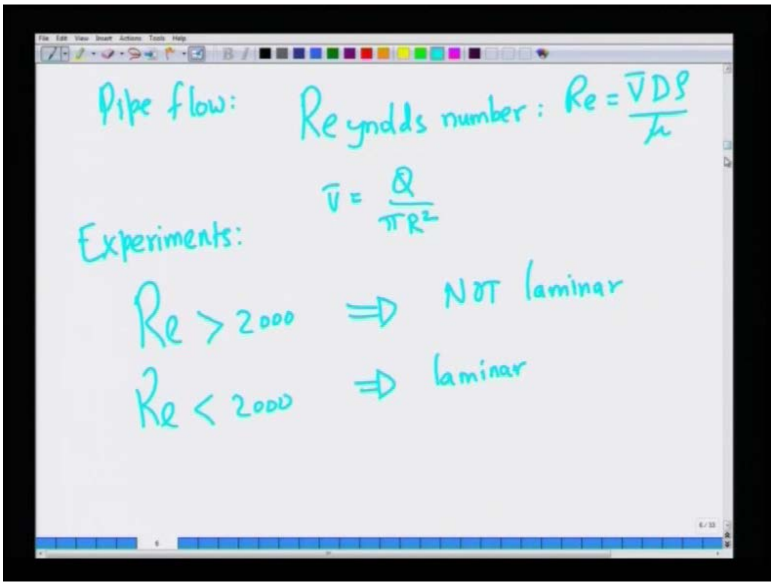

And it turns out that for a pipe flow we will see this in a little while later also for pipe flow based on the Reynolds number which is R e is defined as the average velocity times the diameter of the pipe time's rho by mu if you define the Reynolds number based on average velocity where the average velocity is nothing but volumetric flow rate divided by cross sectional is apart square, then if the Reynolds number is greater than 2000, then the flow is not laminar this is what experiments say.

While if the Reynolds number is less than 2000 the flow is certainly laminar and the above expression is valid, that is this expression is valid for Reynolds number less than 2000, while it is not valid for Reynolds number greater than 2000 that is the series limitation that we have to keep in mind and for Reynolds number greater than 2000 the flow becomes turbulent. Turbulence follows when the Reynolds number is Reynolds 2000 or around that value, so we cannot use the laminar flow predictions under such conditions. We will rest we will return to this problem of flow in a pipe in a slightly different context of looking at this problem from a perspective of a non dimensionalizing the problem, using various non-dimensional groups and in that context we will go in to we are going to revisit this same expression.

But right now before I just complete this I am going to just make two points, namely from here we can calculate the average velocity by dividing this expression by pi R square we will get the average velocity to be delta P R square by 8 mu L is the average velocity.

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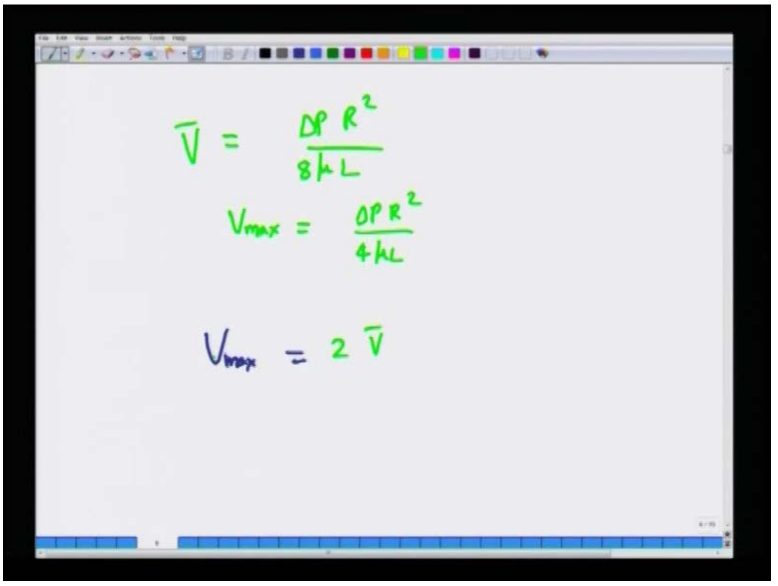

Now, in a pipe flow then, so the so, the average velocity in a pipe flow is delta P R square by 8 mu L in a pipe flow one can also find the maximum velocity and that happens to be delta P R square by 16 mu L this happens directly from the governing laminar for velocity for profile equation which is right here if I write this in terms of average velocity I will get if I eliminate this delta P R square by 8 mu L.

So, this becomes 2 V bar times 1 minus R square by R square, so the maximum velocity occurs at the center of the pipe when small r is 0 it is twice the average velocity. So, this is another important result which we already used in the context of kinetic energy correction factors in a pipe. So, V max sorry this will be 4 mu L, V max is twice v bar this is an important result again which we used which says that the flow velocity inside a pipe is highly non-uniform, that maximum velocity is twice the average velocity.

And in the context of rectangular channel flows we found that the maximum velocity is 3 by 2 times the average velocity, so different velocity different geometries have different relations for the maximum and average velocities, but all these go on to say that, the flow is non-uniform and therefore, the velocity profile vary from a value of 0 at the value to something maximum at the center and how different is the maximum and the average that depends from problem to problem.

So now, we will return to this problem slightly later when we do dimension analysis and we will introduce the notion of factors which are essentially pressure drops written in a non-dimensional way, but right now I am going to move to a slightly different problem that is quoting of a wire.

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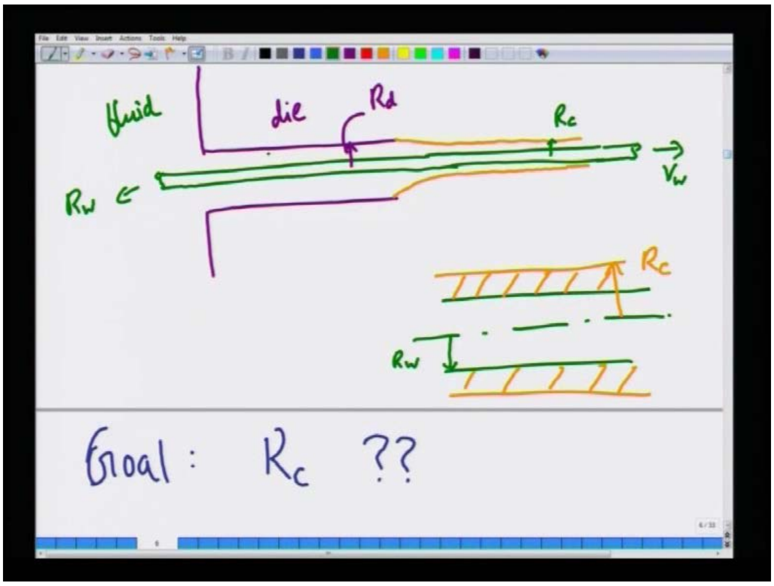

So, so essentially the idea is you have a bath of liquid and you have a die you want to coat a wire a long wire, let us center show the wire is slightly different color we want to coat a long wire and the wire is moving with a velocity you are constantly pulling the wire with a velocity V wall, V wire, so constantly pulling the wire in this direction and there is fluid liquid in this bath.

So, we have a wire very very long wire infinitely long wire that is being pulled from a bath of the liquid, so with the intention of coating this wire. So, essentially you have a long wire which when it comes out, so I and drawing the wire with a green color thing and the fluid, let us draw it with an orange color will be coated as as you pull the wire. Now the radius of the wire is the radius of the wire is R w and the radius of the coated wire from the center of the wire to the coated wire is R c, so this, so let me draw this in an enlarged way.

So, this is the wire surrounding the wire is a coating far away from the exit of a die and this this plated in a concentric way although my diagram is not very accurate in depicting that, this entire distance from the center of the wire is to the coating this is the coating a liquid coating while this distance is the diameter of the wire.

Now, the diameter of the die which is of course, another parameter is from the radius of the die that is R d, so you have 3 R d 1 is the radius of the wire itself other is radius of coated wire which is greater than radius of the wire, because the liquid is going to coat the wire and of course, there is a radius of the die in which the wire is moving.

The goal of our calculation using the differential momentum balance is to find an expression for the coated radius, can you predict what will be the coated radius in terms of parameter such as the radius of the wire, radius of the die, the velocity of the wire, can we say something and that is what we want to sound.

And another thing that we can ask is what is the force that we must exert on the wire, so that the wire moves with a constant velocity this another question of practical interest that is if I continuously want of the pull wire with a constant velocity I have to keep applying a force on the wire and that will be resisted by the fluid motion in the die. So, our goal is to analyze what is going to happen for the fluid flow in the die region in the anode of the die region, so I am going to share it with yellow, so I and going to be interested in the flow distribution between these two in this region I am not going to worry about the flow in this region, because far away downstream that would not be any viscous drag on the liquid that coats the wire and therefore, the all the liquid will move with the constant velocity same as the wire velocity V w.

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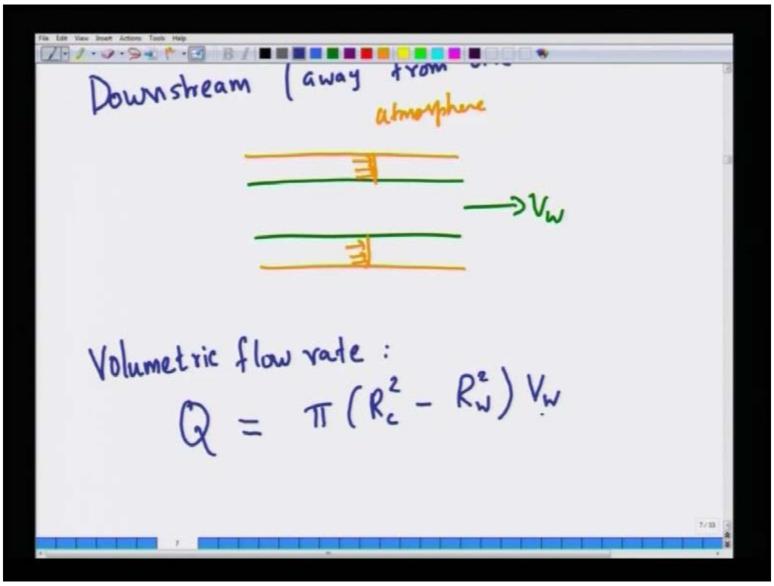

So, far downstream, downstream away from die exit exit of the die, I am going to have suppose I have the die going with the velocity V w and the coated fluid, the coated fluid will have the same velocity it will move like a rigid body along with the and the velocity profile will be a plug like flow, because there is no shear stress exerted by that is negligible shear stress exerted by the air surrounding as on the liquid. So, therefore, there would not be any variation in the velocity in the coated annular liquid faraway downstream, because it is surrounded by atmospheric air which is air, so which is very stationery.

So, I can do a simple mass balance I say that in the downstream side the volumetric flow rate therefore, of the of the liquid is that Q is the area of cross section which is pi times R c square minus R w square times the velocity at which fluid flowing which is V w.

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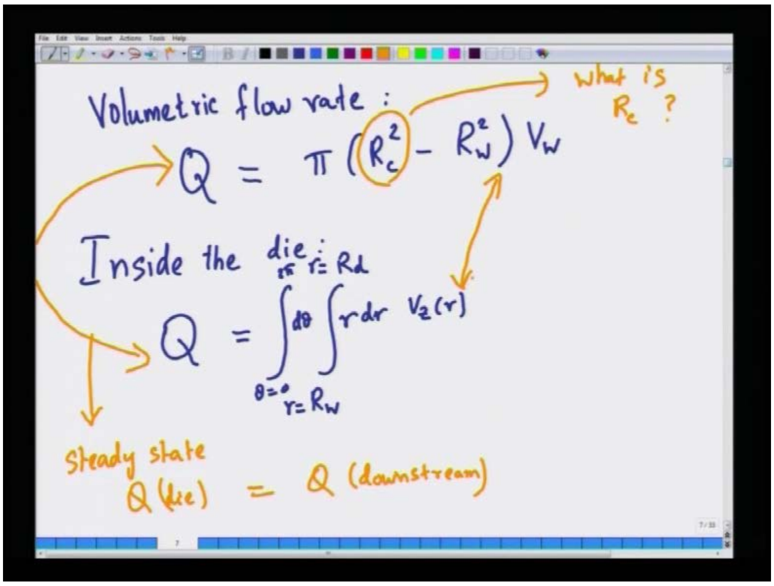

That is the volumetric flow rate that I would expect far away in the form the downstream of the die exit inside the die volumetric flow rate is nothing but integral R w to r d r equals R w 2 r equals r d theta 0 to 2 pi d theta times R d r d r times the velocity profile. So, in order to able, so our goal is to find a relation for r c our goal is to find what is the coated radius, so the strategy for our solution is to equate at steady state the volumetric flow rate inside the die, it should be the bee same as a volumetric flow rate that comes out of the die faraway in the in the downstream.

So, at steady state mass conservation says that these twos these two‘s must be the same at steady state Q in the die is Q at downstream section, so by equating these two we can find by equating this and this, we can find these two expressions we can find an expression the answers for what is the coated radius, but in order to do that we should we know we should know what is the velocity distribution in the gap of the die. So, that is what this is, so what is the velocity distribution here, what the velocity distribution, so let us solve that problem first it is very similar to what we did is just now, that is for the pipe flow, but there are some differences that is the whole point to discussing this application.

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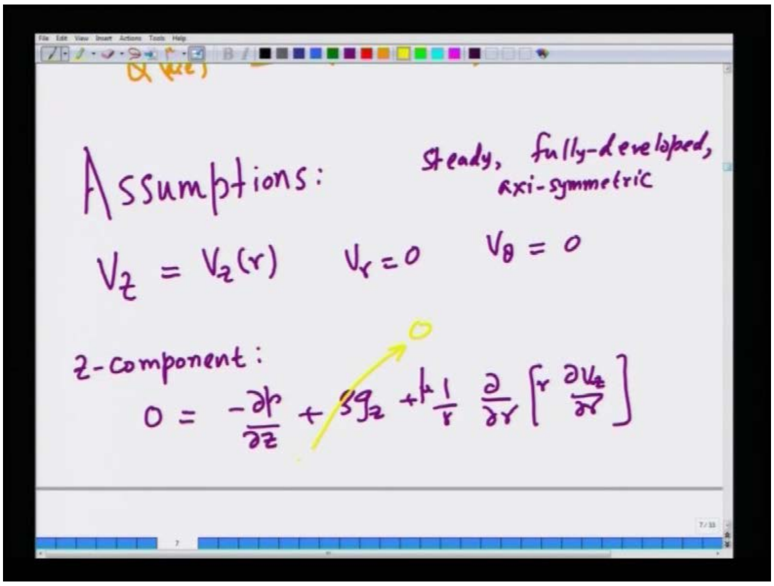

Now, we are going to use a usual assumptions I am going to quickly state them, because we have been explaining the motivation behind this assumptions in far details in the last two lectures in the last two problems that is, so the only velocity is V z the flow steady fully developed and axis symmetric same as before, the only velocity is V z and V z is the function only of the radial coordinate r V r is 0 and V theta is 0. So, z component of the momentum balance will imply 0 is and if you align, so let us align all the three terms first and mu times, if you align your die perpendicular to the gravity direction then this g z are 0 and since there is no pressure drop that a drive a flow in the die it is merely the flow in the die, is merely driven by the fact that the wire is being pulled at a constant velocity this is 0.

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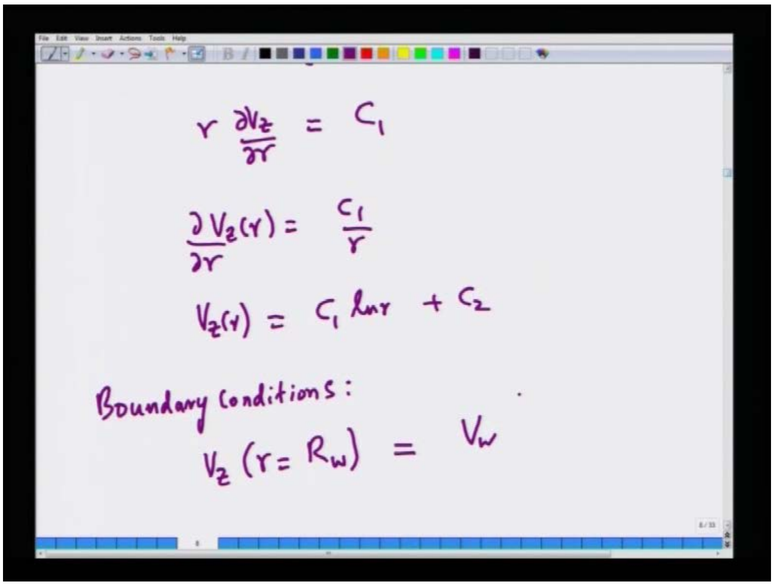

So, the momentum balance simply becomes mu times 1 over r partial partial r of partial r V z partial r is 0. Since mu is a constant that does not matter this implies that r partial v z by partial r is some constant or V z of r is c 1 by r or d z d r c 1 by r or V z of r c 1 log r plus c 2. Now the two constants are determined with by two boundary conditions namely that V z at r equals r wire is V w the velocity of the wire.

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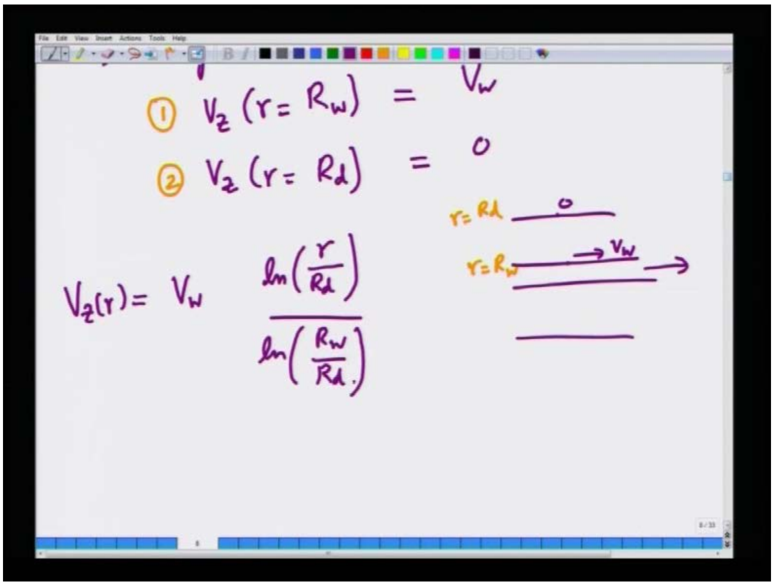

And V z at the possession of the die radius is 0, because if you remember the geometry the wire is being pulled inside the die ok at this velocity is V w while this velocity is 0 these are the two boundary conditions to determine the two constants. Now the key reason for the introduction of this problem is that, in the pie fossil flow we neglected c 1 because at r equals 0 the solution was blowing up to infinity it was diverging to infinity whereas, in this problem we the flow domain does not involve r equal 0, because the flow domain is basically R equals R wire to R equals R die the radius of the wire is not 0, because wire has finite radius therefore, the r equal to 0 which is as point of singularity in the symmetrical co-ordinate system does not appear in the flow domain.

So, there is no reason for us to (()) away c 1 in this problem, so we have to merely retain c 1 and c 2 and fix the two constants by using the two boundary conditions, so we have two equations, two boundary conditions for fixing the two constants, so we have two equations and two unknowns we can solve them to yield V z of r is k is equal to V w time's logarithm of r by R d the radius radial coordinate by radius of the die logarithm of R wire by R d this satisfies the two boundary conditions that at r equals r wire you have velocity of the wire at r equals r d logarithm of one is 0.

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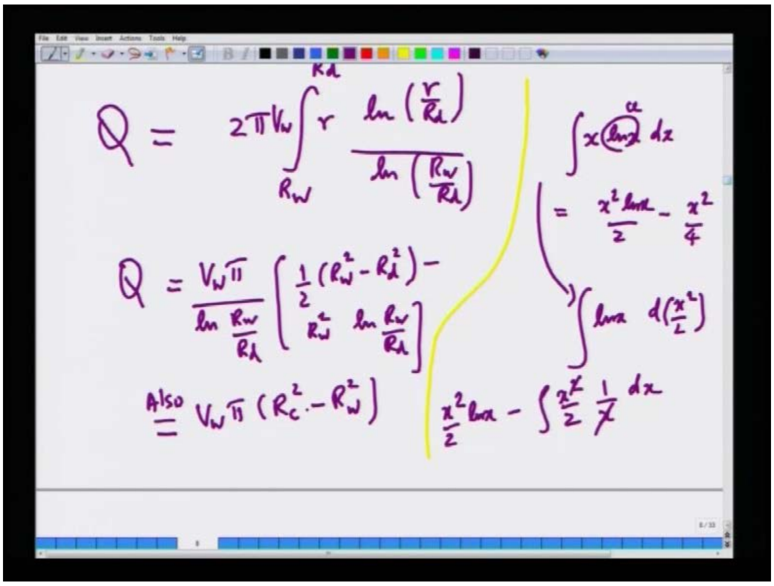

So, we have 0 velocities having done this we can do what is the volumetric flow rate the volumetric flow rate is 2 pi two pi which is the answer from the theta integral R equals R wire to R d r times V z of r which is lon of V w which you can pull log of r by r d wire log of r w by r d.

Once you do the integration, so all in order to do this integration we have to know that. Integral of x lon x of d x, because it is of that form is x square lon x by 2 minus x square by 4, this is the only this is obtained by integrating the parts by using the u d v kind of rule we can use x lon x as u and x d x as you can write it as d of x square by 2 this can be done as lon x d of x square by 2 and integral of u d v is u v minus integral v d u.

So, u v x square by 2 lon x minus integral v d u v is x square by 2 d x of lon x is one over x d x, so you get again x square by 2 x square by 4, because you have x by 2 if you integrate you will get this. So, this is the only relation you have to know to in order to carry out this integration and the rest is just straight forward algebra. So, Q becomes after simplifying V w pi by log R w by R d by times half R w square minus R d square this is volumetric flow rate inside the die, minus R w square lon R w 2 by r 2, but this must also be equal to V w times pi R c square minus r w square, but from where far away from the exit the volume the velocity profile is uniform.

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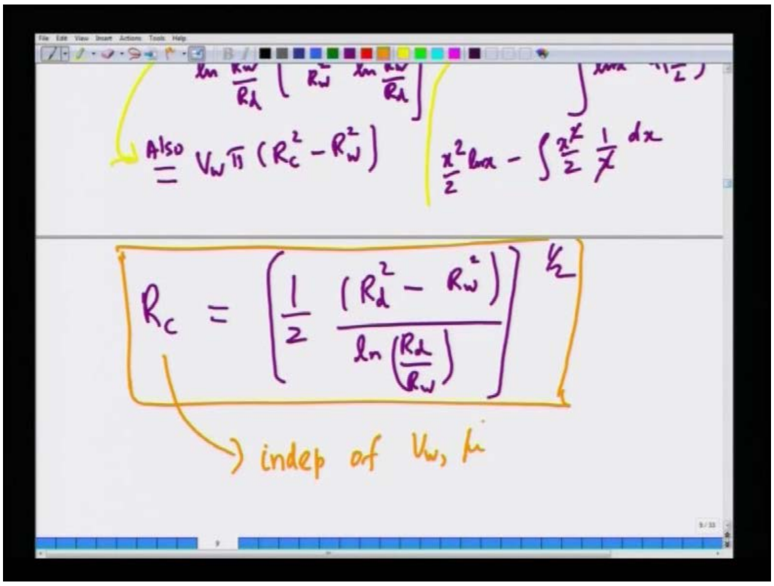

So, we can now equate these two expressions to find what R c is, so R c will then become after cancelling a few terms here half R d square radius of die square one is radius of y square wire square divided by logarithm R d by R w everything raised to the power half to the power half. So, this is the radius of the coating in terms of all the other

parameters in the problem, very interestingly there is no velocity here the radius of coating is independent of velocity, of velocity of the wire and the viscosity of the fluid through which it is flowing and so it is the function only of the wire and die dimensions it is independent of the velocity of the wire and viscosity to the fluid.

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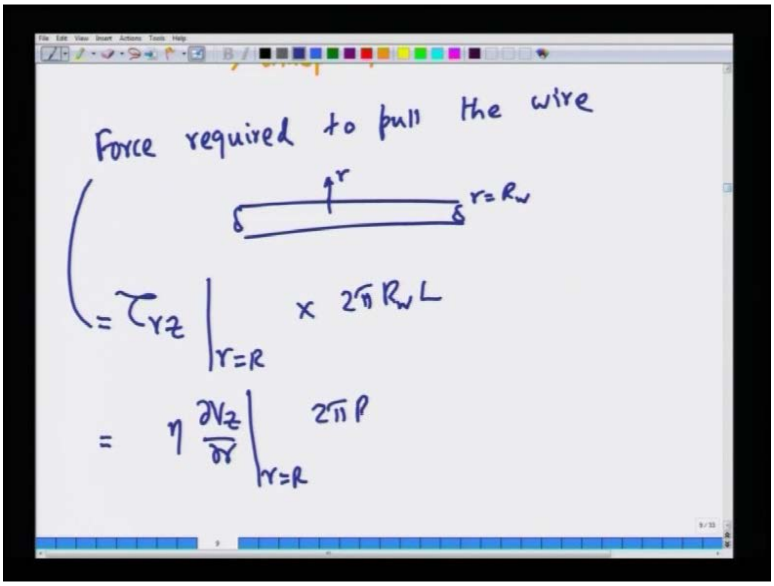

Now, another byproduct of this calculation is the shear stress calculation, so if you want to know what is the force required to pull the wire, you have the wire you have to find what is the sheer stress exerted by the fluid this is the r coordinate at r equals R wire and that component of the shear stress of tau r z at r equals R times the area of the wire which is surface area of the wire which is 2 pi R wire times the length of the wire.

Now, tau this is the force, this tau V z is eta times partial V z partial r evaluated r equals R times 2 pi R w L.

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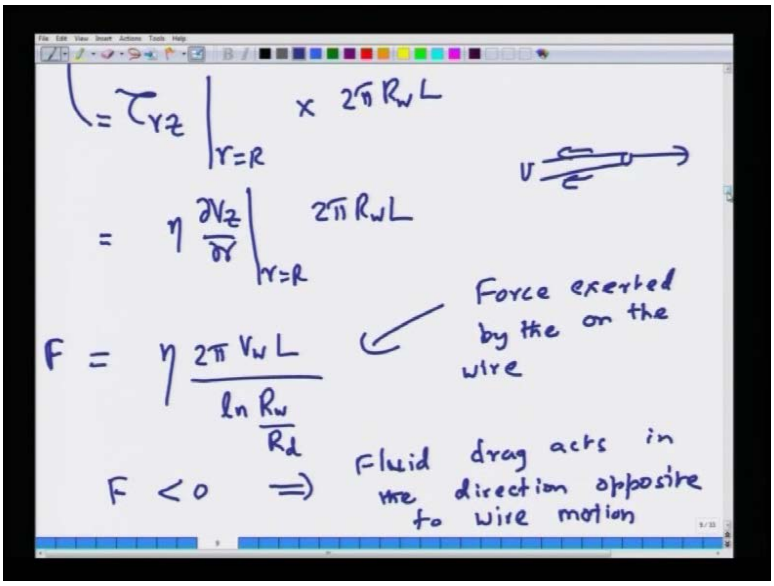

So, this is nothing, but eta times 2 Pi, so after doing this, so if you recall d V z d r is c 1 by r, so c 1 after, so if you use this expression, so this becomes 2 pi the force is eta times 2 pi times V w L divided by logarithm R w by R d.

So, this is the force this is the force exerted by the fluid on the wire, so since r w is less than r die this force is negative, because the fluid drag force acts in the direction opposite to flow, opposite to wire motion.

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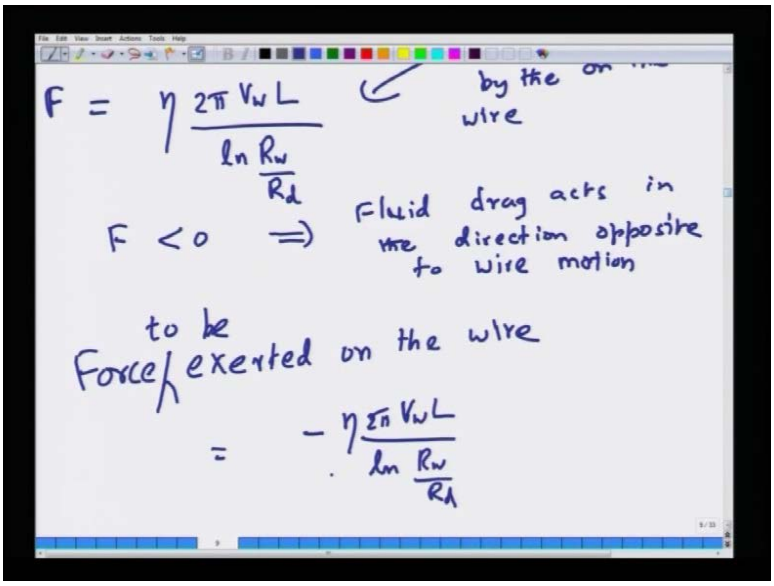

So, the wire is trying to move in the plus z direction the fluid is trying to exert a force in the minus z direction, so in order to make the wire to move in the constant velocity you have to apply force x to be exerted on the wire should be minus of that, because this will now be in the positive x direction.

So, with this we are going to complete our discussion on differential momentum balances, this is a very very vast topic in itself, because the solution of Navier stokes equations using various simplifying approximations or more sophisticated mathematical methods or computational methods is a very very important topic in modern fluid mechanics, but being an introductory course we will have to content ourselves by stopping at this point, we will return to some applications a little later in the course at the far end of the course, but right now we will stop at this point and in the next lecture we are going to start a new topic that is dimensional analysis. So, we will see you in the next lecture.